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On mode localization of a weakly coupled beam system with spring-mass attachments

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Abstract. There are a large number of papers in the literature dealing with the free vibration analysis of single/multi-span uniform beam with multiple spring-mass systems, but that of coupled multi-span beams carrying spring-mass attachments is rare. In this note, free vibration analysis of a weakly coupled beam system with spring-mass attachments is conducted. The mode localization and frequency loci veering phenomena of the coupled beam system are investigated. Studies show that for weakly coupled beam system with spring-mass attachments, the mode localization and frequency loci veering will occur once there is a disorder in the system.

Keywords: vibration; coupled beam system; mode localization; frequency loci veering

1. Introduction

The vibration of beams carrying any number of spring-mass systems has been studied by many researchers. Various techniques have been presented to conduct the free vibration analysis of beam with point mass or single degree-of-freedom spring-mass systems in the past few decades (Gürgöze *et al.* 1996, Wu and Chou 1998, Low 2000, Wu and Chen 2001, Chen and Wu 2002, Lin and Tsai 2007).

The vibrational characteristics of uniform and non-uniform beams carrying two degrees-of-freedom spring-mass systems have been studied by several researchers (Wu and Whittaker 1999, Wu 2004, Qiao *et al.* 2002). Chan *et al.* (1996) investigated the vibratory characteristics of a simply supported Euler-Bernoulli beam with distributed rigid mass. Zhou and Ji (2006) studied the dynamic characteristics of a beam with continuously distributed spring-mass system using the transfer matrix method.

On the other hand, mode localization and frequency loci veering phenomena in weakly coupled systems have been studied extensively. Pierre *et al.* (1987) applied a modified perturbation method

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to study the mode localization on a disordered dual-span Euler-Bernoulli beam. Chen and Ginsberg (1992) investigated the relationship between mode localization and eigenvalue loci veering of nearly periodic structures by applying a perturbation method to a general eigenvalue problem. Lu *et al.* (2006) studied the mode localization and frequency loci veering in a disordered coupled beam system using the finite element method.

In this technical note, the free vibrations analysis for a weakly coupled beam system with single/multiple degrees-of-freedom spring-mass systems are analyzed by means of the finite element method. Vibration characteristics are investigated for the coupled beam systems. Studies in this paper reveal that for weakly a coupled beam system, a disorder in the physical parameter of the beam will lead to the occurrence of mode localization.

2. Theory

2.1 Equation of motion of a coupled beam system with spring-mass attachments

A coupled beam system that consists of two Bernoulli-Euler beams coupled via a linear and rotational sprung is shown in Fig. 1. For convenience, the coupled beam system is called a “bare” beam system if it does not carry any spring-mass systems and is called a “loaded” system if it carries spring-mass systems.

The governing differential equations of the beams and the ith spring mass are represented, respectively by Wu and Chou (1998)

$$EI_j\frac{\partial^4 y_j(x_j,t)}{\partial x^4}+\rho A_j\frac{\partial^2 y_j(x_j,t)}{\partial t^2} = \sum_{i=1}^{s} -k_i(\hat{y}_i-z_i)\delta(x-\hat{x}_i) \qquad (j=1,2) \tag{1}$$

$$m_i\ddot{z}_i+k_i(z_i-\hat{y}_i) = 0 \qquad (i=1,2,\ldots,s) \tag{2}$$

where E is the Young’s modulus, I_1, I_2 are the moment of inertia of the cross-sectional area of the

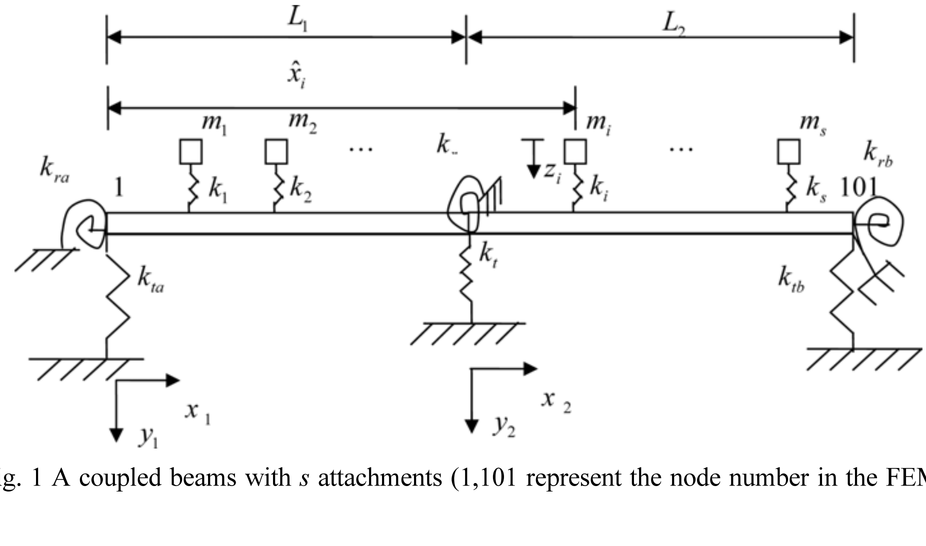

Fig. 1 A coupled beams with s attachments (1,101 represent the node number in the FEM)

