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Response determination of a viscoelastic Timoshenko beam subjected to moving load using analytical and numerical methods

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Abstract. In this paper the dynamic behavior of a viscoelastic Timoshenko beam subjected to a concentrated moving load are studied analytically and numerically. The viscoelastic properties of the beam obey the linear standard model in shear and incompressible in bulk. The governing equation for Timoshenko beam theory is obtained in viscoelastic form using the correspondence principle. The analytical solution is based on the Fourier series and the numerical solution is performed with finite element method. The effects of the material properties and the load velocity are investigated on the responses by numerical and analytical methods. In addition, the results are compared with the Euler beam results.

Keywords: viscoelastic timoshenko beam; moving load; analytical solution; linear standard model; FE

1. Introduction

The moving load on a structure is one of the common problems in engineering. Travelling of a crane on a beam, moving cars on a bridge, transmitted fluid in a polymeric tube, air flow on a composite aircraft wing are some of the practical cases. In the most cases, for discussing the moving load effects, the structure is assumed elastic while the most of the materials are in the viscoelastic field or they have time depended properties. Huang (1976, 1975, 1978) investigated the response of infinite viscoelastic cylindrical shells to a moving pressure using the standard linear model. Fung (1996) investigated the dynamic stability of a viscoelastic beam subjected to harmonic and parametric excitations simultaneously. They extracted the governing equations in both linear and nonlinear forms. Also they used the dimensionless parameters for simplifying the governing equations and results. González *et al.* (2004) analyzed a two-dimensional viscoelastic solid subjected to a moving load along the boundaries without considering the inertia effects. They used the correspondence principle and the results were compared with the boundary element method. Kocatürk *et al*. (2006) and Şimşek *et al.* (2009) studied the behaviour of Timoshenko and Euler-Bernoulli beams under harmonic moving load. They used the energy method for extracting the

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governing equations, the Kelvin-Voigt model as a material property and numerical method for solution. Mofid *et al.* (2010) studied the behaviour of a Kelvin-Voigt beam under a moving mass by using the discrete element technique. Fryba (1972) solved many problems in the elastic form subjected to the moving load. Andersen *et al.* (2001) analyzed an infinite Euler beam under moving load using the finite elements (FE) and analytical method. Zhang *et al.* (2001) investigated the effects of the initial tension and fluid velocity on the natural frequency of a Timoshenko tube conveying fluid with the FE method by considering the Prony series for material behaviour. Tehrani *et al.* (2012) studied the vibrations of a viscoelastic Euler beam subjected to a distributed moving load analytically.

In the most published articles the Kelvin-Voigt model, numerical solution and Euler beam theory has been used for dynamic analysis of viscoelastic beams under moving load. In this paper an analytical solution is presented for analysis of a Timoshenko viscoelastic beam which obeys the linear standard model. Then parametric studies are performed and the effects of the load velocity and the viscoelastic properties are investigated on the response of the beam. The results are compared with the FE method and the results obtained by the Euler theory (Tehrani et al. 2012). Also the ability of the each theory on the prediction of the response of a viscoelastic beam has been discussed.

2. Governing equation

The differential constitutive law for a linear viscoelastic material can write as

$$IP(D)\sigma_{ij}(t) = IQ(D)\gamma_{ij}(t) \tag{1}$$

Where σ = stress, γ = strain and IP, IQ = differential operators which are defined as

$$IQ(D) = \sum_{r=o}^{N} q_r \frac{\partial^r}{\partial t^r} \tag{2}$$

$$IP(D) = \sum_{r=o}^{N} p_r \frac{\partial^r}{\partial t^r} \tag{3}$$

q_r and p_r are independent of time in linear form. The dilatational and deviatoric forms of the constitutive law are

$$P_1\sigma_{ij}^{de}(t) = Q_1\gamma_{ij}^{de}(t) \tag{4}$$

$$P_2\sigma_{KK}^{dil}(t) = Q_2\gamma_{KK}^{dil}(t) \tag{5}$$

The subscript *de* and *dil* designate the deviatoric and dilatational parts and P_1, P_2, Q_1, Q_2 = differential operators similar to Eqs. (2) and (3). By using Lame's coefficients, the tensile modulus of elasticity and the Poisson's ratio are

$$G(D) = \frac{1}{2}\frac{Q_1}{P_1} = \frac{Q^G}{P^G} \tag{6}$$

$$E(D) = \frac{3Q_1Q_2}{P_2Q_1 + 2P_1Q_2} = \frac{Q^E}{P^E} \tag{7}$$

$$\nu(D) = \frac{P_1Q_2 - P_2Q_1}{P_2Q_1 + 2P_1Q_2} = \frac{Q^\nu}{P^\nu} \tag{8}$$

