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An inverse approach for the calculation of flexibility coefficient of open-side cracks in beam type structures

N. Fallah* and M. Mousavi

Civil Engineering Department, University of Guilan, P.O. Box 3756, Rasht, Iran

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Abstract. An inverse approach is presented for calculating the flexibility coefficient of open-side cracks in the cross sectional of beams. The cracked cross section is treated as a massless rotational spring which connects two segments of the beam. Based on the Euler-Bernoulli beam theory, the differential equation governing the forced vibration of each segment of the beam is written. By using a mathematical manipulation the time dependent differential equations are transformed into the static substitutes. The crack characteristics are then introduced to the solution of the differential equations via the boundary conditions. By having the time history of transverse response of an arbitrary location along the beam, the flexibility coefficient of crack is calculated. The method is applied for some cracked beams with solid rectangular cross sections and the results obtained are compared with the available data in literature. The comparison indicates that the predictions of the proposed method are in good agreement with the reported data. The procedure is quite general so as to it can be applicable for both single-side crack and double-side crack analogously. Hence, it is also applied for some test beams with double-side cracks.

Keywords: crack; forced vibration; flexibility coefficient; Euler-Bernoulli beam

1. Introduction

Cracks are one of the most important damage types in both mechanical and civil structures which can cause catastrophic failures. The existence of cracks in a structural member changes the physical characteristics of component which affects the dynamic behavior of the whole structure, e.g. its presence in a member causes a local increase in flexibility which subsequently changes the modal response of structure.

Cracks may take place in structures due to several reasons: as a result of the limited fatigue strength in a structural component, the fatigue cracks may occur due to cyclic loading, a type of crack may also be commenced during the manufacturing processes which are usually small in sizes and another group of cracks may take place due to the mechanical defects.

There are two types of surface cracks which have received attention by researchers. Breathing cracks are those which open and close during vibration of the component. The stiffness of a component having breathing cracks is most affected when it is under tension. The breathing cracks introduce non-linearity in the beam vibration.

*Corresponding author, Ph.D., E-mail: fallah@guilan.ac.ir

Cracks which are always open during the loading state are named gaping cracks or open cracks. They are also more commonly called notches. Gaping cracks are easy to simulate in a laboratory condition and hence most experimental research have been concentrated on this particular crack type.

The dynamics of cracked beam has received much attention in the past two decades due to its importance in mechanical and civil engineering applications such as bridge structures and railways, etc. There are three categories of crack simulation approaches presented in literatures which are stated as: equivalent reduced section model, the local flexibility model based on the fracture mechanics theory and the continuous crack flexibility model.

A study has been done by Friswell and Penny (2002) which compares different approaches in crack modeling. They also studied the non-linearity effects of breathing cracks on the response of a simple beam.

According to the review on various methods dealing with cracked structures by Dimarogonas (1996), only during the 10 years before that survey over 500 papers were published which shows the importance of studying the behavior of cracked members.

The first attempts to simulate the effect of notch on flexibility of the structural elements have been made by Thomson (1943) and Kirmsher (1944). They simulated the crack by a local bending moment or reduction in cross section of component with magnitudes which were evaluated by experiments.

Since 1957, several researchers computed local flexibility for a variety of crack shapes in structural member having different geometries. Using existing results from fracture mechanics Liebowitz *et al.* (1967), Liebowitz and Claus (1968) and Okamura *et al*. (1969) calculated the local rotational flexibility induced by a transverse surface crack with uniform depth in a beam with rectangular cross section. According to the above works, a beam type structure with a crack can be considered as two pinned segments in which a rotational spring placed at the crack location.

Gudmunson (1983), Rauch (1985), Chen and Wang (1986) calculated the local flexibility induced by cracks in structural members using finite element methods. Christides and Barr (1984) developed a continuous crack flexibility model which presents the stiffness reduction in a rectangular cross sectional Euler-Bernoulli beam. The model consists of an exponential function which expresses the flexural rigidity decay in terms of the distance from the crack section and contains a parameter which needs to be determined by experiments.

Zheng and Fan (2003) calculated the local flexibility coefficient of cracked hollow-sectional beams by using the fracture mechanics theory. They derived formulas for calculating the flexibility coefficient of a range of cross sectional geometries in which the derived formulas are dependant to the crack depth ratio. In the second part of the above work the derived flexibility coefficients have been applied to study the vibration and stability of cracked hollow-sectional beams. However, the authors stated that the validity of the proposed formulas depends on the future experimental works. Recently, the effect of double-sided cracks on dynamic characteristics of Timoshenko beam has been studied by Ayatollahi *et al.* (2010).

