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Modified finite element-transfer matrix method for the static analysis of structures

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Abstract. In this paper the Modified Finite Element-Transfer Matrix Method, which is the combination of Transfer Matrix Method and Finite Element Method, is applied to the static analysis of the structures. In the method, the structure is divided into substructures thus the number of unknowns that need to be worked out is reduced due to the transformation process. The static analysis of the structures can be performed easily and speedily by the proposed method. At the end of the study examples are presented for ensuring the agreement between the proposed method and classic Finite Element Method.

Keywords: modified finite element transfer matrix; transformation; static analysis

1. Introduction

Structural analysis can be carried out by different types of methods one of which is Finite Element-Transfer Matrix Method. This method, which is combining Transfer Matrix Method (Xiang *et al.* 2008, Bozdogan and Ozturk 2009) and Finite Element Method, is used in several studies in the literature (Dokanish 1972, Ohga *et al.* 1984, Degen *et al.* 1985, Xue 1997, Bhutani and Loewy 1999, Xue 2003a, b, Xue 2004, Duhamel *et al.* 2006, Tu *et al.* 2008, Abbas *et al.* 2010, Chevillotte and Panneton 2011, Rong *et al.* 2011).

Rong *et al.* (2011) developed an algorithm for eigenvalue problems of structures. In their study, free vibration analysis of the structures was investigated by combining Finite Element and Transfer Matrix Method. The advantage of the method is expressed as solving the problem easily due to having a low order system matrix.

In this paper, the method proposed by Rong *et al.* (2011) is applied to the static analysis of the structures. The material is accepted as linear elastic and the geometric nonlinear effects are ignored in the study.

2. Method

The Modified Finite Element-Transfer Matrix method is presented for the static analysis of the

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structures. In the method, the structure is divided into n number of substructures.

Static equilibrium equation for *i*th substructure is written as below

$$\begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix} \begin{Bmatrix} U_{i-1} \\ U_i \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_i \end{Bmatrix} = \begin{Bmatrix} -Q_{i-1} \\ Q_i \end{Bmatrix} \tag{1}$$

where K_{i1}, K_{i2}, K_{i3} and K_{i4} are the submatrices of stiffness matrix of *i*th substructure, U is the displacement vector, F is the external force vector, Q is the internal force vector, respectively.

The equation for Q_{i-1} and Q_i is considered as

$$Q_{i-1} = T_{i-1}U_{i-1} + P_{i-1} \tag{2}$$

$$Q_i = T_iU_i + P_i \tag{3}$$

Substituting these two equations in Eq. (2) and Eq. (3) into equation Eq. (1) gives the matrix equation system below.

$$\begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix} \begin{Bmatrix} U_{i-1} \\ U_i \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_i \end{Bmatrix} = \begin{Bmatrix} -T_{i-1}U_{i-1} - P_{i-1} \\ T_iU_i + P_i \end{Bmatrix} \tag{4}$$

Eq. (5) and Eq. (6) are obtained from the Eq. (4).

$$K_1U_{i-1} + K_2U_i = -T_{i-1}U_{i-1} - P_{i-1} \tag{5}$$

$$K_3U_{i-1} + K_4U_i + F_i = T_iU_i + P_i \tag{6}$$

From Eq. (5) U_{i-1} can be written in terms of U_i

$$U_{i-1} = -(K_1 + T_{i-1})^{-1}K_2U_i - (K_1 + T_{i-1})^{-1}P_{i-1} \tag{7}$$

When U_{i-1} in Eq. (6) is substituted with the expression in Eq. (7), Eq. (8) is obtained.

$$-K_3(K_1 + T_{i-1})^{-1}K_2U_i - K_3(K_1 + T_{i-1})^{-1}P_{i-1} + K_4U_i + F_i = T_iU_i + P_i \tag{8}$$

Eq. (9) and Eq. (10) are obtained from the Eq. (8).

$$T_i = -K_3(K_1 + T_{i-1})^{-1}K_2 + K_4 \tag{9}$$

$$P_i = -K_3(K_1 + T_{i-1})^{-1}P_{i-1} + F_i \tag{10}$$

T_1 and P_1 are obtained by using boundary conditions. They vary depending on the boundary conditions. For fixed supports, as the boundary conditions are applied to the Eq. (6), U_{i-1} is equal to zero vector.

$$K_4U_1 + F_1 = T_1U_1 + P_1 \tag{11}$$

After obtaining Eq. (11), T_1 and P_1 are denoted as

$$T_1 = K_4 \tag{12}$$

$$P_1 = F_1 \tag{13}$$

Relations in Eq. (9) and Eq. (10) are applied through the structure successively. For *n*th substructure the equation below is obtained.

$$T_n U_n + P_n = Q_n \tag{14}$$

For the *n*th substructure vector of Q_n is equal to zero vector.

$$U_n = -T_n^{-1} P_n \tag{15}$$

After the displacements of *n*th substructure are obtained, other displacements are found successively by using Eq. (16).

$$U_{i-1} = K_3^{-1}(T_i U_i + P_i - K_4 U_i - F_i) \tag{16}$$

Finally by using the displacements and the stiffness matrices of each substructure internal forces are found.

