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A modified particle swarm approach for multi-objective optimization of laminated composite structures

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Abstract. Particle Swarm Optimization (PSO) is a stochastic population based optimization algorithm which has attracted attentions of many researchers. This method has great potentials to be applied to many optimization problems. Despite its robustness the standard version of PSO has some drawbacks that may reduce its performance in optimization of complex structures such as laminated composites. In this paper by suggesting a new variation scheme for acceleration parameters and inertial weight factors of PSO a novel optimization algorithm is developed to enhance the basic version's performance in optimization of laminated composite structures. To verify the performance of the new proposed method, it is applied in two multi-objective design optimization problems of laminated cylindrical. The numerical results from the proposed method are compared with those from two other conventional versions of PSO-based algorithms. The convergancy of the new algorithms is also compared with the other two versions. The results reveal that the new modifications inthe basic forms of particle swarm optimization method can increase its convergence speed and evade it from local optima traps. It is shown that the parameter variation scheme as presented in this paper is successful and can evenfind more preferable optimum results in design of laminated composite structures.

Keywords: cylindrical composite laminated shell; optimization algorithms; particle swarm optimization

1. Introduction

Laminated composites are a group of the most popular structural members in mechanical systems. Their ability of being tailored is a great advantage over the conventional materials. By tailoring the laminates, structures with totally different properties can be designed with a similar production costs. Tailoring increases complexities in the design problems. These complexities exist, not only because of numerous design variables, but also because of having a multimodal and variable-dimensional optimization problem with unattainable or costly derivatives. Usually the most desired structure is the one which has the most compatibility with design limitations and has the lowest production costs. The design domain for laminated composites is quite articulated; therefore, advanced methods are required to search for the optimal design. One of the most important objectives in the design of laminated structures is to achieve an optimum layout, which gives the

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desired mechanical properties while keeping low rates of weight and costs. Various optimization methods have been considered by researchers (Sandhu 1971, Cairo 1970, Lansing *et al.* 1971, Lombardi *et al.* 1992, Callahan and Weeks 1992, Le Riche and Haftka 1995, Ghiasi *et al.* 2009). Some primitive studies have used gradient based methods for optimization of laminated composites (Sandhu 1971). The Steepest Descend method was among the methods which were employed in later studies (Cairo 1970, Lansing *et al.* 1971). Next generations of optimization methods benefited a lot form the computational power of computers. This made the methods like Simulated Annealing (Lombardi *et al.* 1992) and Genetic Algorithm (Callahan and Weeks 1992, Le Riche and Haftka 1995) among the most popular methods for optimization of laminated composites. High computational intensity and premature convergence were stated as the main disadvantages of Genetic Algorithm (GA) (Ghiasi *et al.* 2009). This has led some researcher to look for other optimization algorithms as alternative methods (Zehnder and Ermannim 2006, Jiang *et al.* 2008, Luo *et al.* 2011, NarayanaNaik *et al.* 2011). In recent few years, other heuristic optimization methods have attracted lots of attentions to themselves (David 2010). The Particle Swarm Optimization (PSO) is one of the most successful methods in this category. PSO algorithm is a stochastic population based optimization procedure. This method was originally introduced by Kennedy and Eberhart (1995) for optimization of continuous problems. Moreover PSO has particles driven from natural swarms, with communications based on evolutionary computations. In this algorithm, a candidate solution is presented as a particle. This method combines self-experiences of particles with the swarm's social experiences. A number of advantages with respect to other algorithms make PSO an ideal candidate for optimization tasks. The algorithm is robust and well suited for analysis of non-linear and non-convex design spaces with discontinuities. It can also be applied into continuous, discrete and integer variable types with ease. As compared to other robust design optimization methods, PSO is more efficient, requires fewer numbers of function evaluations, and leads to better or the same quality of results (Hu *et al.* 2003, Hassan *et al.* 2005). PSO's potential power of optimization has been used by multiple researchers. In recent few years this method has been considered for design optimization of composite structures. Suresh and Sujit have (2007) used this algorithm in multi-objective design of box beam structures and compared efficiency of their algorithm with previous solutions obtained by GA. Better performance and computational efficiency of PSO compared to GA have been reported in their study. Kathiravan (2007) compared PSO to a gradient-based method for the maximization of the failure strength of a thin walled composite box-beam. He found that PSO could give results superior or equivalent to the gradient-based method. In comparison with similar evolutionary algorithms, PSO is a population based algorithm however, in PSO; data is not destroyed during iterations. Another advantage of PSO is that the initial assumptions have minor effects on the convergence of algorithm. As a new random search method, PSO has encountered some problems such as premature convergence, slow search speed, and too fast decrease of the variety of the particle swarm, resulting a search failure in some cases (Clerc 2006). Researchers have tried to resolve this problem by introducing new modified optimization algorithms based on basic PSO algorithm. Kennedy and Eberhart used a discrete binary version of particle swarm optimization to resolve combinatorial optimization problems in engineering practice. Zheng (2007) proposed a method of changing velocity rate to enhance searching speed. Test experiments of domain topology were conducted (Kennedy 2007) in which the best form of topology were designed based on actual situation. In order to improve the performance of PSO and maintain the diversities of particles, distancesfrom the global best position to other positions were calculated to adjust the velocity suitably of each particle (Kennedy 2000).

Chen (2010) introduced particle swarm optimizer hybridized with extremal optimization. Extrapolation techniques (Arumugam *et al.* 2009) have also been employed to create algorithms similar to particle swarm optimization. Chen (2009) has also introduced a particle swarm optimization with adaptive population size to improve the basic PSO's performance.

In this study, new scheme for variation of the PSO parameters is suggested to enhance the performance and reduce the chance of entrapping in local optima. This new scheme is experimented in optimization of laminated composite structures. In the suggested method, the acceleration coefficients and inertial weights are prescribed to vary with certain schemes. In order to investigate the performance of the modified PSO, this method is applied to some numerical examples and its performance is compared with other two conventional versions of PSO, the Basic PSO (BPSO) and Repulsive PSO (RPSO) algorithms. The Mechanical APDL software is used as FEM solver in these problems. In order to use the FEM solver, the optimization codes are written in APDL language and linked to the FEM solver. Numerical results and convergence graphs are presented for a thorough investigation.

