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Using integrated displacement method to time-history analysis of steel frames with nonlinear flexible connections

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Abstract. Most connections of steel structures exhibit flexible behaviour under cyclic loading. The flexible connections can be assumed as nonlinear rotational springs attached to the ends of each beam. The nonlinear behaviour of the connections can be considered by suitable moment-rotation relationship. Time-history analysis by direct integration method can be used as a powerful technique to determine the nonlinear dynamic response of the structure. In conventional numerical integration, the response is evaluated for a series of short time increments. The limitations on the size of time intervals can be removed by using Chen and Robinson improved time history analysis method, in which the integrated displacements are used as the new variables in integrated equation of motion. The proposed method permits longer time intervals and reduces the computational works. In this paper the nonlinearity behaviour of the structure is summarized on the connections, and the step by step improved time-history analysis is used to calculate the dynamic response of the structure. Several numerical calculations which indicate the applicability and advantages of the proposed methodology are presented. These calculations illustrate the importance of the effect of the nonlinear behaviour of the flexible connections in the calculation of the dynamic response of steel frames.

Keywords: steel frames; nonlinear analysis; time-history analysis; flexible connection; semi-rigid connection; integrated displacement method

1. Introduction

Experimental researches show that the true behaviour of beam-to-column connections in the steel structures lies in between that of ideally pinned and fully rigid (Nader and Astaneh 1991, Popov and Takhirov 2002). Neglecting the real behaviour of the connections in the analysis and the design of steel structures may lead to unrealistic predictions of the response and reliability. Thus, both of these extreme assumptions may be inaccurate and uneconomic (Hadianfard and Razani 2003, Hadianfard and Rahnema 2010).

In the recent past, extensive experimental and analytical studies have been done on the behaviour of beam-column joints. For example, a comprehensive experimental investigation on cyclic behaviour of two types of semi-rigid double web angle connections (bolted-bolted and welded-

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bolted) is introduced by Abolmali *et al*. (2003). They presented the moment-rotation hysteresis loops and the failure modes for these connections. Yang and Kim (2007) also presented an experimental investigation on the cyclic behaviour of bolted and welded beam-column joints. The main objective of their studies was to directly compare behaviour of beam-column steel joints with high-strength bolted and welded. The results of a series of tests that performed in order to examine the behaviour of blind-bolted angle connections between open beams and tubular columns are presented by Elghazouli *et al.* (2009). Analyzing and designing a semi-rigid frame require a clear understanding of the moment-rotation relations of its connections. This relationship is usually derived by fitting suitable curves to the experimental data. Various types of moment-rotation relations as: linear, polynomial, exponential, power etc. have been proposed by many researchers (Richard and Abbott 1975, Chen and Lui 1991, Kishi *et al.* 1993, Lee and Moon 2002, Pirmoz *et al.* 2009). The linear models are simple to use, but they cannot represent a hysteretic behavior of the connections under cyclic loads as earthquake. More realistic models for moment-rotation relations are usually nonlinear over the entire range of loading. Hence, to consider the real behavior and flexibility of the connections, a nonlinear analysis is necessary.

The dynamic behavior of steel frames with flexible connections may be significantly different from frame with rigid or pinned connections. Therefore, the conventional methods of using ideal connections (fully rigid or pinned) are inadequate and they cannot represent real structural behavior. Then, the nonlinear dynamic analysis is the only appropriate method for verifying the real behavior of the flexible steel frames under earthquake excitation.

Nonlinear response of frames, subjected to strong ground motions, has been recognized by engineers for many years (Elnashai and Izzuddin 1993, Chan and Mingho 1994, Clough and Penzien 2003). The nonlinear dynamic analysis of steel frames with flexible connections has been studied in several papers as well. For example, Sekulovic *et al.* (2002) considered the effects of flexibility and damping in the connections on the dynamic behaviour of plane steel frames. They developed a numerical model that includes both nonlinearity of the connections and geometric nonlinearity of the structure. Nonlinear dynamic analysis of a semi-rigid low-rise portal frame is also done by Silva *et al.* (2008). They developed finite element model includes the geometrical non-linearity and influence of a nonlinear and hysteretic connection stiffness. Sekulovic and Nefovska (2008) developed the numerical model that simultaneously includes nonlinear connection behaviour, member yielding and geometrical nonlinearity of the steel structures. They concluded that in seismically active regions using the steel frames with flexible connections may be a better option than using fully rigid connections.