In the present study, a new approach for determining the local flexibility coefficient of an open crack with different depth is presented. In this procedure two models of a cracked beam are incorporated. Firstly a mathematical model of a cracked beam which relies on Euler-Bernoulli beam theory is used. Secondly a finite element model of a simply supported beam having a V-shaped open surface crack is used for obtaining the output data due to the excitation on the cracked beam.

In order to develop the approach a simply supported beam with an open crack subjected to a sinusoidal concentrated force is considered. It is assumed that the crack is open during the vibration and also a linearly elastic assumption has been made to develop the approach. In the first part of the approach, the cracked beam has been divided into two segments which are connected together by hinges and a rotational spring placed at the crack location. By applying the Euler-Bernoulli beam theory, the governing differential equation of each part is written and then by integrating the differential equation of forced vibration of the beam due to the time, the dynamic problem transformed to the static one. For calculating the crack flexibility coefficient, the time history of transverse displacement of an arbitrary location along the beam is needed to be measured. In the absence of such measurement one can use a finite element model of the cracked beam and obtain the necessary time history response. Hence, in the second part of the approach an in-house finite element software is applied for the simulation of the cracked beam to obtain the desired dynamic response. For finding the effect of finite element model on the performance of the proposed method, both plane stress and 3D model of the cracked beam are used. By applying the present method the local flexibility coefficient of cracked solid rectangular cross sectional beam is calculated and compared with the reported data. This comparison indicates the capability of the proposed method in good predictions. Due to the fact that the proposed method is general in essence and not limited to the single surface crack analysis, it is also applied for the beam with a double-side cracks located on top and bottom surface of the beam.

This paper is organized as follows: in section two the relevant equations regarding to the crack flexibility coefficient are presented from the fracture mechanics theory; in section three the background theory of the proposed approach is discussed. In section four the configuration of simulated cracks and the geometrical and mechanical characteristics of the beam and the applied load are illustrated. Section five presents the finite element model of the beam. In section six the results obtained for the plane stress and 3D models are compared with the reference results. Finally, in section 7 the conclusions are presented.

2. Calculation of the flexibility coefficient of an open-side crack using the fracture mechanics theory

In this section the fracture mechanics based approach for the calculation of the crack flexibility coefficient is presented briefly. Fig. 1 shows a solid rectangular cross section of a beam with a single-side V-shaped open crack having uniform penetration depth a along the width b. The crack surface area is $a \times b$ which is assumed to be constant, in other words the open crack assumption is

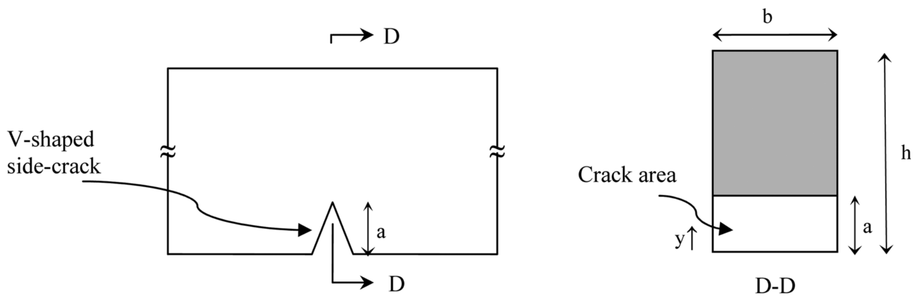

Fig. 1 A V-shaped one sided sectional crack in a rectangular sectional of a beam

applied. According to Paris equation (Tada *et al*. 2000), the relative rotation of the cracked section shown in Fig. 1, carrying bending moment M, can be expressed as

$$\theta = \frac{\partial}{\partial M}\int_0^a bG dy \tag{1}$$

where G is the strain energy release rate function during the crack extension (Anderson 1995) and other parameters of Eq. (1) are shown in Fig. 1. The flexibility coefficient of the cracked section, the reciprocal of the stiffness coefficient, can be obtained as

$$C = \frac{\partial \theta}{\partial M} \tag{2}$$

Substituting Eq. (1) into Eq. (2) gives

$$C = \frac{\partial^2}{\partial M^2}\int_0^a bG dy \tag{3}$$

By expressing the energy release rate function in terms of the stress intensity factor (Anderson, 1995) one can obtain the following relation

$$C = \frac{72\pi}{E'bh^2}\int_0^{a/h} xF^2 dx \tag{4}$$

where $E' = E$ for plane stress problem and $E' = E/(1-\nu^2)$ for plane strain problem, in which E is the Young's modulus and ν is Poisson's ratio.