left and right beam, respectively, ρ is mass density, A_1, A_2 are the area of the cross-section of the left and right beam, respectively, m_i and k_i represent the point mass and spring constant of the ith spring-mass system, s is the total number of the spring-mass system, $z_i, \ddot{z}_i$ denote the displacement and acceleration of the ith spring-mass relative to its static equilibrium position, $\hat{x}_i$ is the location of ith spring-mass, $\hat{y}_i$ is the transverse displacement of the beam at which the ith spring-mass locates. $\delta(\cdot)$ is the Dirac's delta function.

Finite element analysis is adopted for free vibration analysis for the coupled beam system. The Bernoulli-Euler beam element is used in the finite element model. After assembling all the elemental stiffness and mass matrices, we can obtain the systematic mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$ of the bare beam. Then, the spring-mass attachments are introduced into the equation of motion of the beam. Letting $\overline{\mathbf{M}}$ and $\overline{\mathbf{K}}$ be the systematic mass and stiffness matrices of the beam with spring-mass attachments, respectively, the finite element equation of free vibration for the coupled beam system with all attachments takes the form

$$\overline{\mathbf{M}}\ddot{d} + \overline{\mathbf{K}}d = 0 \tag{3}$$

2.2 Free vibration analysis for the coupled beam system

The natural frequencies and associated mode shapes can be obtained from the following generalized eigenvalue problem

$$\overline{\mathbf{K}}V = \lambda\overline{\mathbf{M}}V \tag{4}$$

where $\lambda = \omega^2$, ω is the circular frequency of the beam system, V is the normalized mode shape matrix.

2.3 Mode localization and frequency loci veering in a weakly coupled beam system with attachment

It is well known that for a weakly coupled system, when it is symmetric, each vibration mode of the system is either symmetric or anti-symmetric, thus localized vibration modes will not occur. However, when disorders are introduced in the system, that is to say, the symmetry condition is broken; it is very likely that the mode localization and frequency loci veering will occur. As shown in Fig. 1, three kinds of disorders are introduced into the weakly coupled beam system: (a) disorder in the elemental flexural rigidity EI, (b) disorder in the elemental mass of the beam, and (c) disorder in the length of the beam.

$$EI_i = EI_{i0}(1+\varepsilon) \tag{5}$$

$$m_i = m_{i0}(1+\varepsilon) \tag{6}$$

$$L_i = L_{i0}(1+\varepsilon) \tag{7}$$

where EI_i, L_i and m_i denote the flexural rigidity of the ith element, the length of the beam and the mass of the ith element after perturbation, respectively; EI_{i0}, L_{i0} and m_{i0} denote the flexural rigidity of the ith element, the length of the beam and the mass of the ith element of the original system, respectively. $\varepsilon(|\varepsilon| << 1)$ is the perturbation in the parameters, which governs the disorder in the system. It is worth noting that when $\varepsilon = 0$, there is no disorder.

