Structural Engineering and Mechanics, Vol. 40, No. 1 (2011) 65-84
DOI: http://dx.doi.org/10.12989/sem.2011.40.1.065

A simple mathematical model for static analysis of tall buildings with two outrigger-belt truss systems

Reza Rahgozar*[1], Ali Reza Ahmadi[2a], Omid Hosseini[3b] and Mohsen Malekinejad[3c]

[1]*Department of Civil Engineering, Shahid Bahonar University of Kerman, Kerman, Iran*
[2]*Department of Civil and Mechanical Engineering, International Center for Science and High Technology, Mahan, Iran*
[3]*Department of Civil Engineering, Shahid Bahonar University of Kerman, Kerman, Iran*

(Received June 22, 2010, Revised April 26, 2011, Accepted June 24, 2011)

Abstract. In this paper a simple mathematical model for approximate static analysis of combined system of framed tube, shear core and two outrigger-belt truss structures subjected to lateral loads is presented. In the proposed methodology, framed tube is modeled as a cantilevered beam with a box section and interaction between shear core and outrigger-belt truss system with framed tube is modeled using torsional springs placed at location of outrigger-belt truss; these torsional springs act in a direction opposite to rotation generated by lateral loads. The effect of shear lag on axial deformation in flange is quadratic and in web it is a cubic function of geometry. Here the total energy of the combined system is minimized with respect to lateral deflection and rotation in plane section. Solution of the resulting equilibrium equations yields the unknown coefficients of shear lag along with the stress and displacement distributions. The results of a numerical example, 50 storey building subjected to three different types of lateral loading obtained from SAP2000 are compared to those of the proposed method and the differences are found to be reasonable. The proposed method can be used during the preliminary design stages of a tall building and can provide a better understanding of the effects of various parameters on the overall structural behavior.

Keywords: tall building; framed tube; shear core; outrigger-belt truss; shear lag; static analysis

1. Introduction

There are many structural systems such as rigid frame, braced frame and shear-walled frame, framed tube, braced tube, bundled tube and outrigger-belt truss systems that can be used to improve the lateral resistance in tall buildings (Halis and Emer 2007). The framed tube system is widely accepted as an efficient structural system in resisting lateral loads on tall buildings. This system consists of closely spaced perimeter columns tied at each floor level by spandrel beams to form a tubular structure. Under lateral load, a framed tube acts like a cantilever box beam, and the overturning moment due to the lateral load resisted by axial stresses in the columns of the frame

*Corresponding author, Associate Professor, E-mail: rahgozar@mail.uk.ac.ir
[a]Associate Professor, E-mail: iran.mahan@gmail.com
[b]Postgraduate Student, E-mail: hosseiniomid768@yahoo.com
[c]Ph.D. Candidate, E-mail: mohsen_maleki1363@yahoo.com