In general, a nonlinear dynamic analysis is performed using an incremental formulation, in which the variables are updated incrementally corresponding to successive time steps. The step by step integration procedure is the most effective technique for nonlinear time-history analysis of structures (Clough and Penzien 2003). In the usual calculations, a small time interval is necessary to represent the variation in external loading (earthquake record). Also, when there are some rapid changes in material properties, as in yielding or unloading, the stiffness can even experience a large jump. It is necessary to use a short time interval for considering this rapid change. Then, an accurate solution of the nonlinear dynamic equations can be expected if the time steps are made sufficiently small.

In the majority of the aforementioned studies, the nonlinear dynamic equations are solved by using the conventional step by step integration procedure with short time intervals. These types of analysis consume a lot of computational time. In other words, the computational effort in the time-history analysis will increase roughly inversely to the size of the time interval. Therefore, it is reasonable to find a method to increase the time intervals.

In the present study, by using the improved technique proposed by Chen and Robinson (Chen and Robinson 1993, Robinson and Chen 1993), the limitations on the time interval due to rapidity of excitation to nonlinear material behaviour and to modes with very high frequencies are eliminated. Then, the proposed method permits longer time intervals and reduces the computational works. It is assumed that the material nonlinearity behaviour of the structure is concentrated in the connections, too. A computer program is developed to perform improved time-history analysis of flexible frames. Some examples are worked out using this software. The results show accuracy and efficiency of the proposed method and also show the effects of nonlinear behaviour of the connections on the response of steel frames.

2. Modelling of connection nonlinearity

There are two common forms of nonlinearity that arise in structural analysis, geometric nonlinearity (large deflections) and material nonlinearity (plasticity). The principal material property is the stress-strain curve. For a linear problem, the stress-strain curve is linear and elastic modulus is constant but for a structure with material nonlinearity this curve is nonlinear (or multi-linear) and elastic modulus is variable. Then, the solution must be found by a series of iterations, and the modulus of elasticity and stiffness matrix must be calculated in each iteration.

In the steel structures, most types of the beam-column connections are semi-rigid or flexible. Based on experimental studies, many mathematical models have been developed to show the connection behaviour. This behaviour can be modelled in the finite element analysis of the structures by using moment-rotation (M-θ) diagram for each connection. Numerous test results show that M-θ relations for most connections are nonlinear (Chen and Lui 1991, Abolmaali *et al.* 2003, Yang and Kim 2007).

The connection flexibility is considered by attaching rotational springs at the ends of beams in the present study. These springs have nonlinear behaviour with initial stiffness K_0, tangential stiffness K and ultimate moment capacity M_u.

When a moment is applied to a flexible connection, the connection rotates according to the M-θ curve. However, if the direction of the moment is reversed, the connection will unload and follow a different path which is linear with a slope equal to the initial slope (K_0) of the M-θ curve. This

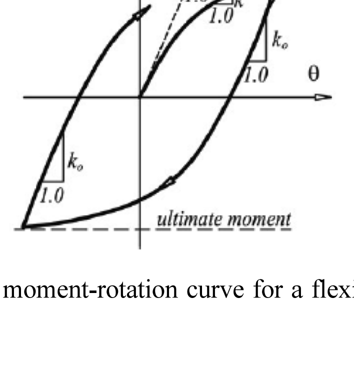

Fig. 1 Cyclic moment-rotation curve for a flexible connection

cyclic moment-rotation curve is schematically shown in Fig. 1.

In this paper, the beam-to-column connections assumed to be with bolted angles. Three types of connections with angles are considered:

1- Single or double web-angle connection (SWA or DWA).

2- Top and seat angle connection (TSA).

3- Top and seat angle with double web-angle connection (TSDWA).

The three-parameter power model is selected to represent the nonlinear M-θ curve. This model firstly proposed by Richard and Abbott (1975). The model can be formulated as

$$M = \frac{K_0 \theta}{\left[1 + \left(\frac{\theta}{\theta_0}\right)\right]^{1/n}} \tag{1}$$

Where K_0 is initial stiffness of the connection, $\theta_0 = M_u/K_0$ is plastic rotation of the connection, M_u is ultimate moment capacity of the connection and n is shape parameter. The values K_0, M_u and n are determined from empirical equations. The tangential stiffness K which is slope of the M-θ curve can be obtained from Eq. (2).

$$K = \frac{dM}{d\theta} = \frac{K_0}{\left[1 + \left(\frac{\theta}{\theta_0}\right)^n\right]^{\frac{1+n}{n}}} \tag{2}$$

The tangential stiffness K decreases from the initial value K_0 (maximum value of stiffness) to zero, when the moment M reaches ultimate moment capacity M_u.