2. Particle Swarm Optimization algorithms

2.1 Basic Particle Swarm Optimization algorithm (BPSO)

A PSO algorithm contains a swarm of particles, where each particle represents a potential solution to the optimization problem. Particles move through a multi-dimensional search space and their positions are adjusted according to its own experience and the experience of the other particles in the swarm. The particle swarm process is stochastic in nature; the particle's status in the search space is characterized by two factors: position and velocity. The position and the velocity of the ith particle in the d-dimensional search space can be represented as $X_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,d})$ and $V_i = (v_{i,1}, v_{i,2}, \ldots, v_{i,d})$, respectively. Regarding to the "memory" gained by each particle, the velocity vector is updated, conceptually resembling an autobiographical memory, as well as the knowledge gained by the swarm as a whole. There are generally two types of memories in PSO: personal best and global best. Personal best remembers the best result each particle has reported since the beginning of the solution. This memory is denoted as $P_i = (p_{i,1}, p_{i,2}, \ldots, p_{i,d})$. Global best memory is the best results that the swarm has found so far. This parameter is denoted as G. The position of each particle in the swarm is updated based on the social behavior of the swarm, which adapts to its environment by returning to the promising regions of the space previously discovered and searching for better positions over time. The updated velocity of particles in ith iteration is stated in Eq. (1).

$$v_{i,j}(t+1) = wv_{i,j}(t) + c_1 r_1 [p_{i,j} - x_{i,j}(t)] + c_2 r_2 [g_j - x_{i,j}(t)], \quad j = 1, 2, \ldots, d \tag{1}$$

In Eq. (1), c_1 and c_2 are constants called acceleration coefficients. Usually r_1 and r_2 are two independent random numbers uniformly distributed in the range [0, 1] and $c_1 = c_2$. Large coefficients increase velocity updates and make the algorithm globally explore the design space. On the other hand small inertia values concentrate the velocity updates to nearby regions of the design space. w is called the inertial weight factor which is often in the range of [0.1, 0.9]. The value of w may be prescribed to change along the solution. As the iteration starts, the velocity of particles is updated then it is used to update the position of particles. The new position is prescribed by Eq. (2).

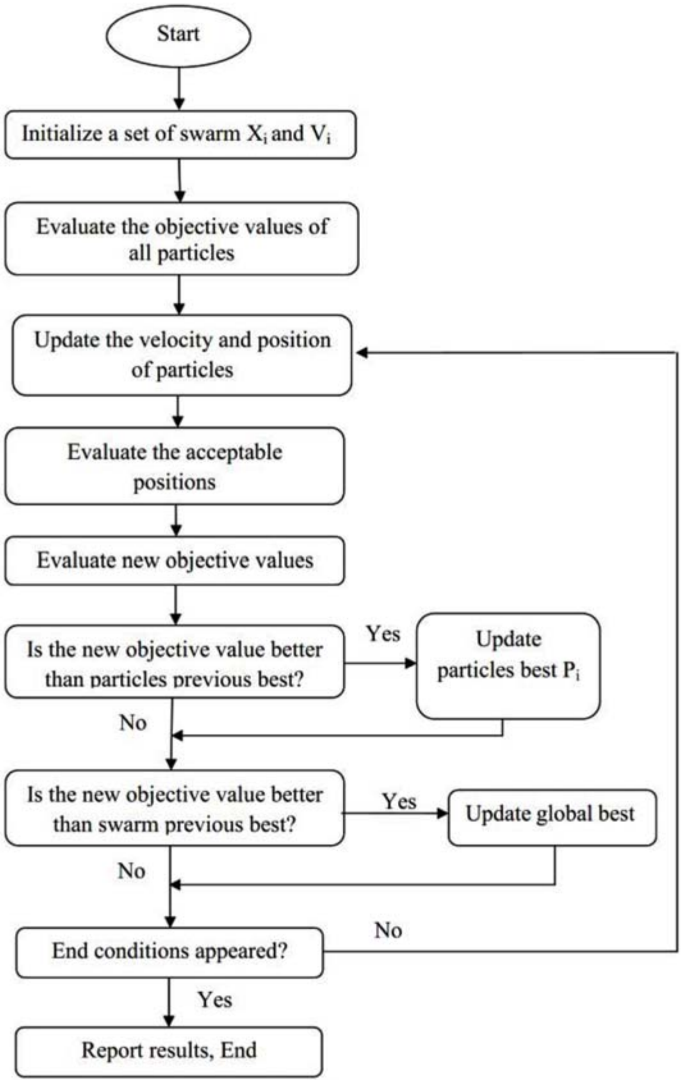

Fig. 1 Schematic flow chart of PSO

General description of PSO based algorithms can be summarized in Fig. 1.

$$x_{i,j}(t+1) = x_{i,j}(t) + v_{i,j}(t+1) \tag{2}$$

2.2 Repulsive Particle Swarm Optimization algorithm (RPSO)

PSO can stop evolution and rather fall into premature convergence especially for complex problems with many optimization parameters and local optima in their design spaces (Ozcan and Mohan 1999). Therefore, various different models of PSO have been developed recently to improve its performance, and increase the diversity of particles of the original PSO (James *et al.* 2001).

RPSO is a particle swarm optimization method in which there is repulsion between particles. This repulsion enhances optimization procedure by preventing particles to be concentrated at one point. By scattering articles through search space the risk of falling into local optima traps will be considerably reduced. The main difference of RPSO with BPSO is the velocity assignment mechanism. A random repulsion term has been considered in RPSO velocity equation. The velocity assignment mechanism is presented in Eq. (3)

$$v_{i,j}(t+1) = w_1 v_{i,j}(t) + c_1 r_1[p_{i,j} - x_{i,j}(t)] + c_2 r_2[g_j - x_{i,j}(t)] + c_3 r_3 w_2, \quad j = 1, 2, \ldots, d \tag{3}$$

The main parameters in Eq. (3) are as described for Eq. (2). The fourth term on left side generates a noise in the velocity of a particle to enhance the exploration to new areas in the search space. r_3 and w_2 are random and repulsion inertia factors respectively. Consequently, RPSO can prevent the swarm from being trapped in local minimum, which would cause a premature convergence and lead to failure in finding the global optimum. Moreover, it can find global optima in more complex search spaces.

