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Analysis of composite frame structures with mixed elements – state of the art

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Abstract. The paper presents a review of the application of the newly proposed mixed finite element model for seismic simulation of different types of composite frame structures. To evaluate the performance of the element, a comparison with displacement-based and force-based models is conducted. The study revealed that the mixed model is superior to the others in terms of both speed of convergence and numerical stability, and is therefore considered the most practical approach for modeling of composite structures. In this model, the element is derived using independent force and displacement shape functions. The nonlinear response of the frame element is based on the section discretization into fibers with uniaxial material models. The interfacial behavior is modeled using an inelastic interface element. Numerical examples to clarify the advantages of the model are presented for the following structural applications: anchored reinforcing bar problems, composite steel-concrete girders with deformable shear connectors, beam on elastic foundation elements, R/C girders strengthened with FRP sheets, R/C beam-columns with bond-slip, and prestressed concrete girders. These studies confirmed that the model represents a major advancement over existing elements in simulating the inelastic behavior of composite structures.

Keywords: composite structures; composite girders; bond; interface element; displacement formulation; force formulation; mixed finite element

1. Introduction

Composite elements are defined as elements made up of more than one material, and connected with either natural or mechanical bond interfaces. A reinforced concrete column is considered a composite element since it is made up of a concrete core bonded naturally to reinforcing bars. Composite steel-concrete girders represent another example since they consist of a solid cast-in-place concrete slab connected to a steel rolled W section or a welded I-shape girder by means of mechanical shear connectors. Other types of composite structures include concrete beams strengthened with Fiber Reinforced Polymer (FRP) sheets, and concrete foundations resting on soil which acts as an interface element that transfer the forces to the support system. The key factor in modeling composite elements is the accurate representation of the interfacial behavior. Several models were developed recently that account for this effect. The paper presents an assessment of the state of the art in modeling composite structures.

The analysis of composite beam elements started with the work of Newmark *et al*. (1951), who were

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the first to study the equilibrium of elastically connected beams. Their work considered slip effects but neglected uplift and friction. Adekola (1968) extended this work by including uplift and frictional effects. He suggested a finite difference procedure for solving the differential equation of uplift and axial forces. Ma (1973) solved the differential equation of the slip problem between steel girder and concrete deck using a finite difference method for the case of nonlinear connector behavior. Robinson and Naraine (1988) further studied the topic of slip and uplift using the same approach to discuss the effect of whether the load is acting on the slab or pulling down on the steel beam. Their work, however, was limited to elastic cases. Cosenza and Mazzolani (1993) proposed a new solution for the same problem for different loading conditions also for elastic cases, as is discussed in Viest *et al*. (1997). Schnabl *et al*. (2006) presented the solution of a three-layer elastic composite beam. Schnabl *et al*. (2007) developed a locking-free linear elastic Timoshenko composite beam element. Foraboschi (2009) developed an exact nonlinear model for a two-layer composite beam with an elastoplastic strain-softening bond-slip law. Faella *et al*. (2010) derived an exact stiffness matrix and load vector for elastic steel-concrete composite beams. Nguyen *et al*. (2011a,b) derived the exact stiffness matrix for elastic Timoshenko composite beams.

As discussed by Spacone and El-Tawil (2004), finite element analysis of steel-concrete composite elements can be grouped into the following two categories: (a) models utilizing plate, shell or brick finite elements to represent in detail the constituents of the composite structural element; and (b) one dimensional inelastic beam elements that capture salient features of the nonlinear behavior of composite girders. Examples of the first type of models include the work by Wegmuller and Amer (1977), who used a layered beam-plate element to study fully bonded composite beams, Hirst and Yeo (1980), who used quadrilateral isoparametric finite elements to study partially connected composite beams, Razaqpur and Nofal (1989), who used a three dimensional bar element to model shear connectors, a quadrilateral element to model the web, and a flat shell element to model the flanges and the slab, and Bursi and Ballerini (1996) used a smeared crack two-dimensional model to study the behavior of composite frames. Bursi *et al*. (2005) proposed a three-dimensional model with shell elements to simulate the seismic behavior of composite steel-concrete frames. Zhou *et al*. (2007) developed a three-dimensional finite element model for composite structures including a fractured steel connection element, for analysis under very large cyclic loads.

Within the second category proposed models can be grouped in two sub-categories: (i) full composite action models; and (ii) models that account for the partial composite action between concrete and steel. Within the first category, Mirza and Skrabek (1991) proposed a model with displacement interpolation functions and fiber discretization of the cross section for the analysis of composite columns under uniaxial bending. El-Tawil *et al*. (1995) also adopted the concept of fiber discretization to evaluate the stress resultants of the composite section under biaxial bending. Later, El-Tawil and Deierlein (2001) proposed a force-based inelastic beam element with a bounding surface plasticity model to describe the biaxial behavior of composite elements under cyclic loads. Liew *et al*. (2001) developed a model for analysis of composite steel-concrete beams based on the quasi-plastic hinge approach proposed by Attala *et al*. (1994). Kim and Engelhardt (2005) adopted a plastic hinge approach with newly proposed Moment-Rotation hysteretic models. Providakis (2008) also used the plastic hinge element in the commercial package ETABS2000 (2002) to simulate the behavior of seismically isolated composite buildings. Within the second category, several formulations have been proposed.

To account for partial interaction, several models were proposed. Displacement-based formulations with fiber section discretization have been proposed by Arizumi *et al*. (1981) and Daniel and Crisinel

(1993) for composite beams under monotonic loads, by Amadio and Fragiacomo (1993) to account for the effect of concrete creep and shrinkage, by Hajjar *et al*. (1997) for the analysis of concrete-filled steel tube columns, and by Sakr and Sakla (2008) to evaluate the long term deflection at the serviceability limit state. Dall'Asta and Zona (2002) proposed a displacement-based model with 10 degrees of freedom, including 2 internal degrees of freedom. Zona *et al*. (2008) used this model to evaluate the effect of different modeling assumptions on the seismic behavior of composite buildings. A displacement-based second order inelastic model for composite beams based on a total Lagrangian formulation was proposed by Pi *et al*. (2006). Ranzi *et al*. (2006) and Gara *et al*. (2006) developed a displacement-based element with both longitudinal interface as well as vertical uplift. Ranzi (2008) developed a displacement-based element for multi-layer composite beams that address locking issues. Battini *et al*. (2009) adopted a higher order corrotational element with a local linear formulation based on the exact stiffness matrix to avoid curvature locking. Gara *et al*. (2009) adopted a displacement approach for the long-term analysis of composite beams and accounting for the shear lag effect in the slab. Erkmen and Bradford (2011) addressed slip locking in displacement-based elements by introducing strategies such as the assumed strain method, the discrete strain gap method, and the kinematic interpolatory technique. Zona and Ranzi (2011) presented and compared the behavior of three displacement-based steel-concrete beam elements with and without shear deformations. Souza and Da Silva (2007) proposed an alternative procedure to model the interface element. In their approach, the relative displacements are independently interpolated on the top and bottom element edges. Da Silva and Souza (2009) extended their work for layered Euler-Bernoulli and Timoshenko beams.

Force-based models with fiber discretization were proposed by Salari *et al*. (1998) for composite beams under cyclic loads, and by Ayoub (2005). Valipour and Bradford (2009) proposed a force-based model with a total secant solution strategy. A piecewise interpolation of the slip strain along the element axis is employed to calculate the slip forces along the element length.

Ayoub and Filippou (2000) proposed a two-field fiber-based mixed model with independent approximation of both the displacement and force fields. Analysis using the two-field mixed model to simulate the cracking of the concrete slab in the hogging moment region was conducted by Nguyen *et al*. (2009). Tort and Hajjar (2010) also adopted a two-field mixed approach to model material and geometrical nonlinearity in concrete-filled steel tube columns. Recently, Dall'Asta and Zona (2004a) proposed a three-field mixed model for composite beams, in which they approximated the displacements, forces, and curvatures with independent interpolation functions. Dall'Asta and Zona (2004b) compared the numerical behavior of the three-field mixed and displacement-based finite element models. They confirmed the efficiency of mixed elements but pointed out that the behavior of displacement elements can be improved if locking-free elements with richer shape functions are used. Dall'Asta and Zona (2004c) also developed locking-free displacement-based composite beam elements.