The parameter F is given as

$$F = (2/\pi x \tan \pi x/2)^{0.5}\left[\frac{0.923 + 0.199(1-\sin \pi x/2)^4}{\cos \pi x/2}\right] \tag{5}$$

in which $x = y/h$.

Zheng and Fan (2003) have computed Eq. (4) numerically and then they used the least-squares technique to find the best fitted expression which resulted in the following formula

$$\begin{aligned} CE'bh^2 = e^{1/(1-\zeta)}(&-0.2314\times 10^{-4}\zeta + 52.3790\zeta^2 - 130.2463\zeta^3 + 308.4111\zeta^4 \\ &-602.1761\zeta^5 + 937.6805\zeta^6 - 1306.7397\zeta^7 + 1398.7523\zeta^8 \\ &-1059.6215\zeta^9 + 388.1628\zeta^{10}), \quad 0 \le \zeta \le 0.5, \quad \zeta = a/h \end{aligned} \tag{6}$$

As can be seen in Eq. (6), the flexibility coefficient is dependent on crack extent and beam's cross-sectional geometries and also beam's mechanical properties.

3. Calculation of crack flexibility coefficient by using beam forced vibration data

In this section an inverse approach for the calculation of the crack flexibility coefficient is presented. The presented approach with similarities has been already applied for the identification of model parameters by Langer and Ruta (1995), Ruta and Szydło (2005), Sieniawska *et al.* (2009) and Jarczewska *et al.* (2011).

Assume that a simply supported beam with length L and rectangular cross section with dimension

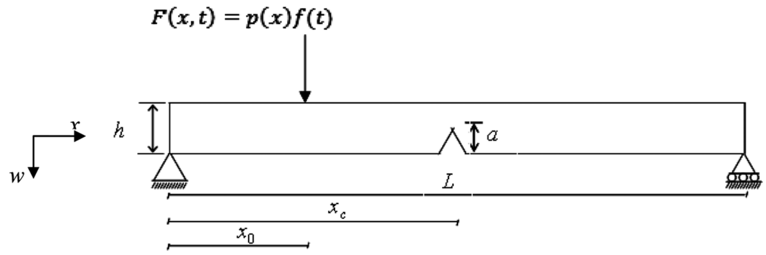

Fig. 2 simply supported beam with an open crack subjected to a consentrated dynamic load

$b \times h$ having an open crack at the distance of x_c from the left end of the beam is subjected to a dynamic pulse at the distance of x_0 from the left end of the beam (Fig. 2).

According to the Euler-Bernoulli beam theory the differential equations governing the vibration of the cracked beam are as follows

$$\begin{cases} EI\dfrac{\partial^4 w(x,t)}{\partial x^4} + c\dot{w}(x,t) + m\ddot{w}(x,t) = F(x,t); & 0 \le x \le x_c \\ EI\dfrac{\partial^4 w(x,t)}{\partial x^4} + c\dot{w}(x,t) + m\ddot{w}(x,t) = 0 \quad ; & x_c < x \le L \end{cases} \tag{7}$$

where the above two differential equations are coupled through the crack boundary conditions. In Eq. (7) EI stands for the flexural rigidity of the beam, c is the damping coefficient, m is the mass per unit length of the beam and $F(x, t)$ represents dynamic load which can be decomposed by time dependent part $f(t)$ and space dependent part $p(x)$. For a concentrated force acting at $x = x_0$, the space dependent part $p(x)$ can be expressed using the Dirac delta function as follows

$$p(x) = \delta(x - x_0) \tag{8}$$

By assumption of rest and un-deformed initial conditions for the beam vibration, we have