2.3 The Modified Particle Swarm Optimization algorithm (MPSO)

As mentioned earlier, BPSO has an excellent performance in search for the optimum value but it may be trapped into local optima points. Particles in RPSO have a repulsion fields that prevents their premature concentration into local optima points. But this repulsion reduces the robustness of algorithms and slows its evolution towards the global best. In this paper a new modified PSO based algorithm is introduced for maintaining the robustness of BPSO and dynamics of RPSO at the same time. This algorithm also benefits from dynamic acceleration coefficients as well as variable inertia factors. The velocity assignment equation for MPSO is identical to Eq. (3) The Velocity inertial factor is considered to linearly decrease during iterations. This variation can be summarized as Eq. (4)

$$w_1 = w_1^{\max} - \frac{(w_1^{\max} - w_1^{\min})}{iter_{\max}} \times iter \tag{4}$$

In this equation the inertial factor is restricted between its prescribed minimum and maximum values which are presented as $w_1^{\max}$ and $w_1^{\min}$ and *iter* and $iter_{\max}$ stand for number of current iteration and total number of iterations respectively. Acceleration coefficients are also considered to linearly vary in MPSO; c_1 which represents the influence of best personal history is linearly decreasing while the global best influence c_2 is increased. These parameters vary in a manner that in each iteration their summation is constant and equal to 4. This summation value is proposed by Clerc (2002) for convergence insurance of PSO. The equations for variation of acceleration coefficients are presented in Eqs. (5) and (6). At first iteration personal best history has the major influence on velocity equation this makes the MPSO particles to look around their neighborhood before traveling to the region of best global optima. After searching particles neighborhoods increasing c_2 attracts attention of particles towards the global best results and they start traveling to that region.

$$c_1 = c_1^{\max} - \frac{(c_1^{\max} - c_1^{\min})}{iter_{\max}} \times iter \tag{5}$$

$$c_2 = c_{2_{\min}} + \frac{(c_2^{\max} - c_2^{\min})}{iter_{\max}} \times iter \tag{6}$$

As it is observed in Eq. (3) like RPSO, the MPSO velocity equation also has a repulsive factor but instead of a constant w_2 the repulsion coefficient is also taken as a variable. A multi-equation scheme is chosen for this variation. The repulsion coefficient is confined between two prescribed maximum and minimum values. This scheme can be summarized as follows

$$w_2 = \begin{cases} 0 & iter < \dfrac{iter_{\max}}{4} \\ w_2^{\max} - \dfrac{iter(w_2^{\max} - w_2^{\min})}{iter_{\max}} & iter \geq \dfrac{iter_{\max}}{4} \end{cases} \tag{7}$$

In first one fourth of iterations, the MPSO acts with variable inertial and acceleration factors, the repulsion term is inactive in these iterations. After one fourth of iterationsin the time when the particles are closing together the repulsion term is activated and linearly increased. This dynamics helps the algorithm to prevent being trapped into local optimal points.

3. Optimization procedure

3.1 Structural analysis

Numerical case studies in this paper mainly involve tensile and torsional cross-sectional stiffness optimization of thin walled laminated cylindrical shells. Fan (1983) and Lin (2001) have proposed methods for evolution of cross-sectional stiffness of laminated cylinders. They have suggested a modified ABD matrix for these types of structures. They have added some additional stiffness terms due to the curvature of the shell that has significant effects on the local stiffness in thick shells. Since in this paper only thin shell structures are considered, these additional terms could be neglected due to their minor contribution to overall cross-sectional stiffness. The usual approach for calculating the cross-sectional stiffness properties in thin shells isto calculate the module of the curved shell as if it were a flat laminate, and then simply apply this data in conjunction with the shell geometry to obtain the overall shell stiffness values (Lemanski and Weaver 2006). In this paper asimilar approach is considered for evaluation of ABD matrix in finite elements calculation of cross-sectional stiffness. Displacement field in first order shear deformation theory (FSDT) is described as Eq. (8) where u_0, w_0 and v_0 are midplane displacements, $\varphi_x = \partial u/\partial z$ and $\varphi_y = \partial v/\partial z$. The displacement-strain relations, taking Eq. (8) intoaccount are presented in Eq. (9).

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\varphi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\varphi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{8}$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} - z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{yz}^{(1)} \\ \gamma_{xz}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial w_0}{\partial y} + \varphi_y \\ \dfrac{\partial w_0}{\partial x} + \varphi_x \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} - z \begin{Bmatrix} \dfrac{\partial \varphi_x}{\partial x} \\ \dfrac{\partial \varphi_y}{\partial y} \\ 0 \\ 0 \\ \dfrac{\partial \varphi_x}{\partial y} + \dfrac{\partial \varphi_y}{\partial x} \end{Bmatrix} \tag{9}$$

The equilibrium equations for FSDT are given by Reddy (2004). In this theory constitutive equations are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{44} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix} \begin{bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{bmatrix} \tag{10}$$

In Eq. (10), $[\overline{Q}_{ij}]$ represents the transformed stiffness matrix (Reddy 2004). Resultant forces, moments and shear forces are defined in Eqs. (11) to (13). Elements of A, B and D matrices in these equations are defined in Eqs. (14) and (15). K in Eq. (13) is the shear correction factor which for a general laminate depends on lamina properties and lamination scheme (Reddy 2004).

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \tag{11}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} \tag{12}$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = K \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} \tag{13}$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}(1, z, z^2) dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)}(1, z, z^2) dz \tag{14}$$

$$(A_{44}, A_{45}, A_{55}) = \int_{-h/2}^{h/2} (\overline{Q}_{44}, \overline{Q}_{45}, \overline{Q}_{55}) dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\overline{Q}_{44}^{(k)}, \overline{Q}_{45}^{(k)}, \overline{Q}_{55}^{(k)}) dz$$

$$= \sum_{k=1}^{N} (\overline{Q}_{44}^{(k)}, \overline{Q}_{45}^{(k)}, \overline{Q}_{55}^{(k)})(z_{k+1} - z_k) \tag{15}$$

3.2 Finite elements implementation

Cross-sectional stiffness for such structures is evaluated in a Finite Elements (FE) approach using first order shear deformation theory (FSDT) (Reddy 2004). These elements have 8 nodes with 5 degrees of freedoms in each node. All degrees of freedoms are estimated by Lagrange interpolation functions as presented in Eqs. (16) to (20).

