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Evaluation of accidental eccentricity for buildings by artificial neural networks

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Abstract. In seismic analyses of structures, additional eccentricity is introduced to take account for oscillations of random and unknown origins. In many codes of practice, the torsion about the vertical axis is considered through empirical accidental eccentricity formulation. Due to the random nature of structural systems, it is very difficult to evaluate the accidental eccentricity in a deterministic way and to specify its effect on the overall seismic response of structures. The aim of this study is to develop a procedure for the evaluation of the accidental eccentricity induced by uncertainties in stiffness and mass of structural members, using the neural network techniques coupled with Monte Carlo simulations. This method gives very interesting results for single story structures. For real structures, this method can be used as a tool to determine the accidental eccentricity in the seismic vulnerability studies of buildings.

Keywords: seismic design; accidental eccentricity; neural networks; Monte Carlo; uncertainty

1. Introduction

In order to take into consideration the torsion effects in buildings under seismic loading, the codes of practice introduce, in addition to lateral loads, two torsional moments due to calculated and accidental eccentricities, respectively. The aim of introducing the accidental eccentricity is to take account for all uncertainties and errors related to: the geometrical and mechanical behaviour of structural members, the random failure of non structural members, the spatial variability of dead loads, the unfavourable distribution of live loads, the torsional vibrations induced by rotational motion of the foundations, and other sources of torsion not explicitly considered in the analysis (Fahjan *et al.* 2006). According to seismic codes, this accidental eccentricity is arbitrary taken as 5% or 10% of the building dimension perpendicular to the direction of ground motion (Lin *et al.* 2001). Many studies considered the effects of accidental eccentricity on the response of single-story buildings. Based on statistical assumptions, it has been shown that the uncertainty in the location of the center of mass and in the stiffness of structural members, represent more than 70% of the total

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increase in the response due to accidental eccentricity (De la Llera and Chopra 1996). This increase, due to the rotational motion of the foundation, may reach 40% for systems with short periods and large torsional flexibility (De la Llera and Chopra 1994a, b). On the basis of analytical expression for accidental eccentricity in this class of systems (Newmark 1969), it has been shown that the calculated accidental eccentricity may reach 7% of the dimension perpendicular to the seismic direction (De la Llera and Chopra 1994b), which is slightly greater than the specified value by many codes, that is 5% (Shakib and Tohidi 2002). In a study based on the recorded accelerations in nominally symmetric-plane buildings, a ratio between the foundation torque and shear was used to calculate the instantaneous accidental eccentricities. However, the large values of accidental eccentricity do not have sense due to the small values of foundation shear (Fischer *et al.* 2002). The work of Pekau and Guimond (1990) investigated the adequacy of code predictions for accidental eccentricity to take account for torsion induced in nominally symmetrical structures due to the variation in the elastic-plastic strength of lateral load resisting members. They proposed a new expression for accidental eccentricity allowing to take into account the variation of member's lateral strength.

In addition, researchers have shown that the results for single-story systems are also applied for a special class of multistory systems, in which the centers of mass lie on a vertical line and the stiffness matrices of the floors show a constant ratio. These results are also applicable in an approximate sense to more general class of buildings which do not fully satisfy the requirements for the special class of buildings (Humar and Kumar 1998). Further works based on probabilistic methods have studied the effect of various parameters on the structural response, without evaluating the accidental eccentricity itself (De-la Colina and Almeida 2004). It is recognized that the various models in the literature investigating the accidental eccentricity and its effect on structural response have led to inconsistencies between the obtained results (Bugeja *et al.* 1999). Therefore, the need for a comprehensive procedure is required for the assessment of real buildings.

The Neural Networks (*NN*) have shown a great capacity to solve problems with large complexity. The *NN* have the ability to learn functional relationships from the training samples and adapt them for new situations. In civil engineering, they are applied in many fields such as structural reliability analysis (Cardoso *et al.* 2008), seismic damage identification in buildings (Zapico and Gonzaléz 2006), prediction of force reduction factor of prefabricated industrial buildings (Hakan Arslan *et al*. 2007), and evaluation of existing bridges (Molina and Chou 2002). Bourahla *et al*. (2006) suggested the use of neural networks to determine the eccentricity from the structural response, but the evaluation methodology and the elaboration of representative database still require more developments for comprehensive use in engineering practice. A representative database should be based on realistic random variations of the structural properties, particularly the mass and the stiffness distributions within the structure.

In view of the uncertain nature of the accidental eccentricity, it is very difficult to assess explicitly and accurately its effect on the overall seismic response of structures. In this context, the aim of the present work is to quantify the accidental eccentricity for single-story, nominally symmetrical and unsymmetrical buildings, due to uncertainties in member rigidities and structural mass distributions, using the neural networks. In the following sections, the neural networks concept is presented. The developed procedure consists in generating a numerical database and performing the learning and the verification of the neural network model. Finally, the proposed procedure is applied to several numerical examples, in order to show their performance.

2. Analysis procedure

The main idea in this work consists in determining the accidental eccentricity due to uncertainties in structural stiffness and mass by knowing the response of the building at floor levels (i.e., displacements or accelerations), as well as the natural frequencies of the structure. To achieve this target, we proceed as following (Fig. 1):

(1) The first step consists in defining the nominal properties and the probabilistic distributions of the material density ρ and Young's modulus E. These distributions are taken as lognormal, which is usually recommended for this type of variables.