$$w(x,0) = 0, \;\; \dot{w}(x,0) = 0 \tag{9}$$

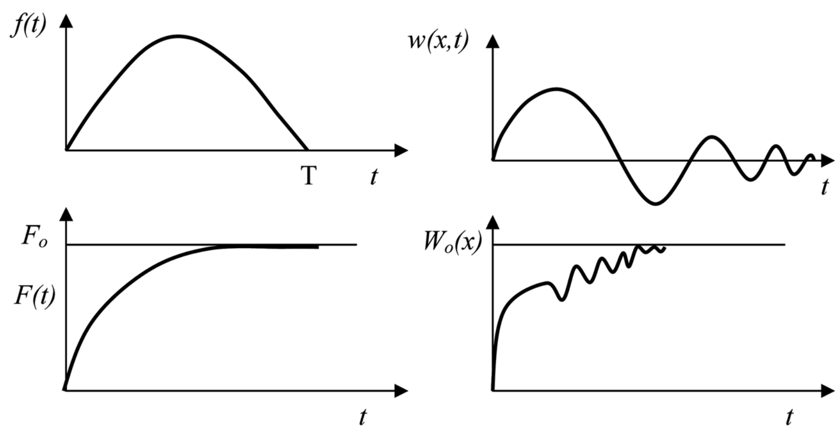

Fig. 3 The shapes of $w(x,t)$, $f(t)$ and their corresponding parameters as $W_0(x)$ and F_0

The boundary conditions at the beam ends with simply supports are as follows

$$w(x,t)|_{x=0} = 0 \qquad w(x,t)|_{x=L} = 0$$
$$EIw''(x,t)|_{x-0} = 0 \qquad EIw''(x,t)|_{x=L} = 0 \tag{10}$$

The boundary conditions at the crack location are:

(1) The continuity of deflection at the crack location is represented as

$$w(x_c^+,t) = w(x_c^-,t) \tag{11a}$$

(2) The slope of deflection of the beam at the crack location is discontinuous and is related to the crack stiffness, κ_τ, as follows

$$w'(x_c^+,t) = w'(x_c^-,t) + \frac{EI}{\kappa_\tau} w''(x_c^-,t) \tag{11b}$$

(3) The internal moment at the both sides of the crack are the same and denoted as follows

$$EIw''(x_c^-,t) = EIw''(x_c^-,t) \tag{11c}$$

(4) The shear consistency at the crack location is represented by

$$EIw'''(x_c^+,t) = EIw'''(x_c^-,t) \tag{11d}$$

Since the actuation time of the applied force is finite, due to the presence of damping, it can be concluded that

$$\lim_{t\to\infty} w(x,t) = 0, \quad \lim_{t\to\infty} \dot{w}(x,t) = 0 \tag{12}$$

Having Eq. (12) in mind and integrating Eq. (7) within the time interval (0, ∞), the static equivalence of Eq. (7) will be obtained as follows (Ruta and Szydło 2005, Sieniawska *et al.* 2009).

$$\begin{cases} EIW_0^{IV}(x) = F_0\, p(x); & 0 \le x \le x_c \\ EIW_0^{IV}(x) = 0 \quad ; & x_c < x \le L \end{cases} \tag{13}$$

where

$$F_0 = \lim_{t\to\infty} F(t) = \lim_{t\to\infty} \int_0^t f(\tau)d\tau = \int_0^T f(\tau)d\tau$$
$$W_0(x) = \lim_{t\to\infty} W(x,t) = \lim_{t\to\infty} \int_0^t w(x,\tau)d\tau \tag{14}$$

As shown in Fig. 3, the parameter T in Eq. (14) is the actuation time of the dynamic pulse. Fig. 3 illustrates $w(x, t)$, $f(t)$ and their corresponding functions of $W_0(x)$ and F_0.

By doing integration within the time interval $(0,\infty)$ on boundary conditions presented in Eqs. (10)-(11) and using second part of Eq. (14), one can analogously get

$$W_0(x)|_{x=0} = 0, \qquad W_0(x)|_{x=L} = 0$$
$$EIW_0''(x)|_{x=0} = 0, \quad EIW_0''(x)|_{x=L} = 0 \tag{15a}$$

And at the crack location, the results are

$$
\begin{aligned}
W_0(x_c^-) &= W_0(x_c^-) \\
EIW_0''(x_c^-) &= EIW_0''(x_c^+) \\
EIW_0''(x_c^-) &= EIW_0''(x_c^+) \\
W_0'(x_c^+) &= W_0'(x_c^-) + \frac{EI}{\kappa_\tau} W_0''(x_c^-)
\end{aligned}
\tag{15b}
$$

Solving Eq. (13a) in which $0 \le x \le x_c$ results in

$$
\begin{aligned}
EIW_0(x) &= a_4 + a_3 x + \frac{1}{2} a_2 x^2 + \frac{1}{6} a_1 x^3 + F_0 H(x - x_0) \left\{ \frac{x^3}{6} - \frac{x^2}{2} x_0 + \frac{x_0^2}{2} x - \frac{x_0^3}{6} \right\} \\
EIW_0'(x) &= a_3 + a_2 x + \frac{1}{2} a_1 x^2 + F_0 H(x - x_0) \left\{ \frac{x^2}{2} - x x_0 + \frac{x_0^2}{2} \right\} \\
EIW_0''(x) &= a_2 + a_1 x + F_0 H(x - x_0) \{ x - x_0 \} \\
EIW_0'''(x) &= a_1 + F_0 H(x - x_0)
\end{aligned}
\tag{16}
$$