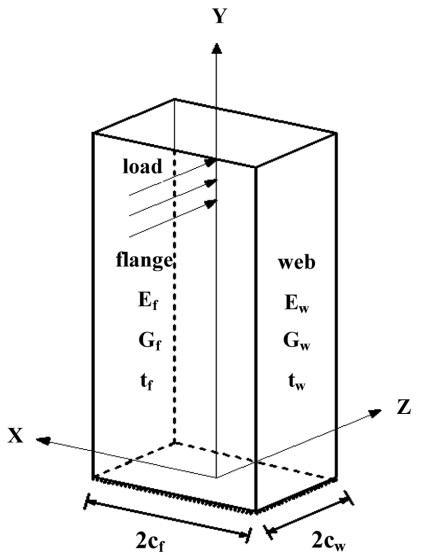

Fig. 1 Orthotropic membrane tube in framed tube

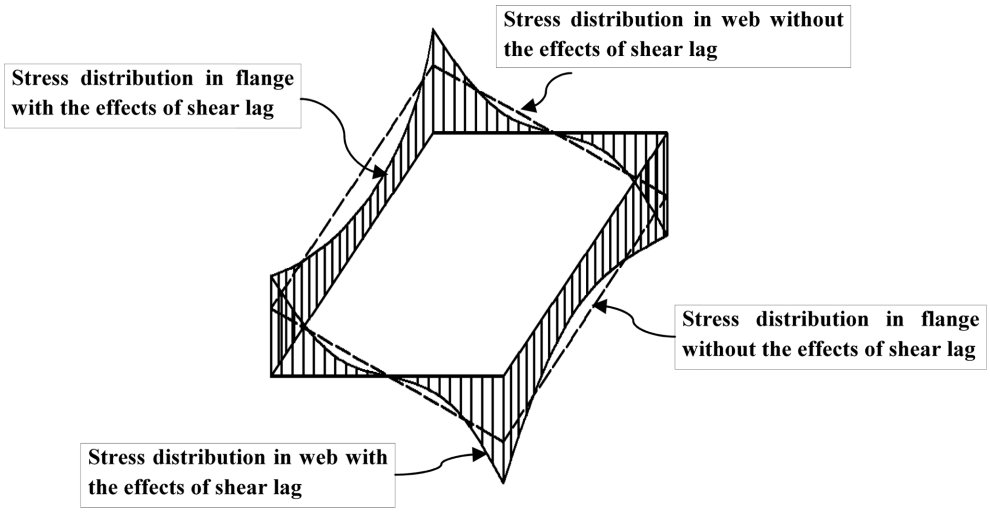

Fig. 2 Stress distributions in web and flange of panels

panels in the perimeter of the structure. The overall bending behavior of the tubular building is similar to that of a box girder, though shear deformations, generally neglected in box girders, play an important role in tubular buildings (Fig. 1). In framed tube structures, the flexibility of the spandrel beams produces a shear lag with the effect of increasing axial stresses in corner columns and of reducing them at the middle of the frame panels (Fig. 2). This effect produces warping of the floor slabs and deformation of secondary structures. Over the past decades many researchers have investigated the behavior of framed tube system subjected to lateral loads (Coull and Subedi 1971, Coull and Bose 1975, 1976, 1977, Coull and Ahmed 1978, Connor and Pouangare 1991, Saffari *et al.* 2003). Considerable effort has been devoted to understanding the shear lag effects in framed

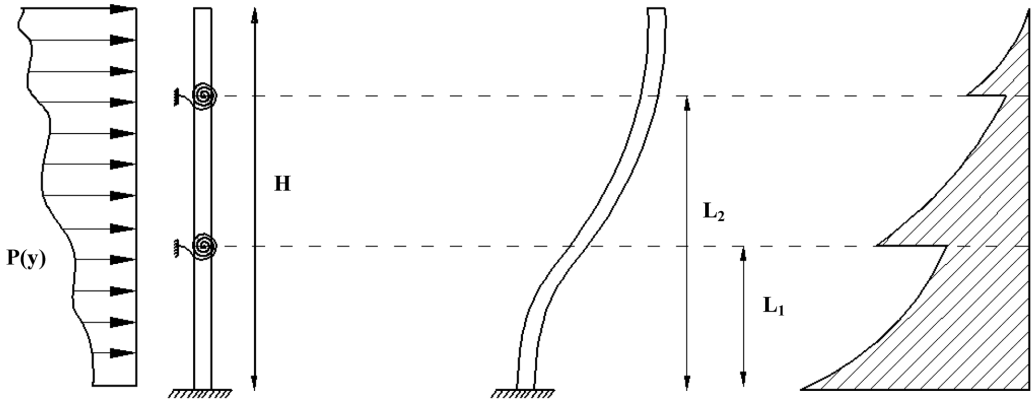

Fig. 3 Behavior of shear core and outrigger-belt truss system

tube structures (Foutch and Chang 1982, Chang and Zheng 1987, Shushkewich 1991, Singh and Nagpal 1993, Takabatake *et al.* 1993, Kwan 1994, 1996).