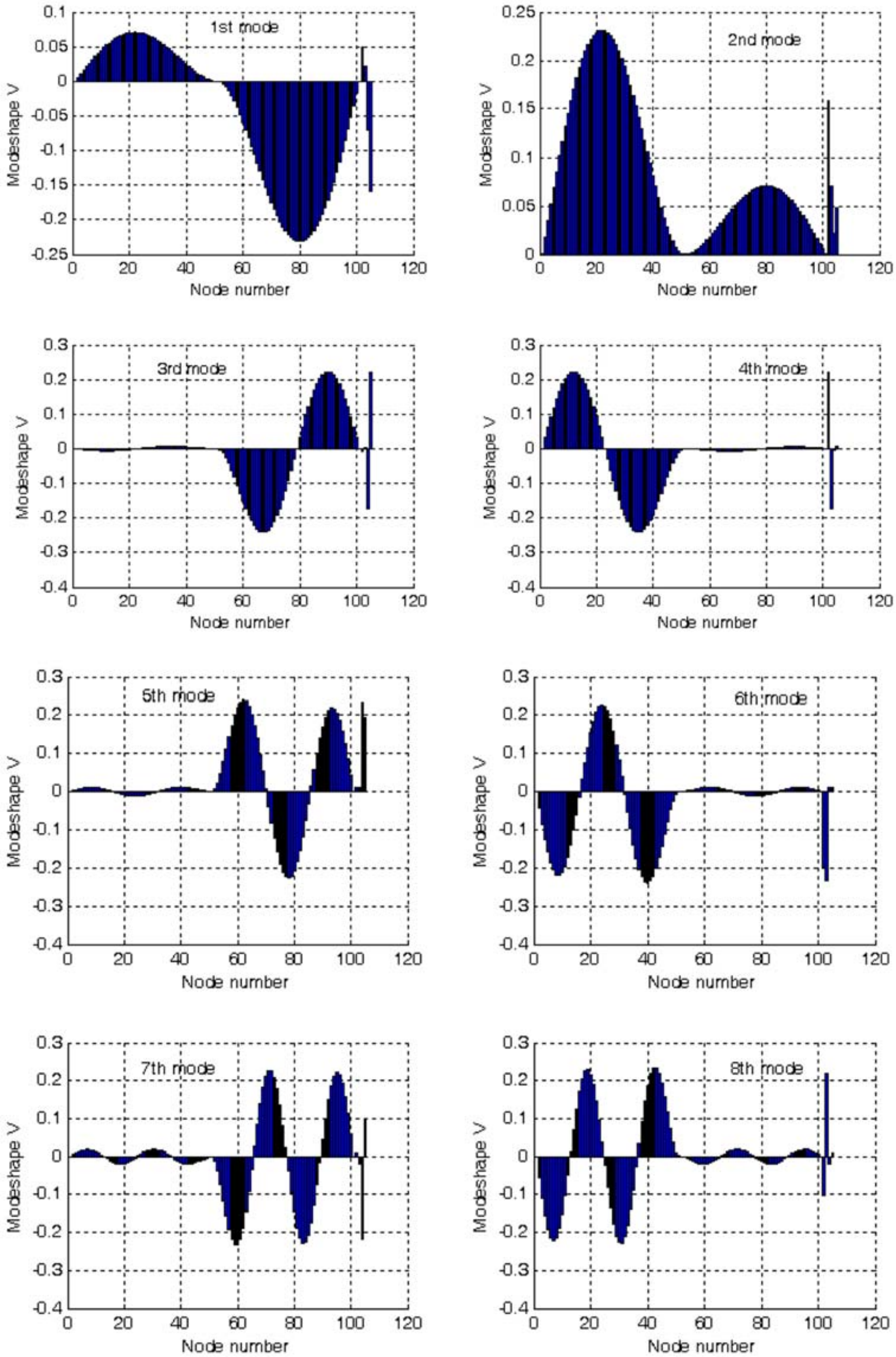

Fig. 2 The first eight modeshapes of the coupled beam system with a disorder in the elemental flexural rigidity

3. Numerical simulations

As shown in Fig. 1, the coefficients of the coupling spring k_t and k_r are taken as: $k_t = 5.1 \times 10^9$ N/m, $k_r = 5.1 \times 10^8$ Nm/rad respectively, and the two beams are weakly coupled. The parameters of the sprung of both ends are $k_{ra} = k_{rb} = 0$, $k_{ta} = k_{tb} = 10^{15}$ N/m. Parameters of the system are: Young's modulus $E = 2.069 \times 10^{11}$ Pa, diameter of the cross-section of the beam $d = 0.05$ m, moment of inertia of the cross-sectional area $I = 3.06796 \times 10^{-7}$ m^4, mass density $\rho = 7837$ kg/m^3, length of the two beam $L_1 = L_2 = 2.5$ m. The bare beam is discretized into 100 two-node Euler beam elements, the total degrees-of-freedom of the coupled beam with spring-mass systems is $2 \times 101 + 4 = 206$. It is assumed that the coupled beam system carries four intermediate spring-mass systems which locate at the 11th, 41st, 61st and 91st node of the beam in the finite element model.

Case Study 1: *Disorders in the elemental flexural rigidity of the beam*

A disorder is introduced into the flexural rigidity of the 2nd and 3rd elements of the left beam to investigate the mode localization and frequency loci veering. The perturbation parameter ε varies from −0.1 to 0.1. Fig. 2 shows the first eight mode shapes of the system. The first eight natural frequencies of the disordered system with $\varepsilon = 0.1$ are listed in Table 1. Fig. 3 shows the frequency loci veering of the first eight modes. From these figures one can see that when there is disorder in the flexural rigidity, the mode localization and frequency loci veering phenomena occur from the fundamental mode.