A new power model with three parameters similar to Richard and Abbott model has been proposed by Kishi and Chen (1990). This model is shown in Eq. (3).

$$\theta = \frac{M}{K_0\left[1 - \left(\frac{M}{M_u}\right)^n\right]^{1/n}} \tag{3}$$

For this model the tangent modulus is as following

$$K = \frac{dM}{d\theta} = K_0\left[1 - \left(\frac{M}{M_u}\right)^n\right]^{\frac{1+n}{n}} \tag{4}$$

The Kishi-Chen power model is a semi-empirical model. In this model, the initial stiffness and ultimate moment capacity are evaluated analytically, while the shape parameter is obtained by a curve-fitting technique (Kishi *et al.* 1993).

To describe the M-θ curve for connections with angles, instead of the three-parameter power model, the two-parameter model of Lee and Moon (2002) may be used. This model can be formulated as Eq. (5).

$$M = \alpha[\ln(n.10^3\theta + 1)]^n \tag{5}$$

Where α and n are shape parameters which are calculated by using the least squares method between the experimental data. The tangent stiffness (K) in this model can be determined from Eq. (6).

$$K = \frac{dM}{d\theta} = n.\alpha[\ln(n.10^3\theta + 1)]^{n-1}\frac{n.10^3}{n.10^3 + 1} \tag{6}$$

This model requires only two parameters α and n, and can provide a good fit with various connection M-θ curve. These two parameters can be determined in terms of initial stiffness (K_0) and plastic stiffness (Kp) from statistical regression analysis.

$$\alpha = f(K_0, K_p), \qquad n = f(K_0, K_p) \tag{7}$$

For example in the semi-rigid connection with double web angle (DWA), the α and n parameters can be calculated from Eq. (8) and Eq. (9).

$$\alpha = 1.499E - 0.3K_0 + 1.499E - 0.3K_p + 0.704 \tag{8}$$

$$n = -3.594E - 0.5K_0 - 3.496E - 0.5K_p + 1.170 \tag{9}$$

Where E is elastic modulus (KN/m^2) and K_0 and K_p are initial and plastic stiffness (KN.m/rad).

Design of steel structures usually is done based on strong columns and weak beams; therefore, the plastic hinges form at the beams before the columns. In the partially restrained or flexible steel frames the moment capacity of the connections is usually less than the moment capacity of the beam. Then, the first plastic hinges form at the beam-column connections.

Besides, experimental studies by Nader and Astaneh (1991) on the shaking table show that the response of the rigid structure is almost elastic and linear. But the response of the flexible and semi-rigid structure is more inelastic and nonlinear. In other words, the cyclic behaviour of the flexible connections is the mainly factor of nonlinear behaviour of the semi-rigid frames. The test results by Yang and Kim (2007) also confirm the above topic. These test results show that for high-strength bolted DWA connection, the development of the deformation in the vicinity of the bolt holes is the primary mode of failure, and then one plastic hinge line formed in the web angles. However, it cannot be seen the significant deformation or plastic hinge in the beams or columns. Also for TSDWA connection, it can be seen that the first major deformation begins at the top and seat angles in the bending, and this deformation was developed into the bolt line of the top and seat angles part attached to the column flange. Similar to DWA connection, it was not observed severe deformation in the beams and columns.

Consequently, it can be assumed that in the steel frames with flexible connections the mainly nonlinear behaviour of the structure is concentrated on the connections. Thus, by using the dynamic analysis of steel frames with nonlinear connections, main nonlinear behaviour of the frame is considered in the analysis.

3. Method of dynamic analysis

The equations of motion for a N-degrees of freedom structure under earthquake loading is

$$M\ddot{U} + C\dot{U} + KU = -M\ddot{U}_g \tag{10}$$

Where M is the mass matrix, C is the damping matrix and K is the stiffness matrix. The time dependent vectors $U(t)$ and $\ddot{U}_g$ are displacement and load (ground acceleration) vectors. These matrices (or vectors) can be calculated for one-dimensional elements by defining the proper interpolation functions (Clough and Penzien 2003). The column stiffness matrix takes the usual form, but the beam stiffness matrix is function of the rotational stiffness of the connections. This rotational stiffness can be calculated at each point from the slope of the M-θ curve (Hadianfard and Razani 2003).

The elements property matrices can be assembled to form the global property matrix for a given structure. In the nonlinear problems, during the time-history analysis, the mass matrix is constant, but the damping and stiffness matrices may be variable (since they are include the nonlinear effects of the connections) and they should be calculated in each step. If nonlinear behaviour of the structural material is described by multi-linear or elasto-plastic diagram, in this case, for each linear portion of the diagram the stiffness matrix is constant and only yielding or unloading changes the stiffness matrix. But for nonlinear materials without linear portion, the tangent stiffness must be calculated in each step, and stiffness matrix should be improved in each step.