(2) For each member of the structure, Monte Carlo simulations generate random configurations which are analyzed by the mechanical model under seismic loading.

(3) For each sample, the output of the mechanical model allows us to determine the structural response (i.e., displacement or acceleration), in order to deduce the maximum value, as well as

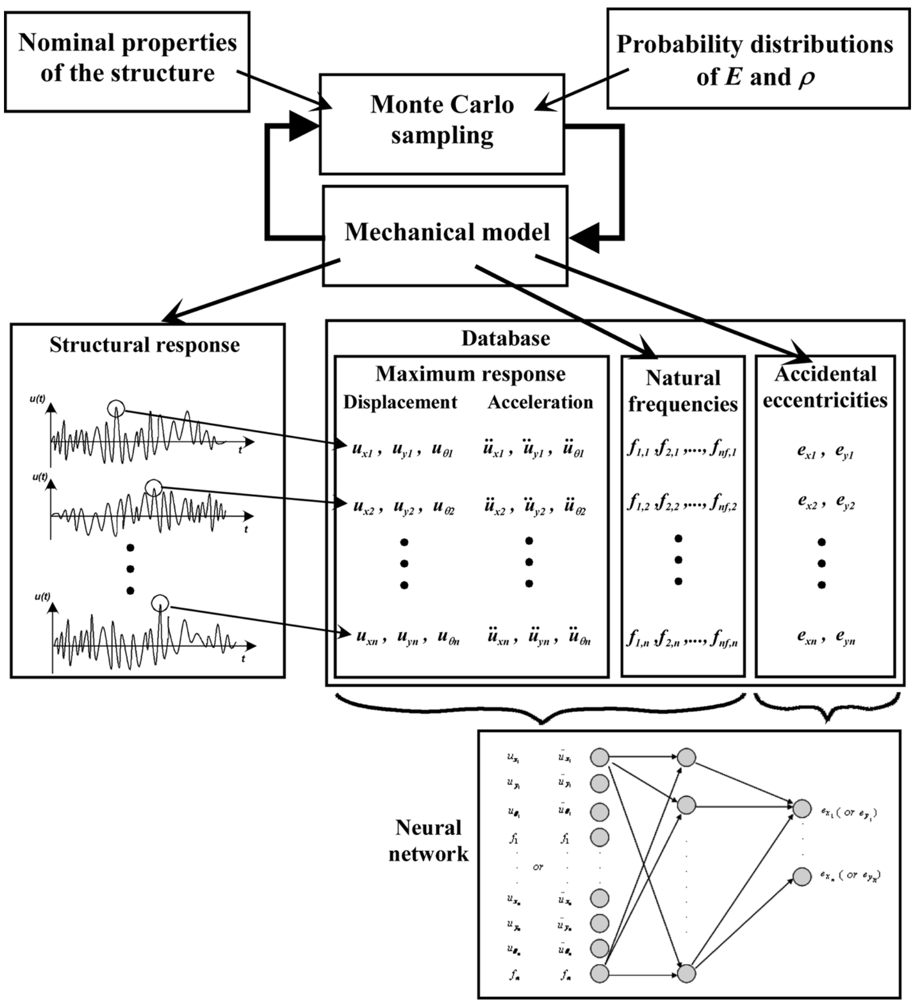


Fig. 1 Analysis procedures for building the NN

the natural frequencies. In addition, by using the mass and stiffness matrices, the mechanical model computes the eccentricities in both directions for each floor.

(4) The database is now constituted of: the sets of maximum responses, the natural frequencies, and the corresponding accidental eccentricities. This database is used for the learning and the validation of the neural network model. The input data for the *NN* are the maximum responses for each floor (i.e., either displacements u_x, u_y and u_θ, namely the translations of the centre of mass CM in *x*- and *y*-directions and the rotation about the vertical axis through CM, respectively, or the three corresponding accelerations $\ddot{u}_x, \ddot{u}_y$, and $\ddot{u}_\theta$), in addition to the natural frequencies of the structure. The output data are the corresponding accidental eccentricities.

(5) The NN learning procedure is applied in order to determine the optimal network architecture; 50% of the database is used for this purpose, the other part being used for verification and validation. When the NN is established, it can predict the real accidental eccentricity for any similar structure subjected to seismic loads or man-made vibrations, by only measuring the maximum response (i.e., displacements or accelerations).

3. Neural networks analysis

3.1 Network design

The neural networks (NN) are numerical models inspired from the architecture and functioning of human brain. They have shown their capacity to solve complex problems in various domains. A neural network is a set of calculation cells forming the nodes of a connection graph as shown in Fig. 2. In this study, we have used the feed-forward multi-layer neural network in which the neurons are distributed in layers in such a way that two consecutive layers are fully connected; all