Case Study 2: *Disorder in the elemental mass of the beam*

In this Case Study, a disorder in the mass of the 2nd and 3rd elements of the beam is introduced into the system. The perturbation parameter ε varies from −0.1 to 0.1. Table 1 shows the first eight natural frequencies of the original system and disordered system with $\varepsilon = 0.1$. Fig. 4 shows the first eight mode shapes of the system. Fig. 5 shows the frequency loci veering of the first eight modes. From these figures one can see that a disorder in the mass of the beam will also lead to the mode localization and frequency loci veering phenomena.

Table 1 The first eight natural frequencies (rad/s) for weakly coupled beams with four spring-mass systems ($\varepsilon = 0.1$)

Mode No.	Original system	Disorder in elemental flexural rigidity	Disorder in mass	Disorder in beam length
1	157.91	157.91	157.88	143.26
2	157.93	157.96	157.92	157.92
3	509.43	509.44	509.05	426.25
4	509.45	509.81	509.44	509.44
5	1061.03	1061.10	1059.54	962.90
6	1061.18	1062.55	1061.51	1061.10
7	1820.16	1820.47	1816.26	1651.77
8	1820.82	1824.36	1820.51	1820.49

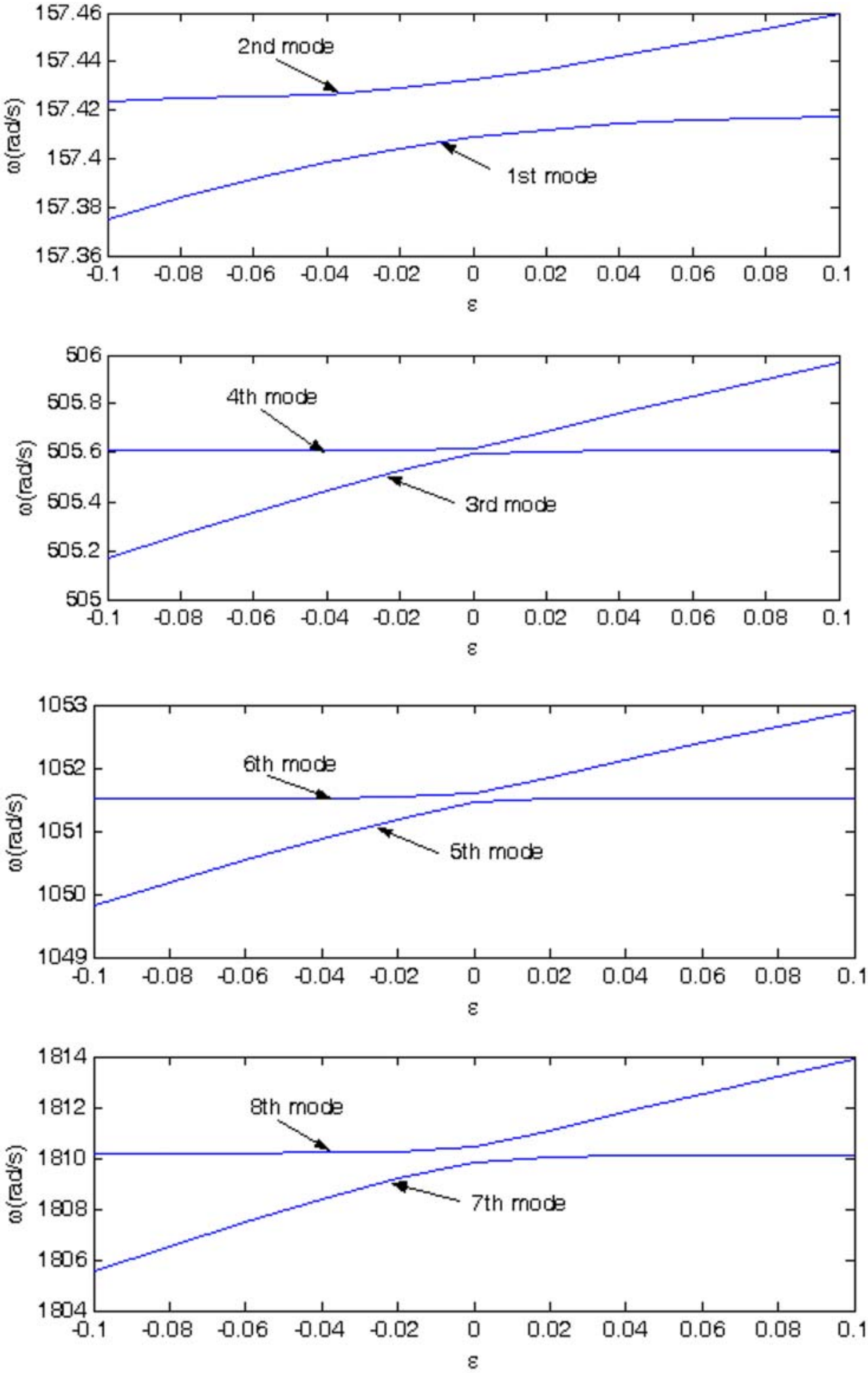

Fig. 3 Frequency loci veering of the coupled beam system with a disorder in the elemental flexural rigidity

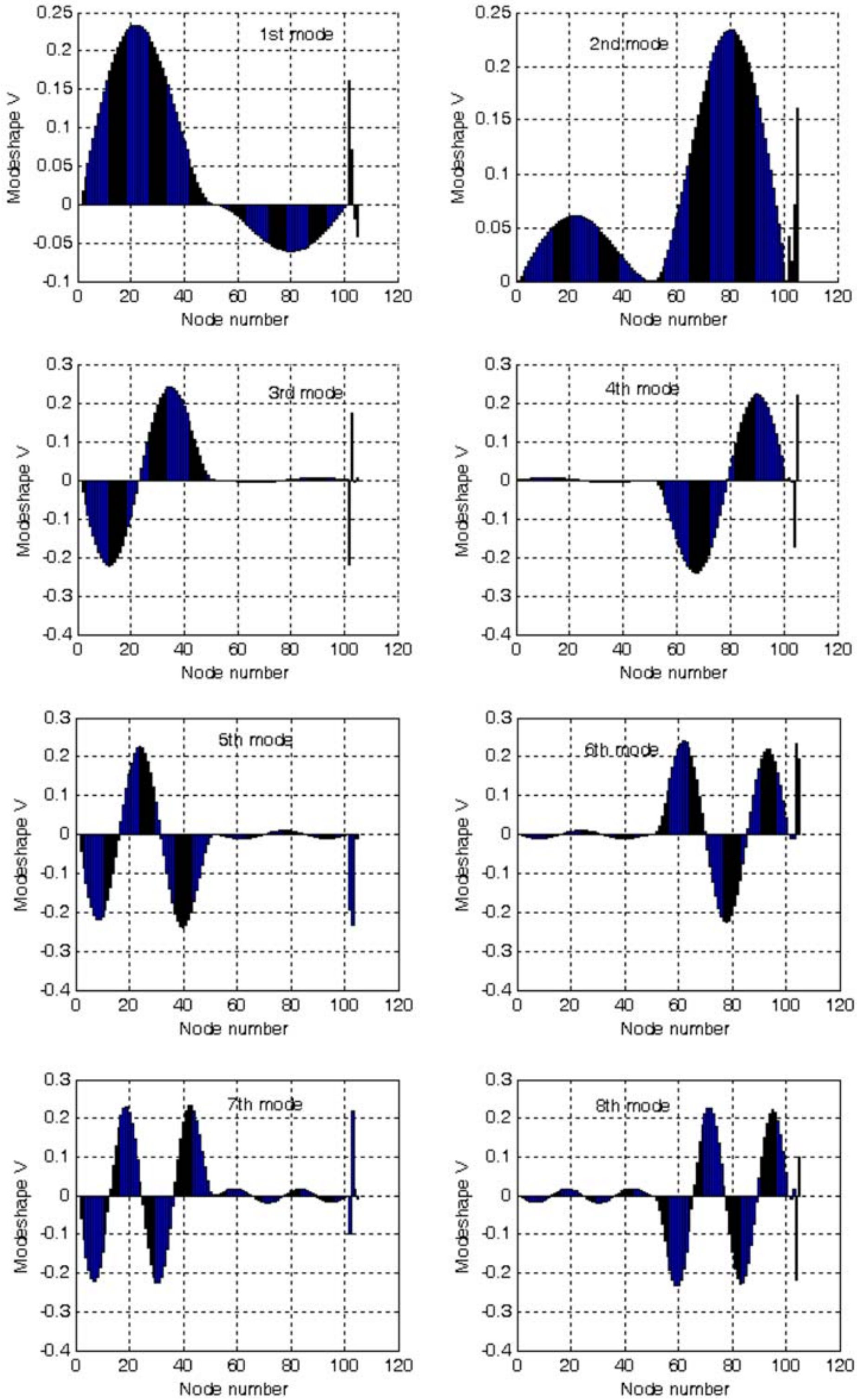

Fig. 4 The first eight modeshapes of the coupled beam system with a disorder in the elemental mass