For a material which behaves incompressible in bulk and viscoelastic in shear, P_2 equals to zero and Eqs. (7) and (8) reforms as

$$E(D) = \frac{3Q_1}{2P_1} = \frac{Q^E}{P^E} \tag{9}$$

$$\nu = \frac{1}{2} = \frac{Q^\nu}{P^\nu} \tag{10}$$

The transverse shear force and the bending moment for a Timoshenko beam can be expressed as

$$V = G\lambda A(w_x - \psi) \tag{11}$$

$$M = EI\psi_x \tag{12}$$

Where $w(x, t)$ = lateral deflection, V = transverse shear force, M = bending moment, A = cross section area, I = Area moment of inertia, ρ = density, λ = shear correction factor which equals to 5/6 for rectangular cross section, ψ = angular deformation, and $f(x, t)$ = force per unit length. By considering the Newton's second law for an element results

$$\rho A w_{tt} = V_x + f(x, t) \tag{13}$$

$$\rho I\psi_{tt} = V + M_x \tag{14}$$

By using the correspondence principle, Eqs. (11) and (12) reforms as

$$P^G V = Q^G \lambda A(w_x - \psi) \tag{15}$$

$$P^E M = Q^E I\psi_x \tag{16}$$

From Eqs. (13) and (14), one can results

$$\rho I\psi_{ttx} - \rho A w_{tt} = M_{xx} - f(x, t) \tag{17}$$

By applying the operators P^G and P^E in to Eqs. (13) and (17) and by using Eqs. (15) and (16)

$$P^E(\rho I\psi_{ttx} - \rho A\psi_{tt} + f) = Q^E I\psi_{xxx} \tag{18}$$

$$P^G(\rho A\psi_{tt} - f) = \lambda A Q^G(w_{xx} - \psi_x) \tag{19}$$

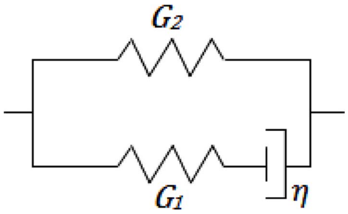

Fig. 1 Linear standard model

And by combining Eqs. (18) and (19), the equation of motion obtains as the following

$$P^E\left[I\rho Q^G w_{xxtt}-\frac{I\rho^2}{\lambda}P^G w_{tttt}+\frac{I\rho}{\lambda A}P^G f_{tt}\right]-\rho A Q^G P^E w_{tt}+P^E Q^G f =$$

$$Q^E I\left[-\frac{\rho}{\lambda}P^G w_{ttxx}+\frac{1}{\lambda A}P^G f_{xx}+Q^G w_{xxxx}\right] \tag{20}$$

The lateral deflection for a simply supported beam with the boundary conditions $w(0,\ t)$ = $w(L,t) = M(0,t) = M(L,t) = 0$ is assumed as

$$\psi(x,t) = \sum_{n=1}^{\infty} b_n(t)\cos\left(\frac{n\pi x}{L}\right) \tag{21}$$

$$w(x,t) = \sum_{n=1}^{\infty} a_n(t)\sin\left(\frac{n\pi x}{L}\right) \tag{22}$$

For a concentrated moving load, the Fourier series can be written as

$$f(x,t) = \frac{2f_0}{L}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi ut}{L}\right)\sin\left(\frac{n\pi x}{L}\right) \tag{23}$$

Where u = load velocity, f_0 = magnitude of the load and L = length of the beam.

A linear standard viscoelastic model, as shown in Fig. 1, is considered for the material property. The constitutive equation is in the following form

$$\dot{\sigma}+\frac{G_1}{\eta}\sigma = 2\left((G_1+G_2)\dot{\gamma}+\frac{G_1G_2}{\eta}\gamma\right) \tag{24}$$

From Eqs. (24) and (4), Q_1 and P_1 are extracted as

$$Q_1 = 2\left((G_1+G_2)\frac{d}{dt}+\frac{G_1G_2}{\eta}\right) \qquad P_1 = \frac{d}{dt}+\frac{G_1}{\eta} \tag{25}$$

From Eqs. (6), (9), (10), (21), (22), (23) and (25), the time part of Eq. (20), has to satisfy the following differential equation

$$\frac{d^6a(t)}{dt^6}+B_5\frac{d^5a(t)}{dt^5}+B_4\frac{d^4a(t)}{dt^4}+B_3\frac{d^3a(t)}{dt^3}+B_2\frac{d^2a(t)}{dt^2}+B_1\frac{da(t)}{dt}+B_0a(t) = F_1\cos(n\omega t)+F_2\sin(n\omega t)$$

$$B_5 = 2\frac{G_1}{\eta}, \quad B_4 = \left(\frac{G_1}{\eta}\right)^2 + \frac{G_0}{\rho L^2}\left(\lambda\frac{AL^2}{I} + (\lambda+3)n^2\pi^2\right), \quad B_3 = \frac{G_1}{\eta}\frac{G_0+G_2}{\rho L^2}\left(\lambda\frac{AL^2}{I} + (\lambda+3)n^2\pi^2\right)$$

$$B_2 = \left(\frac{G_1}{\eta}\right)^2 + \frac{G_2}{\rho L^2}\left(\lambda\frac{AL^2}{I} + (\lambda+3)n^2\pi^2\right) + 3\lambda n^4\pi^4\left(\frac{G_0}{\rho L^2}\right)^2, \quad B_1 = 6\lambda n^4\pi^4\frac{G_1}{\eta}\frac{G_0}{\rho L^2}\frac{G_2}{\rho L^2}$$

$$B_0 = 3\lambda n^4\pi^4\left(\frac{G_1}{\eta}\right)^2\left(\frac{G_2}{\rho L^2}\right)^2, \quad F_1 = \frac{2P_0}{\rho AL}\frac{G_1}{\eta}\left(\frac{G_0+G_2}{\rho L^2}\left(3n^2\pi^2 + \lambda\frac{AL^2}{I}\right)n\omega - 2n^3\omega^3\right)$$

$$F_2 = n^4\omega^4 - \left(\frac{G_1}{\eta}\right)^2 n^2\omega^2 - \frac{G_0}{\rho L^2}\left(3n^2\pi^2 + \lambda\frac{AL^2}{I}\right)\left(\frac{G_0}{\rho L^2}n^2\omega^2 - \left(\frac{G_1}{\eta}\right)^2\frac{G_2}{\rho L^2}\right) \tag{26}$$