Control of drift is important matter in design of tall buildings. Many researchers have investigated drift of tall buildings (Lam *et al.* 2010, Takewaki and Tsujimoto 2011). In order to control drift, stresses at structure's base and the effects of shear lag in framed tube system, combined system of framed tube, shear core and outrigger-belt truss can be used. A method of obtaining additional stiffness is to specially design two or three floor levels in order to provide stiff points of resistance in the structure. In these levels, stiff outrigger arms are used to activate a perimeter truss which in turn causes the axial resistance of exterior columns. It has been pointed out by Taranath (1988) that the partial fixity of outrigger truss pulls the moment diagram back at these levels (Fig. 3); consequently, the bending moment at the base of the building is further reduced because of the greater translation of lateral forces to axial forces. The effect of outrigger flexibility on the drift and moments, and their optimum locations in multi-outrigger systems were investigated by Smith and Salim (1981). Hoenderkamp and Bakker (2003) presented a graphical method of analysis for the preliminary design of tall building structures comprising braced frames with outrigger trusses subjected to horizontal loading. The analysis allows a simple procedure for obtaining the optimum location of the outrigger up the height of the structure and a rapid assessment of the impact of the outrigger on the behavior of the high-rise structure. Gerasimidis *et al.* (2009) implemented a basic design optimization technique of tall steel structures for lateral loads, mainly wind, into trying to find the optimum locations and number of outriggers for a specific high-rise building. The geometry produced by stress based design, although below the stress limit, was very flexible and exhibits horizontal displacements and interstory drifts much above the acceptable limits, due to the wind loading. Then the structure was analyzed with all the possible outrigger locations monitoring important factors, such as the drift of the building or the moments on the core. Static analysis and optimum location of one or more outrigger and belt truss structures are considered in different types of tall buildings by other researchers such as Taranath (1975), McNabb and Muvdi (1975), Ding (1991), Zhang *et al.* (1996), Gao (1998), Fu (1999). A simple mathematical model for static and dynamic analysis of a combined system of framed tube, shear core and one outrigger-belt truss have been developed (Rahgozar and Sharifi 2009, Rahgozar *et al.* 2010, Malekinejad and Rahgozar 2011).

In this article a simple approximate model for the combined system of framed tube, shear core and two outrigger-belt truss structure based on research work conducted by (Rahgozar and Sharifi 2009, Rahgozar *et al.* 2010) is presented. Framed tube is modeled as a cantilevered hollow section beam with orthotropic plates and equivalent properties (Ha *et al.* 1978, Takabatake *et al.* 1993, Kwan 1994). Effects of outrigger-belt truss systems is modeled as torsional springs with constant rotational stiffness which act as concentrated moment positioned at different levels (Fig. 3). Stress distributions in web-frame and flange-frame systems are assumed to be cubic and quadratic functions as shown in (Fig. 2). Here total potential energy is minimized to derive equations of structural deformation and stresses at the base of the structure. The approximate method is used for static analysis of a 50 storey building that is subjected to three types of lateral loading and results are compared with SAP2000 software program; so to investigate the efficiency and accuracy of the proposed analytical procedure. Good agreements between the results have been observed; and it may be concluded that the proposed method can be utilized during the preliminary design stages.

2. Analytical modeling and basic assumptions of combined system

The research carried out by Rahgozar *et al.* (2010) was a useful introduction to analytic work on a combined system of framed tube, shear core, and outrigger-belt truss. The following assumptions have been made to simplify the modeling process and the associated analysis (Kwan 1994, Rahgozar and Sharifi 2009, Rahgozar *et al.* 2010, Malekinejad and Rahgozar 2011): (a) Behavior of the structure is linear and elastic, (b) Floor slabs are rigid diaphragms, (c) Core is fully fixed at the base, (d) Connection between outrigger and shear core is rigid, (e) Structure is assumed to be symmetric in plan and height and hence it cannot twist, (f) Beam and column spacings are uniform through the building height, (g) Outrigger-belt truss members have the same cross sectional area, (h) Beams and columns have uniform cross sections, (i) Effects of interaction between the outrigger-belt truss system with shear core and framed tube structure is modeled as a torsional spring at outrigger-belt truss location and finally, (j) The stress concentration occurring in the location of belt truss is neglected.