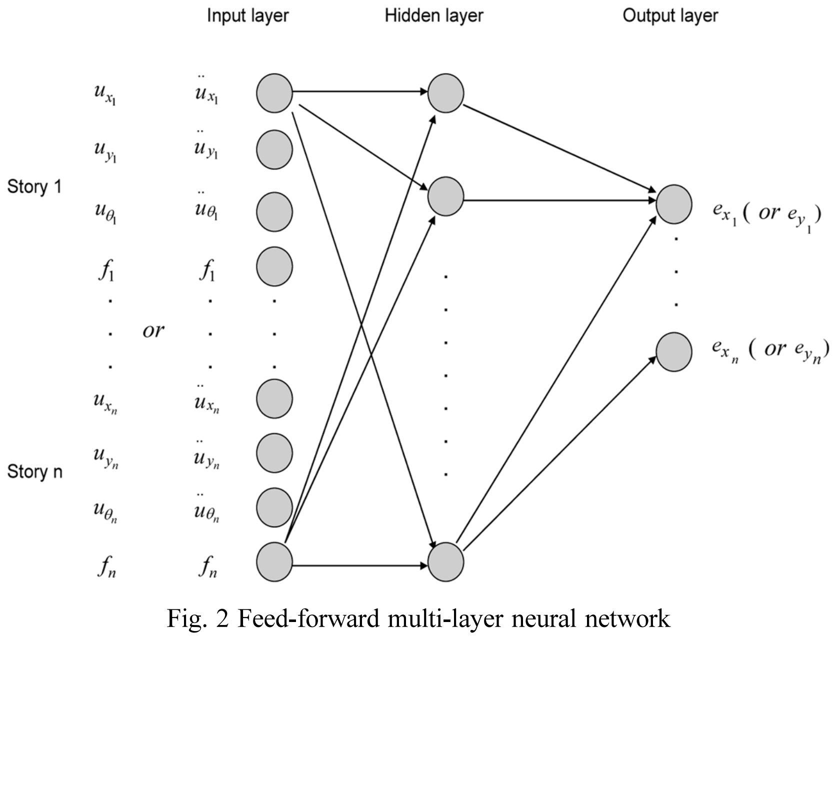


Fig. 2 Feed-forward multi-layer neural network

the neurons of an input layer receive the outputs of all neurons in the previous layer. A signal propagates from the input layer to the output layer through several hidden layers. For each set of input signals, a cell performs a weighted sum in which a transfer function is applied, and the output is transmitted to the following layer. The number of hidden layers, the number of cells per layer and their connections define the architecture of the neural network. The transfer function allowing to calculate the cell output is often a linear sigmoidal function.

For a building with N floors and n_f significant natural frequencies, the adopted neural network in this work is composed of "$3N + n_f$" neurons in the input layer, corresponding to the maximum responses (i.e., either displacements or accelerations) at the center of mass of the floors, in addition to the natural frequencies. The output layer is constituted by N neurons, corresponding to the eccentricity in each story. The neurons n_f are introduced because the use of only three displacements (or accelerations) cannot ensure uniqueness between the input and the output of the network. In other words, the same maximum displacement may correspond to different values of eccentricity. It is thus necessary to introduce more information about the structural dynamics, in order to ensure that the output corresponds to a single input set, which is mandatory for correct calibration of the *NN* parameters. For this reason, we have introduced the lowest natural frequencies in order to provide additional information concerning the mechanical behaviour of the structure.

3.2 Databases

To build the database, we choose, as input parameters, either the maximum displacements $(u_{x_i}, u_{y_i}, u_{\theta_i})$ or the maximum accelerations $(\ddot{u}_{x_i}, \ddot{u}_{y_i}, \ddot{u}_{\theta_i})$ at the mass center of each floor in the building, in addition to the first n_f natural frequencies f_k.

The output parameter corresponds to the eccentricity in the direction perpendicular to the seismic excitation. That means, two cases are considered: the eccentricity e_{x_i} in the x-direction is observed when the seismic excitation is applied in the y-direction, and the eccentricity e_{y_i} in the y-direction is observed when the seismic excitation is applied in the x-direction. In this paper, only the results for e_{x_i} are presented, as the calculation of e_{y_i} is carried out in a similar way.

To take into account the uncertainties in stiffness and mass, we have defined the Young's modulus and the density of each structural member by random sampling according to the log-normal probability density functions, with coefficients of variation of 0.14 and 0.40 respectively (De la Llera and Chopra 1994b, De-la Colina and Almeida 2004). Even for nominally symmetrical structure, the randomness of the Young's modulus and the density of each member leads to accidental eccentricities in both x- and y-directions.

3.3 Neural network training

The learning procedure aims at calibrating the weighting coefficients in the neural network in order to fit as good as possible the observed behaviour. In this study, MATLAB Neural Network Toolbox has been applied to the random sampling database described in the above sections. In order to give stable solution, the input and the output vectors have been standardized in such a way that they have a zero mean and a unit standard deviation.

The error back-propagation algorithm has been used for the training of the neural network model. Among the possible back propagation algorithms, the Levenberg-Marquardt algorithm has been chosen in this study because it provides fast convergence. The performance function used to train the neural network is the Mean Square Error (MSE) given by:

$$MSE = Mean\left(\sum_k (e_{NN_k} - e_{T_k})^2\right)$$

where e_T is the target accidental eccentricity, e_{NN} is the accidental eccentricity calculated by the NN, and k is the sample number. The data are entered simultaneously in batch mode. To avoid the over-fitting of the network, the method of early stopping is used.

The database has been divided into three parts: 50% of the records are used for the training procedure allowing to determine the weighting coefficients of the neural network, 25% of the records has been used for the validation of the obtained coefficients, allowing to provide a measure of the network generalization and to give a stopping criterion for the training when the generalization cannot be improved anymore, and the last 25% is used to verify that the generalization is correct.