The damping is assumed to be Rayleigh damping. The damping matrix is proportional to mass and stiffness matrix and it can be calculated in each step from the following relation

$$C(t) = \alpha M + \beta K(t) \tag{11}$$

Where α and β are Rayleigh damping factors (Clough and Penzien 2003).

In the Rayleigh damping, the relation between damping ratio (ξ) and natural frequency (ω) of each mode is described as Eq. (12).

$$\xi_n = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2} \tag{12}$$

Hence, the α and β factors can be determined by the solution of simultaneous equations if the damping ratios ξ_m and ξ_n associated with two specific modes m and n (ω_m, ω_n) are known. These simultaneous equations in matrix form leads to

$$\begin{Bmatrix} \xi_m \\ \xi_n \end{Bmatrix} = 0.5 \begin{bmatrix} 1/\omega_m & \omega_m \\ 1/\omega_n & \omega_n \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} \tag{13}$$

The factors α and β can be found from above equation as

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \frac{2\omega_m\omega_n}{\omega_n^2 - \omega_m^2} \begin{bmatrix} \omega_n & -\omega_m \\ -1/\omega_n & 1/\omega_m \end{bmatrix} \begin{Bmatrix} \xi_m \\ \xi_n \end{Bmatrix} \tag{14}$$

In this study, it is assumed that the same damping ratio applies to all frequencies ($\xi_m = \xi_n = \xi$). And the factors can be determined from simplified equation as below

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \frac{2\xi}{\omega_m + \omega_n} \begin{Bmatrix} \omega_m\omega_n \\ 1 \end{Bmatrix} \tag{15}$$

Direct integration method is applied to solve the nonlinear Eq. (10). The iterative method in each step is based on evaluating the tangent stiffness matrix which depends on the rotational stiffness of the connections.

The most powerful technique for nonlinear analysis is the step-by-step integration procedure. In this approach, the response is evaluated for a series of short time increments. The incremental equation of motion can be expressed as

$$M\Delta\ddot{U}(t)+C(t)\Delta\dot{U}(t)+K(t)\Delta U(t) = \Delta P(t) \tag{16}$$

Where $\Delta U(t)$ and $\Delta P(t)$ are incremental vectors of displacements and external loads respectively.

In the conventional methods, the time intervals are chosen short enough to give an accurate representation of such a rapid varying function of time. But in the method proposed by Chen and Robinson (Chen and Robinson 1993, Robinson and Chen 1993), the limitations on the size of time intervals are removed, and it permits longer time intervals.

4. Elimination the limits on the size of time intervals

In the conventional time-history analysis, there are three major factors limiting the size of the time intervals in the numerical integration. The first one is the rapid changes in the excitation (ground acceleration). The second one is the rapid changes in the resistance due to yielding or unloading (material nonlinearity) and the third one is the presence of modes with very high frequencies. A set of methods developed to handle these limitations as described below.

4.1 Rapid changes in the ground acceleration

In the time-history analysis, the input usually is ground acceleration. The time intervals should be selected short enough in the conventional methods to provide an accurate representation of such a rapidly varying function of time. One of the best methods to eliminate this limitation on the size of time intervals is based on the smoothing effect of time integration which can be used to smooth out the variation in the dynamic loading (ground acceleration). The usual equations of motion are integrated twice with respect to time to establish the smoothing effect. And by introducing integrated displacements as new variables in the integrated equations of motion, the limitation on the size of time intervals may be removed. The modified equations of motion can be calculated by carrying out once integration and twice integration of Eq. (10) as below

$$M(\ddot{P}-\ddot{P}_{t^*})+\int_{t^*}^{t}C\dot{U}d\xi+\int_{t^*}^{t}KUd\xi = -M\dot{U}_g \tag{17}$$

$$M[\ddot{Q}-\ddot{Q}_{t^*}-\ddot{P}_{t^*}(t-t^*)]+\int_{t^*}^{t}\int_{t^*}^{\xi}C\dot{U}d\eta d\xi+\int_{t^*}^{t}\int_{t^*}^{\xi}KUd\eta d\xi = -MU_g \tag{18}$$

Where, two new variables P and Q are as following

$$P(t) = \int_{t^*}^{t}U(\xi)d\xi \tag{19}$$

$$Q(t) = \int_{t^*}^{t}P(\xi)d\xi = \int_{t^*}^{t}\int_{t^*}^{\xi}U(\xi)d\eta d\xi \tag{20}$$

Where, t^* is any instant during the time of interest.