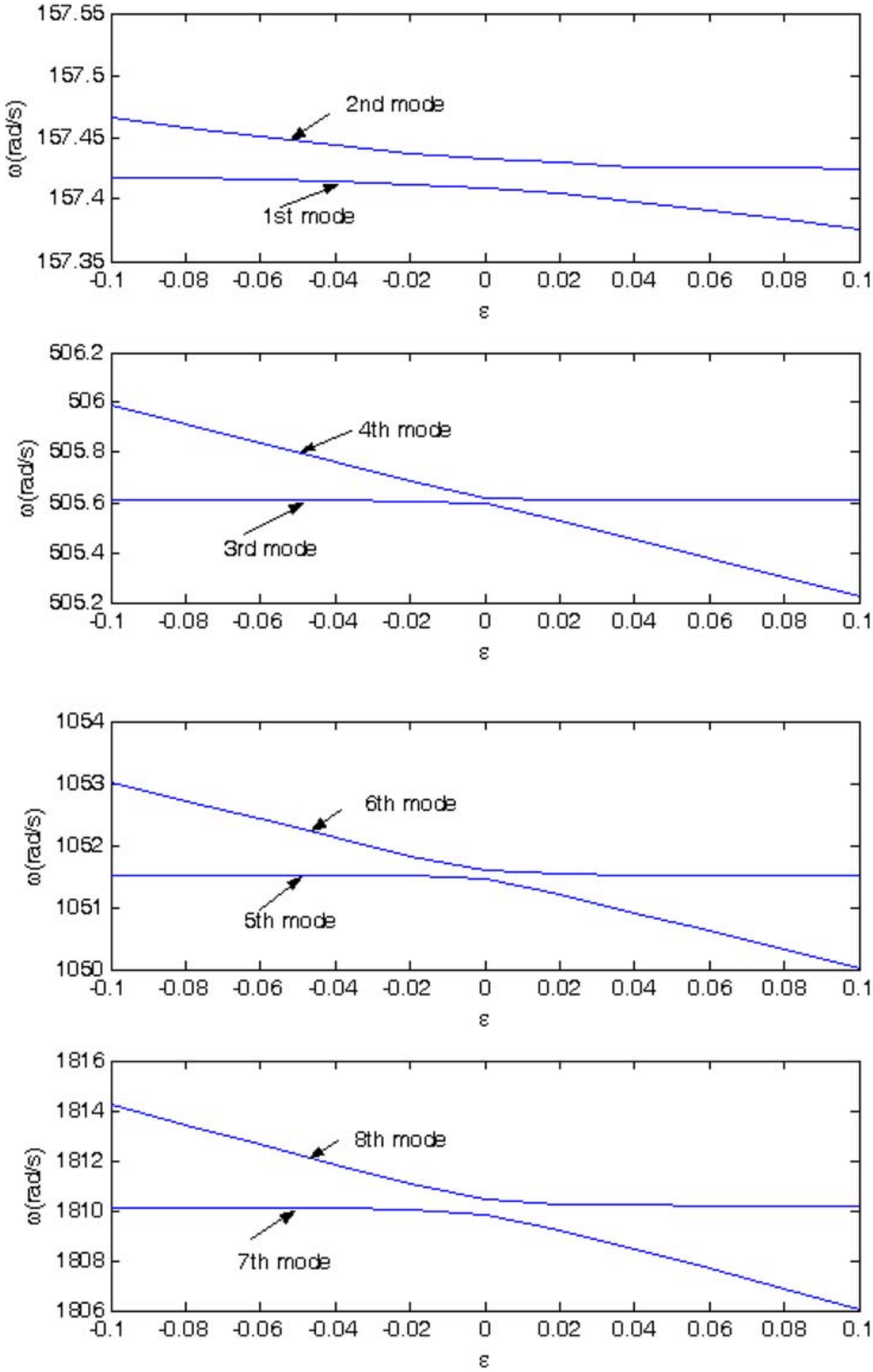

Fig. 5 The frequency loci veering of the coupled beam system with a disorder in the elemental mass