$$u_0(x, y, t) = \sum_{j=1}^{m} u_j(t) \psi_j^e(x, y) \tag{16}$$

$$v_0(x, y, t) = \sum_{j=1}^{m} v_j(t) \psi_j^e(x, y) \tag{17}$$

$$w_0(x, y, t) = \sum_{j=1}^{n} w_j(t) \psi_j^e(x, y) \tag{18}$$

$$\varphi_x(x, y, t) = \sum_{j=1}^{p} S_j^1(t) \psi_j^e(x, y) \tag{19}$$

$$\varphi_y(x, y, t) = \sum_{j=1}^{p} S_j^2(t) \psi_j^e(x, y) \tag{20}$$

For each element a linear system of equations as Eq. (21) is obtained which should be assembled with respect to loadings and boundary conditions. Final response of the composite structures to proposed loading is obtained from solution of the assembled system of linear equations.

$$[K^e]\{\Delta^e\} = \{F^e\} \tag{21}$$

3.3 Optimization computer code implementation

Specialized computer codes are developed in current study for optimization of composite structures with the PSO-based algorithms. In these codes, FEM is used to find the response of the system to different loadings and boundary conditions. The results are then reported to PSO for fitness function evaluation. Later PSO assigns new velocities for each particle. New particles positions are introduced to FEM solver again. Mechanical APDL is used for FEM analysis on the basis of theories described in Sec. Finite Elements Implementation. Design parameters such as geometry, laminations and loads are given to the software to evaluate the deformation of composite structures under the prescribed loadings. This data is sent back to PSO to evaluate the FIT function.

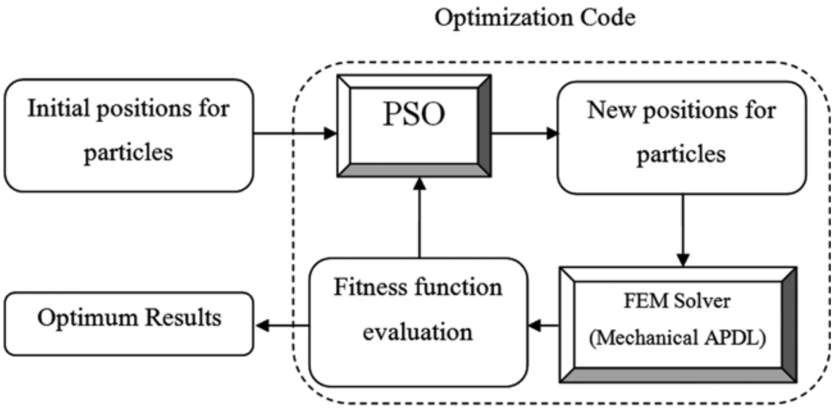

Fig. 2 Optimization computer implemented code

This cycle continues for a certain number of iterations. As chematic flow chart of optimization computer implemented code is demonstrated in Fig. 2.

4. Optimal design of laminated composite shell for maximum tensile stiffness and minimum weight

4.1 Problem description

In this section a numerical example of the optimal design of laminated composite structures is presented and the performances of different PSO algorithms introduced in this paper are compared to each other. A cylindrical shell of four laminated layers with the mean radius of 0.4 m and length of 2.0 m is considered to be optimally designed for maximum tensile stiffness and lightest structural weight. One end of the shell is clamped and the other is left free. A concentrated tensile force of $F = 1000$ N is exerted to the center of the rigid cap at the end of the cylindrical shell (Fig. 3).

The maximum longitudinal deformation is taken as the shell's response to loading. The optimal design is recognized to have minimum longitudinal deformation and lightest structural weight. The fiber direction angles, the layers thickness and the stacking sequence were considered as design

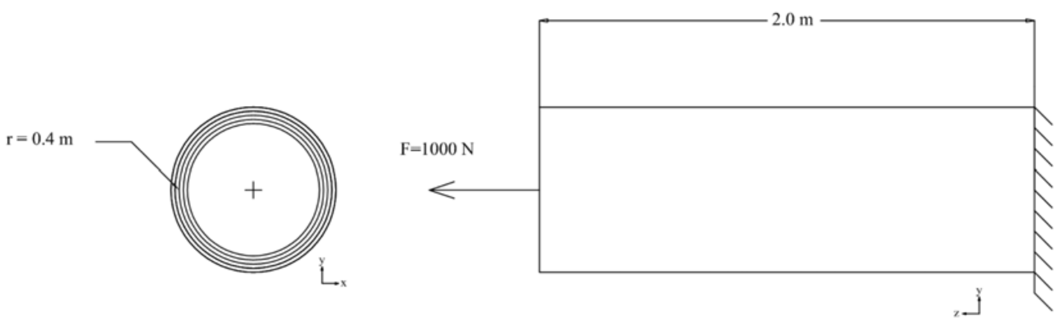

Fig. 3 Schematics of the geometry and loading for design optimization under tensile loading (the drawing is not scaled)

Table 1 Mechanical properties of Graphite-Epoxy laminates

E_1	E_2	E_{12}	X_t	X_c	Y_t	Y_c	S	ρ
181.00 Gpa	10.34 Gpa	7.17 Gpa	1500.00 Mpa	1500.00 Mpa	40.00 Mpa	68.00 Mpa	246.00 Mpa	15.70 kN/m^3

variables. The shell is made up of 4 fiber reinforced laminates of Graphite-Epoxy T300/5208 with mechanical properties as stated in Table 1. These data are taken from Tsai (1987). Also in Table 1, E_1 and E_2 are young modulus in directions 1 and 2. E_{12} is shear modulus and v_{12} is the Poisson's ratio. ρ is the specific weight of graphite epoxy laminates. Composite's strength parameters in tension and compression for longitudinal and transversal directions are represented as X_t, X_c, Y_t and Y_c, respectively. S is shear strength of the laminate.

The allowable thickness for the laminates is confined between 0.25 to 2.0 mm in 0.25 mm intervals and the allowable angle for the direction of fibers in the laminates is also between −75 to 90 degrees in 5 degrees intervals. Any other values beyond these definitions are considered to be unacceptable. Eq. (22) represents the fitness evaluation function defined for this multi-objective optimization problem. This function is used to reduce weight and deflection simultaneously. The primary optimization parameters (i.e., structural deformation and weight) have conflicting natures; the heavier structures would have smaller deflections. This would make the process a multi-objective optimization. α is introduced as the weight factor of each objective. When α = 0.0, only the displacement is reduced and the result of optimization is a design that has the smallest displacement. When α = 1.0, PSO algorithms try to obtain the lightest structure, which may have a large displacement. To investigate the integrity of MPSO with the suggested parameters, multiple runs in a range of α starting from 0.0 to 1.0 are performed (Almeida and Awruch 2009). In order to equalize the effect of maximum deformation of the structure and its weight, these parameters are normalized using their maximum and minimum possible values. The equations used for normalization of the variables are given by Eq. (23). In this equations the maximum and minimum values for weight of structure i.e., $W_{\max}$ and $W_{\min}$ are evaluated by assuming maximum and minimum layer thicknesses respectively. As it can be concluded the heaviest structures with the best fiber orientations would have the least deformations ($D_{\min}$) under applied loading. The largest deformations ($D_{\max}$) also will be seen in the lightest structures with the least appropriate fiber orientations. The PSO algorithms are used to minimize FIT in different values of α. In each case the smallest possible FIT is sought. It should be noted that each α defines a new optimization problem. By changing the value of α, the FIT function will change. This makes it possible to study the performance of MPSO in multiple cases for each problem.