We use the following dimensionless parameters for discussion of results

$$\tau = \Omega t \qquad E_1 = \frac{G_2}{G_0} \qquad K = \frac{G_1}{\eta}\frac{1}{\Omega} \qquad \beta = \frac{G_0}{E_0}\frac{AL^2}{I} \qquad \alpha = \pi\frac{u}{L\Omega} \tag{27}$$

Where

$$G_0 = G_1 + G_2 \quad E_0 = 3G_0 \quad \Omega^2 = \frac{\pi^2}{2}\frac{E_0}{\rho L^2}\left(J - \sqrt{J^2 - \frac{4\lambda G_0}{E_0}}\right) \quad J = 1 + \lambda/3 + \lambda\beta/\pi^2$$

Eq. (26) is a linear differential equation. For solving this equation analytically, one can transform the equation to Laplace domain. This resultant algebraic equation is solved in Laplace domain and by calculating the inverse Laplace, it is possible to find the solution in time domain. The authors have prepared a code in Maple 12 package to perform these calculations. The Euler beam results have presented in Tehrani and Eipakchi (2012).

3. FE model

For FE analysis, Ansys 11 Package is used. This software supports *Maxwell* viscoelastic model and for the other models, one has to use the Prony series for simulation. The Prony series for relaxation function is defined as

$$G(t) = G_\infty + \sum_{i=1}^{n_1} G_i e^{\left(\frac{-t}{\zeta_i^G}\right)} \tag{28}$$

Where G_∞ = shearing modulus, ζ_i^G = relaxation time for each member of the series. The parameters of Eq. (28) can set by comparing the response of a linear standard model to a step strain function as the following

$$\zeta = \eta/G_1 \quad G_\infty = G_2 \quad n_1 = 1 \tag{29}$$

For modelling the viscoelastic beam element, Beam188 element was selected. This element which is formulated by using Timoshenko beam theory, contains two nodes in the global coordinate system. The Shear deformation effects are included. For each node, six degrees-of-freedom is defined, translations in x, y, and z directions and rotations about these axis. It is suitable for analyzing slender to moderately thick beam structures. Also it is well-suited for linear, large rotation, and large strain nonlinear applications. Ansys does not support any wizard for modelling the moving load and one should use the Ansys Parametric Design Language (APDL), which is a programming environment for fast and efficient modelling of moving load. An APDL code was prepared for FE analysis.

4. Comparison of analytical and numerical results

4.1 Velocity effects

Before starting analysis we set some of the parameters as the following:

$$A = 0.01 \text{ m}^2, \rho = 3000 \text{ kg/m}^3$$

All the response graphs have two parts. The first part relates to the existence of the load on the beam (forced vibrations) i.e., until $t = L/u$ where u is the loads velocity and the second part stands

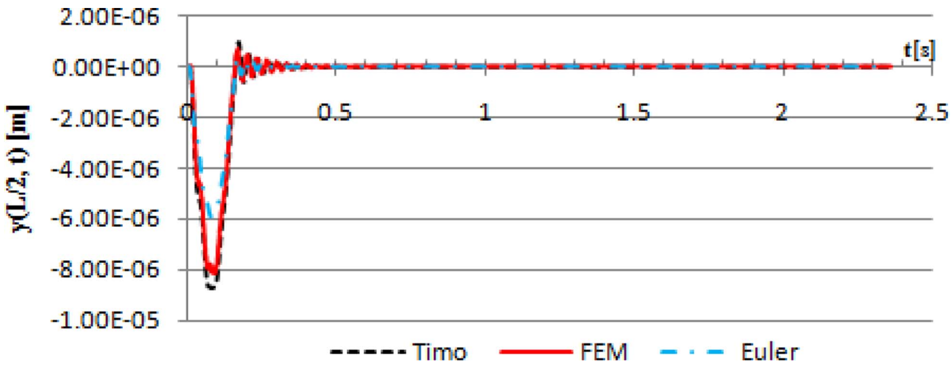

Fig. 2 Deflection of mid-span in terms of time for L = 0.25[m], u = 1.590 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 3752.804 [MPa.s] and K = 1, E_1 = 0.75, β = 25, α = 0.1

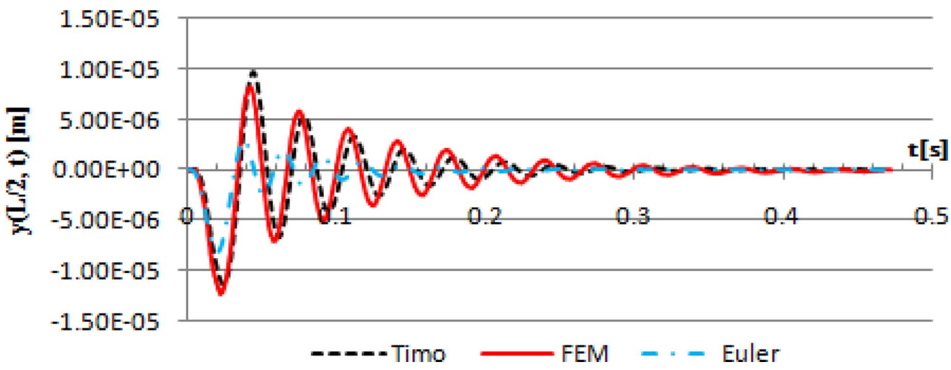

Fig. 3 Deflection of mid-span in terms of time for L = 0.25 [m], u = 7.952 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 3752.804 [MPa.s], and K = 1, E_1 = 0.75, β = 25, α = 0.5