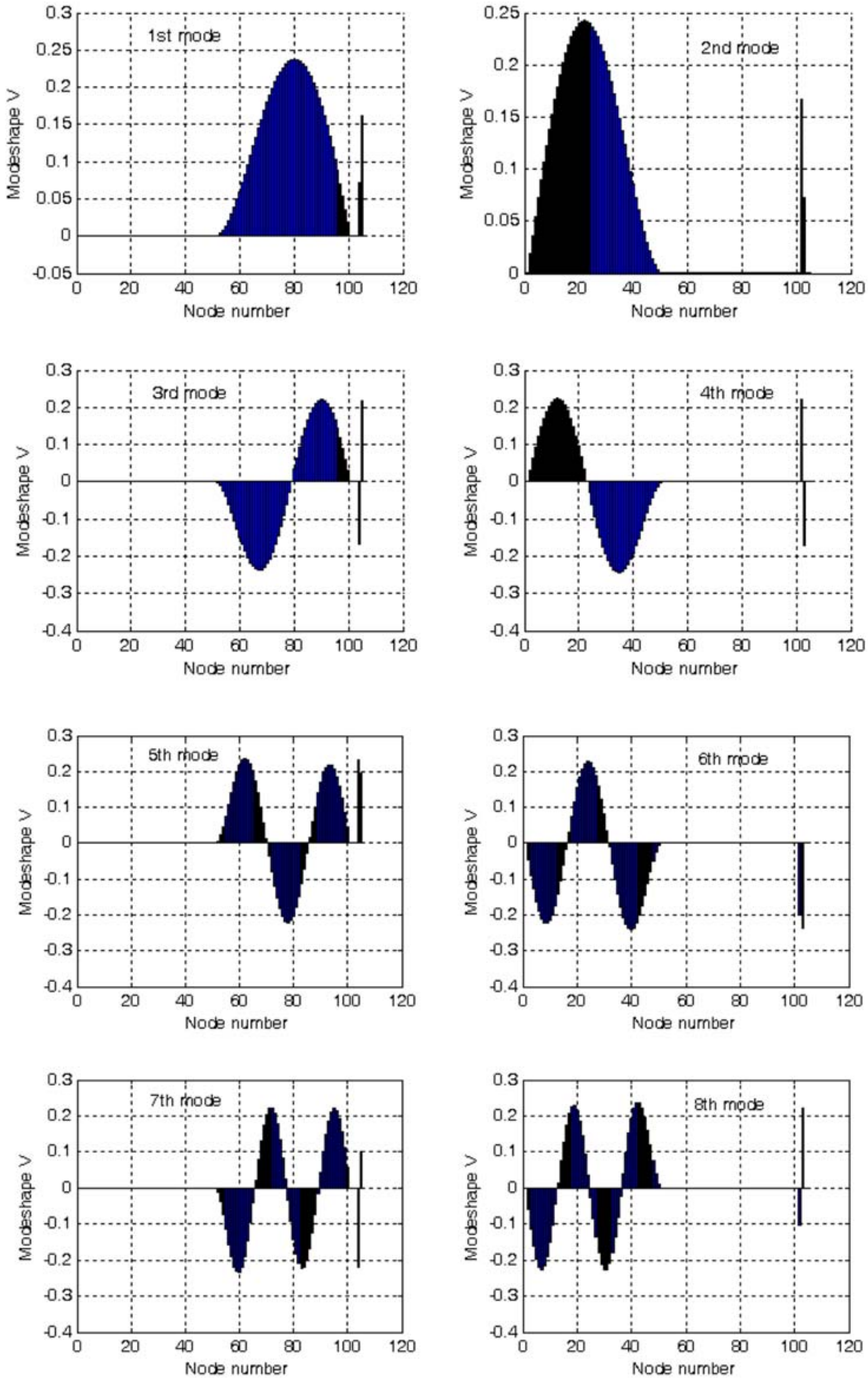

Fig. 6 The first eight modeshapes of the coupled beam system with a disorder in the length of the beam