In this paper a review of finite element models for composite beam elements made-up of more than one material is presented. Detailed numerical comparisons between the mixed formulation, and both displacement and force-based models are discussed. The study confirmed that the mixed formulation overcomes the limitations associated with displacement and force models, while combining most of their advantages. Force-based models produce a non-consistent tangent stiffness at the global level as discussed by Hjelmstad and Taciroglu (2002) and confirmed by Sun and Bursi (2005), which results in poor numerical performance. Displacement-based models, on the other hand, require a very fine mesh for inelastic analysis which renders the problem computationally expensive. The mixed model, however, proved to overcome these problems, and is therefore considered the most practical approach for nonlinear analysis of composite beam elements. Ayoub and Filippou (1999, 2000, 2010), Ayoub (2001, 2003, 2006, 2011), and Lu and Ayoub (2008, 2011) developed the mixed formulation to consider several structural applications: anchored reinforcing bars, composite steel-concrete beams, beam on Winkler supports, reinforced concrete beam-columns with bond-slip, prestressed concrete beams, and reinforced concrete girders strengthened with FRP laminates. The new mixed model for composite elements was implemented in the general purpose finite element program FEAP developed by R.L. Taylor and described in Zienkiewicz and Taylor (1989). A brief description of the basic equations of the finite element formulations is described next followed by a detailed discussion of the numerical performance of each model.

2. Finite element formulation

In a nonlinear finite element formulation, the structure is subdivided into discrete elements. Consistent linearization of the governing differential equations for a single element is used to derive the element stiffness matrix and resisting load vector. The structure stiffness matrix and resisting load vector is then assembled from the element contributions following well established principles of structural analysis. The resulting system of equations is solved by an iterative solution strategy, commonly of Newton-Raphson type. In this solution strategy, the linearized system of equations about the current state of the structure is solved for the unknown increments of the primary variables. The iteration continues until convergence is satisfied within a specified tolerance. The solution then advances to the next load step.

By considering the equilibrium of a composite beam segment as shown in Fig. 1, the displacement formulation at a Newton-Raphson iteration step i leads to the following equation

$$(\mathbf{K}^{i-1} + \mathbf{K}_b^{i-1})\delta\mathbf{V}^i = \mathbf{P} - \mathbf{q}^{i-1} - \mathbf{q}_{rb}^{i-1} \tag{1}$$

where $\mathbf{q}$ and $\mathbf{V}$ are the element end forces and displacements respectively, δ represents an increment,

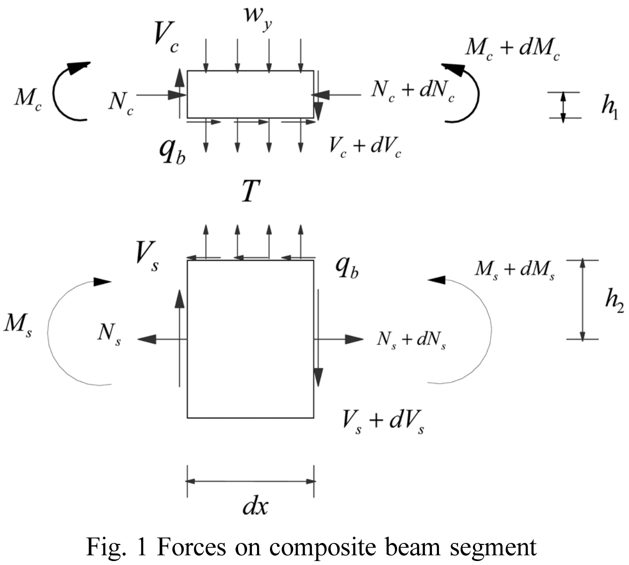


Fig. 1 Forces on composite beam segment

$\mathbf{K}^{i-1} = \int_0^L \mathbf{B}^T(x)\boldsymbol{k}^{i-1}(x)\mathbf{B}(x)dx$ is the section element stiffness matrix

$\mathbf{K}_b^{i-1} = \int_0^L \mathbf{B}_b^T(x)k_b^{i-1}(x)\mathbf{B}_b(x)dx$ is the bond element stiffness matrix

$\mathbf{q}^{i-1} = \int \mathbf{B}^T\mathbf{D}^{i-1}(x)dx$ is the section element resisting load vector

$\mathbf{q}_{rb}^{i-1} = \int \mathbf{B}_b^T(x)D_b^{i-1}(x)dx$ is the bond element resisting load vector

and **P** is the vector of applied external loads.

$\mathbf{B}(x) = \mathbf{La}(x)$, where $\mathbf{a}(x)$ is the vector of displacement interpolation functions, and **L** is a differential operator that includes the first derivative of the axial displacements, and second derivatives of the vertical displacements, $\mathbf{B}_b(x) = \mathbf{L}_b\mathbf{a}(x)$, $\mathbf{L}_b$ is a differential operator needed for calculation of the interface slip between top and bottom beams, $\boldsymbol{k}(x)$ is the section stiffness at a distance x typically evaluated through fiber integration, $k_b(x)$ is the interface stiffness coefficient, **D** is the vector of section forces, and D_b is the interface resisting force.

The force formulation as derived by Salari *et al.* (1998) results in the following equation

$$\mathbf{F}^{i-1}\delta\mathbf{q}^{i-1} = \mathbf{V}^{i-1} \tag{2}$$

Where **F** is the composite element flexibility matrix and is given by: $\mathbf{F} = \mathbf{f}_{\mathbf{QQ}} - \mathbf{f}_{\mathbf{QQ}_b}\mathbf{f}^{-1}_{\mathbf{Q}_b\mathbf{Q}_b}\mathbf{f}_{\mathbf{Q}_b\mathbf{Q}}$. $\mathbf{f}_{\mathbf{QQ}}$ is the section element flexibility matrix, $\mathbf{f}_{\mathbf{Q}_b\mathbf{Q}_b}$ is the bond element flexibility matrix, and $\mathbf{f}_{\mathbf{Q}_b\mathbf{Q}}$ is a coupling term. The element flexibility matrix **F** must be inverted to obtain the element stiffness matrix.

The mixed formulation as developed by Ayoub and Filippou (2000) yields the following system of equations

$$\begin{bmatrix} -\mathbf{F}^{i-1} & \mathbf{T} \\ \mathbf{T}^T & \mathbf{K}_b^{i-1} \end{bmatrix}\begin{bmatrix} \delta\mathbf{q}^i \\ \delta\mathbf{V}^i \end{bmatrix} = \begin{bmatrix} \mathbf{V}_r^{i-1} \\ \mathbf{P}-\mathbf{T}^T\mathbf{q}^{i-1}-\mathbf{q}_{rb}^{i-1} \end{bmatrix} \tag{3}$$

where $\mathbf{T} = \int_L \mathbf{b}^T(x)\mathbf{B}(x)dx$, $\mathbf{b}(x)$ represents the vector of force interpolation functions,

$\mathbf{F}^{i-1} = \int_L \mathbf{b}^T(x)\boldsymbol{f}^{i-1}(x)\mathbf{b}(x)dx$ is the element flexibility matrix, $\boldsymbol{f}(x) = \boldsymbol{k}^{-1}(x)$ is the section flexibility matrix,

$\mathbf{V}_r^{i-1} = \int_L \mathbf{b}^T(x)\boldsymbol{e}^{i-1}(x)dx - \mathbf{T}\mathbf{V}^{i-1}$ is the element deformation residual, and $\boldsymbol{e}$ is the vector of section deformations.

The state determination of mixed models requires an internal element iteration in order to zero the residual deformation vector $\mathbf{V}_r$. The implementation of the mixed formulation in a general purpose finite element program is discussed in details in Ayoub (1999, 2001). The relationship between the order of the displacement and force interpolation functions is determined according to the Babuska-Brezzi (B-B) stability condition (1973, 1974) and De Veubeke's principle of limitation (1965). Accordingly, the order of the axial displacement interpolation function should be higher by one than the order of the axial force interpolation function. Typically, a linear force and quadratic displacement functions are assumed for the axial behavior.

