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An efficient approach to structural static reanalysis with added support constraints

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Abstract. Structural reanalysis is frequently used to reduce the computational cost during the process of design or optimization. The supports can be regarded as the design variables in various types of structural optimization problems. The location, number, and type of supports may be varied in order to yield a more effective design. The paper is focused on structural static reanalysis problem with added supports where some node displacements along axes of the global coordinate system are specified. A new approach is proposed and exact solutions can be provided by the approach. Thus, it belongs to the direct reanalysis methods. The information from the initial analysis has been fully exploited. Numerical examples show that the exact results can be achieved and the computational time can be significantly reduced by the proposed method.

Keywords: structural static reanalysis; Cholesky factorization; increase of support constraints; stiffness matrix; computational cost

1. Introduction

The design or optimization of a structure is usually an iterative process, and some of the design variables are varied during each iterative step. Each variation of the design variables requires a fresh analysis for stresses and displacements. Thus, repeated and tremendous calculations may be involved. To reduce the high computational cost, structural static reanalysis problem has been proposed. The purpose of static reanalysis is to evaluate accurately structural responses under a given static load of the structure after modifications by using the original information which has already been known from the initial analysis as much as possible so that the computational cost can be greatly reduced (Abu Kasim and Topping 1987). Static reanalysis techniques are significant for large structures, especially for finite element analysis (FEA) where only a small part of the structure is progressively modified (Hassan *et al*. 2010, Terdalkar and Rencis 2006).

So far, many static reanalysis methods have been proposed. Generally speaking, the various methods can be classified into the following two categories: direct reanalysis methods and approximate reanalysis methods. Direct methods provide exact closed-form solutions and are

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efficient for the modifications where the changes in design variables are large in magnitude, yet only affect a relative small part of the structure. Most of these methods are based on Sherman-Morrison-Woodbury formulae (Sherman and Morrison 1949, Woodbury 1950). Various improvements were developed and the readers are referred to Akgün *et al*. (2001). Approximate reanalysis methods are suitable for the minor modifications to a large part or the whole of the structure, these methods give the approximate solutions of the response of the modified structure by using the information which has already been obtained during the full analysis of the original structure. In these methods, the accuracy of the approximate solution and the rate of the convergence are extremely important. Approximate reanalysis methods can be divided into the following four classes: local approximations, global approximations, combined approximations (CA), and preconditioned conjugate gradient (PCG) approximations. For the detailed derivation of the above four methods, we refer the readers to Kirsch (2008), Li *et al*. (2007).

In general, the design variables used in structural design or optimization can be categorized as follows (Olhoff and Taylor 1983): cross-sectional design variables, material design variables, geometrical design variables, topological design variables, shape design variables, and support design variables. For the detailed meaning of these design variables, we refer the readers to Olhoff and Taylor (1983). The structural modifications corresponding to the variation of the above design variables are called cross-sectional modifications, geometrical modifications, topological modifications, layout modifications and supporting modifications, respectively. In recent years, the reanalysis of the first four modifications have been extensively studied, and some progress has been made (Li and Wu 2007, Kirsch 2008). However, structural supports as the optimization variables have been widely used (Akesson and Olhoff 1988, Wang and Chen 1996, Takezawa *et al*. 2006, Tanskanen 2006, Zhu and Zhang 2010), especially in the field of building construction, aircraft structures and printed circuit boards (Wang *et al*. 2004). In addition, elastic contact problems, such as normal, tangential, and rolling contacts, can be transformed into multiple point constraints for nodal displacements in the FEA method (Liu *et al*. 2010). The reanalysis studies on supporting modifications are relatively few. Thus, it is eager to study the reanalysis method for such a modification. The supporting modifications include the variations of the location, the number, and the type of the structural support (Olhoff and Taylor 1983). A small change in support can influence the structural performance significantly, especially in the displacements of the nodes under a given load and the natural frequency. At the same time, these modifications often result in the variations of the number of the degrees of freedom (DOFs). Therefore, the reanalysis on the supporting modifications are challenging problems in the field of structural reanalysis.

This paper investigates the static reanalysis problem with added supports where some node displacements along axes of the global coordinate system are specified. A new approach is proposed. The algorithm preserves the ease of implementation and can be used with a general finite element system. The remainder of the paper is organized as follows. The problem is formulated in Section 2 and the algorithm is presented in Section 3. Numerical examples are given to validate the effectiveness of the proposed method in Section 4. Conclusions are drawn in Section 5.

2. Problem formulation

The static reanalysis problem for addition of support constraints, where some node displacements along axes of the global coordinate system are specified, can be stated as follows.

Given an initial design and the corresponding stiffness matrix $\mathbf{K}_0 \in R^{m \times m}$, the displacements vector $\mathbf{x}_0$ can be calculated by the following equilibrium equations

$$\mathbf{K}_0 \mathbf{x}_0 = \mathbf{R}_0 \tag{1}$$

where $\mathbf{R}_0$ denotes the load vector. The stiffness matrix $\mathbf{K}_0$ is symmetric and positive definite (SPD). From the initial analysis, the Cholesky factorization of $\mathbf{K}_0$ has already been known

$$\mathbf{K}_0 = \mathbf{L}_0 \mathbf{L}_0^T \tag{2}$$

where $\mathbf{L}_0$ is a lower triangular matrix, $\mathbf{L}_0^T$ represents the transpose of $\mathbf{L}_0$.