3. Procedure

In the study a code is developed for the proposed method by using Matlab. The procedure of the code is explained, briefly.

- The structure is divided into substructures.
- Stiffness matrix of each substructure is assembled according to the finite element method.
- T matrix and P vector are obtained for each substructure.
- The displacements at the top of the structure are obtained by Eq. (15).
- The displacements for each level are obtained by Eq. (16).
- The internal forces are obtained by using the displacements and stiffness matrices of each substructure.

4. Numerical examples

The proposed method in this study can be applied to various structures like trusses, frames and shear walls. In the first example the method is applied to a planar coupled shear wall and the obtained results are presented. Afterwards two examples of planar frame systems are studied to investigate the method.

4.1 Example 1

To investigate the proposed method two-bay coupled shear wall in Fig. 1 is considered. The height of the coupled shear wall is 78 m, the lengths of the shear walls are 7.32 m and the thicknesses are 0.305 m. The heights of connecting beams are 0.7 m and the thicknesses are 0.305 m. Modulus of Elasticity is considered as $2.5 * 10^7$ kN/m^2. The loads acting at each floor

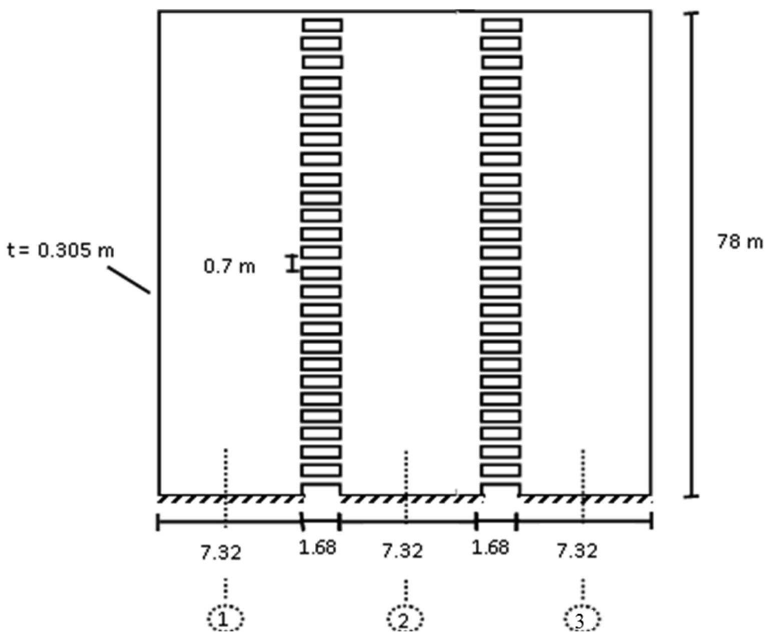

Fig. 1 Two-bay coupled shear wall.

Table 1 The loads acting at each floor level

Storey	Load (kN)	Storey	Load (kN)
1	30	14	420
2	60	15	450
3	90	16	480
4	120	17	510
5	150	18	540
6	180	19	570
7	210	20	600
8	240	21	630
9	270	22	660
10	300	23	690
11	330	24	720
12	360	25	750
13	390	26	780

Table 2 Comparison of the lateral displacements

Storey	Proposed Method (m)	Sap2000 (m)
1	0.0011	0.0011
3	0.0059	0.0059
5	0.0131	0.0132
7	0.0219	0.0220
10	0.0372	0.0374
12	0.0484	0.0487
15	0.0660	0.0664
18	0.0840	0.0845
20	0.0959	0.0964
23	0.1131	0.1137
26	0.1295	0.1303

Table 3 Forces at base level of first shear wall

	Proposed Method	Sap2000
F_x	3300.6 kN	3253.2 kN
F_y	25312 kN	25347 kN
M	35074 kNm	35064 kNm

Table 4 Forces at base level of third shear wall

	Proposed Method	Sap2000
F_x	3003 kN	3158 kN
F_y	25297 kN	25263 kN
M	34899 kNm	34294 kNm

level is given in Table 1. The static analysis of the coupled shear wall system is made by the proposed method and the results are compared with the ones obtained by Sap2000 Program. The displacements of each floor are presented in Table 2. The forces at base level of the first shear wall are presented in Table 3 and the base internal forces of the third shear wall are presented in Table 4.