The distribution of axial displacements in the web, D_w, and flange, D_f, of framed tube are considered to be cubic and parabolic respectively and can be denoted by following equations (Kwan 1994)

$$D_w = \psi c_w\left[(1-\xi)\frac{z}{c_w} + \xi\left(\frac{z}{c_w}\right)^3\right] \tag{1}$$

$$D_f = \psi c_w\left[(1-\eta) + \eta\left(\frac{x}{c_f}\right)^2\right] \tag{2}$$

where ψ is the rotation of a plane section that joins the four corners of a tubular structure which initially lies on the same horizontal plane; ξ and η are dimensionless shear lag coefficients representing the degrees of shear lag in the web and flange panels, respectively (Kwan 1994). $2c_w$ and $2c_f$ are the width of web and flange dimensions for box section respectively; x and z are coordinates of any plane along the height of structure (Fig. 1). The axial strains in web, ε_w, and flange, ε_f, of structure are as follows (Kwan 1994)

$$\varepsilon_w = \frac{\partial D_w}{\partial y} \tag{3}$$

$$\varepsilon_f = \frac{\partial D_f}{\partial y} \tag{4}$$

Similarly the shear strains in web panels, γ_{zy}, and flange panels, γ_{xy}, can be expressed by following equations (Kwan 1994)

$$\gamma_{zy} = \frac{\partial w}{\partial y} + \frac{\partial D_w}{\partial z} \tag{5}$$

$$\gamma_{xy} = \frac{\partial D_f}{\partial x} \tag{6}$$

where w is lateral displacement in z direction.

It is suggested by Rahgozar and Sharifi (2009) that the total potential energy of a combined system may be calculated by summing the strain energy (U) and the work (V) done by external forces

$$\Pi = U + V \tag{7}$$

Strain energy of web and flange in the combined system can be stated by the following relations (Rahgozar and Sharifi 2009)

$$U_w = \frac{1}{2}\int_v (E_w \varepsilon_w^2 + G_w \gamma_{zy}^2) dv \tag{8}$$

$$U_f = \frac{1}{2}\int_v (E_f \varepsilon_f^2 + G_f \gamma_{xy}^2) dv \tag{9}$$

From these axial and shear strain equations, strain energy of the framed tube can be evaluated by the following relation (Rahgozar and Sharifi 2009)

$$\Pi_e = \int_0^H \int_{-c_f}^{c_f} t_f (E_f \varepsilon_f^2 + G_f \gamma_{xy}^2) dx dy + \int_0^H \int_{-c_w}^{c_w} t_w (E_w \varepsilon_w^2 + G_w \gamma_{zy}^2) dz dy + \int_0^H 2 E_r A_r \varepsilon_r^2 dy \tag{10}$$

where, E_f, G_f, E_w and G_w are the elastic and shear modulus of flange and web frames respectively; E_r is modulus of elasticity for the four corner columns, A_r is the cross sectional area of each corner column, ε_r is the axial strain of each corner column, t_f and t_w are equivalent thickness for flange and web frames and H is total height of the frame. Potential energy for concentrated, uniformly, and triangularly distributed loads is given by the following equations respectively (Kwan 1994)

$$\Pi_p = -Cw(H) \tag{11}$$

$$\Pi_p = -\int_0^H U_o w(y) dy \tag{12}$$

$$\Pi_p = -\int_0^H T_o \frac{y}{H} w(y) dy \tag{13}$$

In Eqs. (11)-(13), $w(y)$ is the lateral displacement at height y as measured from the base of the structure. C, U_o and T_o, are concentrated load, intensity of uniformly distributed load, and intensity of the triangular load, respectively. Furthermore, potential energy due to action of outrigger-belt truss can be expressed as (Rahgozar *et al.* 2010, Malekinejad and Rahgozar 2011)

$$\Pi_p = K_i\theta_i^2 \qquad (i = 1, 2) \tag{14}$$

where K_i is the rotational stiffness of outrigger-belt truss system and θ_i is the rotation of the frame at outrigger-belt truss location.