And also solving Eq. (13b) in which $x_c < x \le L$ results in

$$
\begin{aligned}
EIW_0(x) &= \frac{1}{6} b_1 x^3 + \frac{1}{2} b_2 x^2 + b_3 x + b_4 \\
EIW_0'(x) &= \frac{1}{2} b_1 x^3 + b_2 x + b_3 \\
EIW_0''(x) &= b_1 x + b_2 \\
EIW_0'''(x) &= b_1
\end{aligned}
\tag{17}
$$

where in Eqs. (16), $H(x - x_0)$ is the Heaviside function which is defined as follows

$$
H(x - x_0) = \begin{cases} 1; & x \ge x_0 \\ 0; & x < x_0 \end{cases}
\tag{18}
$$

As can be seen in Eqs. (16)-(17) the number of unknowns are 8 $(a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$ and the number of equations from the boundary conditions (Eq. (15)) are also 8 but κ_τ as an unknown is included in them, so one more equation is needed to be considered for finding the final solution of Eq. (13). The 9^{th} equation is deduced from the time history transverse response of an arbitrary location along the beam, $x = x_e$. By having that response and applying Eq. (14b) one can obtain $W_o(x_e)$ corresponding to that location. In the absence of such experiments, one can rely on the vibration simulation of the finite element model of the cracked beam and obtain the desired response data. In this work the above data is obtained from the vibration simulation of cracked beam using a finite element software.

Based on the location of measuring point, we will have:

If $0 \le x_e \le x_c$, then

$$W_0(x_e) = 1/EI\left\{a_4 + a_3x_e + \frac{1}{2}a_2x_e^2 + \frac{1}{6}a_1x_e^3 + F_0H(x_e - x_0)\left(\frac{x_e^3}{6} - \frac{x_e^2}{2}x_0 + \frac{x_0^2}{2}x_e - \frac{x_0^3}{6}\right)\right\} \tag{19}$$

If $x_c < x_e \le L$, then

$$W_0(x_e) = 1/EI\left\{\frac{1}{6}b_1x_e^3 + \frac{1}{2}b_2x_e^2 + b_3x_e + b_4\right\} \tag{20}$$

Depending on the measuring location, by using Eq. (19) or Eq. (20) and solving the resulted simultaneous equations, all unknowns can be calculated. The inverse of parameter κ_τ, known as the crack flexibility coefficient is denoted by C which will be used thereinafter.

For the case $x_c < x_e \le L$, κ_τ can be calculated from the following equations

$$\begin{bmatrix} -x_c & x_c & 1 \\ 0 & L & 1 \\ 0 & x_e & 1 \end{bmatrix} \times \begin{bmatrix} a_3 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} F_0\left(\frac{x_c x_0^2}{2} - \frac{x_0^3}{6}\right) \\ F_0\frac{x_0 L^2}{3} \\ W_0 EI - F_0\frac{x_0 x_e^3}{6L} + F_0\frac{x_0 x_e^2}{2} \end{bmatrix} \tag{21}$$

By solving Eq. (15), finally we have

$$\kappa_\tau = \left(EIF_0x_0\frac{x_c - L}{L}\right) \Big/ \left(F_0\frac{x_0^2}{2} + a_3 - b_3\right) \tag{22}$$

Similarly, an equation can be obtained for the calculation of κ_τ in the case of $0 \le x_e \le x_c$.

4. Crack configurations and applied load properties

In the following sections, the flexibility coefficient of the V-shaped side crack is calculated using simply supported prismatic beam based on the procedure presented in the previous sections. The beam has the following geometrical properties: length $L = 300$ cm, cross sectional dimensions are width $b = 6$ cm and height $h = 12$ cm.

The material properties of the beam are: the Young's modulus (E) is 210 (GPa), the density (ρ) is 7850 (kg/m^3) and Poisson's ratio (υ) is 0.3.