Suppose that the structure is modified by addition of some support constraints where k node displacements along axes of the global coordinate system are specified. The structures of truss shown in Fig. 1 are used to illustrate the capability of the proposed method. Case of adding support constraint like Fig. 1(b) is considered in this paper, while case of adding skew support constraint like Fig. 1(c) is not dealt with in this paper.

Compared with the number of the original DOFs, the number of the added support constraints is usually very small, i.e., $k << m$. The equilibrium equation of the modified structure is

$$\mathbf{K}\mathbf{x} = \mathbf{R} \tag{3}$$

where $\mathbf{K} \in R^{(m-k)\times(m-k)}$ is the stiffness matrix of the modified structure and is also SPD, $\mathbf{x}$ denotes

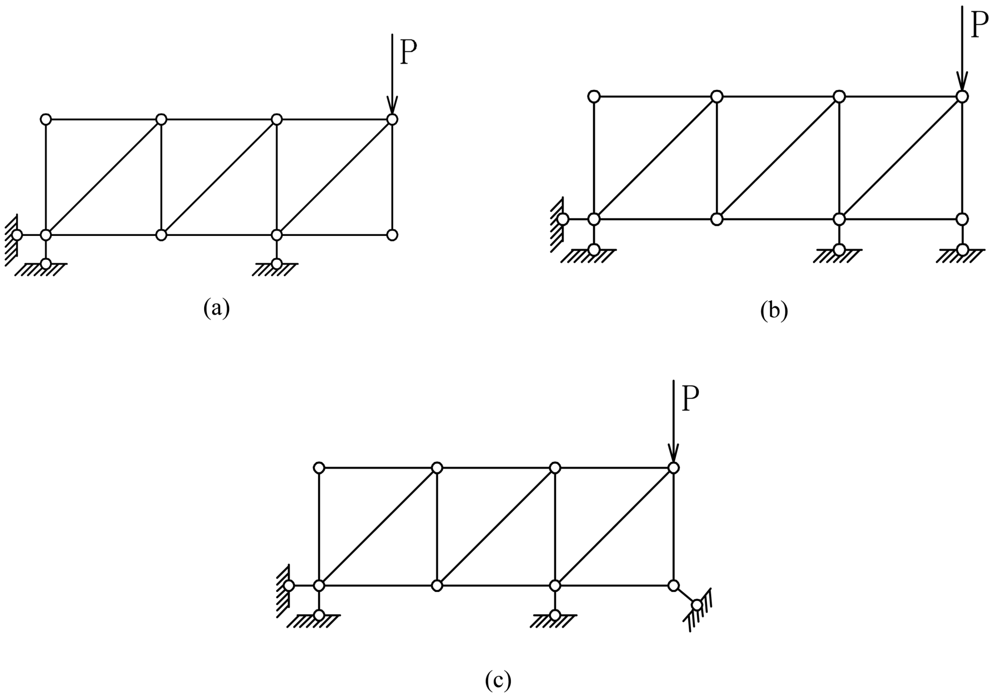

Fig. 1 Structures of truss. (a) initial design, (b) modified design with one added support along vertical axis of the global coordinate system, (c) modified design with one added skew support

the displacements vector and **R** represents the load vector of the modified structure. The purpose of static reanalysis is to calculate the displacements vector **x** by using the information of the initial analysis so that the computational cost can be significantly reduced. Once the displacements are obtained, the stresses can be readily determined by utilizing explicit stress-displacement relations.

To illustrate our approach, the relationship between $\mathbf{K}_0$ and $\mathbf{K}$ is given as follows. Suppose

$$\mathbf{K}_0 = \begin{bmatrix} k_{11} & \cdots & k_{1i_1-1} & k_{1i_1} & k_{1i_1+1} & \cdots & k_{1i_k-1} & k_{1i_k} & k_{1i_k+1} & \cdots & k_{1m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{i_1-11} & \cdots & k_{i_1-1i_1-1} & k_{i_1-1i_1} & k_{i_1-1i_1+1} & \cdots & k_{i_1-1i_k-1} & k_{i_1-1i_k} & k_{i_1-1i_k+1} & \cdots & k_{i_1-1m} \\ k_{i_11} & \cdots & k_{i_1i_1-1} & k_{i_1i_1} & k_{i_1i_1+1} & \cdots & k_{i_1i_k-1} & k_{i_1i_k} & k_{i_1i_k+1} & \cdots & k_{i_1m} \\ k_{i_1+11} & \cdots & k_{i_1+1i_1-1} & k_{i_1+1i_1} & k_{i_1+1i_1+1} & \cdots & k_{i_1+1i_k-1} & k_{i_1+1i_k} & k_{i_1+1i_k+1} & \cdots & k_{i_1+1m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{i_k-11} & \cdots & k_{i_k-1i_1-1} & k_{i_k-1i_1} & k_{i_k-1i_1+1} & \cdots & k_{i_k-1i_k-1} & k_{i_k-1i_k} & k_{i_k-1i_k+1} & \cdots & k_{i_k-1m} \\ k_{i_k1} & \cdots & k_{i_ki_1-1} & k_{i_ki_1} & k_{i_ki_1+1} & \cdots & k_{i_ki_k-1} & k_{i_ki_k} & k_{i_ki_k+1} & \cdots & k_{i_km} \\ k_{i_k+11} & \cdots & k_{i_k+1i_1-1} & k_{i_k+1i_1} & k_{i_k+1i_1+1} & \cdots & k_{i_k+1i_k-1} & k_{i_k+1i_k} & k_{i_k+1i_k+1} & \cdots & k_{i_k+1m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{m1} & \cdots & k_{mi_1-1} & k_{mi_1} & k_{mi_1+1} & \cdots & k_{mi_k-1} & k_{mi_k} & k_{mi_k+1} & \cdots & k_{mm} \end{bmatrix}_{m\times m} \tag{4}$$