4.2 Rapid changes in the resistance

For a structure with nonlinear or inelastic behaviour, hysteretic behaviour gives rise to rapid variations or jumps in the stiffness. On the other hand, the structural stiffness terms in the equation of motion are not smooth function of time. Then, in the conventional methods, the time intervals should be small enough to permit the accurate representation of rapid variation in the stiffness. This limitation may be removed by using integrated equations together with a simple interpolation procedure make it possible to pass right through points of yielding or unloading. The interpolation scheme is based on two procedures: The first one is assumption of parabolic variations of velocities and displacements in a time interval. The second procedure is application of smoothing by time integration, as described in section 4.1 for smoothing the rapid changes in the ground acceleration.

The time of yielding is determined from a parabolic variation of displacement and the time of unloading is found from a parabolic variation of velocity. Hence, out of three values of displacements (beginning, middle and end points), the time of zero displacement (yielding) can be found out by solution of a quadratic equation. Similarly, the time of zero velocity (unloading) can be recognized by a parabolic curve passing through the three velocities in an interval. For example, to find the time of unloading, the velocity interpolation function in term of the velocities at the beginning, middle and end points can be calculated from Eq. (21) as follows

$$\dot{U}(\zeta) = \dot{U}_1\Psi_1(\zeta) + \dot{U}_2\Psi_2(\zeta) + \dot{U}_3\Psi_3(\zeta) \tag{21}$$

Where $\dot{U}(\zeta)$ is the velocity at time ζ, and ζ is the time measured from the beginning of the time interval. The values $\dot{U}_1, \dot{U}_2, \dot{U}_3$ are the velocities at the beginning, middle and end points of each time interval. The functions Ψ_1, Ψ_2, Ψ_3 are Lagrange interpolate functions as expressed by Eq. (22) to (24).

$$\Psi_1(\zeta) = (2\zeta^2 - 3\zeta h + h^2)/h^2 \tag{22}$$

$$\Psi_2(\zeta) = (4\zeta h - 4\zeta^2)/h^2 \tag{23}$$

$$\Psi_3(\zeta) = (2\zeta^2 - \zeta h)/h^2 \tag{24}$$

Where h is the time increment. Then, we determine the time of unloading by setting $\dot{U}(\zeta) = 0$.

An improved integration scheme with one intermediate point as Successive Symmetrical Quadratures (SSQ) method provides all the information needed for parabolic interpolations (Chen and Robinson 1993).

The SSQ method is a part of predictor-corrector methods and permits well accurate calculation of the time history of dynamical system, even for very long time intervals. This method, in reality, is a generalization of the Newmark method for $\beta = 1/4$. For a single intermediate point at the centre of the interval, we can calculate the time integration of function $f(t)$ by passing a parabola through the three points (beginning, middle and end points) through following equations

$$\int_{tn0}^{tn1} f(t)dt = h(f_0 a_{01} + f_1 a_{11} + f_2 a_{21}) \tag{25}$$

$$\int_{tn0}^{tn2} f(t)dt = h(f_0 a_{02} + f_1 a_{12} + f_2 a_{22}) \quad (26)$$

Where t_{n0} is the time of beginning point of each interval, t_{n1} is the time of middle point and t_{n2} is the time of end point. The values $f_i = f(t_{ni})$, h is time step and the a's are dimensionless weights as following

$$a_{01} = 5/24,\ a_{11} = 8/24,\quad a_{21} = -1/24,\quad a_{02} = 1/6,\quad a_{12} = 4/6,\quad a_{22} = 1/6 \quad (27)$$

The method is unconditionally stable and conserves energy exactly (Healey and Robinson 1984).

4.3 Presence of modes with very high frequencies

In the step by step integration method, the presence of modes with very high frequencies often causes severe numerical problems. The time intervals should be small enough to avoid these numerical problems. General sources of modes with very high frequency are degrees of freedom with very small masses or degrees of freedom with very large stiffness. The masses associated with rotational degrees of freedom and the stiffnesses related with columns extensions are examples of additional degrees of freedom.

This limitation is handled by taking advantage of a new type of perturbation procedure which is an important improvement compared to the conventional method of static condensation. The method allows using the large number of degrees of freedom, but treats most of them as non-dynamical variables. The additional degrees of freedom as the small rotational masses and the column extensions can be removed by using the perturbation procedure, and very high frequency modes are not introduced (Robinson and Chen 1993).