To assess the performance of the mixed model as compared to the displacement and force models, a series of numerical studies of different structures under different loading conditions is conducted. Numerical studies for anchored reinforcing bars, composite steel-concrete beams, beams on Winkler foundations, reinforced concrete girders strengthened with Fiber Reinforced Polymer (FRP) laminates, reinforced concrete beam-columns with bond-slip, and prestressed concrete beams are all presented.

3. Anchored reinforcing bar element

An anchored reinforcing bar element is an inelastic steel truss element connected to a rigid concrete block through a nonlinear interfacial bond element. Due to the presence of the interfacial element connecting the two materials, the element belongs to the family of composite structures. To assess the relative performance of the three formulations in simulating the behavior of anchored bars, namely the standard displacement model, the force (flexibility) model developed by Monti and Filippou (1997) for anchored bars, and the proposed mixed model (Ayoub and Filippou 1999), an analytical study was performed on a sample model for all three formulations. The Viwathanatepa bar (1979) is selected for the study. The bar is a straight #8 (25 mm diameter) reinforcing bar embedded in a confined concrete block with an anchorage length of 25 bar diameters. The study was conducted for a monotonic pull-out loading condition under displacement control. The material properties were kept the same for all three formulations. The steel constitutive law followed the model by Menegotto and Pinto (1973). Its Young's modulus = 205,000 MPa, yield strength = 470 MPa, and hardening ratio = 1.4%. The bond constitutive law followed the model by Eligehausen *et al*. (1983). The bond strength = 13.5 MPa. A refined displacement model with 40 elements using quadratic displacement shape functions was used as a benchmark for the analysis. A comparison with a mesh of equally spaced 6 elements is performed for the displacement, force, and mixed models respectively. Five Gauss-Lobatto integration points were used for the displacement and force models, and five Gauss-Lobatto steel integration points along with three reduced Gauss-Lobatto bond integration points were used for the two-field mixed model. The finite element model is shown in Fig. 2.

The global end load-displacement response for both the force and mixed models compared to the refined model is shown in Fig. 3. Good agreement with the refined model is obtained, the mixed model giving almost identical results. The distribution of the steel stress and strain for the quadratic displacement formulation is shown in Figs. 4 and 5. Distributions along the bar length for the refined and coarse meshes, for four loading stages are shown in the figures: A) a load in the elastic

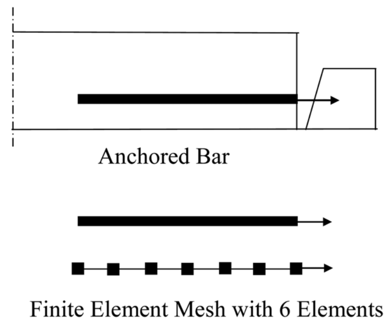


Fig. 2 Finite element model of anchored reinforcing bar

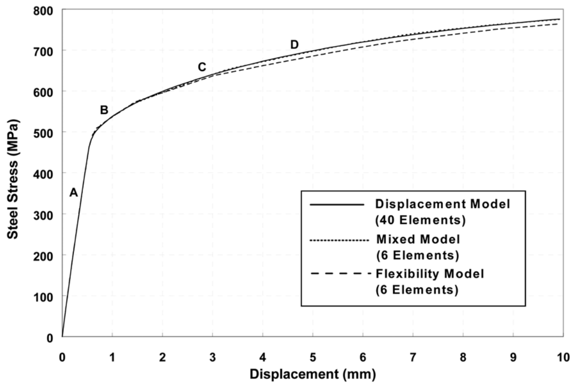


Fig. 3 Global response of viwathanatepa specimen

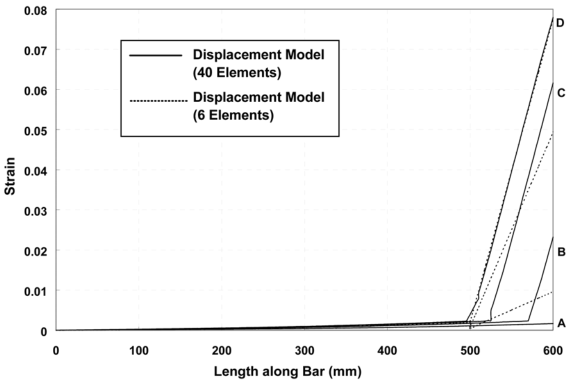


Fig. 4 Steel strain distribution of viwathanatepa specimen

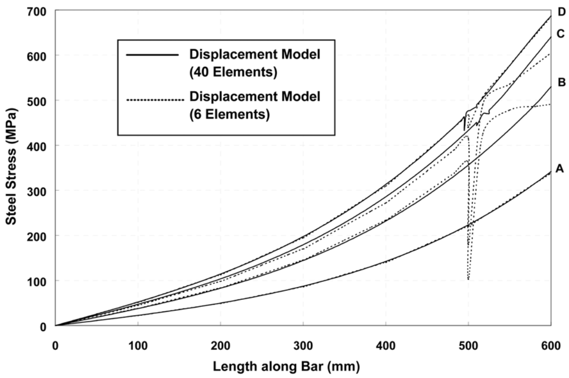


Fig. 5 Steel stress distribution of viwathanatepa specimen

range, B) a load shortly after steel yielding, C) a load after bond yielding, and D) a load after bond softening. The four loading stages A-D are identified in the global response of Fig. 1. Fig. 4 shows that the large size mesh yields accurate results for strain distributions in the elastic range. However, in the nonlinear range, it fails to describe the strain localization at the loaded bar end accurately. This is attributed to the assumption of a linear strain field within the element, while the exact strain distribution is actually very steep in the plastic region. Fig. 4 also shows a jump in strains at element boundaries. That jump is amplified by Young's modulus in Fig. 5 that shows the stress distribution, resulting in poor behavior. From the preceding discussion, it is clear that displacement models could not be used with large elements size. A very fine mesh is needed for a converging solution. For that reason, displacement models are considered computationally costly. The distribution of the steel stress and strains for the force (flexibility) and mixed models is shown in Figs. 6 and 7, and compared to the refined mesh. Fig. 6 shows that both the force and mixed models yield identical results for strains in the elastic range. They also describe the strain localization successfully in the non-linear range, the mixed model being closer to the refined mesh. Fig. 7 also shows that both models yield identical stress distributions in the elastic range, and close results in the non-linear range, the mixed model being closer to the refined model. These results confirm the superiority of the force and mixed models in describing the inelastic behavior of the anchored bar problem. The

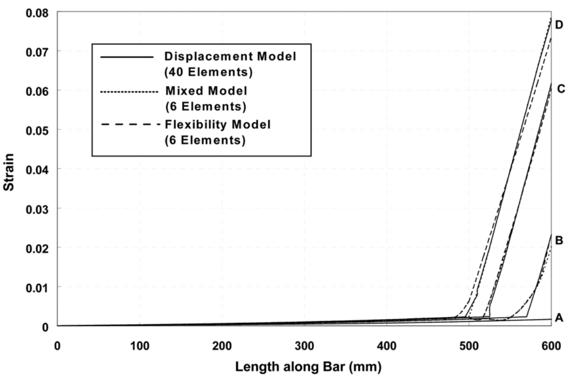


Fig. 6 Steel strain distribution of viwathanatepa specimen

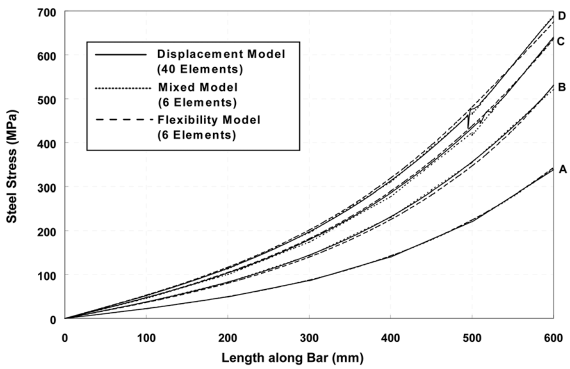


Fig. 7 Steel stress distribution of viwathanatepa specimen

force model, however, was shown to produce a non-consistent tangent stiffness at the global level (Hjelmstad and Taciroglu 2002), which results in poor numerical performance, especially under severe loading conditions. In addition, the stiffness matrix derived in the model by Monti and Filippou (1997) was found to be un-symmetric, which renders the solution more computationally demanding. In addition, the force model fails to produce accurate results for specimens with complex bond stress-slip relationship since it attempts to approximate the resulting irregular bond distribution. The mixed model, on the other hand, has shown stable numerical behavior irrespective of the complexity of the bond stress-slip relationship since it attempts to approximate the smooth slip distribution. The mixed model has therefore proved to overcome the limitations associated with both the displacement and force models while combining most of their advantages, and therefore is considered the preferred approach for modeling of these types of structures.