$$FIT = (\alpha W^* + (1-\alpha)D^*) \times (1+\beta) \tag{22}$$

$$W^* = \frac{W - W_{\min}}{W_{\max} - W_{\min}}; \qquad D^* = \frac{D - D_{\min}}{D_{\max} - D_{\min}} \tag{23}$$

β is referred as the violation of the limits in contiguous plies thickness with the same fiber orientation. This parameter is equal to the exceeding value violating the limit fixed to the thickness of contiguous plies with the same fiber orientation. As an example, if the thickness of each one of two contiguous plies with the same fiber orientation is equal to 1.5 mm, the exceeding value

Table 2 Comparison of the parameter configurations for BPSO, RPSO and MPSO algorithms

	PSO Parameters									
Algorithm	w_1^{max}	w_1^{min}	w_2^{max}	w_2^{min}	c_1^{max}	c_1^{min}	c_2^{max}	c_2^{min}	Iterations	Particles
BPSO	0.5	0.5	-	-	2.0	2.0	2.0	2.0	150	20
RPSO	0.5	0.5	0.1	0.01	2.0	2.0	2.0	2.0	150	20
MPSO	0.5	0.1	0.1	0.00	1.5	0.5	2.5	3.5	150	20

violating the limit (i.e., β) is 1.0, since this work adopts a limit of 2.0 mm. When $\beta = 0$ the value FIT in Eq. (22) may become zero thus regardless of the lamination *FIT* would behave its minimum value; to avoid this unwanted situation 1 is added to β. This optimization problem can be summarized into the problem of maximizing the fitness function in the design space by suggesting the primary variables of fiber orientation angles and thickness of laminates. Three algorithms of BPSO, RPSO and the new MPSO where applied to minimize the FIT function. Details of optimal solutions were reported for α starting from 0.0 to 1.0. The algorithm parameters were chosen for best chances of convergancy according to Ref. and (Clerc and Kennedy 2002) presented in Table 2.

4.2 Numerical results and discussion

In separate approaches PSO codes linked to Mechanical APDL FEM solver were developed. In order to clearly compare the optimization performance of BPSO, RPSO and MPSO, similar finite element approaches were chosen for evaluation of structural response to the proposed loading. Number of particles and iterations were also maintained constant in each approach. In independent efforts, all three algorithms were employed in search for the minimum value of the FIT function. To experiment the performance of the proposed algorithm multiple cases of optimization would be required. Numerical results for optimization for α starting from 0.0 to 1.0 are reported in Table 3. All of the applied algorithms could find acceptable results for optimum stacking sequence of laminated cylindrical shell within the defined constrains and boundary conditions. Each algorithm was used to find the minimum value for FIT in constrained design space. In Fig. 4, the minimum values for FIT retrieved by each algorithm are presented. It can be observed that BPSO has shown an appropriate robustness and efficiency to find the optimum FIT value. Despite the good dynamics of RPSO, the noisy term in its velocity assignment mechanism has made a little obstruction for

Table 3 Optimum stacking sequences obtained for tensile loading of cylindrical laminated shell

Weighting factor (α)	BPSO	RPSO	MPSO
0	$[90^{1.75}, -75^{2.0}, -85^{1.0}, -75^{1.75}]$	$[85^{1.5}, 0^{1.75}, -90^{1.75}, -85^{1.5}]$	$[75^{1.0}, 80^{2.0}, 90^{2.0}, -85^{1.75}]$
0.2	$[80^{1.0}, -15^{0.25}, 20^{0.25}, 80^{0.25}]$	$[90^{1.0}, 85^{1.0}, 90^{1.0}, 85^{1.0}]$	$[-80^{0.5}, 90^{1.0}, 20^{0.25}, 0^{0.75}]$
0.4	$[-50^{0.5}, 85^{0.25}, 90^{0.25}, 5^{0.5}]$	$[-85^{0.5}, 85^{0.5}, 35^{0.5}, -85^{0.5}]$	$[90^{1.75}, 40^{1.75}, -45^{1.0}, 15^{1.0}]$
0.6	$[-85^{0.5}, 40^{0.75}, -85^{0.25}, -5^{0.25}]$	$[-80^{0.5}, -85^{0.75}, -90^{0.25}, 75^{0.25}]$	$[-55^{0.25}, 60^{0.25}, -15^{0.25}, -65^{0.25}]$
0.8	$[-80^{0.5}, 55^{0.5}, 55^{0.25}, -60^{0.5}]$	$[-55^{0.25}, 35^{0.5}, 30^{0.25}, 80^{0.75}]$	$[55^{0.25}, -85^{0.5}, 80^{0.25}, 20^{0.25}]$
1	$[25^{0.5}, 0^{0.25}, -65^{0.25}, -5^{0.25}]$	$[20^{0.25}, -35^{0.25}, -75^{0.75}, -5^{0.25}]$	$[90^{0.25}, 0^{0.25}, 75^{0.25}, 85^{0.25}]$

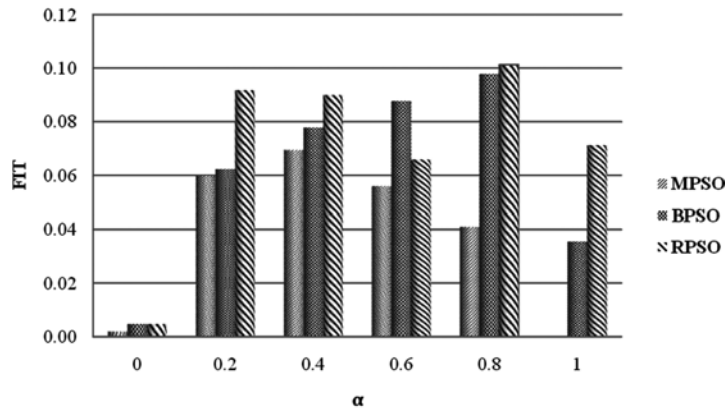

Fig. 4 Minimum values for FIT function

Table 4 Comparison of weights and deflections of the optimum results retrieved by each algorithm