4.2 Example 2

The frame system in Fig. 2 is investigated in this example. The beams and the columns are W24 × 146 in 20-storey frame system with cross sectional area of 0.0277 m^2 and moment of inertia of 1.906×10^{-3} m^4. Modulus of elasticity is 199.948 GPa in the example. The shear deformation is ignored. The loads are applied to the system as illustrated in Fig. 2 and the proposed method is tested against Sap2000 Program.

The comparison of the top displacements is presented in Table 5 while the comparison of the forces at base level is presented in Table 6. The computational time comparison using the proposed method and the Finite Element Method is given in Table 7.

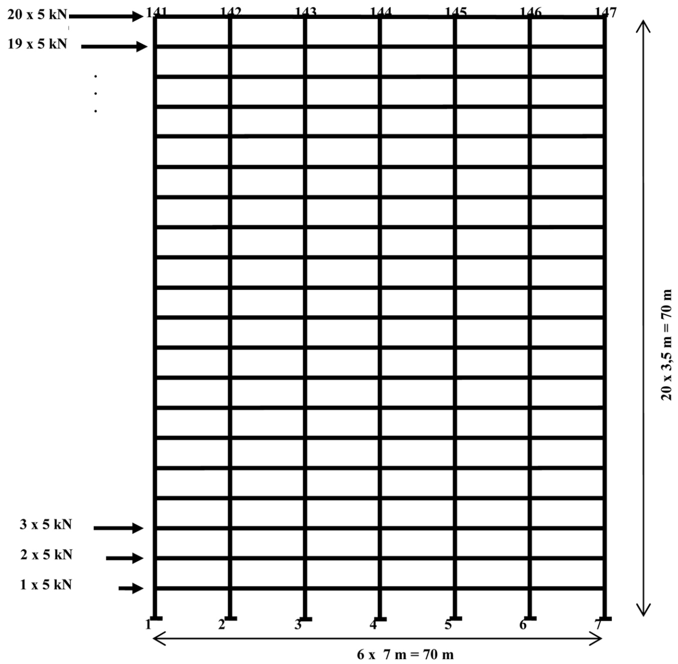

Fig. 2 Frame system in Example 2

Table 5 The comparison of the top displacements

Node	Displacement	Proposed Method	Sap2000
141	Δ_x	0.0746 m	0.0746 m
	Δ_y	0.0045 m	0.0045 m
	θ	0.0004 rad	0.0004 rad
142	Δ_x	0.0745 m	0.0745 m
	Δ_y	0.0018 m	0.0018 m
	θ	0.0003 rad	0.0003 rad
143	Δ_x	0.0744 m	0.0744 m
	Δ_y	0.0005 m	0.0005 m
	θ	0.0002 rad	0.0002 rad
144	Δ_x	0.0743 m	0.0743 m
	Δ_y	0.0000 m	0.0000 m
	θ	0.0002 rad	0.0002 rad
145	Δ_x	0.0743 m	0.0742 m
	Δ_y	0.0006 m	0.0006 m
	θ	0.0002 rad	0.0002 rad
146	Δ_x	0.0743 m	0.0742 m
	Δ_y	0.0018 m	0.0018 m
	θ	0.0003 rad	0.0003 rad
147	Δ_x	0.0743 m	0.0743 m
	Δ_y	0.0044 m	0.0044 m
	θ	0.0004 rad	0.0004 rad

Table 6 The comparison of the forces at base level

Node	Force	Proposed Method	Sap2000
1	F_x	119.5847 kN	119.5885 kN
	F_y	973.9283 kN	974.0460 kN
	M	318.4856 kNm	318.4877 kNm
2	F_x	161.9905 kN	161.9908 kN
	F_y	212.5780 kN	212.4368 kN
	M	369.5126 kNm	369.5086 kNm
3	F_x	162.6488 kN	162.6456 kN
	F_y	66.3979 kN	66.3259 kN
	M	371.1050 kNm	371.0959 kNm
4	F_x	163.3333 kN	163.3295 kN
	F_y	0.9318 kN	0.9309 kN
	M	372.0566 kNm	372.0467 kNm
5	F_x	162.3242 kN	162.3214 kN
	F_y	68.0414 kN	67.9680 kN
	M	370.3892 kNm	370.3809 kNm
6	F_x	161.3221 kN	161.3231 kN
	F_y	213.3343 kN	213.1926 kN
	M	368.0525 kNm	368.0502 kNm
7	F_x	118.7963 kN	118.8010 kN
	F_y	970.5966 kN	970.7772 kN
	M	316.5285 kNm	316.5328 kNm