4. Application to buildings

Two nominally symmetrical and unsymmetrical single-story structures are considered in this study. These structures have the geometrical configurations shown in Figs. 3(a) and 3(b), respectively. The structures are subjected to seismic excitation of Morgan Hill signal. The dimensions in the plan are defined by: $a = 6$ m, $b = 12$ m and the story height is 3 m. The total dead and live loads are represented by an equivalent slab thickness of 0.60 m for the floor. The column cross-sections have the dimensions of 0.25×0.25 m. The mean density ρ, the Young's modulus E and the Poisson's ratio v are respectively 2400 kg/m^3, 2.7×10^{10} N/m^2 and 0.2. The critical damping rate is equal to 0.05.

4.1 Network architecture

The choice of the network architecture is very important, as it affects both the model precision and the computing time. It is not obvious to say that complex networks with many cells and layers lead to better results for the whole range of application of the model, as numerical noise and local instability may be observed. In order to determine the optimal architecture, we have considered

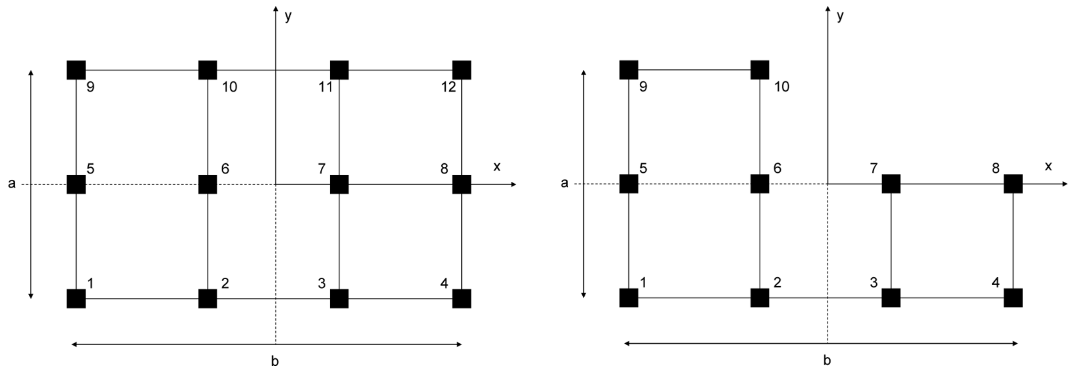


Fig. 3 (a) Nominally symmetric-plan system, (b) unsymmetric-plan system

Table 1 Mean square error (MSE) of different neural architectures

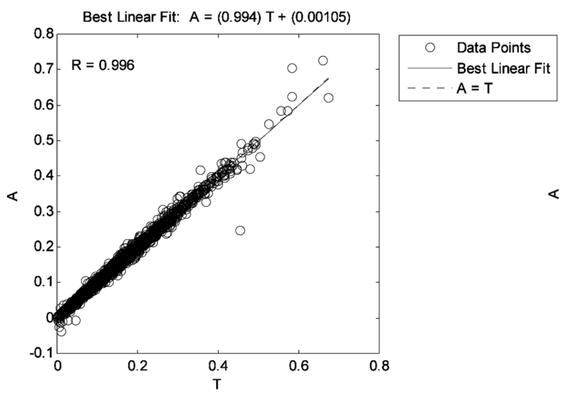
Syst.	Freq. in input	NN Model	Mean square error (MSE). Recorded displacements		$\varepsilon \leq 1\%$	Mean square error (MSE). Recorded accelerations		$\varepsilon \leq 1\%$
			Training	Testing		Training	Testing	
1 story Sym.	f_1	4-4-1	0.013404	0.017027	10.45%	0.012694	0.017471	11.05%
		4-12-1	0.006945	0.017027	11.75%	0.010578	0.017693	11.20%
		4-22-1	**0.005823**	**0.013360**	**16.70%**	**0.008023**	**0.017157**	**10.70%**
		4-32-1	0.003398	0.018809	15.80%	0.007804	0.023308	11.35%
	f_1-f_3	6-4-1	9.87×10^{-5}	0.000293	74.50%	0.000108	0.000197	77.95%
		6-12-1	9.51×10^{-5}	0.000346	77.50%	8.83×10^{-5}	0.000143	80.60%
		6-22-1	$\mathbf{6.44 \times 10^{-5}}$	**0.000158**	**80.40%**	$\mathbf{7.20 \times 10^{-5}}$	**0.000604**	**80.60%**
		6-32-1	4.77×10^{-5}	0.000154	82.90%	3.24×10^{-5}	0.000337	87.15%
	f_1-f_6	9-4-1	9.91×10^{-5}	0.000306	76.25%	9.82×10^{-5}	0.000161	76.60%
		9-12-1	9.21×10^{-5}	0.000216	78.35%	4.96×10^{-5}	0.000129	79.60%
		9-22-1	$\mathbf{4.93 \times 10^{-5}}$	**0.000095**	**86.90%**	$\mathbf{2.48 \times 10^{-5}}$	**0.000094**	**82.75%**
		9-32-1	7.79×10^{-5}	0.000237	79.40%	4.27×10^{-5}	0.000138	81.70%
1 story Unsym	f_1	4-4-1	0.024710	0.021387	09.60%	0.019191	0.019573	08.60%
		4-12-1	0.020807	0.021935	08.90%	0.019238	0.020883	08.55%
		4-22-1	**0.020554**	**0.023709**	**08.75%**	**0.017988**	**0.021777**	**08.30%**
		4-32-1	0.016889	0.023489	07.65%	0.015889	0.023945	07.55%
	f_1-f_3	6-4-1	0.000251	0.000313	43.60%	0.000300	0.000303	40.40%
		6-12-1	9.90×10^{-5}	0.000146	52.95%	9.81×10^{-5}	0.000153	54.35%
		6-22-1	$\mathbf{5.96 \times 10^{-5}}$	**0.000147**	**58.95%**	$\mathbf{6.77 \times 10^{-5}}$	**0.000137**	**59.55%**
		6-32-1	8.11×10^{-5}	0.000122	57.65%	7.35×10^{-5}	0.000137	57.45%
	f_1-f_6	9-4-1	0.000223	0.000293	43.60	0.000464	0.000639	32.35%
		9-12-1	8.41×10^{-5}	0.000126	57.30	9.95×10^{-5}	0.000188	52.40%
		9-22-1	$\mathbf{6.41 \times 10^{-5}}$	**0.000116**	**59.30**	$\mathbf{8.85 \times 10^{-5}}$	**0.000167**	**55.40%**
		9-32-1	5.69×10^{-5}	0.000150	59.40	9.17×10^{-5}	0.000180	55.95%