Weighting factor α	BPSO		RPSO		MPSO	
	Weight (N)	Deflection (mm)	Weight (N)	Deflection (mm)	Weight (N)	Deflection (mm)
0.0	266.34	1.82E-03	256.48	1.82E-03	266.34	1.39E-03
0.2	69.05	8.97E-03	157.837	2.27E+00	98.65	4.38E-03
0.4	59.19	1.37E-02	78.92	9.54E+00	69.05	7.94E-03
0.6	69.05	1.02E-02	59.19	6.16E-03	39.46	2.27E-02
0.8	69.05	-1.06E-02	69.05	1.32E-02	49.32	-1.07E-02
1.0	49.32	-4.99E-02	59.19	1.63E-02	39.46	-2.02E-02

obtaining better solutions this unwanted effect can be seen for α = 0.2, 0.4 and 1 in Fig. 4. In all six cases, the MPSO has demonstrated excellent robustness and dynamics in pursue of the optimum value. In primitive iterations, this method has focused on local neighborhoods and later it has led the particles to the globally optimum region. These modifications in MPPSO resulted in better performance compared with BPSO and RPSO. Weights and deflections of the optimum designs retrieved from each method are presented in Table 4. As shown in this table, for α = 0 and α = 1 where only stiffness and weight of the structure is considered as the design objectives, MPSO has suggested the best designs. Even in other cases where both weight and deflection are considered in multi-objective approach, MPSO has suggested the best results. For example for α = 0.8 the result retrieved by MPSO has a deflection as the same as BPSO but 19.7 N lighter. Convergancy curves of the three algorithms for α = 0 to 1 are presented in Fig. 5. It is seen that MPSO has converged faster than the other proposed methods. For cases α = 0 and α = 0.8 where BPSO and RPSO are trapped in a local optima, the MPSO has managed to escape this traps and has converged to better results. Performance of MPSO can be clearly observed in Fig. 5.

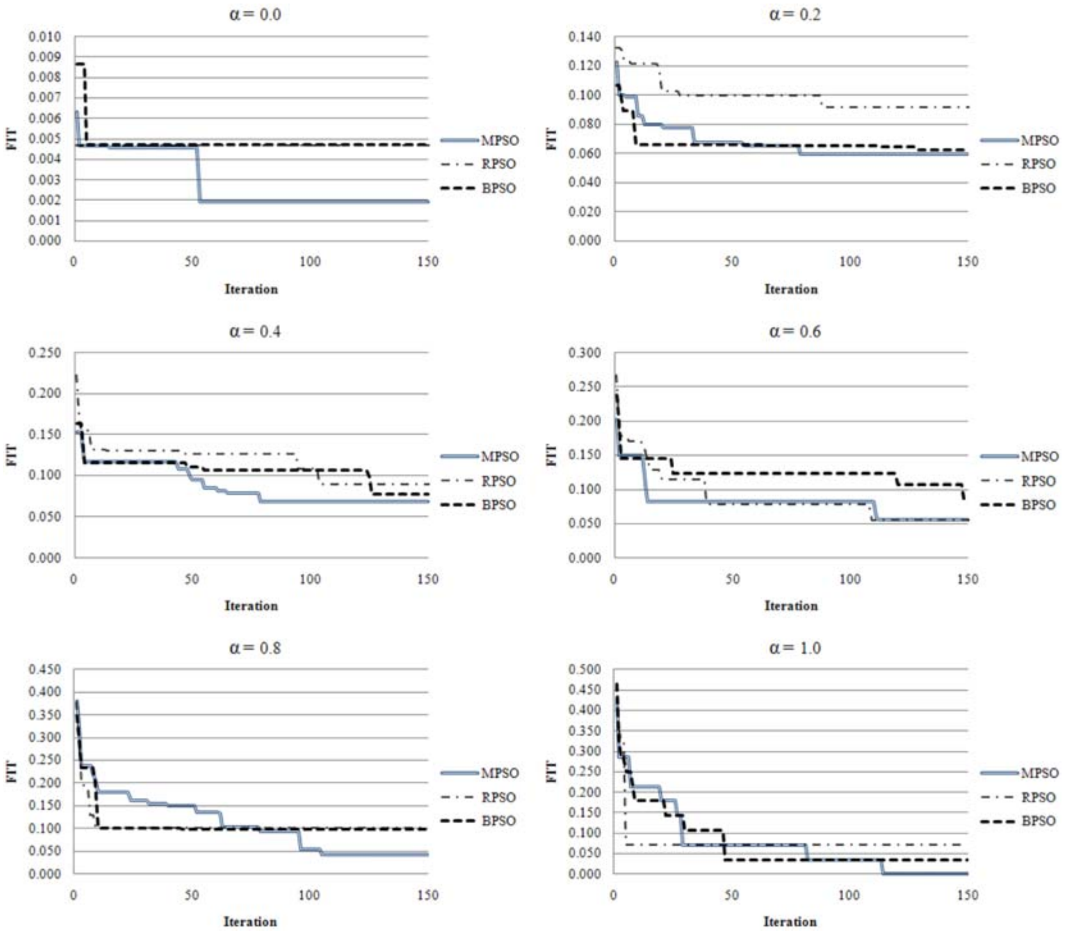

Fig. 5 Comparison of convergancy of MPSO, BSPO and RPSO for tensile loading of the laminated cylindrical shell

5. Optimal design of laminated composite shell for maximum torsional stiffness and minimum weight

5.1 Problem description

For better investigation of the MPSO's performance compared with the other two methods, another multi-objective problem of design optimization of laminated composites is studied. Here a cylindrical shell with geometry and boundary conditions similar to the structure mentioned in section 3 is considered to be optimally designed for maximum torsional stiffness and lightest possible structural weight. In this case a torque of $T = 1000$ N.m is exerted at the free end of cylinder. Maximum rotational angle is evaluated as the structural response to the proposed loading condition. This rotation and the total structural weight arethe two independent variables that are considered to be minimized. The material properties and the mechanical data for this problem are given in Table 1. The allowable thickness for laminates is taken between 0.25 to 2.0 mm in 0.25