Total potential energy is the sum of the potential energy of applied force and strain energy of the structure. Equilibrium equations are obtained by minimizing the total potential energy with respect to ψ, w, and shear lag coefficients, resulting in a system of four second order partial differential equations.

Minimization of the total potential energy with respect to ψ yields the moment equilibrium equation (Kwan 1994)

$$M = EI\frac{\partial \psi}{\partial y} \tag{15}$$

where EI is the effective bending stiffness of tubular structure and M is the overturning moment caused by lateral loading at position y.

Since the value of EI varies with height, the resulting expressions for Eq. (15) are rather complicated. However, since as most of the bending deformations occur near the base and exact values of EI near the top do not really affect structure's response, hence, variation of EI along the height may be neglected and the value of EI at the base is used throughout building height. This is equivalent to assuming that the structure behaves like a cantilevered beam with a constant bending stiffness of EI. Therefore bending rotation ψ can be expressed as follows (Kwan 1994)

$$\psi = \frac{1}{EI}\int_0^y M dy \tag{16}$$

If the total potential energy is minimized with respect to unknown parameter, w, the following equation is obtained (Kwan 1994)

$$F = 4G_w t_w c_w\left(\frac{\partial w}{\partial y} + \psi\right) \tag{17}$$

where F is the shear force caused by of lateral loads. From this equation, displacement of structure along the direction of lateral loading can be determined as follows (Kwan 1994)

$$w = \int_0^y \left(\frac{F}{4G_w t_w c_w} - \psi\right) dy \tag{18}$$

Substituting Eqs. (16) and (18) into total potential energy and minimizing with respect to unknown shear lag parameters, ξ and η, two more equations are obtained. For sake of simplicity these parameters ξ and η may be estimated using a quadric polynomial function with unknown coefficients that have to satisfy the given boundary conditions. Concentrated moment at the outrigger-belt truss locations requires $d\psi/dy = 0$ which leads to $d\xi/dy = 0$ and $d\eta/dy = 0$ for $y > L_i$; the following equations for ξ and η for concentrated moment at the location of outrigger-belt truss are as follows

$$\xi_{M_i} = \xi_1\left(1-\frac{y}{L_i}\right)^2 + \xi_2\left[2\left(\frac{y}{L_i}\right)-\left(\frac{y}{L_i}\right)^2\right] \tag{19}$$

$$\eta_{M_i} = \eta_1\left(1-\frac{y}{L_i}\right)^2 + \eta_2\left[2\left(\frac{y}{L_i}\right)-\left(\frac{y}{L_i}\right)^2\right] \tag{20}$$

Subscript M in ξ_{M_i} and η_{M_i} in Eqs. (19) and (20) demonstrates the shear lag coefficients created by concentrated moment.

where ξ_1, ξ_2, η_1 and η_2 are unknown coefficients to be solved. Substituting the foregoing equations into the expression for total potential energy and minimizing it with respect to ξ_1, ξ_2, η_1 and η_2, and solving the resulting set of algebraic equations, shear lag coefficients due to concentrated moments at outrigger-belt truss locations are obtained. ξ and η parameters for concentrated loads applied at the top of structure and for uniformly and triangularly distributed loadings have been given by Kwan (1994) and are used herein. These parameters for concentrated moment at outrigger-belt truss location are

$$\xi_{M_1} = \frac{1.03-1.92r_w}{r_w^2+2.49r_w+0.59} \tag{21}$$

$$\xi_{M_2} = \frac{1.03+0.77r_w}{r_w^2+2.49r_w+0.59} \tag{22}$$

$$\eta_{M_1} = \frac{12.92-5.77r_f}{r_f^2+10.46r_f+10.34} \tag{23}$$

$$\eta_{M_2} = \frac{12.92+2.31r_f}{r_f^2+10.46r_f+10.34} \tag{24}$$

where in Eqs. (21)-(24), the coefficients r_w and r_f are

$$r_w = \frac{L_i^2 G_w}{E_w c_w^2} \quad (i=1,2) \tag{25}$$

$$r_f = \frac{L_i^2 G_f}{E_f\, c_f^2} \quad (i=1,2) \tag{26}$$

In above equations L_i is the location of outrigger-belt truss system as measured from the base of the structure. It should be noted from the formulae given above, that the shear lag coefficients of a frame panel and concentrated moment at outrigger-belt truss location are dependent only on elastic properties of that particular panel and not on any other panel.