K can be obtained by deleting some rows and columns of $\mathbf{K}_0$. Assume the $i_1\text{th}, \ldots, i_k\text{th}$ rows and the $i_1\text{th}, \ldots, i_k\text{th}$ columns are deleted, where $i_1, \ldots, i_k$ are determined by the numbers of the nodes on which the support constraints are added. Thus

$$\mathbf{K} = \begin{bmatrix} k_{11} & \cdots & k_{1i_1-1} & k_{1i_1+1} & \cdots & k_{1i_k-1} & k_{1i_k+1} & \cdots & k_{1\,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{i_1-11} & \cdots & k_{i_1-1\,i_1-1} & k_{i_1-1\,i_1+1} & \cdots & k_{i_1-1\,i_k-1} & k_{i_1-1\,i_k+1} & \cdots & k_{i_1-1\,m} \\ k_{i_1+1\,1} & \cdots & k_{i_1+1\,i_1-1} & k_{i_1+1\,i_1+1} & \cdots & k_{i_1+1\,i_k-1} & k_{i_1+1\,i_k+1} & \cdots & k_{i_1+1\,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{i_k-1\,1} & \cdots & k_{i_k-1\,i_1-1} & k_{i_k-1\,i_1+1} & \cdots & k_{i_k-1\,i_k-1} & k_{i_k-1\,i_k+1} & \cdots & k_{i_k-1\,m} \\ k_{i_k+1\,1} & \cdots & k_{i_k+1\,i_1-1} & k_{i_k+1\,i_1+1} & \cdots & k_{i_k+1\,i_k-1} & k_{i_k+1\,i_k+1} & \cdots & k_{i_k+1\,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ k_{m\,1} & \cdots & k_{m\,i_1-1} & k_{m\,i_1+1} & \cdots & k_{m\,i_k-1} & k_{m\,i_k+1} & \cdots & k_{mm} \end{bmatrix}_{(m-k)\times(m-k)} \tag{5}$$

3. The algorithm of adding rows and columns

Based on the structures of $\mathbf{K}_0$ and $\mathbf{K}$ in Eqs. (4) and (5), respectively, it can be shown (Sun and Yuan 2006)

$$\mathbf{K} = \tilde{\mathbf{L}}\tilde{\mathbf{L}}^T \tag{6}$$

where $\tilde{\mathbf{L}} \in R^{(m-k)\times m}$ is obtained by deleting the i_1th, …, i_kth row of $\mathbf{L}_0$. The equilibrium Eq. (2) of the modified structure, however, can not be calculated by solving the following two equations

$$\tilde{\mathbf{L}}\mathbf{y} = \mathbf{R} \tag{7}$$

$$\tilde{\mathbf{L}}^T\mathbf{x} = \mathbf{y} \tag{8}$$

since the coefficient matrices of Eqs. (7) and (8) are not square.

The proposed method in this paper can be described as follows. The column partition of $\mathbf{L}_0^T$ can be written as

$$\mathbf{L}_0^T = [\mathbf{L}_1 \;\; \mathbf{L}_2 \; \ldots \; \mathbf{L}_m] \tag{9}$$

where $\mathbf{L}_i \in R^m$ denotes the ith column of $\mathbf{L}_0^T$, thus

$$\mathbf{L}_0 = \begin{bmatrix} \mathbf{L}_1^T \\ \mathbf{L}_2^T \\ \vdots \\ \mathbf{L}_m^T \end{bmatrix} \tag{10}$$