Table 7 The computational time

	Proposed Method	Finite Element Method
Time (s)	0.270601	2.758461

4.3 Example 3

In this example the proposed method is applied to the frame system in Fig. 3. The beams having cross-sectional dimensions 0.25 m × 0.50 m are same for two storeys. For the first and second storey the columns are 0.50 × 0.50 m and 0.30 × 0.30 m in cross section, respectively. Modulus of elasticity is 3×10^6 (3000000) GPa. The shear deformation is ignored. The load acting on the first storey level is 30 kN and the one acting on the second storey level is 60 kN.

The analysis made by the proposed method and the results are presented in the tables. In Table 8 the top displacements are compared and in Table 9 the base forces are compared.

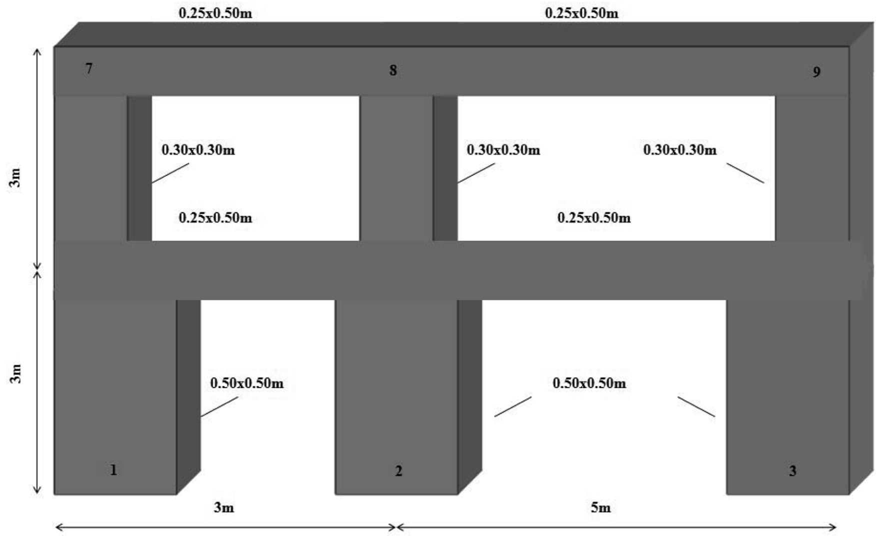

Fig. 3 Frame system in Example 3

Table 8 The comparison of the top displacements

Node	Displacement	Proposed Method	Sap2000
7	Δ_x	0.0414 m	0.0414 m
	Δ_y	0.0004 m	0.0004 m
	θ	0.0028 rad	0.0028 rad
8	Δ_x	0.0411 m	0.0411 m
	Δ_y	0.0002 m	0.0002 m
	θ	0.0008 rad	0.0008 rad
9	Δ_x	0.0408 m	0.0408 m
	Δ_y	0.0002 m	0.0002 m
	θ	0.0038 rad	0.0038 rad

Table 9 The comparison of the forces at base level

Node	Force	Proposed Method	Sap2000
1	F_x	29.1107 kN	29.1107 kN
	F_y	49.6219 kN	49.6219 kN
	M	63.4767 kNm	63.4767 kNm
2	F_x	38.8787 kN	38.8787 kN
	F_y	27.7431 kN	27.7431 kN
	M	72.6790 kNm	72.6790 kNm
3	F_x	22.0106 kN	22.0106 kN
	F_y	21.8788 kN	21.8788 kN
	M	55.5844 kNm	55.5844 kNm

5. Conclusions

In this paper, a method that is combining Transfer Matrix Method and Finite Element Method is investigated. The Modified Finite Element-Transfer Matrix Method, which was previously proposed for the free vibration analysis of structures, is developed for the static analysis of the structures. In the method, the structure is divided into substructures thus the number of the unknowns that need to be worked out in the structure is reduced due to the transformation process.

Three examples are presented for ensuring the agreement between the proposed method and classic Finite Element Method at the end of the study. Initially the method is applied to a coupled shear wall and the analysis results are presented in tables. Afterwards the proposed method is applied to two different frame systems and the results are presented. As the results obtained by using the proposed method and by using the classical method are compared, it is obviously seen that the displacements and forces values given in the tables displayed good agreements. Consequently, the Modified Finite Element-Transfer Matrix Method provides to make the static analysis of the structures easily and speedily.

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