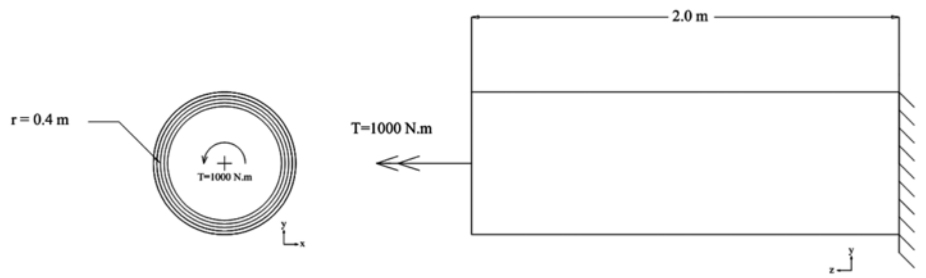

Fig. 6 Schematics of the geometry and loading for design optimization under torsional loading

mm intervals and the allowable angle for direction of fibers in the laminates is also between −75 to 90 degrees in 15 degrees intervals. Any other values beyond these definitions are considered to be unacceptable. Eq. (24) represents the fitness evaluation function defined for this multi-objective optimization problem. This function is used to reduce weight and deflection simultaneously. Minimum value for this function is sought during optimization process. A weighting factor αsimilar to previous problem is introduced as the weight factor of each objective. When α is equal to 0.0, only the rotation is reduced and optimization process gives a design with smallest rotation. The PSO algorithms tries to obtain the lightest structure when α is equal to 1.0, which may have a large rotation. This value is used as the maximum value in the normalization. The normalization of the variables is given by Eq. (24). The PSO-based algorithms are applied to minimize FIT for different values of α. In each case the smallest possible FIT is sought. In Eq. (24), θ and W represent for axial rotation and total structural weight of cylindrical shell. These variables are normalized in a manner described in previous section.

$$FIT = (\beta + 1)(\alpha W^* + (1-\alpha)\theta^*) \tag{24}$$

$$W^* = \frac{W - W_{\min}}{W_{\max} - W_{\min}}; \qquad \theta^* = \frac{\theta - \theta_{\min}}{\theta_{\max} - \theta_{\min}} \tag{25}$$

As stated in previous section, β is referred to the violation of the limit of contiguous plies thickness with the same fiber orientation.

5.2 Numerical results

For design optimization of the cylindrical shell under torsional loading, three PSO based optimization codes (i.e., BPSO, RPSO and MPSO) linked to the Mechanical APDL FEM solver were developed. In order to compare the optimization performance of BPSO, RPSO and MPSO, similar finite elements approaches were chosen for evaluation of structural response under the proposed loadings and boundary conditions. In independent efforts all of the algorithms were employed to minimize the FIT function. The acceleration and weight parameters were chosen according to Table 2. Optimum stacking sequences for six optimization cases with different values of α are reported in Table 5 and optimum weights and rotations retrieved by each method are given in Table 6. As shown for α = 0 and α = 1, the structures introduced by MPSO are respectively stiffer and lighter than the other structures obtained by the other methods. For α = 0.2, MPSO has

Table 5 Optimum stacking sequences obtained from different methods for torsional loading condition

Weighting factor (α)	Laminate		
	BPSO	RPSO	MPSO
0. 0	$[50^{1.75}, 65^{0.15}, -45^{2.0}, -65^{1.75}]$	$[35^{1.75}, -40^{1.5}, -45^{2.0}, -65^{1.75}]$	$[-40^{1.75}, 50^{1.75}, 45^{2.0}, -45^{1.5}]$
0.2	$[50^{1.0}, 40^{1.0}, -50^{1.75}, -50^{1.0}]$	$[35^{0.75}, 45^{2.0}, -45^{1.0}, -40^{1.25}]$	$[45^{0.75}, 45^{1.25}, -45^{0.75}, -45^{1.75}]$
0.4	$[40^{0.25}, -50^{1.5}, -40^{1.5}, 10^{0.25}]$	$[35^{0.5}, 45^{0.75}, 50^{0.25}, -45^{1.25}]$	$[40^{0.25}, -50^{0.75}, -45^{0.75}, 45^{1.25}]$
0.6	$[-35^{0.5}, -55^{0.5}, 50^{1.0}, -5^{0.25}]$	$[35^{0.75}, -40^{1.25}, 45^{0.25}, -25^{0.5}]$	$[40^{0.75}, -50^{0.5}, -50^{0.75}, 65^{0.25}]$
0.8	$[-55^{0.25}, 45^{0.5}, 85^{0.25}, -20^{0.5}]$	$[30^{0.75}, -35^{0.5}, 25^{0.5}, 70^{0.5}]$	$[25^{0.25}, 55^{0.5}, -50^{0.75}, 25^{0.25}]$
1.0	$[-30^{0.5}, 20^{0.25}, 35^{0.25}, -70^{0.25}]$	$[50^{0.5}, -90^{1.0}, -45^{0.25}, -5^{0.25}]$	$[50^{0.25}, 45^{0.25}, 5^{0.25}, 30^{0.25}]$

Table 6 Comparison of weights and deflections of the optimum results retrieved by each algorithm

Weighting factor (α)	BPSO		RPSO		MPSO	
	Weight (N)	Rotation (Rad)	Weight (N)	Rotation (Rad)	Weight (N)	Rotation (Rad)
0.0	276.21	1.49E-04	276.21	1.49E-04	276.21	1.24E-04
0.2	187.43	1.99E-04	197.29	1.76E-04	177.56	1.92E-04
0.4	138.10	2.65E-04	108.51	2.86E-04	118.38	2.88E-04
0.6	88.78	4.45E-04	108.51	2.83E-04	88.78	4.05E-04
0.8	59.19	1.06E-05	88.78	3.19E-04	69.05	5.90E-04
1.0	49.32	1.33E-03	78.92	4.89E-04	39.45	1.50E-04