Let

$$\overline{\mathbf{R}} = (\mathbf{R}_1, \ldots, \mathbf{R}_{i_1-1}, 0, \mathbf{R}_{i_1}, \ldots, \mathbf{R}_{i_k-1}, 0, \mathbf{R}_{i_k}, \ldots, \mathbf{R}_{m-k})^T \in R^m \tag{11}$$

that is to say, the i_1th, …, i_kth components of $\overline{\mathbf{R}}$ are all 0, the remaining components are the same as the components of $\mathbf{R}$ and the order is also kept unchanged. Using the definition of $\overline{\mathbf{R}}$ in Eqs. (11) and (7), we have

$$\mathbf{L}_0\mathbf{y} = \begin{bmatrix} \mathbf{L}_1^T\mathbf{y} \\ \mathbf{L}_2^T\mathbf{y} \\ \vdots \\ \mathbf{L}_m^T\mathbf{y} \end{bmatrix} = \overline{\mathbf{R}} + (\mathbf{L}_{i_1}^T\mathbf{y}) \begin{array}{l} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \leftarrow \text{the } i_1\text{th row} \\ \\ \\ \\ \\ \\ \\ \end{array} + \cdots + (\mathbf{L}_{i_k}^T\mathbf{y}) \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \\ \\ \\ \\ \\ \\ \leftarrow \text{the } i_k\text{th row} \\ \\ \\ \end{array} \tag{12}$$

Let

$$\begin{aligned} \mathbf{b}_1 &= \begin{bmatrix} 0 & \cdots & 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \end{bmatrix}^T \\ &\vdots \\ \mathbf{b}_k &= \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 & \cdots & 0 \end{bmatrix}^T \end{aligned} \tag{13}$$

where the i_jth component of $\mathbf{b}_j \in R^n (j = 1, \ldots, k)$ is 1 and zero entries elsewhere. Then Eq. (12) can be rewritten as

$$\mathbf{L}_0\mathbf{y} = \overline{\mathbf{R}} + (\mathbf{L}_{i_1}^T\mathbf{y})\mathbf{b}_1 + \ldots + (\mathbf{L}_{i_k}^T\mathbf{y})\mathbf{b}_k \tag{14}$$

Pre-multiplying the equation above by $\mathbf{L}_0^{-1}$ on both sides yields

$$\mathbf{y} = \mathbf{L}_0^{-1}\overline{\mathbf{R}} + (\mathbf{L}_{i_1}^T\mathbf{y})\mathbf{L}_0^{-1}\mathbf{b}_1 + \ldots + (\mathbf{L}_{i_k}^T\mathbf{y})\mathbf{L}_0^{-1}\mathbf{b}_k \tag{15}$$

Let

$$\overline{\mathbf{x}} = (\mathbf{x}_1, \ldots, \mathbf{x}_{i_1-1}, 0, \mathbf{x}_{i_1}, \ldots, \mathbf{x}_{i_k-1}, 0, \mathbf{x}_{i_k}, \ldots, \mathbf{x}_{m-k})^T \in R^m \tag{16}$$

i.e., the $i_1\text{th}, \ldots, i_k\text{th}$ components of $\overline{\mathbf{x}}$ are all 0, the remaining components are the same as the components of $\mathbf{x}$ and the order is also kept unchanged. Using the definition of $\overline{\mathbf{x}}$ in Eq. (16), the following relation can be readily verified

$$\mathbf{L}_0^T\overline{\mathbf{x}} = \tilde{\mathbf{L}}^T\mathbf{x} \tag{17}$$

By Eqs. (8), (15) and (17), we obtain the relation

$$\mathbf{L}_0^T\overline{\mathbf{x}} = \mathbf{L}_0^{-1}\overline{\mathbf{R}} + (\mathbf{L}_{i_1}^T\mathbf{y})\mathbf{L}_0^{-1}\mathbf{b}_1 + \ldots + (\mathbf{L}_{i_k}^T\mathbf{y})\mathbf{L}_0^{-1}\mathbf{b}_k \tag{18}$$

Pre-multiplying the two sides of Eq. (18) by $\mathbf{L}_0^{-T}$ yields

$$\overline{\mathbf{x}} = \mathbf{L}_0^{-T}\mathbf{L}_0^{-1}\overline{\mathbf{R}} + (\mathbf{L}_{i_1}^T\mathbf{y})\mathbf{L}_0^{-T}\mathbf{L}_0^{-1}\mathbf{b}_1 + \ldots + (\mathbf{L}_{i_k}^T\mathbf{y})\mathbf{L}_0^{-T}\mathbf{L}_0^{-1}\mathbf{b}_k \tag{19}$$

Let

$$\mathbf{u}_0 = \mathbf{L}_0^{-T}\mathbf{L}_0^{-1}\overline{\mathbf{R}}, \quad \mathbf{u}_i = \mathbf{L}_0^{-T}\mathbf{L}_0^{-1}\mathbf{b}_i \ \ (i = 1, \ldots, k) \tag{20}$$

Eq. (19) can then be written as

$$\overline{\mathbf{x}} = \mathbf{u}_0 + (\mathbf{L}_{i_1}^T\mathbf{y})\mathbf{u}_1 + \ldots + (\mathbf{L}_{i_k}^T\mathbf{y})\mathbf{u}_k \tag{21}$$