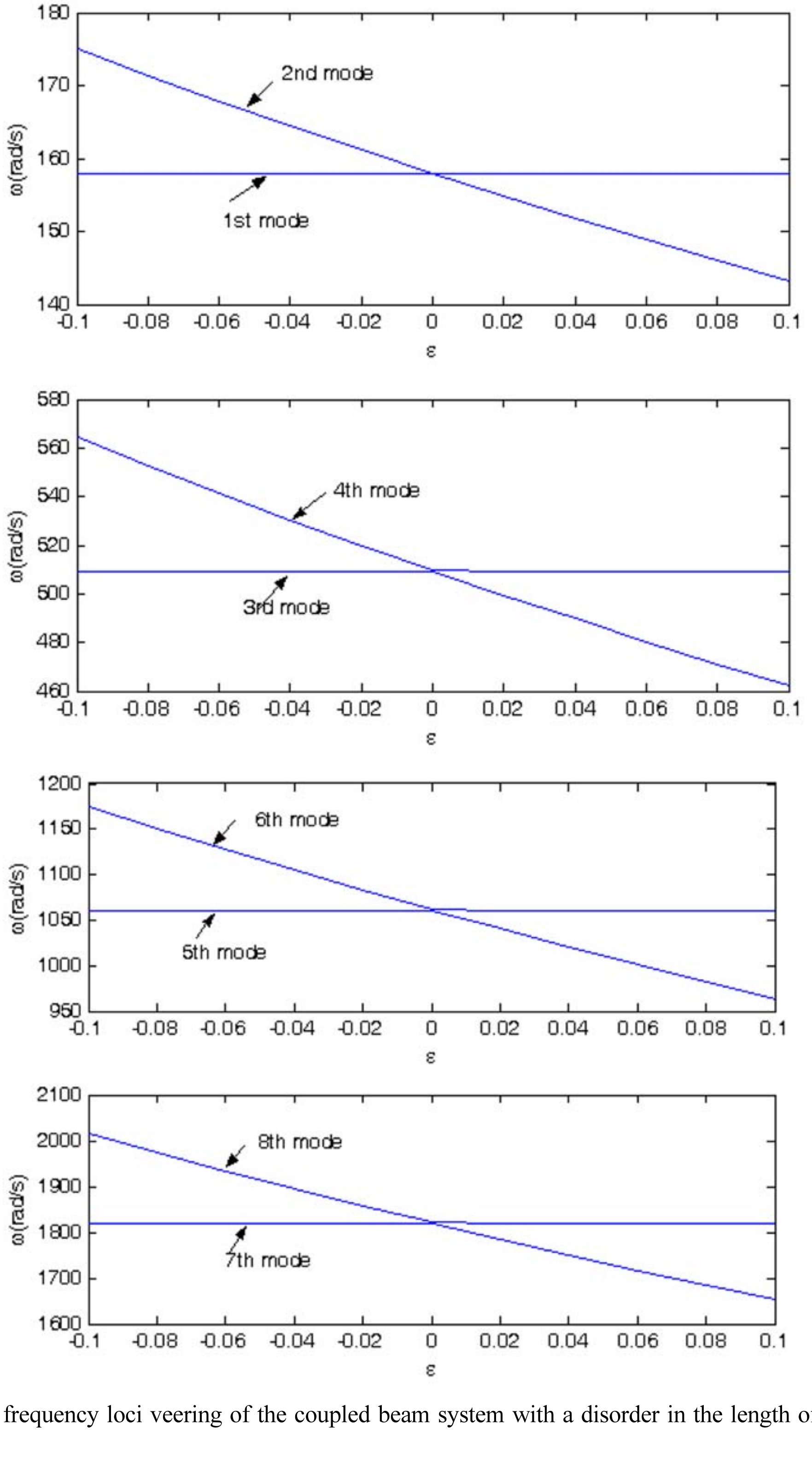

Fig. 7 The frequency loci veering of the coupled beam system with a disorder in the length of the beam

Case Study 3: *Disorder in the length of the beam*

A disorder in the length of the second beam is introduced into the system to investigate the mode localization and frequency loci veering. The perturbation parameter ε varies from −0.1 to 0.1. Table 1 shows the first eight natural frequencies of the original system and disordered system with $\varepsilon = 0.1$. Fig. 6 shows the first eight mode shapes of the system. Fig. 7 shows the frequency loci veering of the first eight modes. From these figures one can see that when there is a disorder in the spring coefficient of the attachment, the mode localization and frequency loci veering phenomena will occur.

From three cases studied above, one can see, for different kinds of disorders in the weakly coupled beam system with attachments, mode localization and frequency loci veering are observed. It is worth noting that for such system, when there is a small damage either in the bare beam, mode localization will occur. If this is the case, the mode shapes of the damaged system will be dramatically different from those of undamaged system as no mode localization phenomena will occur in the undamaged system. Special attention may be paid for damage detection for such weakly coupled system when frequency domain method is used as the mode shapes of the damaged system are much different from the undamaged one, it is very likely that one may not obtain a physically meaningful solution in damage detection.

4. Conclusions

Finite element analysis is used to determine the natural frequencies and mode shapes of weakly coupled beams with of spring-mass systems. The mode localization and frequency loci veering phenomena of weakly coupled beams with spring-mass systems are investigated. Studies show that for a system consisting of weakly coupled beams and attachments, the mode localization and loci veering will occur once there is a disorder in the system.

Acknowledgements

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