various numbers of neurons in the hidden layer. For comparison purpose, the same set of input data is kept unchanged, while the training is performed on different network architectures for single-story symmetrical and unsymmetrical buildings shown in Fig. 3. For this purpose, we have considered the number of neurons in the hidden layer to be equal to 4, 12, 22 and 32; the network structure is noted: n_1-n_2-n_3, where n_k is the number of neurons in layer k. For the input layer, the number of neurons varies with the number of the considered natural frequencies; three cases are considered: 1) only the first frequency (f_1), 2) the three first frequencies (f_1- f_3) and 3) all the six frequencies (f_1- f_6).

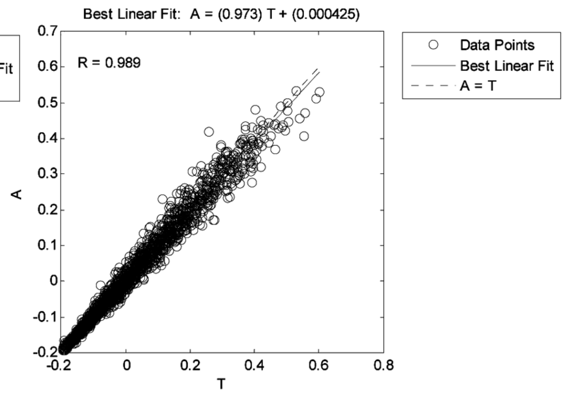
For each network architecture and input frequencies, Table 1 gives the mean square errors (MSE) and the percentage of the population with absolute relative error (ARE) $\varepsilon_k = |e_{NN_k} - e_{T_k}/e_{T_k}|$ less than 1%, according to the maximum response. For example, the configuration 4-22-1 expresses a neural network of 4 neurons in the input layer, 22 neurons in the hidden layer and 1 neuron in the output layer. The threshold of 1% for the ARE ensures high precision of the network architecture. The percentage of records with low errors is an interesting criterion to measure the goodness of fit. The number of records is chosen to be 1000 for training, 500 for validation and 500 for verification. The acceptance criterion for the mean square error is set to 10^{-5}.

The results in Table 1 show that the neural networks 4-22-1, 6-22-1 and 9-22-1 represent the best configurations in terms of data fitting. It can be observed that the use of the first natural frequency alone cannot ensure high precision of the network predictions. However, the use of at least three frequencies improves largely the *NN* performance. When the six frequencies are used, the percentage of closely fitted records (relative error less than 1%) is 86.9% for the symmetrical building and 59.3% for the unsymmetrical building, instead of 16.7% and 8.75% when only the first frequency is used. The precision for unsymmetrical buildings is much lower than for symmetrical ones because of the torsional modes of vibrations.

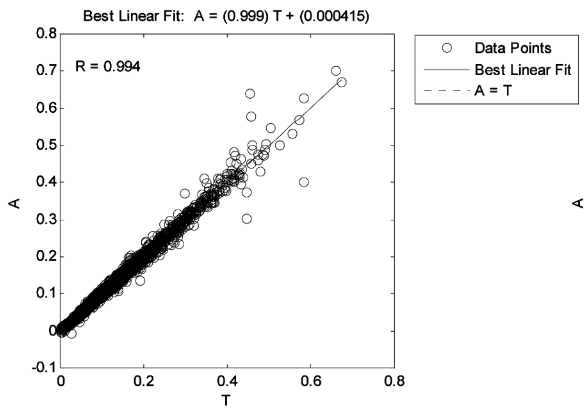
It is also observed that the use of the maximum accelerations as input is globally less precise than the use of the maximum displacements (except for the case f_1-f_3); however, the use of accelerations still gives good results and can be valid for practical use of the method, which is interesting for measurements on real buildings.