In the perturbation procedure we assume that the response of the full degrees of freedom (FDOF) system is function of ε as shown in Eq. (28)

$$U(t, \varepsilon) = U^{(0)} + U^{(1)}\varepsilon + U^{(2)}\varepsilon^2 + \dots \quad (28)$$

Where ε is a small parameter (perturbation parameter) can be determined by the material properties and the geometry of the structure. The displacement vector $U(t, \varepsilon)$ for FDOF system can be partitioned to translational displacements and extra displacements (rotation of small masses and column extensions). The masses and stiffnesses of extra degrees of freedom can also be expressed in terms of a parameter ε as shown in Eq. (29) and Eq. (30).

$$k = \varepsilon^{-r} k^* \quad (29)$$

$$m = \varepsilon^{s} m^* \quad (30)$$

Where k is a large stiffness related to column extension and m is small mass related to rotational degree of freedom. The k^* and m^* are stiffness and mass of translational degree of freedom. The r and s are chosen so that k^* and m^* are of the order of the most important stiffnesses and masses for the lower modes. In this research r and s are selected as $r = 1$ and $s = 2$.

Furthermore, the mass and stiffness matrices for the FDOF system $(M(\varepsilon), K(\varepsilon))$ can be partitioned as below

$$M(\varepsilon) = M^{(0)} + \varepsilon^2 M^{(2)} \quad (31)$$

$$K(\varepsilon) = K^{(0)} + \varepsilon^{-1}K^{(-1)} \tag{32}$$

Where $M^{(0)}$ and $K^{(0)}$ are mass matrix and stiffness matrix of the system without considering extra degrees of freedom, $M^{(2)}$ and $K^{(-1)}$ are the mass and stiffness matrices associated to extra degree of freedom. Then, extra degrees of freedom such as column extensions and rotation of joints with small masses are eliminated by a generalization of the method of static condensation (Robinson and Chen 1993).

5. Improved nonlinear time-history analysis by integrated displacement method

Two factors are very important in the nonlinear time-history analysis of structures: modelling of material nonlinearity and numerical method of integration. In the steel structures the main sources of nonlinearity are often connection behaviour and material yielding. These two sources are located in the joints or at the ends of the members. Combined effects of the connection nonlinearity and the member yielding are possible only if the connection moment capacity is larger than the initial yield moment of the connected member. In the partially restrained steel frames, the ultimate moment capacity of the connection is usually less than the initial yielding moment of the connected member. Hence, member yielding does not exist and moments at the joints are bonded by the moment capacity of the connections (Sekulovic and Nefovska 2008). Consequently in the present study, the nonlinear behaviour of the flexible steel frames is concentrated on the connections (section-2). The numerical integration method is based on the integrated equations of motion and elimination the limits on the size of time intervals. Therefore, combination of the connection nonlinearity with elimination the limits on the size of time intervals (secion-4), organizes the improved nonlinear time-history analysis. A computer program has been developed to fulfil this purpose. This program analyzes 2D semi-rigid steel frames with nonlinear connection behaviour and permits the use of fairly long time intervals while preserves high accuracy. The time saving and considering the real behaviour of the flexible connections in the dynamic analysis are practical advantages of the proposed method.

6. Numerical calculations

Some examples are solved to illustrate the proposed method; these examples show both the applicability of the proposed method in the selection of large time intervals, and the importance of the consideration of nonlinear behaviour of the semi-rigid connections in the time-history analysis of steel frames.

6.1 One storey one bay steel frame with linear-rigid joints

A one storey portal rigid frame as shown in Fig. 2 is considered to show the capability of the proposed method to increase the time intervals. The properties of the columns and beam are given in Table 1. The elastic modulus of steel is $E = 2.1 \times 10^{11}$ N/m^2 and constant coefficients of Eq. (11) for calculating the damping matrix are $\alpha = 0.30$ and $\beta = 0.005$. The materials are linear and elastic

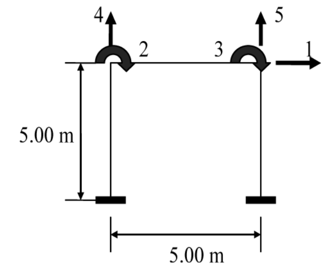

Fig. 2 One storey steel fram

Table 1 Member properties of one story rigid steel frame (section 6.1)

Members	Area (Cm2)	Moment of Inertia (Cm4)	Mass per length (Kg/m)
Columns	65	3830	52
Beam	28	1940	2000

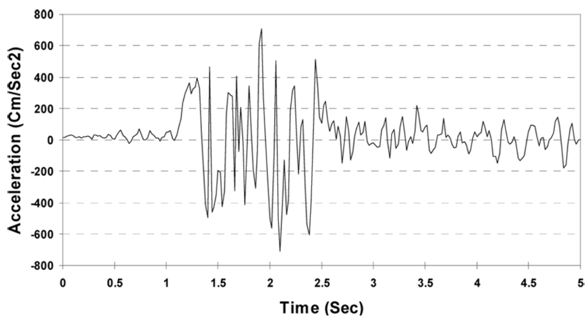

Fig. 3 Naghan earthquake record (Iran 1977)