To study the effect of crack depth on the crack flexibility, five crack depths from 1.2 cm to 6 cm by increasing 1.2 cm at each step are considered. The location of the crack is considered at the middle of the beam span ($x_c = 150$ cm) (see Fig. 2). A concentrated dynamic sinusoidal load with the amplitude of 20 (kN) and period of 1 sec applies to the beam at the location 75 cm from the left end of the beam. The actuation time, T, is considered 0.5 sec. The numerical errors hidden in the procedure are inevitable in which the numerical calculations for finding the parameter W_o (see Eq. (14b)) is the main source of it. Hence, to decrease the effects of such errors on the results accuracy, the transverse response $w(t)$ of different locations including, $x_e = 180$, 195, 210, 225 cm are used in the procedure and the calculated flexibility coefficients are averaged.

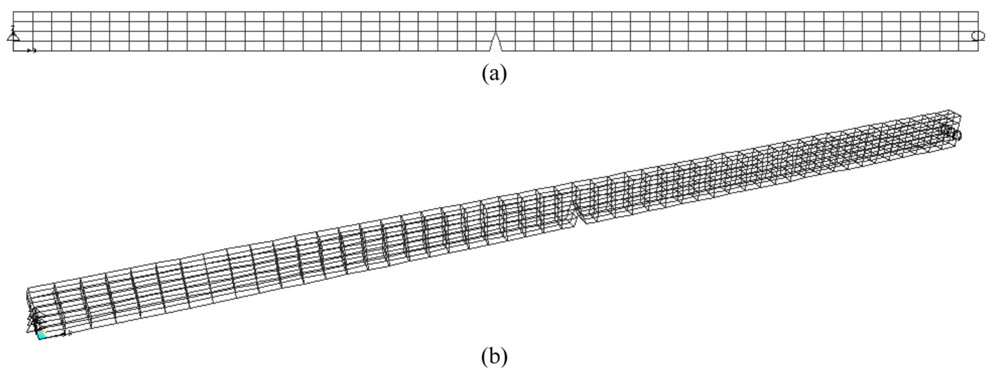

Fig. 4 Finite element mesh, (a) 2D model; (b) 3D model

5. Finite element modeling

An in-house finite element software is applied to find the transverse response $w(t)$ which is needed for solving the simultaneous equations involving unknown crack flexibility coefficient. Hence, a simple beam model with V-shaped edge-crack and with the aforementioned geometries and material properties is developed. Both plane stress (2D) and 3D models are considered. For 2D analysis, 4-node quadrilateral elements are used in discretization and also for 3D analysis 8-node brick elements are used, see Fig. 4.

6. Results and discussion

6.1 Single-side crack

The flexibility coefficient of crack in simply supported beam is calculated using the two and three dimensional modeling of the beam with the assumed properties. In order to validate the procedure, the results obtained are compared with the results reported in literature.

Okamura *et al.* (1969) presented the following formula based on the fracture mechanics theory for calculating the flexibility coefficient of cracked rectangular cross sectional beam

$$C = (6h/E'I)f(\zeta) \tag{23a}$$

where function $f(\zeta)$ is given as

$$\begin{aligned} f(\zeta) = {} & 1.98\zeta^2 - 3.227\zeta^3 + 14.43\zeta^4 - 31.26\zeta^5 + 63.56\zeta^6 - 103.36\zeta^7 \\ & + 147.52\zeta^8 - 127.69\zeta^9 + 61.50\zeta^{10} \end{aligned} \tag{23b}$$

Rizos *et al.* (1990) presented the following formula for calculating the flexibility coefficient

$$C = (5.346h/EI)f(\zeta) \tag{24a}$$

in which

$$f(\zeta) = 1.8624\zeta^2 - 3.95\zeta^3 + 16.375\zeta^4 - 37.226\zeta^5 + 76.81\zeta^6 - 126.9\zeta^7 + 172.\zeta^8 - 143.97\zeta^9 + 66.56\zeta^{10} \tag{24b}$$

In the above equations, h is the height and b is the width of the cross-section, EI is the flexural rigidity, $\zeta = a/h$ and a is the crack depth. For plane-stress problem $E' = E$ and for plane-strain problem $E' = E/(1-\upsilon^2)$, in which υ is Poisson's ratio.

As presented in section two, Zheng and Fan (2003) have also obtained a formula for calculating the flexibility coefficient of cracks in rectangular cross sectional beams which has been expressed in Eq. (6).

By applying the present method and using the data obtained from the plane stress finite element modeling of the beam described above, the flexibility coefficient of cracks with different depths are obtained and the corresponding dimensionless parameter as $CEbh^2$ are calculated. In Fig. 5 the results obtained are presented and compared with the reference results. A good agreement between the results of the present approach and the results of Okamura (1969) and those of the Zheng and Fan (2003) can be observed.