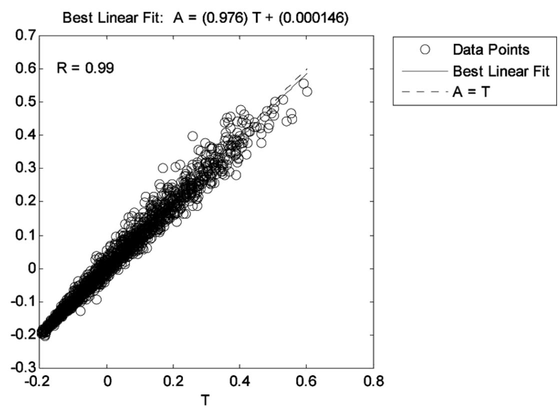
(a) Maximum displacements considered in input for symmetrical structure



(b) Maximum displacements considered in input for unsymmetrical structure



(c) Maximum accelerations considered in input for symmetrical structure



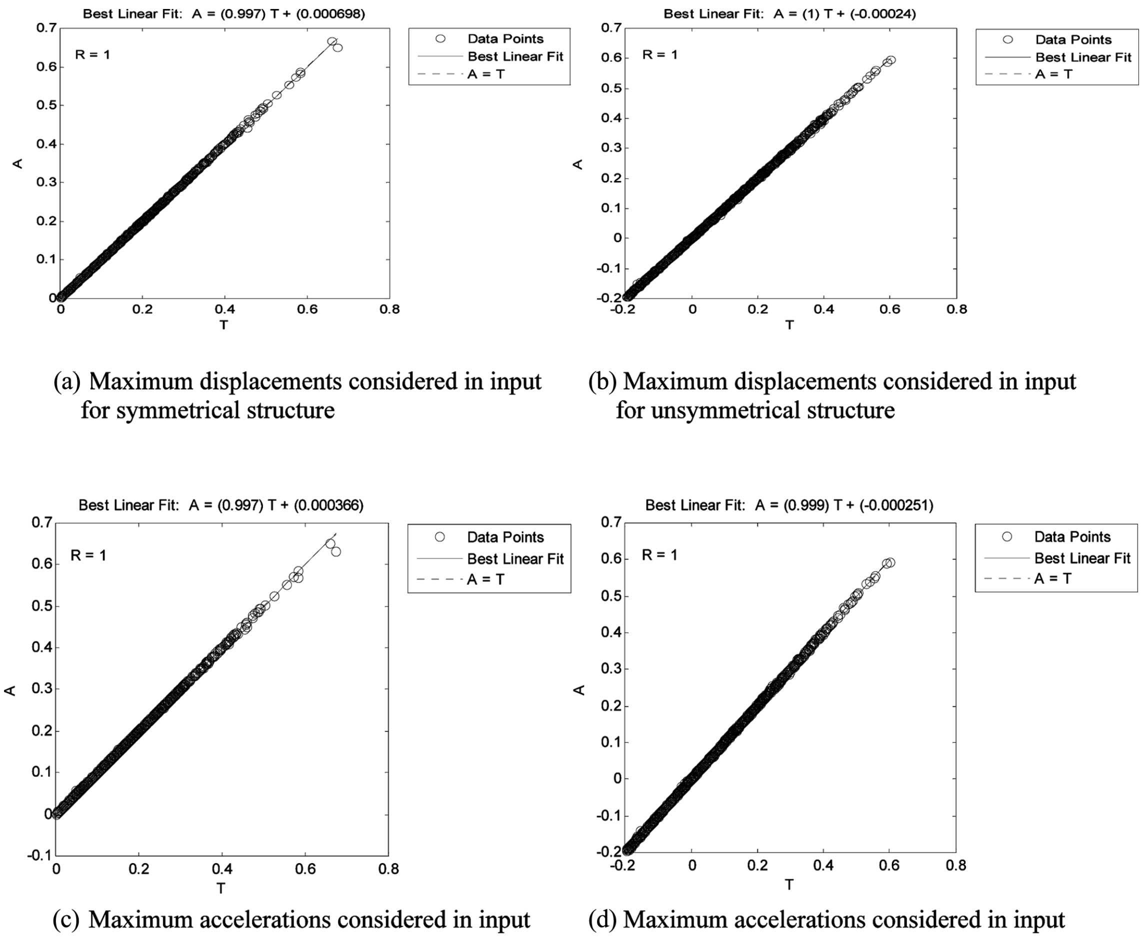
(d) Maximum accelerations considered in input for unsymmetrical structure

Fig. 4 Neural networks versus target eccentricities for single input frequency

4.1.1 Correlation analysis

In order to verify the quality of the selected network (with 22 cells in the hidden layer), the entire set of data (i.e., data used for learning, validation and testing) has been passed through the network to perform linear regression between the network outputs A and the corresponding targets T. The correlation coefficient R allows us to measure the quality of the network prediction; a perfect prediction suggests that all the points are aligned along the diagonal $A = T$ and the correlation coefficient is $R = 1$.

For these structures (Figs. 4 and 5), the fitting lines are practically superposed with the diagonal, and the correlation coefficient is very close to unity, which means that the neural network gives very accurate predictions of the accidental eccentricities. When only one frequency is used for network training, the unsymmetrical structure presents more dispersion than the symmetrical one. It is also noted that the number of input natural frequencies affects significantly the accuracy of the network predictions, regardless the structure geometry and the nature of records considered in the input (maximum displacements or accelerations).