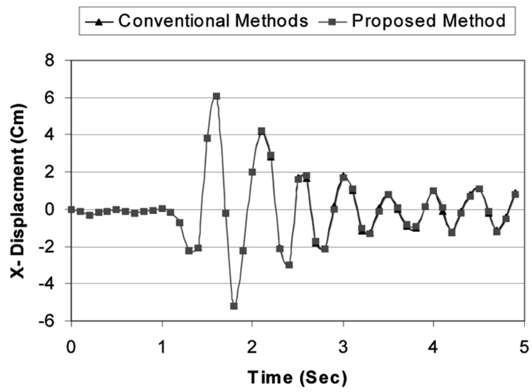

Fig. 4 Comparison of the proposed method with the conventional methods

and the beam-column connections are considered to be fully-rigid with linear behaviour. The natural periods of this frame are: $T1 = 0.499$ Sec, $T2 = 0.163$, $T3 = 0.133$, $T4 = 0.0195$, $T5 = 0.0195$.

In conventional methods, the time intervals should be selected based on the smallest period ($T5 = 0.0195$ Sec, $\Delta t \approx 0.1T \approx 0.002$). But in the proposed method it is selected based on the period of basic modes (in this example $T1 = 0.499$ Sec).

Then, by omitting the modes with high frequency (by perturbation procedure), the time intervals can be increased at least by the coefficient $T1/T5 = 25$. In addition, by using the integrated equations together the SSQ integration technique, the time intervals can be increased again. The Naghan earthquake (Iran, Naghan 1977) as shown in the Fig. 3 is considered as ground acceleration. The response of this frame is given in Fig. 4. Two different analyses; the first by the conventional methods with $\Delta t = 0.002$ Sec, and the other by the proposed method with $\Delta t = 0.1$ Sec are compared in this figure. The results show the accuracy of the proposed method. Also the computational time in the proposed method is about 10% of the conventional methods.

6.2 One storey one bay steel frame with nonlinear-flexible joints

The frame shown in Fig. 2 is considered again as second example. The properties of the members are given in Table 2. The beam-column connections are made of top and seat angles with double web angles (TSDWA). All of the angles are $L120 \times 120 \times 12$ mm. This type of connection behaves as semi-rigid connection. The Kishi and Chen's power model (See Eq. (3)) can be used for describing the nonlinear behaviour of this connection. The connection parameters for this model are:

$$K_0 = 5.871 \times 10^7 \text{ N.m/rad}, \ M_u = 1.02 \times 10^5 \text{ N.m}, \ n = 0.827$$

The response of this frame for two cases, rigid frame with linear behaviour and semi-rigid frame with nonlinear behaviour, subjected to Naghan earthquake, are shown in Fig. 5. This figure shows that, the maximum displacement of the nonlinear-flexible frame is about 2.6 times of the maximum displacement of the linear-rigid frame. Furthermore, the bending moments at the beam end for two different cases are compared in Fig. 6. This figure shows that the maximum moment of the nonlinear-flexible frame is about 0.68 times of the maximum moment of the linear-rigid case. In other words, in the flexible frames, the moments at joints are limited by connections moment capacity, and nonlinear behaviour of the structure is controlled by connection behaviour.

6.3 Two storey one bay steel frame with nonlinear-flexible joints

A two storey steel frame, as shown in Fig. 7, is assumed to be under Naghan earthquake. The member properties are given in Table 3. The beam B1 is connected to the columns by the TSDWA

Table 2 Member properties of one story partially restrained steel frame (section 6.2)

Members	Section type	Area (Cm2)	Moment of Inertia (Cm4)	Mass per length (Kg/m)
Columns	IPB 220	91	8090	71.5
Beam	IPE 270	45.9	5790	2000

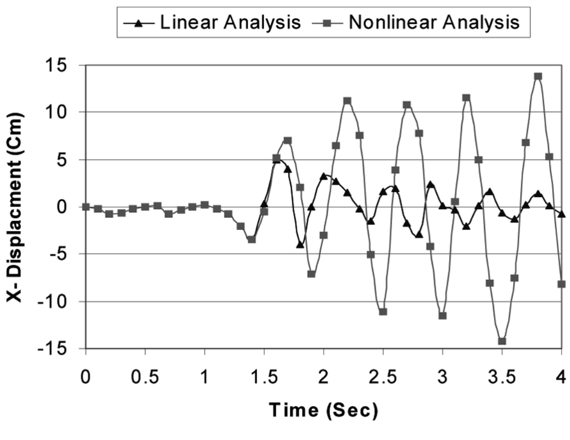

Fig. 5 One storey steel frame, comparison of drift in linear and nonlinear analysis