4. Composite steel-concrete beam element

To evaluate the performance of the mixed model described earlier, a composite beam element under third point bending is used for the comparison. The beam was tested by McGarraugh and Baldwin (1971) as a part of a series of tests to investigate the effect of lightweight concrete on the behavior of composite beams. The beam consists of a W 14×30 steel section connected to a reinforced concrete slab by shear connectors. The slab depth is 4.5 in (114.3 mm) and its width is 36 in (914.4 mm). Five 3/4” (19.05 mm) × 3” (76.2 mm) shear studs per half span were used. The steel yield stress is 36.6 (252.4) ksi and the concrete compressive strength is 5.56 ksi (38.3 MPa). A bond force slip relation is used in the analytical study with a yield force that equals 17.6 kips (78.28 kN) and an initial stiffness that equals 440 kips/in (77 kN/mm). The dimensions, loading arrangements, and cross section dimensions of the specimen are shown in Fig. 8. The finite element model used for the analysis consists of a number of composite elements and is shown in Fig. 9. Five integration points for the beam elements and three for the bond element were used. The cross section of the steel beam is made up of 12 fibers while the cross section of the concrete beam is made up of 4 fibers.

In analyzing steel-concrete composite elements with shear lag, Sun and Bursi (2005) considered all three models: the standard displacement model, the force-base model developed by Salari *et al.*

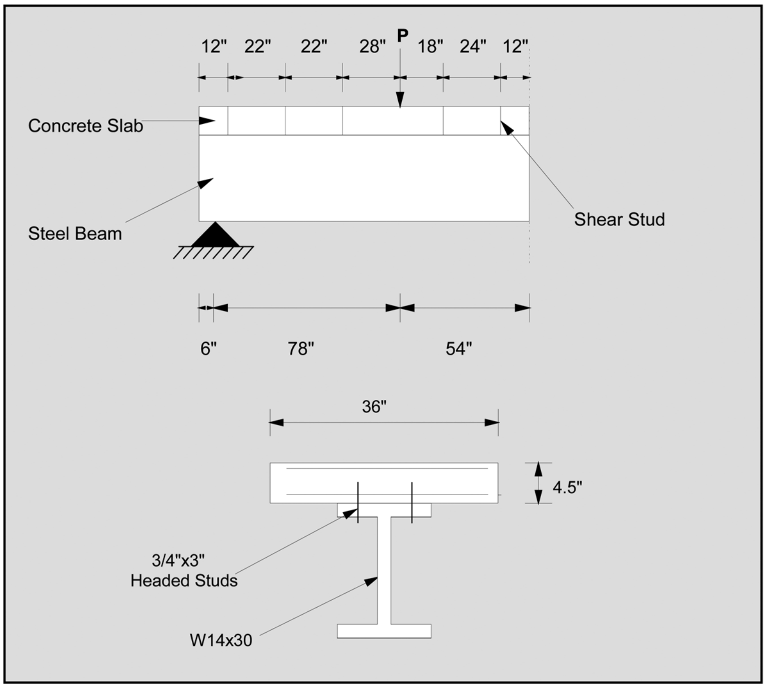


Fig. 8 McGarraugh specimen
(1 in = 25.4 mm)

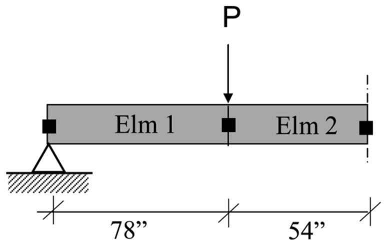


Fig. 9 Finite element model of McGarraugh beam with 4 elements

(1998), and the two-field mixed model developed by Ayoub *et al*. (2000). They ended up omitting the force model due to its non-consistent global tangent stiffness matrix. In addition, in the implementation of the force model by Salari *et al*. (1998), slip compatibility at element boundaries is not satisfied, which can create numerical instabilities especially under cyclic loading conditions. The authors concluded their study by affirming that “greater benefits of mixed composite beam elements with shear lag and flexible shear connection are expected especially for very coarse discretization.” In the following sections, the force model is therefore no longer discussed and emphasis will be placed on the advantages of the mixed model.

The mixed formulation with linear axial force interpolation function and quadratic displacement interpolation function is evaluated for the McGarraugh and Baldwin specimen. The response for a mesh of 4 mixed elements is compared to a displacement model for a mesh of 4 elements and a mesh of 16 elements. The global response of all three cases gives essentially the same results and agrees with the experimental response as shown in Fig. 10. The response for the case of no and full composite action is also shown in Fig. 10. The axial force and bending moment distributions at a load point beyond yielding that corresponds to an external force of 35 kips are shown in Figs. 11 and 12 respectively. The displacement model produces jumps at element boundaries even for the case with 16 elements, while the mixed model gives smooth variations of the forces. In addition, the mixed model was able to produce the exact bending moment diagram. These results confirm the superiority of the mixed model in describing the behavior of composite steel-concrete girders.

Another model for composite steel-concrete girders worth mentioning is the model by Dall'Asta and Zona (2004a), who proposed a three-field mixed model for composite beams. In this model, the displacements, forces, and curvatures are all approximated with independent interpolation functions. The author believes the three-field mixed model is inferior to the two-field mixed model since it attempts to interpolate the beam curvature with an independent linear interpolation function. Since

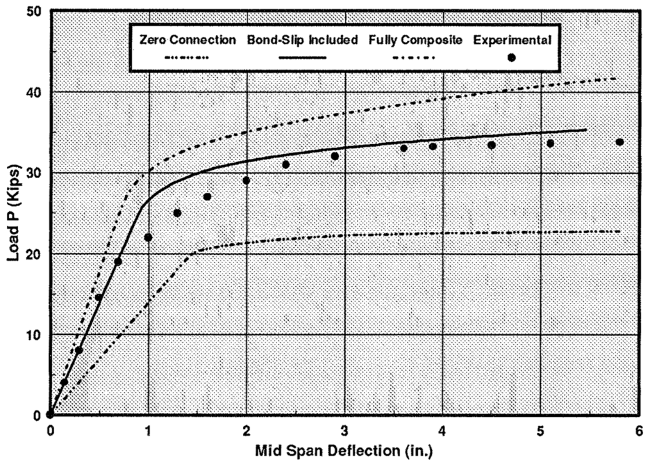


Fig. 10 Global response of McGarraugh specimen
(1 in = 25.4 mm)

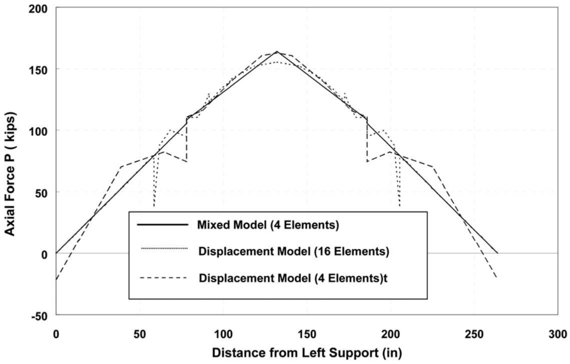


Fig. 11 Axial force distribution of McGarraugh specimen

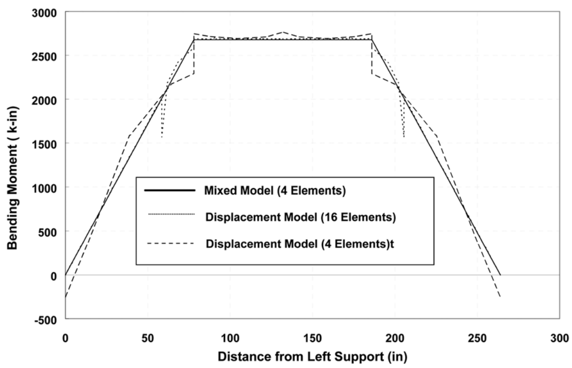


Fig. 12 Bending moment distribution of McGarraugh specimen

the curvature distribution is typically very steep in the plastic zone, this model requires a fine mesh in this region similar to the displacement model.