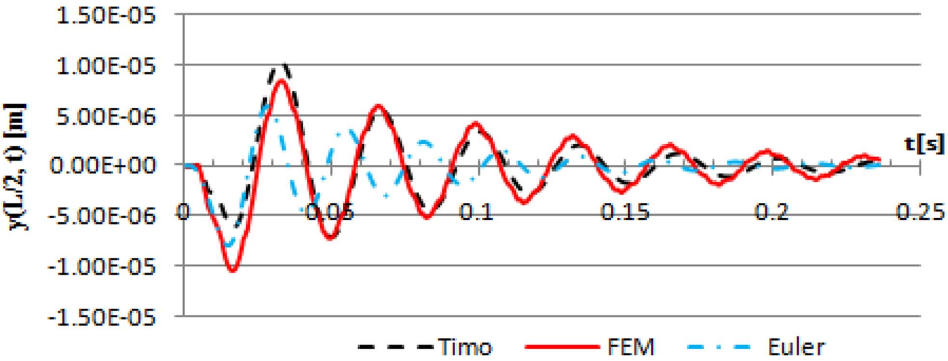

Fig. 4 Deflection of mid-span in terms of time for L = 0.25 [m], u = 15.903 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 3752.804 [MPa.s], and K = 1, E_1 = 0.75, $\beta = 25$, $\alpha = 1$

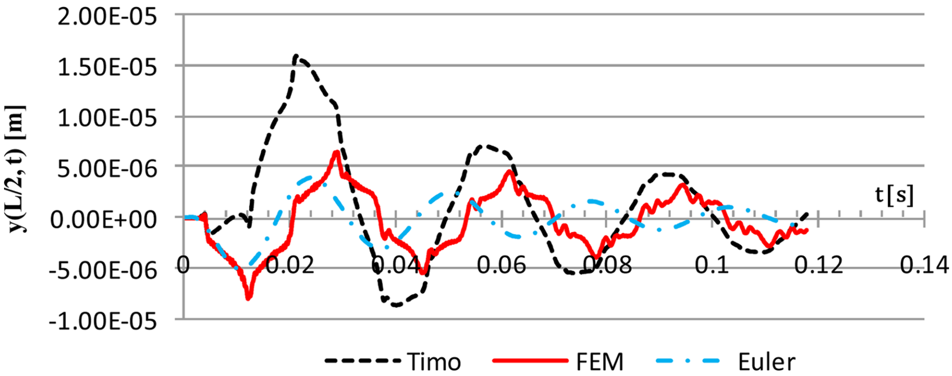

Fig. 5 Deflection of mid-span in terms of time for L = 0.25 [m], u = 31.806 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 3752.804 [MPa.s], and K = 1, E_1 = 0.75, $\beta = 25$, $\alpha = 2$

for the free vibration or departure of the load from the beam i.e., $t > L/u$. Figs. 2-5 show the effect of the velocity on the dynamic displacement of the mid-span of the beam with the analytical (Timoshenko and Euler) and FE methods. By increasing the velocity, the differences between the analytical results increase or Euler theory is not suitable for prediction of the amplitude and frequency of response at high velocities.

4.2 Material properties effects

Figs. 6-8 show the effects of the parameter η on the results. For heavy dampers, the difference between the numerical and analytical results increases. In addition, the damping behaviour in Timoshenko theory is more or Timoshenko theory is more sensitive to damping parameter η.

From Figs. 8-9, when $K \to 0$ or $\eta \to \infty$ the difference between Timoshenko case and two other cases increases (Fig. 8). Also for $K = 1$ (Fig. 4) the response has the maximum damping.When K increases the responses of Timoshenko and FE are very close but the Euler case does not have a good match and the behavior of the beam response approaches to the elastic response (Fig. 9).

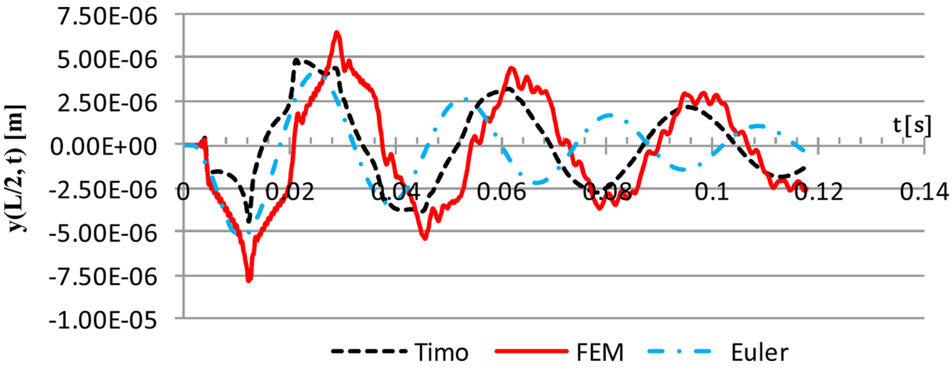

Fig. 6 Deflection of mid-span in terms of time for L = 0.25 [m], u = 31.806 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 1876.402 [MPa.s], and K = 2, E_1 = 0.75, β = 25, α = 2