Analogously, the response of the 3D finite element model is incorporated with the present approach and the flexibility coefficients of cracks are found and then the corresponding dimensionless parameter is calculated. The results are presented in Fig. 6 and compared with the reference results. It can be observed that by applying 3D model, the predictions are also in good agreement with the results from (Okamura 1969, Zheng and Fan 2003). However, the closeness of the plane stress results and the results predicted by Okamura (1969) is noticeable. Also the same observation can be seen between the results of 3D model and those of the Zheng and Fan (2003).

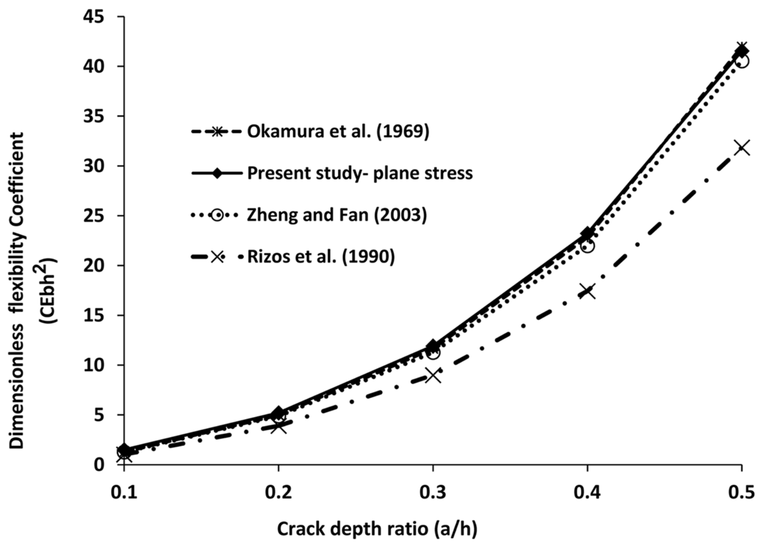

Fig. 5 Dimensionless local flexibility coefficient of a solid rectangular cross sectional beam with a single-side crack of the different extents

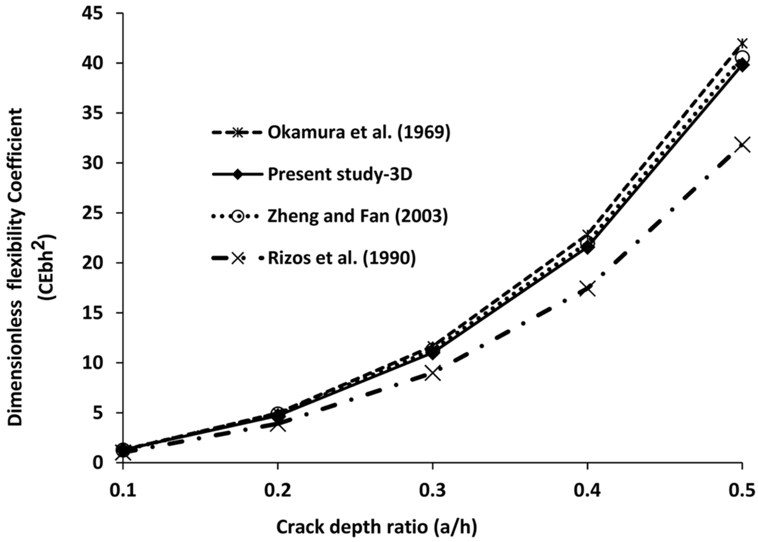

Fig. 6 Dimensionless local flexibility coefficient of a solid rectangular cross sectional beam with a single-side crack of the different extents

6.2 Double-side crack

The presence of crack in one side of the beam, so called as single-side crack, or in both sides, so called as double-side crack, affects crack induced flexibility. Work by Mahmoud (2001) investigated the effect of crack geometries on stress intensity factor (SIF) and also Orhan (2007) investigated the effect of crack geometries on vibration of beam with side cracks. Fig. 7 shows both single-side crack and double-side one. In this section the local flexibility coefficient of a double-side crack is investigated based on the present approach incorporating the plane stress and 3D finite element models. It is assumed that the crack occurred on both sides of a rectangular cross sectional beam with the same extent. The flexibility coefficient of a double-side crack with $\zeta = 0.1$ on each side is calculated and compared with a single-side crack having $\zeta = 0.2$. Also this study is performed for a double-side crack with $\zeta = 0.2$ for both upper and lower sides of the beam and for the corresponding single-side crack with $\zeta = 0.4$.