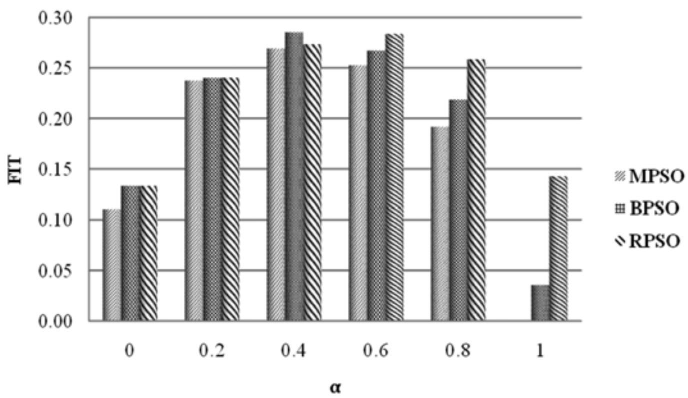

Fig. 7 Minimum reported values for FIT function in torsional loading condition

introduced a lighter structure with higher stiffness than the structure introduced by BPSO. Minimum FIT values reported by each of three methods are presented in Fig. 7. It can be seen that satisfactory results were obtained by all three algorithms and in all cases, MPSO managed to retrieve the best results. It is also seen that the proposed particle dynamics in MPSO has found more preferable solutions in all cases. This fact can also be observed in the convergancy curves given in Fig. 8 in which the convergence history for each case is presented. For example, MPSO shows excellent convergence for α = 0 while the other methods were trapped in local optima and experienced

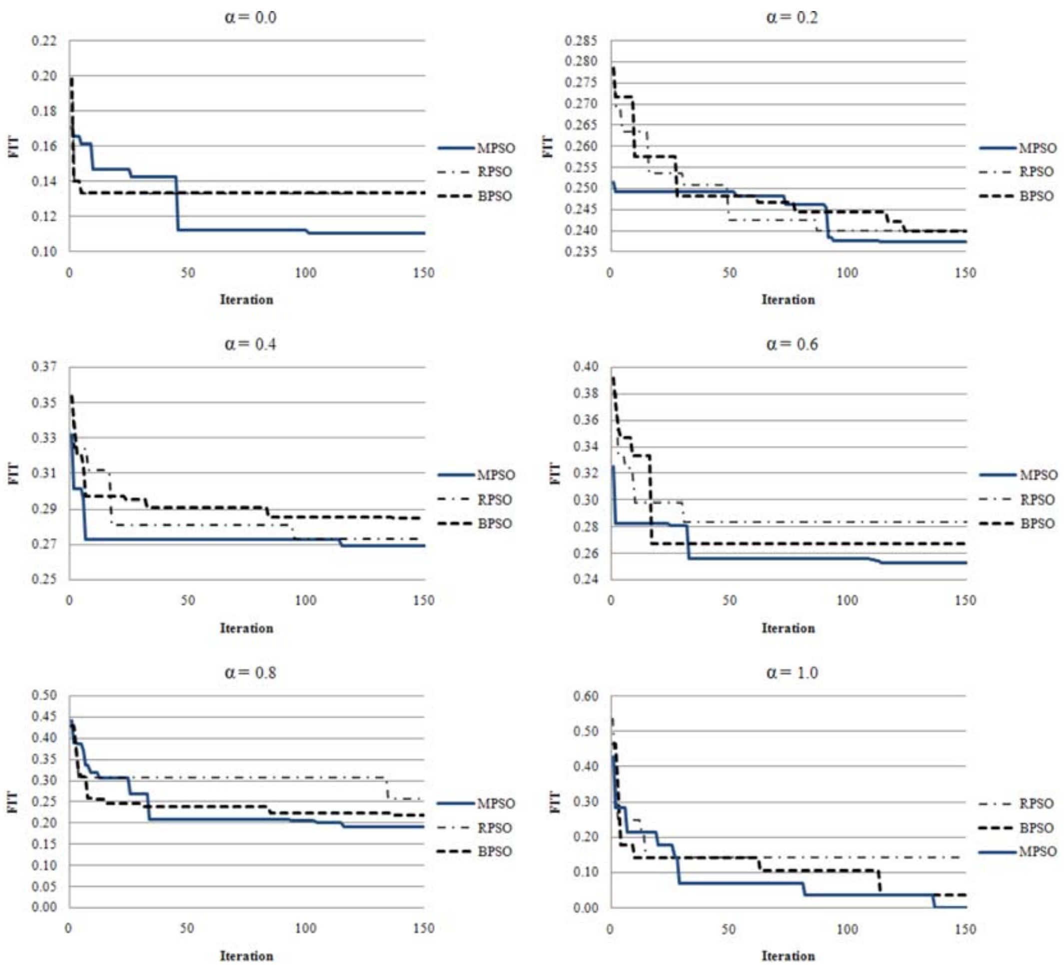

Fig. 8 Comparison of convergancy of MPSO, BSPO and RPSO for torsional loading of the laminated cylindrical shell

premature convergence. Fig. 8 also shows that the new variation scheme suggested in this paper has made it possible for the algorithm to search the design space more effectively and find more appropriate results. It can be concluded that the MPSO method has performed satisfactory performance in torsional stiffness optimization of cylindrical shell.

6. Conclusions

In the present study, convenient versions of particle swarm optimization algorithms were successfully applied to optimize the design of laminated composite cylindrical shells. The results obtained from these methods were compared with a suggested modified version of PSO algorithm called MPSO. The suggested modifications include a new variation scheme for acceleration and inertial weight parameters defined in the basic and repulsive PSO algorithms. As it was reported from the literature, BPSO has major drawbacks like smooth particle dynamics which may put the optimization procedure in danger of being trapped by local optimal points. In order to escape from these traps, some researchers have employed RPSO instead. Although RPSO can effectively survive local optima traps and perform search in more complex design spaces, but it also has the risk of losingthe focus on global and local best results. The noise in RPSO may divert it from moving to optimal solution. Thismay lead the problem to inappropriate solutions. Based on current needs for robust and dynamic optimization algorithms, in this paper a novel dynamic pattern for particle variationsis suggested. The modified PSO algorithm introduced in this paper has the appropriate dynamics to survive the local optima in discreet and complex design spaces. MPSO also has maintained the robustness of BPSO. To compare the performance of MPSO with two otherpopular PSO based methods, it was experimented in two numerical optimization problems of cylindrical laminated composite shells. All three algorithms of BPSO, RPSO and MPSO were employed to search for optimal solutions in the design space. Numerical results and convergence datawere reported for each case. MPSO showed excellent performance in searching the design space and better convergence to theoptimal solution. This new algorithm has the ability to successfully search for the optimum designsfor problems with similar natures. In addition, the comparisons revealed that application of this method wouldmake it possible to obtain more preferable results in optimization. Finally, based on the presented results the application of MPSO can be advised for design optimization of similar laminated structures.

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