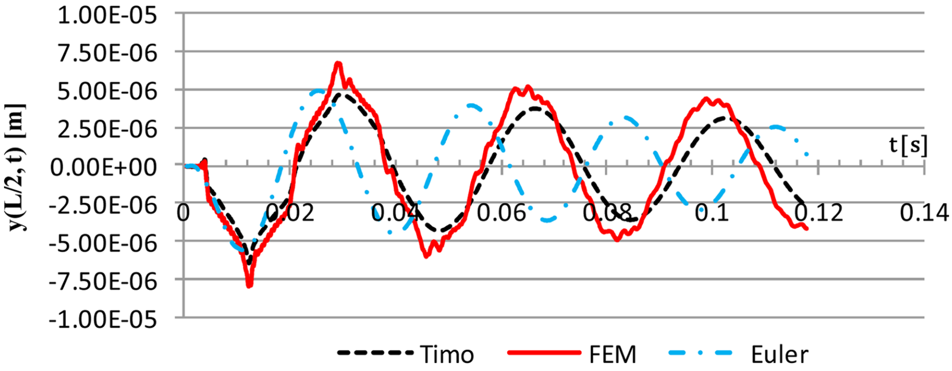

Fig. 7 Deflection of mid-span in terms of time for L = 0.25[m], u = 31.806 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 750.561 [MPa.s], and K = 5, E_1 = 0.75, β = 25, α = 2

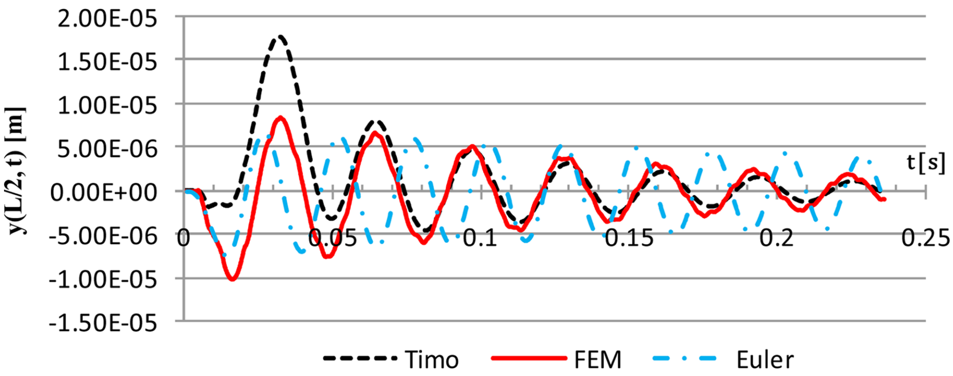

Fig. 8 Deflection of mid-span in terms of time for L = 0.25[m], u = 15.903 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 7505.606 [MPa.s], and K = 0.5, E_1 = 0.75, β = 25, α = 1

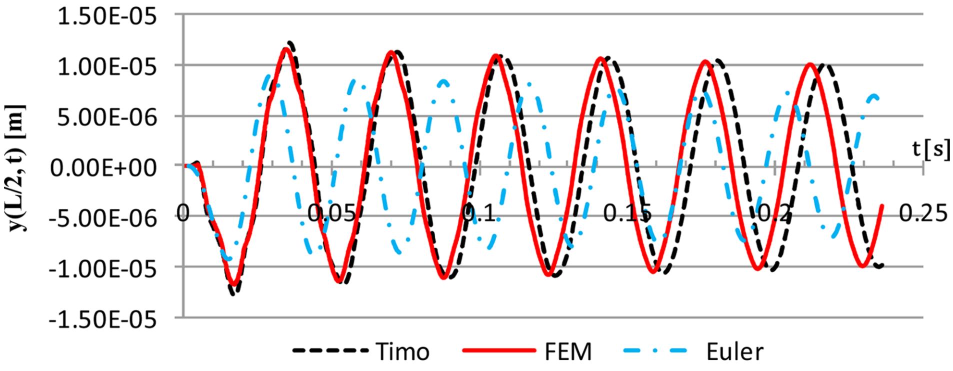

Fig. 9 Deflection of mid-span in terms of time for L = 0.25 [m], u = 15.903 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 125.092 [MPa.s], and K = 30, E_1 = 0.75, β = 25, α = 1

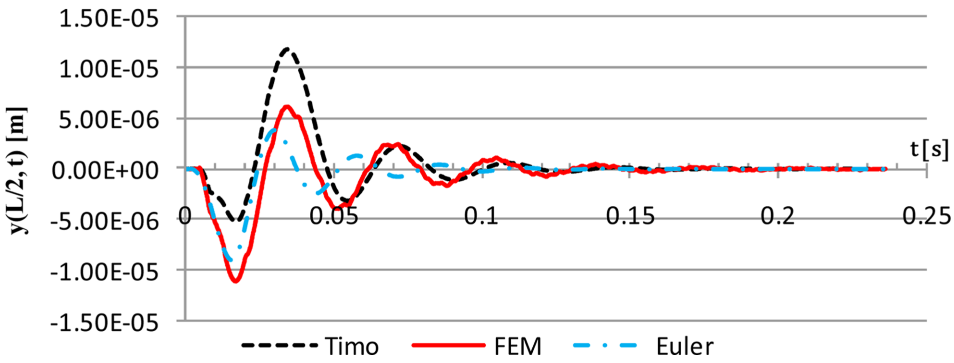

Fig. 10 Deflection of mid-span in terms of time for L = 0.25 [m], u = 15.903 [m/s], G_1 = 1500000 [MPa], G_2 = 1500000 [MPa], η = 7505.606 [MPa.s], and K = 1, E_1 = 0.5, β = 25, α = 1