5. Beam on foundation element

A beam on foundation element is a beam resting on a lateral distributed interface element representing the underlying soil. This problem is different from the two previously discussed applications since the interface element here represents transverse behavior rather than axial. The force-based formulation was developed by Limkatanyu and Spacone (2006) and has the same deficiencies as the ones for the previous applications with axial interfacial behavior. The mixed formulation was developed by Ayoub (2003) and extended by Mullapudi and Ayoub (2010 a,b) for beams on two-parameter foundations. The condition of stability for the mixed model in this case as determined by the B-B condition and De Veubeke's principle requires that the order of the transverse displacement function be higher by two than the order of the bending moment function. Typically, hermitian polynomials are used to approximate the transverse displacements along with a linear moment distribution, although a higher order formulation with cubic and fifth order polynomial approximations for the moments and displacements respectively is also feasible. The mixed model with these choices of interpolation functions satisfies the inf-sup condition as discussed by Bathe (2001) and represents the optimal or best formulation for the problem since it always attempts to approximate the parameters with the most smooth distributions.

The example used to clarify the advantages of the mixed model is an inelastic beam with tensionless elastic support as shown in Fig. 13. The beam length is 6 m, and the cross section is square with 60 cm dimension. The beam uniaxial stress-strain relation is elasto-plastic with young's modulus E=25 GPa, yield strength that equals 17.5 MPa, and a hardening slope that equals 1.6%. The beam section is subdivided into 12 fibers. The elastic foundation stiffness equals 100 MPa. The loading condition

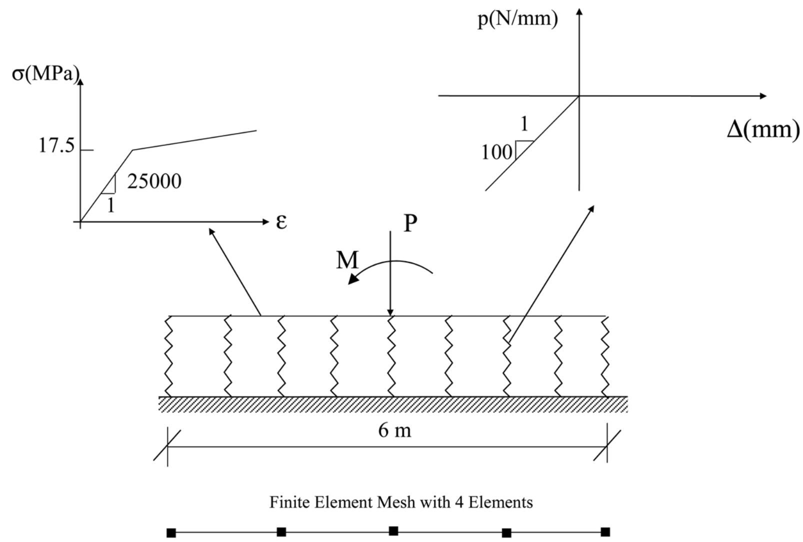


Fig. 13 Beam on winkler support

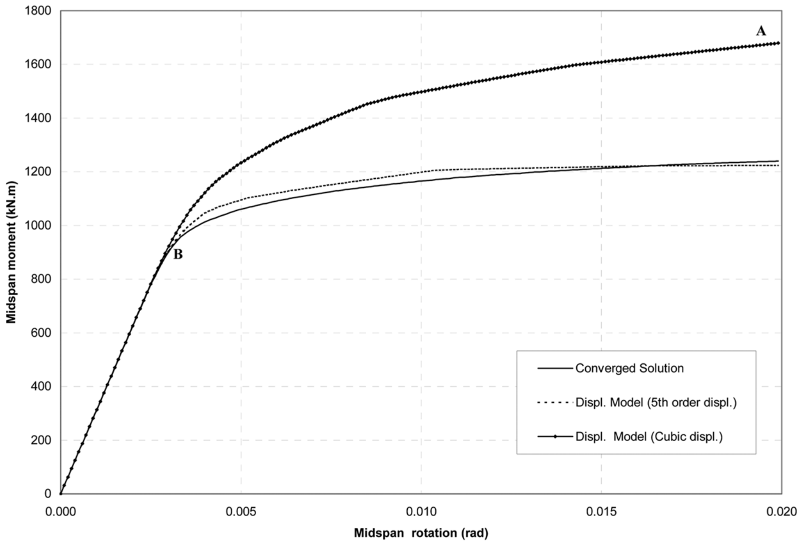


Fig. 14 Global response of beam on winkler support

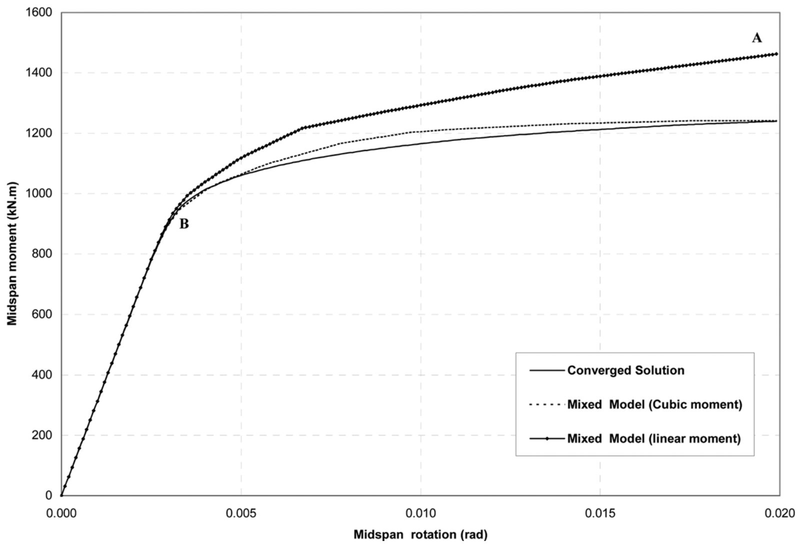


Fig. 15 Global response of beam on winkler support

consists of an axial force and a moment acting at midspan, which is typical of foundation structures. The axial force equals 4000 kN, and is applied under load control, while the moment is applied incrementally under displacement control. The finite element model is also shown in Fig. 13.

The midspan moment rotation behavior of the beam is shown in Figs. 14 and 15 for the displacement and mixed models respectively using 4 finite elements. A mesh consisting of 32 displacement-based elements with 5th order polynomials represents the converged solution. While the lower order models show a large error in the inelastic region, the higher order models captured the global behavior rather