(a) Maximum displacements considered in input for symmetrical structure

(b) Maximum displacements considered in input for unsymmetrical structure

(c) Maximum accelerations considered in input for symmetrical structure

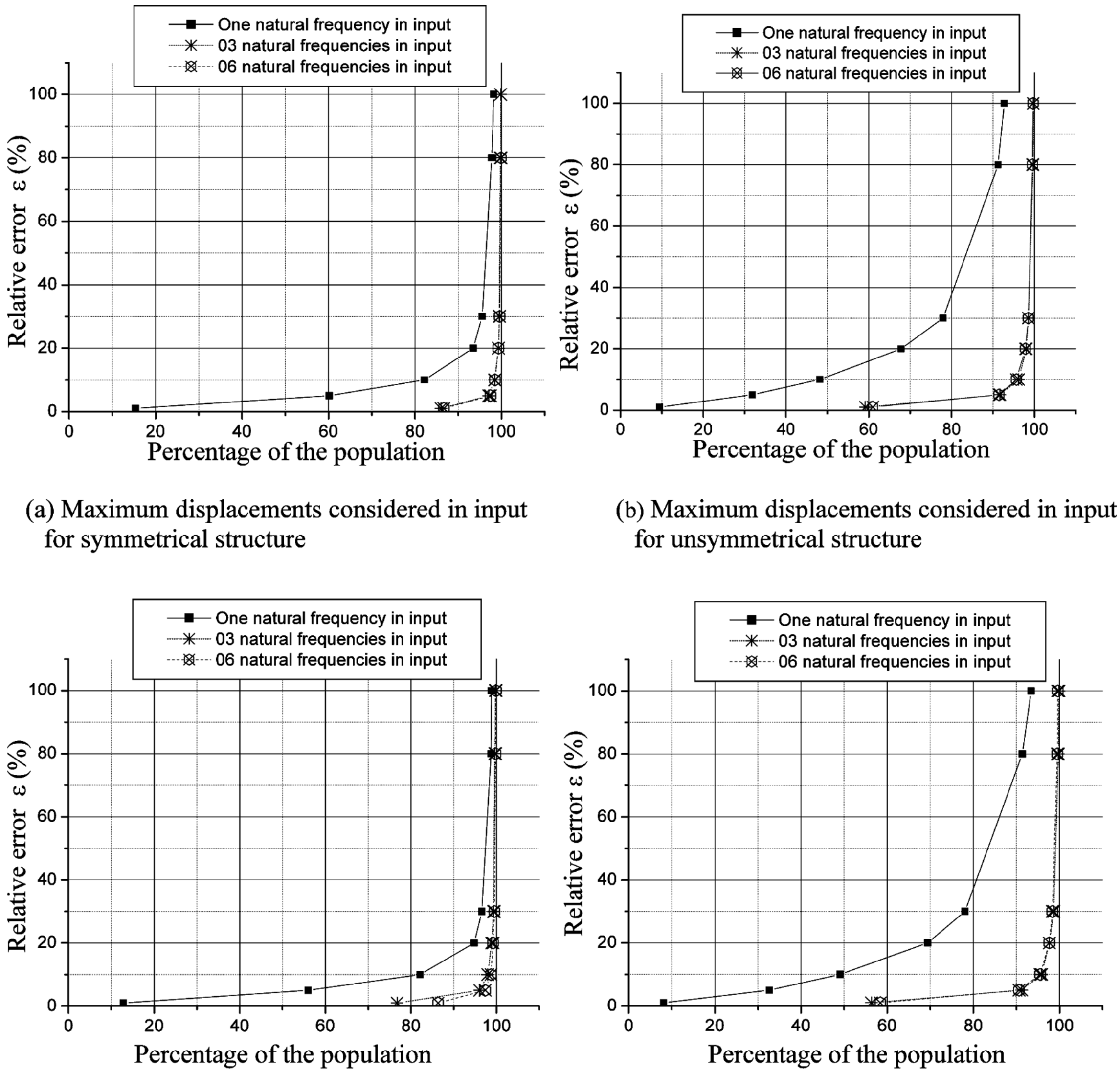
(d) Maximum accelerations considered in input for unsymmetrical structure

Fig. 5 Neural networks versus target eccentricities for three natural frequencies

4.1.2 Error distribution

For both symmetrical and unsymmetrical buildings, Fig. 6 gives the Pareto plots for the absolute relative error (ARE) in terms of the percentage of the records with lower errors. For all cases, more than 95% of accidental eccentricities calculated by NN do not exceed an error rate of 30%, except for the case of unsymmetric building with only one input frequency. When several input frequencies are considered, the number of predicted accidental eccentricities with ARE lower than 5% is 95% for symmetrical building and 90% for unsymmetrical one.

We can therefore conclude that for systems with single-story, symmetrical or unsymmetrical,



(a) Maximum displacements considered in input for symmetrical structure

(b) Maximum displacements considered in input for unsymmetrical structure

(c) Maximum accelerations considered in input for symmetrical structure

(d) Maximum accelerations considered in input for unsymmetrical structure

Fig. 6 Relative error in terms of the record percentage whether displacement or acceleration records are used, the neural network model gives very precise results. The predictive capacity of the neural network model can be controlled and depends on the number of the input parameters. By increasing the number of input frequencies, we can ensure more accurate predictions of the eccentricities (i.e., with errors less than 5%) in more than 90% of the situations.