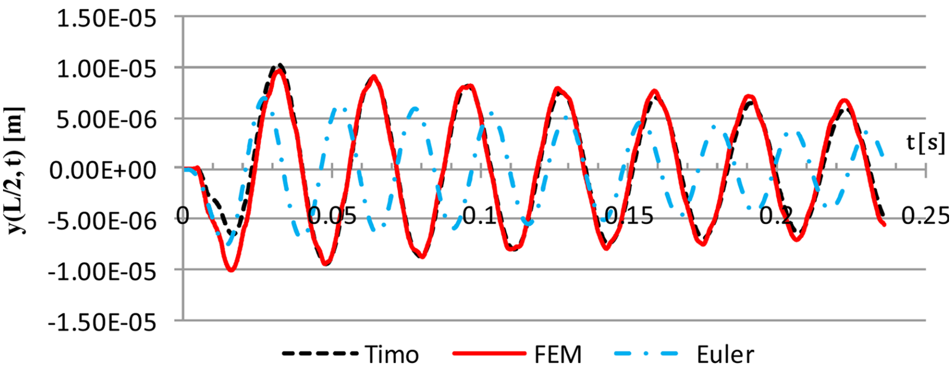

Fig. 11 Deflection of mid-span in terms of time for L = 0.25 [m], u = 15.903 [m/s], G_1 = 150000 [MPa], G_2 = 2850000 [MPa], η = 750.561 [MPa.s], and K = 1, E_1 = 0.95, β = 25, α = 1

In Figs. 10-11 the effects of parameter E_1 has been investigated. Approximately when $E_1 > 0.5$ Timoshenko case is closer than Euler case with the FE response but when $E_1 < 0.5$ the Euler case is nearer to the FE response. When E_1 approaches to one the elastic behaviour is more significant (Fig. 11).

4.3 Effects of parameter β

From Figs. 4, 12, 13, by increasing the parameter β (or the length) the responses of Timoshenko, Euler and FE cases approach to each other. In addition, since with increasing the parameter β the parameter K increases too, and as a result the behaviour of the beam approaches to elastic behaviour.

4.4 Comparison of viscoelastic and elastic responses

For comparison of the elastic and viscoelastic responses, it is sufficient to survey the response when $\eta \to 0$ or $K \to \infty$ and $E_1 \to 1$ or $G_0 \to G_2$. From Eq. (26)

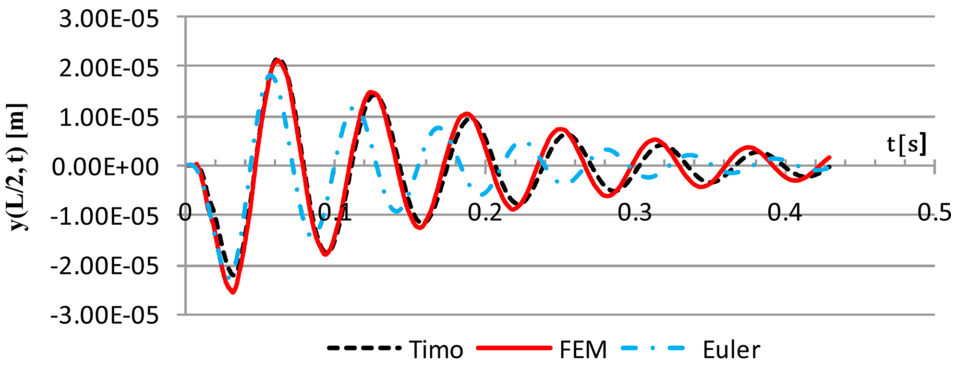

Fig. 12 Deflection of mid-span in terms of time for L = 0.353 [m], u = 12.366 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 3752.804 [MPa.s], and K = 1.819, E_1 = 0.75, β = 50, α = 1

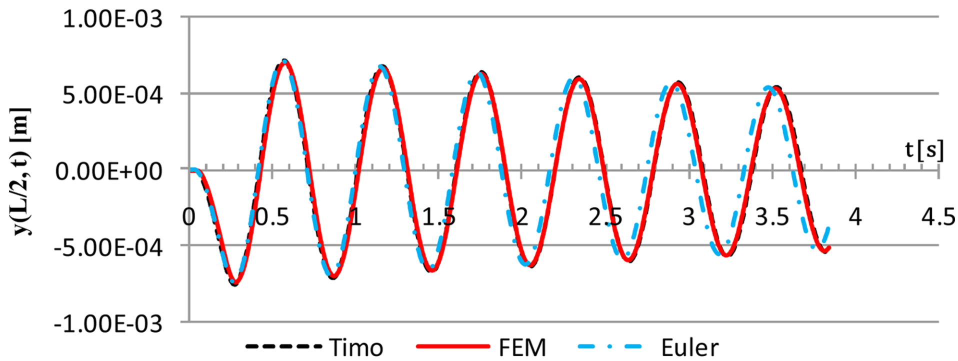

Fig. 13 Deflection of mid-span in terms of time for L = 1.116 [m], u = 4.376 [m/s], G_1 = 75e3 [MPa], G_2 = 22.5e5 [MPa], η = 3752.804 [MPa.s], and K = 16.248, E_1 = 0.75, β = 50, α = 1