Comparing the $i_1\text{th}, \ldots, i_k\text{th}$ components on both sides of Eq. (21) and noting that the $i_1\text{th}, \ldots, i_k\text{th}$ components of $\overline{\mathbf{x}}$ are all 0, we get the following linear system

$$\mathbf{A}\begin{bmatrix} \mathbf{L}_{i_1}^T\mathbf{y} \\ \mathbf{L}_{i_2}^T\mathbf{y} \\ \vdots \\ \mathbf{L}_{i_k}^T\mathbf{y} \end{bmatrix} = \begin{bmatrix} -u_{0\ i_1} \\ -u_{0\ i_2} \\ \vdots \\ -u_{0\ i_k} \end{bmatrix} \tag{22}$$

where

$$\mathbf{A} = \begin{bmatrix} u_{1i_1} & u_{2i_1} & \cdots & u_{k\,i_1} \\ u_{1i_2} & u_{2i_2} & \cdots & u_{k\,i_2} \\ \vdots & \vdots & \vdots & \vdots \\ u_{1i_k} & u_{2i_k} & \cdots & u_{k\,i_k} \end{bmatrix}$$

and u_{ij} denotes the jth component $(j = i_1, i_2, \ldots, i_k)$ of vector $\mathbf{u}_i(i = 0, 1, 2, \ldots, k)$. Eq. (22) is a system of linear equations with k unknowns, where k is the number of the added support constraints. We will prove that the coefficient matrix of Eq. (22) is nonsingular, thus Eq. (22) has a unique solution. Once $\mathbf{L}_{i_1}^T\mathbf{y}, \mathbf{L}_{i_2}^T\mathbf{y}, \ldots, \mathbf{L}_{i_k}^T\mathbf{y}$ have been calculated, substitution of them into Eq. (21) yields $\bar{\mathbf{x}}$. Then the solution $\mathbf{x}$ of the equilibrium Eq. (3) can be easily obtained by utilizing the relationship between $\mathbf{x}$ and $\bar{\mathbf{x}}$ in Eq. (16).

Theorem The coefficient matrix $\mathbf{A}$ of Eq. (22) is nonsingular.

Proof From Eq. (2), we obtain

$$\mathbf{K}_0^{-1} = \mathbf{L}_0^{-T}\mathbf{L}_0^{-1} \tag{23}$$

Use of Eq. (20) yields

$$\mathbf{u}_i = \mathbf{K}_0^{-1}\mathbf{b}_i \quad (i = 1, \ldots, k) \tag{24}$$

By the definition of $\mathbf{b}_i$ $(i = 1, \ldots, k)$ in Eq. (13), we know that $\mathbf{u}_1, \ldots, \mathbf{u}_k$ are the i_1th, $\ldots$, i_kth columns of $\mathbf{K}_0^{-1}$, respectively. Recall that u_{ij} denote the jth component $(j = i_1, \ldots, i_k)$ of vector $\mathbf{u}_i$ $(i = 1, \ldots, k)$, thus the matrix $\mathbf{A}$ is a k - by - k principal submatrix of $\mathbf{K}_0^{-1}$. Note that $\mathbf{K}_0$ is a SPD matrix, thus $\mathbf{K}_0^{-1}$ is also a SPD matrix, any principal submatrix of $\mathbf{K}_0^{-1}$ is also a SPD matrix. Therefore, $\mathbf{A}$ is a SPD matrix, and it is nonsingular.

Based on the derivation above, the following algorithm, which we call the algorithm of adding rows and columns, is presented as follows.

The algorithm of adding rows and columns

1. Determine the location of the nodes on which the support constraints are added, i.e., $i_1, \ldots, i_k$.
2. Solve the $k+1$ linear systems $\mathbf{K}_0\mathbf{u}_0 = \overline{\mathbf{R}}, \mathbf{K}_0\mathbf{u}_1 = \mathbf{b}_1, \ldots, \mathbf{K}_0\mathbf{u}_k = \mathbf{b}_k$, where $\overline{\mathbf{R}}, \mathbf{b}_1, \ldots, \mathbf{b}_k$ are defined by Eqs. (11) and (13), respectively.
3. Form and solve the following system of linear equations

$$\begin{bmatrix} u_{1i_1} & u_{2i_1} & \cdots & u_{k\,i_k} \\ u_{1i_2} & u_{2i_2} & \cdots & u_{k\,i_2} \\ \vdots & \vdots & \vdots & \vdots \\ u_{1i_k} & u_{2i_k} & \cdots & u_{k\,i_k} \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_k \end{bmatrix} = \begin{bmatrix} -u_{0i_1} \\ -u_{0i_2} \\ \vdots \\ -u_{0i_k} \end{bmatrix}$$

4. Calculate $\bar{\mathbf{x}}$ by $\bar{\mathbf{x}} = \mathbf{u}_0 + t_1\mathbf{u}_1 + t_2\mathbf{u}_2 + \ldots + t_k\mathbf{u}_k$.
5. Delete the 0 elements at the i_1th, …, i_kth components, and move forward the remaining elements to obtain the displacements vector $\mathbf{x}$ of the modified structure.

A number of remarks should be made about the algorithm above. The main computational cost of the algorithm is spent in step 2, where $k+1$ linear systems need to be solved. The Cholesky factorization of the common coefficient matrix has already been known from the initial analysis, these linear systems can thus be easily solved by utilizing the forward and backward substitutions. In step 3, only one low-dimension linear system need to be solved, the computational cost of implementing this step is thus inexpensive.