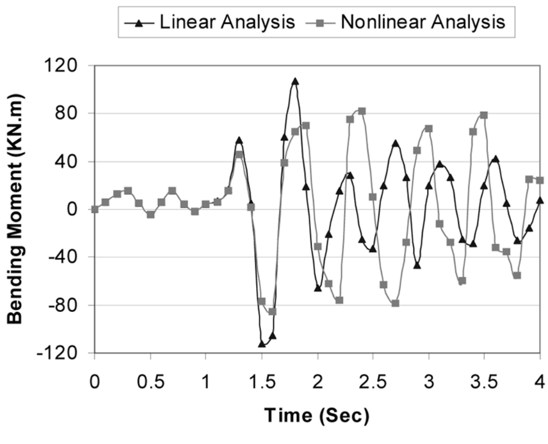

Fig. 6 One storey steel frame, comparison of beam bending moments in linear and nonlinear analysis

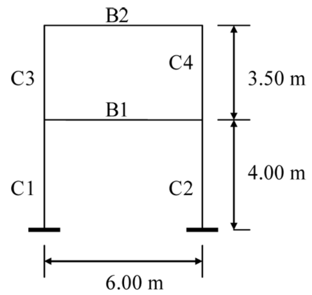

Fig. 7 Two storey steel frame

connection with angles: $L120 \times 120 \times 12$ mm. The Kishi and Chen's parameters for this connection are:

$$K_0 = 5.871 \times 10^7 \text{ N.m/rad}, \; M_u = 1.02 \times 10^5 \text{ N.m}, \; n = 0.827$$

The beam B2 is connected to the columns by the TSDWA connection with angles: $L100 \times 100 \times 10$ mm and parameters as:

$$K_0 = 2.03 \times 10^7 \text{ N.m/rad}, \; M_u = 0.649 \times 10^5 \text{ N.m}, \; n = 1.143$$

The results of analysis of linear-rigid frame and nonlinear-flexible frame are given in Table 4. This table shows that the storey drift of nonlinear analysis is about 1.40 times of the storey drift of linear analysis. But ratio of the beam moments for two cases is about 0.40. From this example, it can be concluded that the flexibly connected frames under ground acceleration, absorb great part of input energy through the connections, and consequently, damage of beam members will be occurred after the connections (damage of beams can be almost negligible). On the contrary, the linear-rigid frames absorb great part of input energy through the members, and consequently experience considerable damage at the beam ends. Hence, by modelling the actual behaviour of the structure in the nonlinear analysis, the realistic response of the structure is evaluated.

Table 3 Member properties of two storey frame

Members	Section type	Area (Cm2)	Moment of Inertia (Cm4)	Mass per length (Kg/m)
C1, C2	IPB 240	106.0	11260	84
C3, C4	IPB 220	91.0	8090	72
B1	IPE270	45.9	5790	4000
B2	IPE270	45.9	5790	3000

Table 4 Comparison of linear and nonlinear analysis of two storey frame

Type of analysis	Maximum drift of first storey (Cm)	Maximum drift of second storey (Cm)	Maximum moment of beam B1 (KN.m)	Maximum moment of beam B2 (KN.m)
Rigid-frame linear (L)	5.7	11	190	160
Flexible-frame nonlinear (NL)	7.2	17	83	52
Ratio of results (NL/L)	1.2	1.5	0.44	0.32

7. Conclusions

In this paper, a simple and practical method is proposed to consider the nonlinear behaviour of the flexible connections in the time-history analysis of steel frames. The flexible connections are modelled as rotational springs attached at the beam ends, and the nonlinear behaviour of the steel frames is concentrated on these connections. By using the integrated equations of motion and integrated displacements as new variables, the improved nonlinear time-history analysis as proposed in this paper permits the use of large time intervals while preserves high accuracy. The numerical analyzes illustrate the potential of the proposed method to model the nonlinear behaviour of the connections. On the basis of the theoretical considerations and results of the numerical examples, it is evident that connection flexibility significantly influences dynamic response of steel frames. Also the flexible connections usually increase the lateral displacements of the frame but they reduce the bending moments in the members. Thus, the more realistic nonlinear behaviour of flexible connections should be considered in the time-history analysis of the steel frames.

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