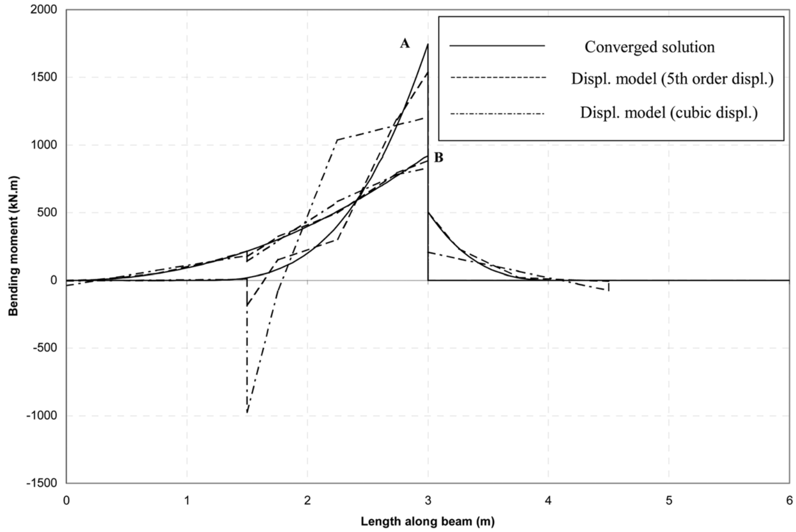


Fig. 16 Moment distribution of beam on winkler support

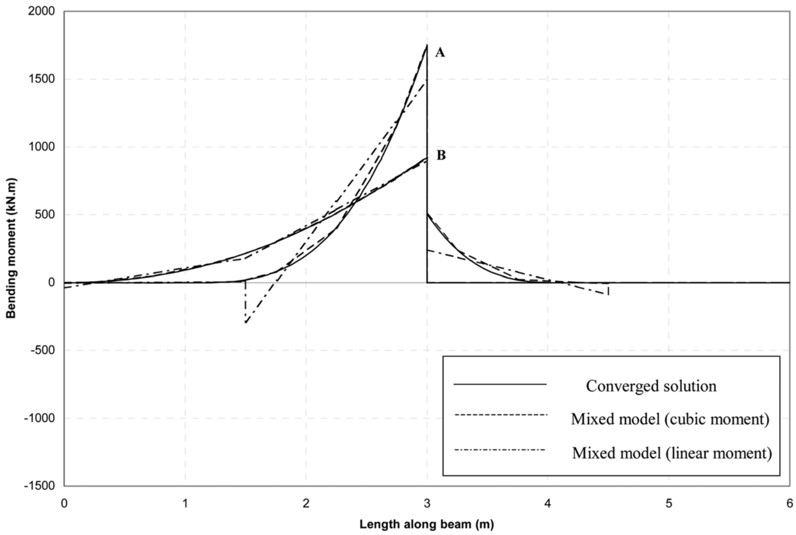


Fig. 17 Moment distribution of beam on winkler support

well. From the plot, it is clear that mixed models have a much higher rate of convergence. Figs. 16 and 17 show the bending moment distribution at load stages A and B identified in the global plot for both the displacement and mixed models respectively. The displacement model shows big jumps at element boundaries as shown in Fig. 16, even with higher order polynomials. The mixed model, on the other hand, successfully describes the bending moment, with the higher order polynomial capturing almost the exact behavior as shown in Fig. 17, confirming the superiority of mixed models.

6. RC girder strengthened with FRP sheets

In a finite element model of a reinforced concrete beam strengthened with FRP sheets, the concrete is modeled with a beam element, the FRP sheets are lumped as truss elements, and a distributed axial interface is used to simulate the behavior of the epoxy resin used to bond the FRP sheets to the concrete surface. A displacement formulation was developed earlier by Aprile *et al*. (2001). The proposed formulation (Lu and Ayoub 2008, 2011) is used to analyze a beam tested by Zarnic *et al*. (1999), which has the geometry shown in Figs. 18 and 19. The Carbon FRP plates width w = 100 mm and their thickness t = 1.2 mm. The concrete compressive strength equals 25 MPa, the steel yield equals 460 MPa, the CFRP modulus equals 140 GPa, and the CFRP strength equals 1800 GPa. A total of 6 finite elements per half span were used along the beam length in order to obtain accurate and detailed interfacial results, as shown in Fig. 20. The experimental and analytical load-deformation results of the CFRP strengthened RC beam are shown in Figs. 21 and 22 respectively.

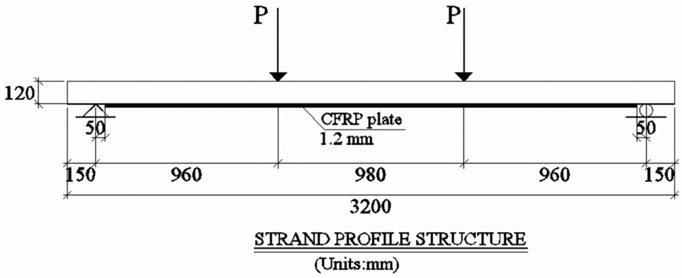


Fig. 18 Zarnic test setup

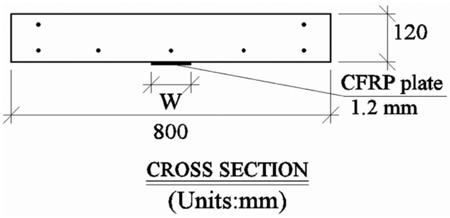


Fig. 19 Cross section of Zarnic specimen

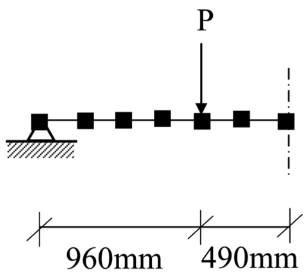


Fig. 20 Finite element model of Zarnic beam with 6 elements

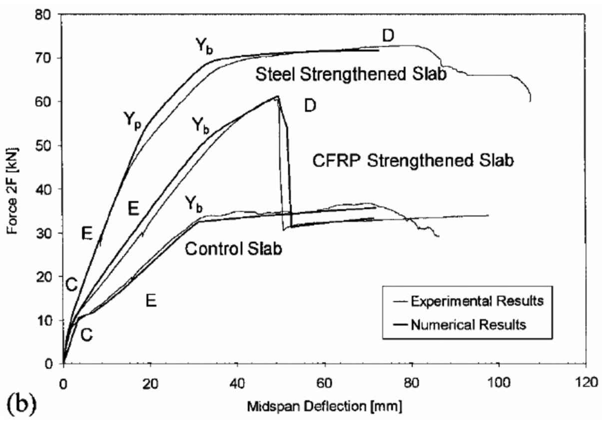


Fig. 21 Experimental global response of Zarnic specimen by Zarnic *et al*. (1999)