The computational cost of our proposed algorithm can be quantified by the number of floating point operations (flops). Suppose the number of DOFs of the original structure is m, and k is the number of the added support constraints. Assume the half-band widths of the initial stiffness matrix and the modified stiffness matrix are the same, and let b denote the half-band width. The case of $b \ll m$ is considered. Solving one linear system of order m with half-band width b by utilizing the forward and backward substitutions requires $4mb - 2b^2 - 2b + m$ flops (Golub and Van Loan 1996). Thus, the total computational cost of the proposed algorithm is $(k+1)(4mb - 2b^2 - 2b + m)$ flops since the computational cost of step 3 is negligible. Direct analysis method costs $(m-k)(b^2 + 8b + 1)$ flops (Golub and Van Loan 1996) since the Cholesky factorization of the modified stiffness matrix is required. The theoretical speed up S_t is defined as the ratio of the flops using the Cholesky method to that using the proposed method (Leu and Tsou 2000), that is

$$S_t = \frac{(m-k)(b^2 + 8b + 1)}{(k+1)(4mb - 2b^2 - 2b + m)} \tag{25}$$

Eq. (25) can be approximated by

$$S_t \approx \frac{(m-k)(b^2 + 8b)}{(k+1)(4mb - 2b^2)} = \frac{(m-k)(b+8)}{(k+1)(4m - 2b)} \tag{26}$$

From Eq. (26), it can be seen that the smaller k is, the larger S_t is. Using Eq. (27) and noting that $b \ll m$ yield $S_t \geq 1$, if

$$k \leq \frac{mb + 4m + 2b}{4m - b + 8} \approx \frac{1}{4}(b+4) \tag{27}$$

i.e., when the number of the added support constraints k satisfies the inequality (27), the computational cost of the proposed method is equal to or less than that of the direct analysis method.

4. Numerical examples

In this section, two examples are presented to demonstrate the effectiveness of the proposed method. All the computations are completed on a PC: Pentium 4, quad-core CPU with 2.66 GHz, 2 GB RAM. Compaq Visual Fortran 6.5 is used.

Example 1

Consider the ceiling structure of a building shown in Fig. 2. The length and the width of the ceiling are 32 m and 24 m, respectively. The ceiling is discretized into a finite element model with 3128 elements and 825 nodes. Every node has 6 DOFs except the 32 supporting nodes (shown in Fig. 3), the top view of the ceiling structure, where ○ denotes the supporting nodes) and the number of DOFs of the structure is 4758. It includes two types of elements: 768 plate elements and 3990 beam elements. The material modulus of elasticity and the Poisson's ratio for the plate are $E_1 = 7 \times 10^{10}$ Pa and $\upsilon_1 = 0.3$. The thickness of the plate is 3×10^{-3} m. The size of each plate element is 1 m × 1 m. The modulus of elasticity and the Poisson's ratio for the beam are $E_2 = 2.07 \times 10^{11}$ Pa and $\upsilon_2 = 0.31$, respectively. The cross-section of the beam is 0.045 m × 0.045 m. Fig. 4 shows the beams below the plate. Every node of the ceiling is subjected to a vertical load $P = 1000$ N. In order to reinforce the ceiling (modification), two unconstrained nodes are supported, and 12 new support constraints are added, as shown in Fig. 3 (□ represents the added supporting nodes). Thus, the number of DOFs of the modified structure is 4746.

Table 1 gives the maximum vertical displacements (two nodes denoted by △, as shown in Fig. 3, have the maximum vertical displacements) of the modified structure calculated by the proposed algorithm and the direct analysis. It can be observed that, for the eight-digit accuracy, the results of the two methods are identical. The computational time for the ceiling structure is presented in Table 2.