The results obtained are presented in Figs. 8-9. It can be seen that the single-side crack is more flexible than the corresponding double-side one.

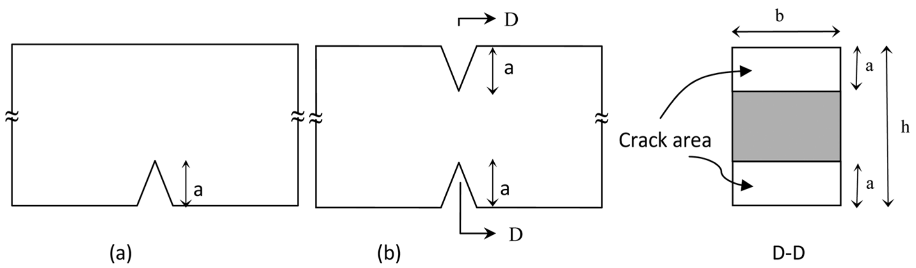

Fig. 7 (a) single-side crack; (b) double-side crack

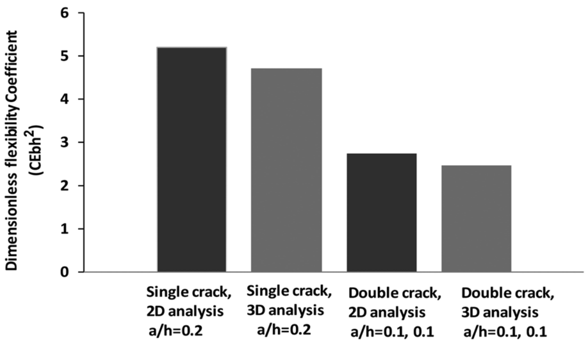

Fig. 8 Predicted dimensionless local flexibility coefficients of a solid rectangular cross sectional beam with a single-side crack and a double-side crack using 2D and 3D finite element models

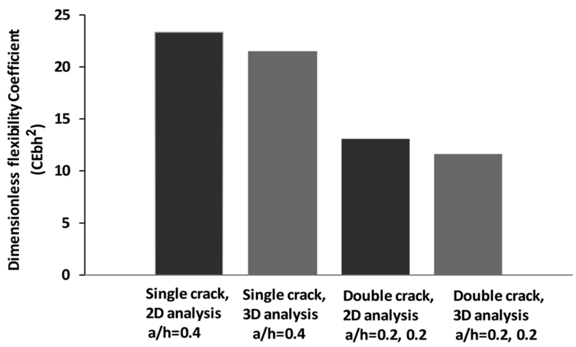

Fig. 9 Predicted dimensionless local flexibility coefficients of a solid rectangular cross sectional beam with a single-side crack and a double-side crack using 2D and 3D finite element models

Also it can be seen that plane stress model predicts larger flexibility coefficients than the 3D model for both single-side and double-side cracks. It should be noted that these different predictions can be due to the fact that the plane stress model of a beam is more flexible than the corresponding 3D model. Although this is a classical issue, however this point can be observed in the results of some recent works performed by Krueger *et al.* (2002) and Narmashiri and Jumaat (2011).

7. Conclusions

In this work an inverse method based on the forced vibration of cracked beams has been presented for the calculation of the local flexibility coefficient of open-side cracks in solid rectangular cross-sectional beams. The following conclusions may be drawn:

- The calculated flexibility coefficients for the single-side cracks are in good agreement with the published results from references (Okamura *et al.* 1969, Zheng and Fan 2003) which are based on the fracture mechanics theory.
- It is found that by incorporating the plane stress finite element simulation of the vibrating cracked beam and the present procedure; the results obtained are noticeably close to the results predicted by Okamura *et al.* (1969).
- Furthermore, it is observed that by incorporating the 3D finite element simulation of the vibrating cracked beam and the present procedure; the results obtained are noticeably close to the results predicted by Zheng and Fan (2003).
- By applying the present procedure for the calculation of flexibility coefficients of double-side cracks in rectangular cross sectional beams, it is found that when the crack depth of a single-side crack is equal to the total crack depth of a double-side crack having equal crack depth on both sides, the single-side crack shows more flexibility than the double-side one.

The presented approach can be extended to calculate the flexibility coefficients of the cracked beams with different cross-sectional geometries which is the current objective of the authors.

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