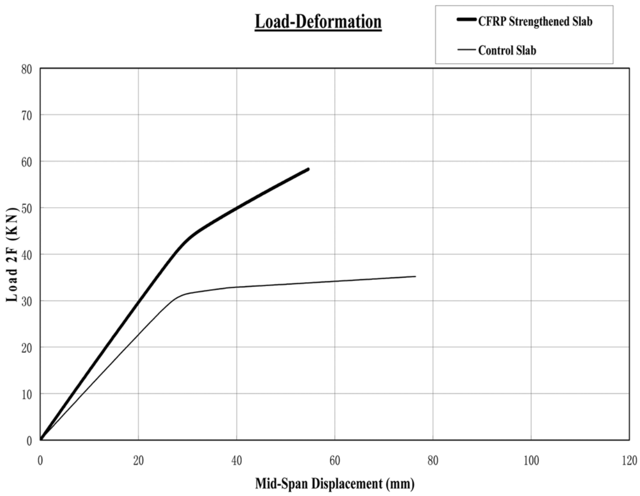


Fig. 22 Analytical global response of Zarnic specimen

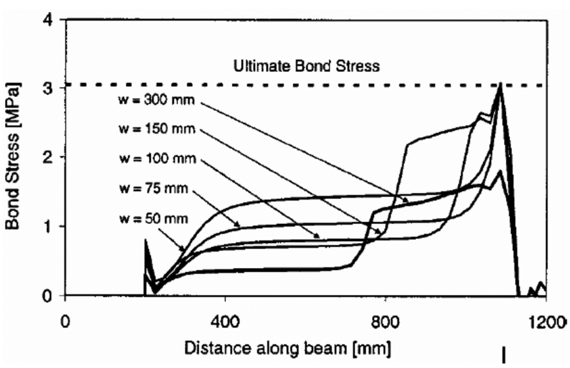


Fig. 23 Bond distribution by Thomsen *et al*. (2004) of Zarnic specimen

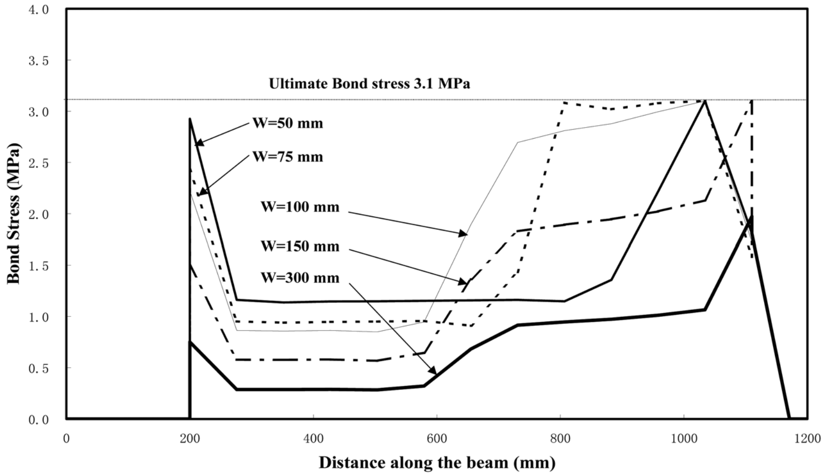


Fig. 24 Bond distribution using proposed model of Zarnic specimen

From the figures, it is clear that the analytical results match well with the experimental ones. The bond stress distribution of Thomsen *et al*. (2004) is shown in Fig. 23. The bond stress distribution from the finite element model is shown in Fig. 24 assuming a bond stiffness of 2,384 MPa/mm and an ultimate bond stress of 3.1 MPa. The results agree with those of Thomsen *et al*. (2004).

7. RC beam element with bond-slip

A reinforced concrete beam element with bond-slip behavior is considered a composite element, since the concrete beam and steel bars are connected by a distributed interfacial bond element. The first model for a beam with bond-slip was proposed by Ngo and Scordelis (1967) followed by Nilson (1971) and Scordelis *et al*. (1974). Fiber beam elements were first proposed by Yassin (1994), who used a force-based approach for RC concrete beams with bond-slip. Monti and Spacone (2000), and Limkatanyu and Spacone (2002) also adopted a force-based approach. Ayoub (2005) adopted a mixed formulation, in which he accounted for pull-out cracks in addition to bond-slip effects.

The proposed model is used to simulate the behavior of the reinforced concrete beam-column specimen tested by Bousias *et al*. (1995) at the ISPRA lab in Europe. The dimensions and loading arrangement of the test column are shown in Fig. 25. The concrete compressive strength is equal to 30.75 MPa, the yield strength of the steel rebars is equal to 460 MPa, and the steel elastic modulus is equal to 210,000 MPa. An anchorage length of 30 bar diameter is used to model the anchorage zone. A mesh of 6 finite elements is used to model the anchorage zone and another 6 elements are used to model the column, as shown in Fig. 26. The experimental, and analytical with bond-slip, tip global load-deflection response of the test column is shown in Fig. 27. The global analytical response obtained with the new model agrees well with the experimentally observed behavior. Fig. 28 shows the top steel force distributions along the column length for different points in the loading history. The different load stages are identified in the global response of Fig. 27. Fig. 29 shows the bond stress distribution along the length for the same load stages identified before. In the anchorage zone, the bond stresses have a low value in the unconfined region due to the formation of a pull-out cone.

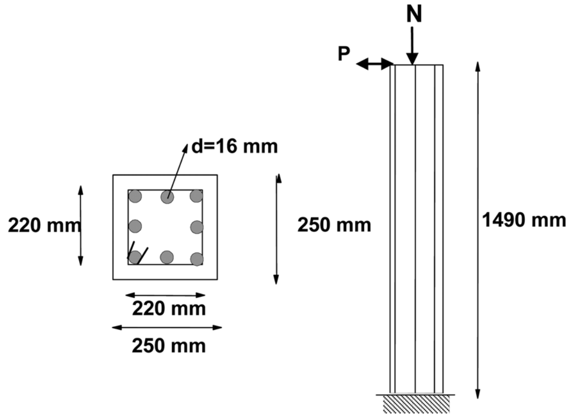
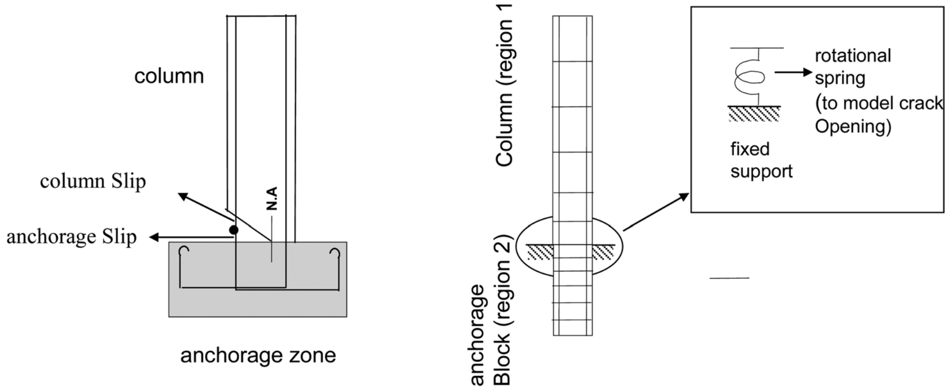


Fig. 25 Bousias test specimen



Column with Crack at Base Finite Element Model

Fig. 26 Finite element model of bousias column

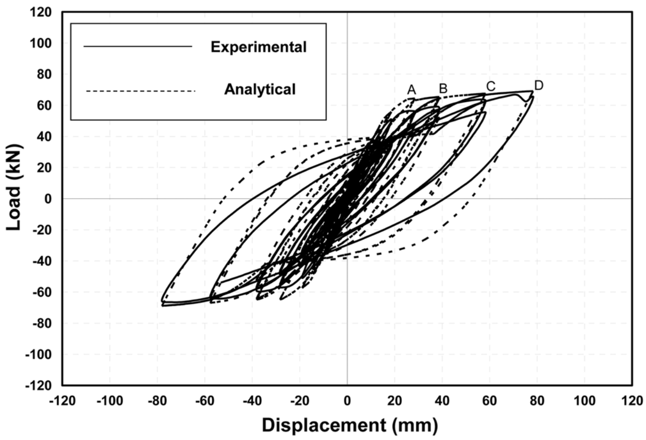


Fig. 27 Global response of bousias specimen

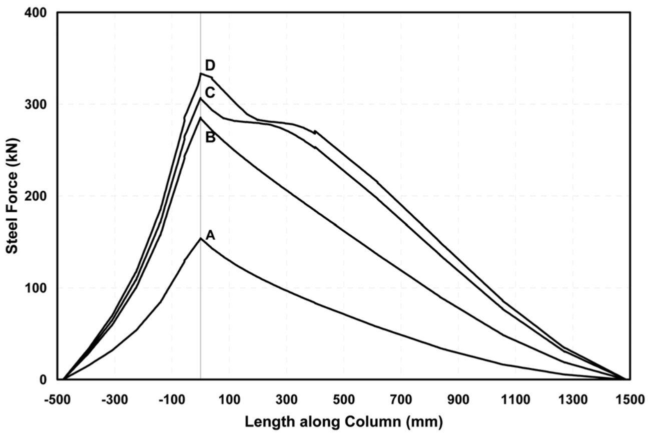


Fig. 28 Steel force distribution of bousias specimen

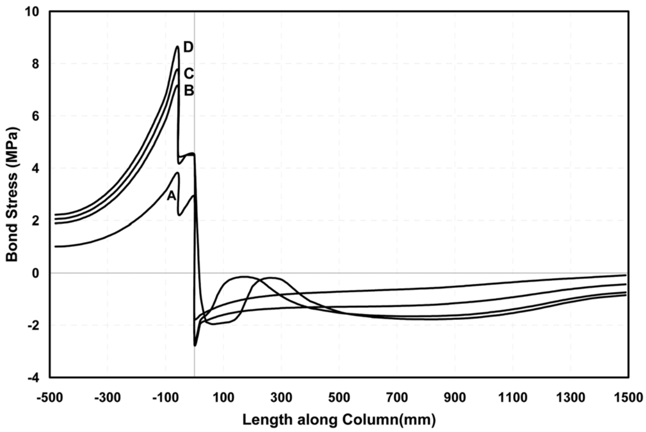


Fig. 29 Bond stress distribution of bousias specimen

A sudden increase is then observed at the start of the confined region, and then an exponential decrease is observed until the end of the anchorage zone. The bond stresses change sign at the support location where the steel stresses are maximum, and are nearly constant along the column length where the steel stresses are almost linear.