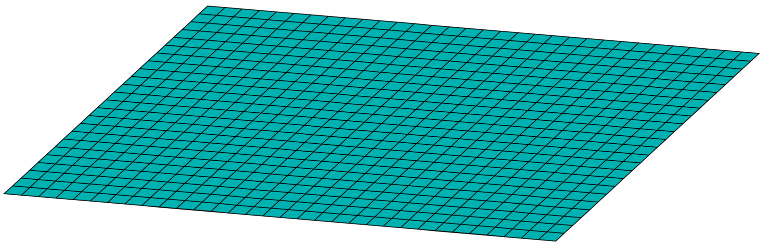

Fig. 2 A ceiling structure

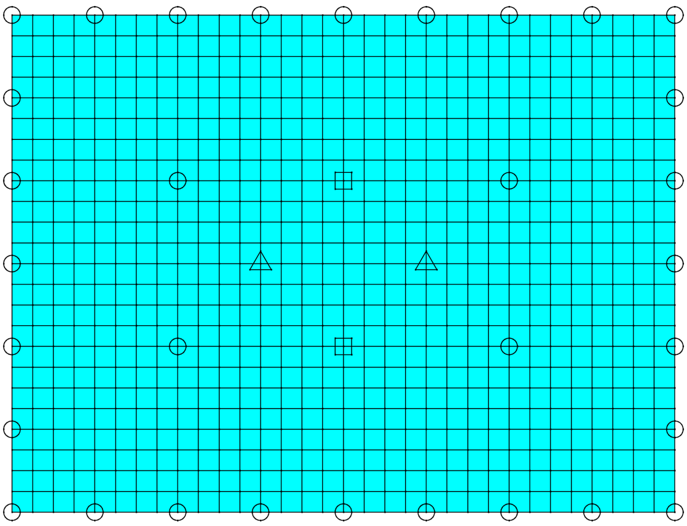

Fig. 3 The top view of the ceiling structure

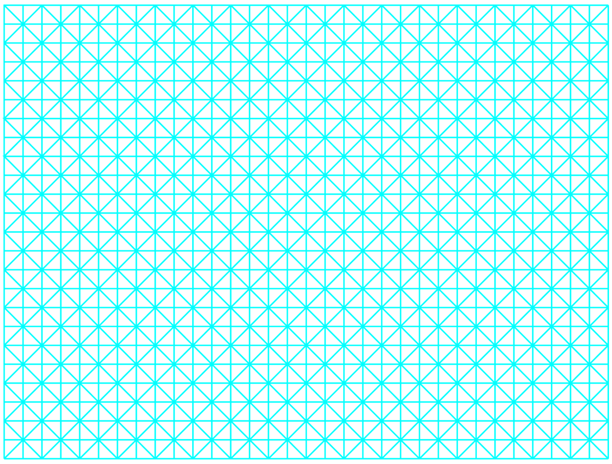

Fig. 4 The beams below the plate

Table 1 The maximum vertical displacements calculated by the proposed algorithm and the direct analysis for the modified ceiling structure

	Proposed algorithm	Direct analysis
The maximum vertical displacements	-0.21343876 m	-0.21343876 m

Table 2 The computational times for the modified ceiling structure

	Proposed algorithm	Direct analysis
The computational times	0.273438 s	1.853125 s

It is obvious from Table 2 that the computational time of the proposed algorithm is much less than that of the direct analysis.

Example 2

Consider a space truss structure shown in Fig. 5. The member cross-sectional areas are equal to $1.3823 \times 10^{-3}\ \mathrm{m}^2$, and the material modulus of elasticity is $E = 2.07 \times 10^{11}\ \mathrm{Pa}$. The spherical radius of the structure, height and span are 20 m, 10 m and 34.642 m, respectively. The structure has 3750 space truss elements and 1291 nodes. Every node has 3 DOFs except the 45 supporting nodes that locate at the lowest circumference (as shown in Fig. 6, the top view of the structure, ○ denotes the supporting nodes), and the total number of DOFs is 3738. Every node of the structure is subjected to a vertical load $P = 3000$ N. To reinforce the structure (modification), five unconstrained nodes are supported, they also locate at the lowest circumference, as shown in Fig. 6 (□ represents the newly added support constraints). Thus, the modified structure has 3723 DOFs.

Table 3 presents the maximum vertical displacements (the highest node of the structure) of the modified structure calculated by the proposed algorithm and the direct analysis. The computational times for the modified structure are given in Table 4. From Table 3, it can be seen that the results of the two methods are the same for the eight-digit accuracy. It is obvious from Table 4 that the computational time of our proposed algorithm is much less compared with the direct analysis.

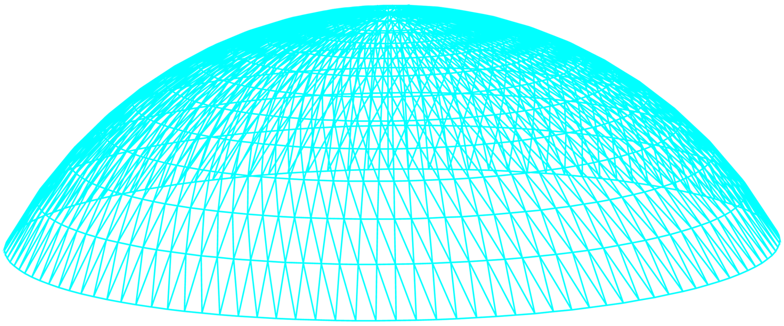

Fig. 5 A space truss structure

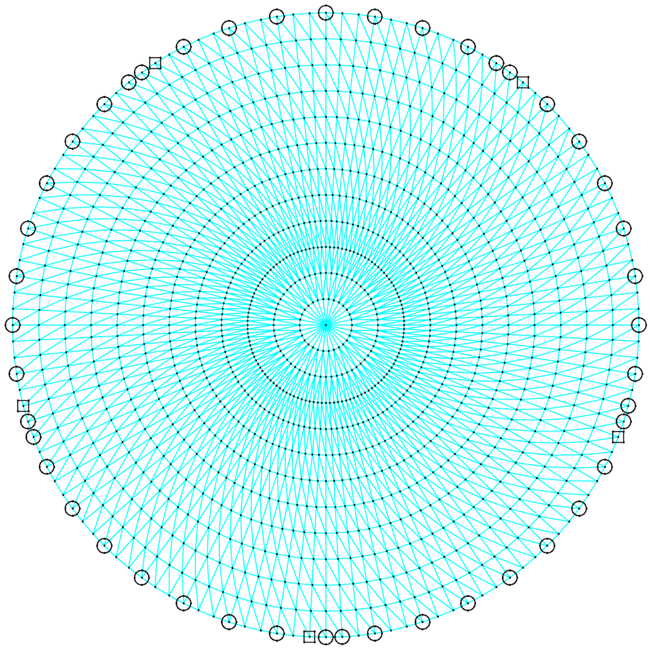

Fig. 6 The top view of the space truss structure

Table 3 The maximum vertical displacements calculated by the proposed algorithm and the direct analysis for the modified space truss structure