5. Conclusions

This investigation puts forward a new method for the evaluation of the accidental eccentricity in buildings, due to uncertainties in stiffness and mass properties. This method is based on neural networks training, coupled with mechanical modelling and Monte Carlo simulations. Knowing the maximum displacement or acceleration in both directions of the floor planes, in addition to few natural frequencies of the structure, a database can be generated for structures subjected to seismic excitation. This database is then used for the training of the Neural Network, in order to determine the accidental eccentricity.

According to this study, we can address the following remarks:

- The calculation of accidental eccentricities by NN for a nominally symmetrical structure is more accurate compared to unsymmetrical structure.
- The accuracy of NN accidental eccentricity increases by increasing the number of input parameters, especially the number of natural frequencies.
- The calculation of accidental eccentricities using the displacement records is more accurate than the calculation using the acceleration records. However, the difference is not significant for practical applications.
- For a nominally symmetrical single-story structure, 95% of accidental eccentricities calculated by NN have an error less than 5%. For unsymmetrical structure, this error level is satisfied for 90% of accidental eccentricities calculated by NN.

The great advantage of this method lies in its ability to determine the accidental eccentricities from the real structural responses recorded during seismic events or the ambient vibration tests for both symmetrical and unsymmetrical structures. This will serve as a tool in seismic vulnerability studies of existing buildings. This procedure can also be applied to improve the empirical formula of the accidental eccentricity recommended by the codes. Finally, the proposed method can be extended in order to evaluate the accidental eccentricity for multi-story structures.

References

Bourahla, N., Boukhamacha, T. and Tafraout, S. (2006), "Detection of the eccentricity variation in nonlinear response using artificial neural networks", *Proceedings of the First European Conference on Earthquake Engineering and Seismology*, Geneva, September.

Bugeja, M.N., Thambiratnam, D.P. and Brameld, G.H. (1999), "The influence of stiffness and strength eccentricities on the inelastic earthquake response of asymmetric structures", *Eng. Struct.*, **21**, 856-863.

Cardoso, J.B., de Almeida, J.R., Dias, J.M. and Coelho, P.G. (2008), "Structural reliability analysis using Monte Carlo simulation and neural networks", *Adv. Eng. Softw.,* **39**, 505-513.

De-la Colina, J. and Almeida, C. (2004), "Probabilistic study on accidental torsion of low-rise buildings", *Earthq. Spectra*, **20**(1), 25-41.

De la Llera, J.C. and Chopra, A.K. (1994a), "Accidental torsion in buildings due to stiffness uncertainty", *Earthq. Eng. Struct. D.,* **23**, 117-136.

De la Llera, J.C. and Chopra, A.K. (1994b), "Accidental torsion in buildings due to base rotational excitation", *Earthq. Eng. Struct. D.,* **23**, 1003-1021.

De la Llera J.C. and Chopra, A.K. (1996), "Accidental and natural torsion in earthquake response and design of buildings", *Proceedings of the Eleventh World Conference on Earthquake Engineering,* Acapulco, Mexico.

Fahjan, Y.M., Tuzun, C. and Kubin, J. (2006), "An alternative procedure for accidental eccentricity in dynamic model analyses of buildings", *Proceedings of the First European Conference on Earthquake Engineering and Seismology*, Geneva, September.

Fischer, T., Alvarez, M., De la Llera, J.C. and Riddell, R. (2002), "An integrated model for earthquake risk assesment of buildings", *Eng. Struct.,* **24**, 979-998.

Hakan Arslan, M., Murat Ceylan, M., Kaltakci, Y., Ozbay, Y. and Gulten Gulay, F. (2007), "Prediction of force reduction factor (R) of prefabricated industrial buildings using neural networks", *Sruct. Eng. Mech.*, **27**(2), 117-134.

Humar, J.L. and Kumar, P. (1998), "Torsional motion of buildings during earthquake. I. Elastic Response", *Can. J. Civ. Eng.,* **125**, 898-916.

Lin, W.H., Chopra, A.K. and De la Llera, J.C. (2001), "Accidental torsion in buildings: Analysis versus earthquake motions", *J. Struct. Eng.,* **127**(5), 475-481.

Molina, A.V. and Chou, K.C. (2002), "Evaluation of existing bridges using neural networks", *Struct. Eng. Mech.,* **13**(2), 187-209.

Newmark, N.M. (1969), "Torsion in symmetrical buildings", *Fourth World Conference on Earthquake Engineering*, Santiago, Chile.

Pekau, O.A. and Guimond, R. (1990), "Accidental torsion in yielding symmetric structures", *Eng. Struct.*, **12**(2), 98-105.

Shakib, H. and Tohidi, R.Z. (2002), "Evaluation of accidental eccentricity in buildings due to rotational component of earthquake", *J. Earthq. Eng.*, **6**(4), 431-445.

Zapico, J.L. and Gonzaléz, M.P. (2006), "Numerical simulation of a method for seismic damage identification in buildings", *Eng. Struct.*, **28**, 255-263.