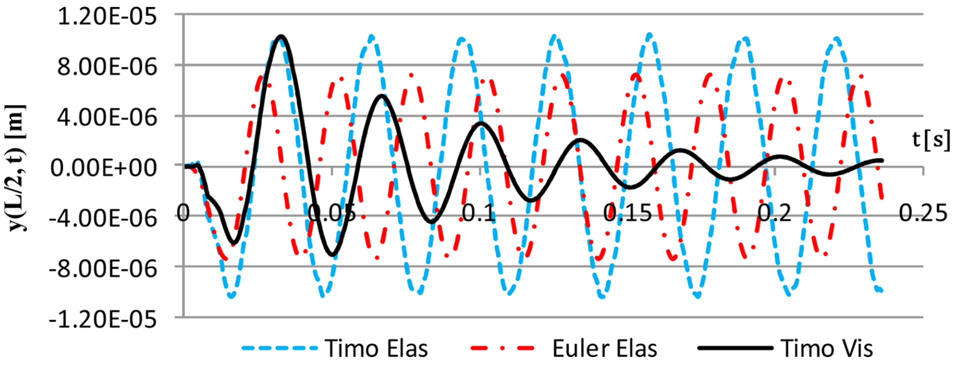

Fig. 14 Deflection of mid-span in terms of time for u = 15.903 [m/s] and G_2 = 3000000 [MPa] for Elastic cases G_1 = 750000 [MPa], G_2 = 2250000 [MPa], η = 3752.804 [MPa.s], and K = 1, E_1 = 0.75, β = 25, α = 1

$$\frac{d^4a(t)}{dt^4}+\left(\frac{G_2}{\rho L^2}\right)\left(\lambda\frac{AL^2}{I}+(\lambda+3)n^2\pi^2\right)\frac{d^2a(t)}{dt^2}+3\lambda n^4\pi^4\left(\frac{G_2}{\rho L^2}\right)^2a(t)=$$

$$\frac{2P_0}{\rho AL}\left(-n^2\omega^2+\left(\frac{G_2}{\rho L^2}\right)\left(\lambda\frac{AL^2}{I}+3n^2\pi^2\right)\right)\sin(n\omega t) \tag{30}$$

In Fig. 14 a comparison between the elastic and viscoelastic cases have been performed. The amplitude of the viscoelastic case is smaller than of the elastic cases. The first peak for the viscoelastic case (Timoshenko) is about 40% its elastic case approximately. The viscoelastic case shows a damping behaviour as expected and its damped oscillation period is longer than the elastic cases studies.

5. Convergence

Fig. 15 shows the response for different time steps where fr1 is the first natural frequency. This Fig changes for different values of time step sizes. In Fig. 16, the effect of number of element on

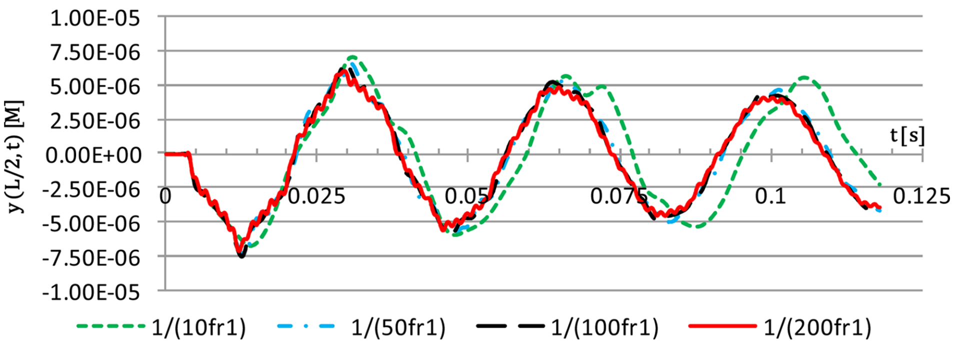

Fig. 15 Response for different time step. L = 0.25 [m], u = 31.806 [m/s], G_1 = 750000[MPa], G_2 = 2250000 [MPa], η = 750.561 [MPa.s], number of element = 30

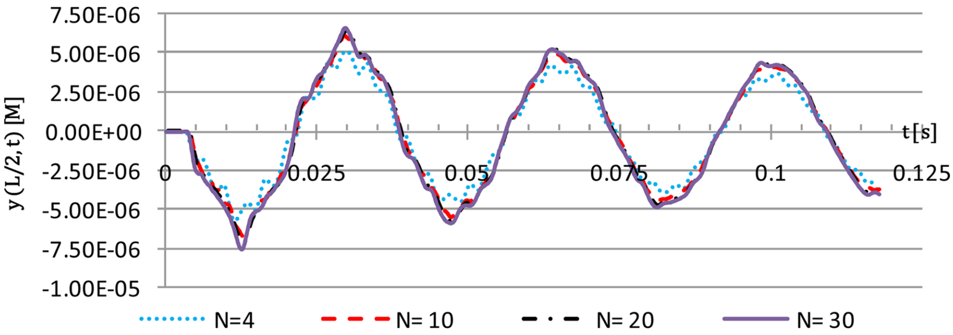

Fig. 16 Effect of number of elements on response. L = 0.25 [m], u = 31.806 [m/s], G_1 = 750000 [MPa], G_2 = 2250000 [MPa], $\eta = 750.561$ [MPa.s], the time step size = 1/100fr1