	Proposed algorithm	Direct analysis
The maximum vertical displacements	0.01457823 m	0.01457823 m

Table 4 The computational times for the modified space truss structure

	Proposed algorithm	Direct analysis
The computational times	0.903125 s	17.818750 s

5. Conclusions

The study in this paper has been focused on the static reanalysis problem with added support constraints whose orientations are the same as the orientations of some axes of the global coordinate system. A new approach for such modifications has been proposed. It provides exact solutions, thus the method belongs to the direct reanalysis methods. The new proposed algorithm is easy to implement and the computational time can be significantly reduced. Numerical examples have demonstrated the advantages of the proposed approach. However, the proposed method can only deal with a special case of support modifications. Future work is to study the static reanalysis problem for the general supports modifications (addition or deletion of some support constraints, and the orientations of the added or deleted support constraints are arbitrary).

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References

Abu Kasim, A.M. and Topping, B.H.V. (1987), "Static reanalysis: a review", *J. Struct. Eng.-ASCE*, **113**(5), 1029-1045.

Akesson, B. and Olhoff, N. (1988), "Minimum stiffness of optimally located supports for maximum value of beam eigenfrequencies", *J. Sound. Vib.*, **120**(3), 457-463.

Akgün, M.A., Garcelon, J.H. and Haftka, R.T. (2001), "Fast exact linear and non-linear structural reanalysis and the Sherman-Morrison-Woodbury formulas", *Int. J. Numer. Meth. Eng.*, **50**(7), 1587-1606.

Golub, G.H. and Van Loan, C.F. (1996), *Matrix Computations,* 3rd Edition, The Johns Hopkins University Press, Baltimore, London.

Hassan, M.R.A., Azid, I.A., Ramasamy, M., Kadesan, J., Seetharamu, K.N., Kwan, A.S.K. and Arunasalam, P. (2010), "Mass optimization of four bar linkage using genetic algorithms with dual bending and buckling constraints", *Struct. Eng. Mech.*, **35**(1), 83-98.

Kirsch, U. (2008), *Reanalysis of Structures*, Springer, Dordrecht.

Leu, L.J. and Tsou, C.H. (2000), "Application of a reduction method for reanalysis to nonlinear dynamic analysis of framed structures", *Comput. Mech.*, **26**(5), 497-505.

Li, Z.G. and Wu, B.S. (2007), "A preconditioned conjugate gradient approach to structural reanalysis for general layout modifications", *Int. J. Numer. Meth.. Eng.*, **70**(5), 505-522.

Liu, C.H., Cheng, I., Tsai, A.C., Wang, L.J. and Hsu, J.Y. (2010), "Using multiple point constraints in finite element analysis of two dimensional contact problems", *Struct. Eng. Mech.*, **36**(1), 95-110.

Olhoff, N. and Taylor, J.E. (1983), "On structural optimization", *J. Appl. Mech.-ASME*, **50**(4), 1139-1151.

Sherman, J. and Morrison, W.J. (1949), "Adjustment of an inverse matrix corresponding to changes in the elements of a given column or a given row of the original matrix", *Ann. Math. Stat.*, **20**(4), 621.

Sun, W.Y. and Yuan, Y.X. (2006), *Optimization Theory and Methods*, Springer, New York.

Takezawa, A., Nishiwaki, S., Izui, K. and Yoshimura, M. (2006), "Structural optimization using function-oriented elements to support conceptual designs", *J. Mech. Des.-ASME*, **128**(4), 689-700.

Tanskanen, P. (2006), "A multiobjective and fixed elements based modification of the evolutionary structural optimization method", *Comput. Meth. Appl. M.*, **196**(1-3), 76-90.

Terdalkar, S.S. and Rencis, J.J. (2006), "Graphically driven interactive finite element stress reanalysis for machine elements in the early design stage", *Finite. Elem. Anal. Des.*, **42**(10), 884-899.

Wang, B.P. and Chen, J.L. (1996), "Application of genetic algorithm for the support location optimization of beams", *Comput. Struct.*, **58**(4), 797-800.
Wang, D., Jiang, J.S. and Zhang, W.H. (2004), "Optimization of support positions to maximize the fundamental frequency of structures", *Int. J. Numer. Meth. Eng.*, **61**(10), 1584-1602.
Woodbury, M. (1950), "Inverting modified matrices", Memorandum Report 42. Statistical Research Group, Princeton University, Princeton.
Zhu, J.H. and Zhang, W.H. (2010), "Integrated layout design of supports and structures", *Comput. Meth. Appl. M.*, **199**(9-12), 557-569.