8. Prestressed concrete beam element

Reinforced concrete inelastic beam elements can be extended to account for prestressing effects. Modeling of prestressing tendons in finite element analysis can be performed using two methods, as discussed by Aalami (2000), either as a loading applied to the member, or as an additional structural element that contributes to the resistance to the applied loads. In the first method, the load is applied either using load balancing techniques (e.g., Aalami 1990) or through applied primary and secondary moments (e.g., Picard *et al*. 1995). In the second method, Kang (1977), Mari (1984), and Cruz *et al*. (1998) used the displacement formulation, while Kodur and campbell (1990) and Yassin

(1994) used the force formulation. The model by Yassin (1994) was the first to consider bond-slip effects for prestressed members though. Ayoub (1999, 2011) and Ayoub and Filippou (2010) followed Yassin's approach but adopted a two-field mixed formulation.

The mixed formulation was used to model the specimens of Mitchell *et al*. (1993), who tested a series of pretensioned concrete beams to investigate the influence of concrete strength on the transfer length and development length of pretensioning strands. The dimensions and loading arrangements of the test beams are shown in Fig. 30. The beam contained a single 15.7 mm diameter low relaxation strand having an ultimate strength of 1793 MPa and a yield strength of 1639.3 MPa. The elastic modulus was 204.9 GPa, the compressive strength of concrete was 31 MPa, the area and perimeter of the strand were 146.4 mm^2 and 50 mm respectively, the initial prestress in the strand prior to transfer was 1286 MPa. The finite element models used for the analysis consist of 6 prestressed concrete finite elements per half span as shown in Fig. 30. The cross section of the beam-column element is made up of 10 concrete layers at 5 control sections. The bond parameters used for the analysis are based on approximate data given by Tabatabai and Dickson (1993). Fig. 31 shows the

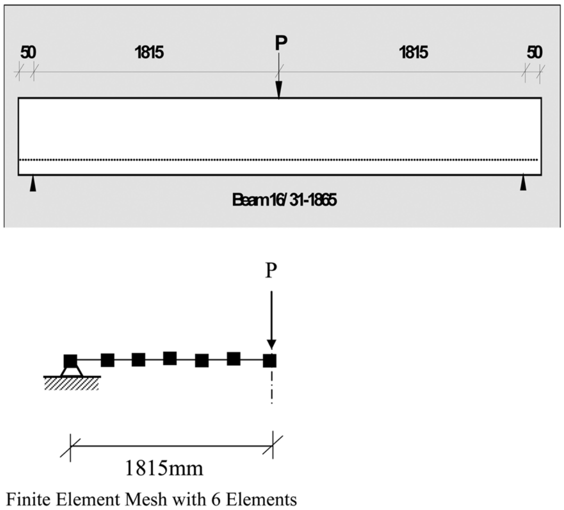


Fig. 30 Mitchell test setup and finite element model

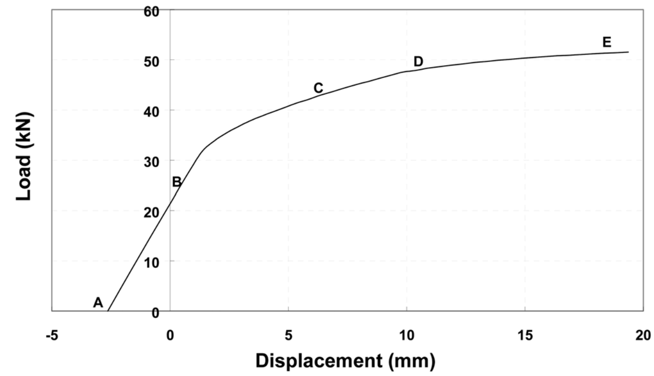


Fig. 31 Global response of mitchell specimen

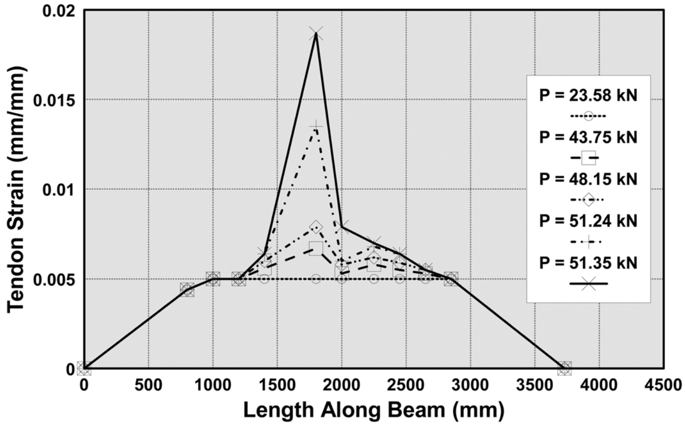


Fig. 32 Experimental tendon strain distribution of mitchell specimen

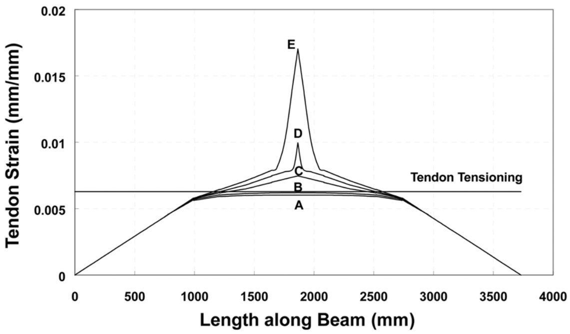


Fig. 33 Analytical tendon strain distribution of mitchell specimen

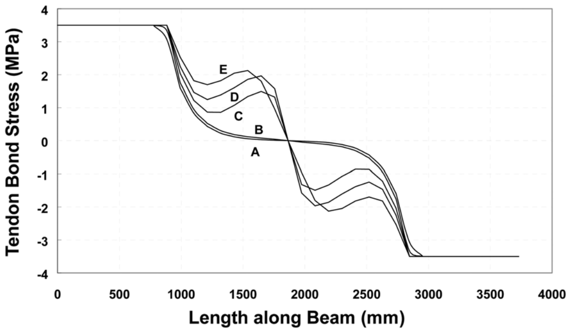


Fig. 34 Bond stress distribution of mitchell specimen

global Load-Displacement response for the beam. Before the application of the load, the displacement shows a negative value, referred to as a camber, which is due to the prestressing force. Yielding of the tendon starts at a load value equals 32 kN. The specimen then shows a ductile behavior. Fig. 32 shows the experimental strain values at different load stages identified in the global response. The maximum measured strand strain was 0.0187, which corresponds to a stress in the strand of 1716 MPa. Fig. 33 shows the analytical strand strain distribution. The analytical results match well with the experimental ones. Fig. 34 shows the bond stress distribution. The bond stress distribution after transfer shows almost constant maximum bond stress occurring within the transfer length, and low values for the rest of the strand. Therefore, bond slip during transfer is confined to the portion of the strand within the transfer length. The bond stress at ultimate loads shows a considerable increase due to the increase in tendon strain in the region near the midspan.

9. Conclusions

The paper presents a discussion of the state of the art in modeling composite frames made-up of more than one material. The recently developed mixed formulation proved to provide the best solution. The element in this case is derived from a two-field mixed approach where both forces and deformations are simultaneously approximated. The bond between the different materials of the structure is accounted for through an interface model with distributed force transfer characteristics. Numerical studies were conducted to compare the model with both the classical displacement and the force formulations. The studies confirmed that the new mixed formulation overcomes the limitations associated with both models, while combining most of their advantages, and is therefore considered the preferred approach for modeling of composite structures. The paper presents the application of the newly developed element to modeling of different structural problems: anchored reinforcing bar problems, composite steel-concrete girders with deformable shear connectors, beam on elastic foundation elements, R/C girders strengthened with FRP sheets, R/C beam-columns with bond-slip, and prestressed concrete girders. These studies confirmed the superiority and practicality of the proposed approach in modeling all types of composite structures under inelastic loading conditions.

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