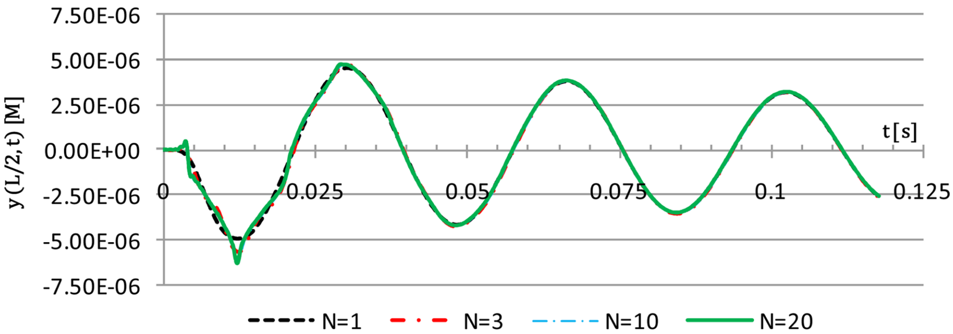

Fig. 17 Effect of numbers of terms in Fourier series on response. L = 0.25 [m], u = 31.806 [m/s], G_1 = 750000 [MPa], G_2 = 2250000 [MPa], $\eta = 750.561$ [MPa.s], the time step size = 1/100fr1

the response is shown and Fig. 17 shows the effect of number of terms in Fourier series on response. In the presented results, we set the time step equal to 1/200fr1 for $\alpha > 1$, 1/100fr1 for $\alpha < 1$, 30 elements and three terms of series.

6. Conclusions

- By increasing the velocity, the difference between the analytical and numerical response increases too.
- For Euler or Timoshenko theory the maximum damping corresponds to K = 1.
- When the parameter K increases the behavior of the beam response approaches to the elastic form.
- Approximately when $E_1 > 0.5$ Timoshenko formulation is closer to the FE response but for $E_1 < 0.5$ the Euler case is nearer to the FE graph.
- The effect of parameter η in Timoshenko case is more significant than Euler case. Also analytical Timoshenko response for high damping is unsatisfactory.
- When $E_1 \rightarrow 1$ this represents an elastic behaviour and when it approaches to zero the viscose behaviour is dominant.

- For large values of β the responses of Timoshenko, Euler and FE are similar to each others.
- Due to shearing effects, the Timoshenko theory shows more damping effects than the Euler theory.

References

Andersen, L., Nielsenm, S.R.K. and Kirkegaard, P.H. (2001), "Finite element modelling of infinite Euler beams on Kelvin foundations exposed to moving loads in convected co-ordinates", *J. Sound Vib.*, **241**(4), 587-604.

Ansys users help manual 11.

Fung, R.F., Hang, J. and Echen, W. (1996), "Daynamic stability of a viscoelastic beam subjected to harmonic and parametric excitations simultaneously", *J. Sound Vib.*, **198**, 1-16.

Fryba, L. (1972), *Vibration of Solids and Structures Under Moving Loads*, Noordhoff International, The Netherlands.

González, J.A. and Abascal, R. (2004), "Linear viscoelastic boundary element formulation for steady state moving loads", *J. Eng. Anal. Bound. Elem.*, **28**, 815-823.

Huang, C.C. (1975), "Forced motions of viscoelastic cylindrical", *J. Sound Vib.*, **39**(3), 273-286.

Huang, C.C. (1976), "Forced motions of viscoelastic thick cylindrical shells", *J. Sound Vib.*, **45**(4), 529-537.

Huang, C.C. (1978), "Moving loads on viscoelastic cylindrical shells", *J. Sound Vib.*, **60**(3), 351-358.

Hagedorn, P. and DusGupta, A. (2007), *Vibration and Wave in Continuous Mechanical Systems*, John Wiley & Sons, Ltd.

Kocatürk, T. and Şimşek, M. (2006), "Vibration of viscoelastic beams subjected to an eccentric compressive force and a concentrated moving harmonic force", *J. Sound Vib.*, **291**, 302-322.

Kocatürk, T. and Şimşek, M. (2006), "Dynamic analysis of eccentrically prestressed viscoelastic Timoshenko beams under a moving harmonic load", *Comput. Struct.*, **84**, 2113-2127.

Mofid, M., Tehranchi, A. and Ostadhossein, A. (2010), "On the viscoelastic beam subjected to moving mass", *Adv. Eng. Softw.*, **41**, 240-243.

Riande, E., Calleja, R.D., Prolongo, M.G., Masegosa, R.M. and Salom, C. (2000), *Polymer Viscoelasticity Stress And Strain in Practice*, Marcel Dekker Inc.

Şimşek, M. and Kocatürk, T. (2009), "Nonlinear dynamic analysis of an eccentrically prestressed damped beam under a concentrated moving harmonic load", *J. Sound Vib.*, **320**, 235-253.

Tehrani, M. and Eipakchi, H.R. (2012), "Analysis of shearing viscoelastic beam under moving load", *Shock Vib.*, **19**, 447-458.

Zhang, Y.L., Gorman, D.G. and Reese, J.M. (2001), "A modal and damping analysis of viscoelastic Timoshenko tubes conveying fluid", *Int.J. Numer. Meth. Eng.*, **50**, 419-433.