3. Axial stresses in web and flange of the combined system

Here, first by calculating the shear lag coefficients, axial displacement and axial stresses in flange and web are then obtained. Using axial stress distribution in y direction, axial strains are derived

next. Then, by considering the stiffness properties of the outrigger-belt truss structure along with the axial strain and equivalent elastic modulus for each panel, equations related to distribution of axial displacement at the outrigger-belt truss location can be derived. For two locations of L_1 and L_2 equations of axial stress distribution in web and flange are derived as follows

for $y > L_2$

$$\sigma_w = E_w \frac{d\psi}{dy} c_w \left[(1-\xi)\frac{z}{c_w} + \xi\left(\frac{z}{c_w}\right)^3 \right] \tag{27}$$

$$\sigma_f = E_f \frac{d\psi}{dy} c_w \left[(1-\eta) + \eta\left(\frac{x}{c_f}\right)^2 \right] \tag{28}$$

for $L_1 < y \le L_2$

$$\sigma_w = E_w \frac{d\psi}{dy} c_w \left[(1-\xi) c_f \frac{z}{c_w} + \xi\left(\frac{z}{c_w}\right)^3 \right] - E_w \frac{K_2\theta_2}{EI_2} c_w \left[(1-\xi_{M_2})\frac{z}{c_w} + \xi_{M_2}\left(\frac{z}{c_w}\right)^3 \right] \tag{29}$$

$$\sigma_f = E_f \frac{d\psi}{dy} c_w \left[(1-\eta) + \eta\left(\frac{x}{c_f}\right)^2 \right] - E_f \frac{K_2\theta_2}{EI_2} c_w \left[(1-\eta_{M_2}) + \eta_{M_2}\left(\frac{x}{c_f}\right)^2 \right] \tag{30}$$

for $y < L_1$

$$\begin{aligned}\sigma_w = {} & E_w \frac{d\psi}{dy} c_w \left[(1-\xi)\frac{z}{c_w} + \xi\left(\frac{z}{c_w}\right)^3 \right] - E_w \frac{K_2\theta_2}{EI_2} c_w \left[(1-\xi_{M_2})\frac{z}{c_w} + \xi_{M_2}\left(\frac{z}{c_w}\right)^3 \right] \\ & - E_w \frac{K_1\theta_1}{EI_1} c_w \left[(1-\xi_{M_1})\frac{z}{c_w} + \xi_{M_1}\left(\frac{z}{c_w}\right)^3 \right]\end{aligned} \tag{31}$$

$$\begin{aligned}\sigma_f = {} & E_f \frac{d\psi}{dy} c_w \left[(1-\eta) + \eta\left(\frac{x}{c_f}\right)^2 \right] - E_f \frac{K_2\theta_2}{EI_2} c_w \left[(1-\eta_{M_2}) + \eta_{M_2}\left(\frac{x}{c_f}\right)^2 \right] \\ & - E_f \frac{K_1\theta_1}{EI_1} c_w \left[(1-\eta_{M_1}) + \eta_{M_1}\left(\frac{z}{c_f}\right)^2 \right]\end{aligned} \tag{32}$$

In Eqs. (27)-(32), σ_w and σ_f are axial stress distribution in web and flange, respectively and ξ_{M_1}, ξ_{M_2}, η_{M_1} and η_{M_2} are shear lag coefficients in web and flange of the framed tube subjected to concentrated moments at locations L_1 and L_2. At the upper levels of this structure, ψ is not zero, hence, the effect of this parameter in axial stress distribution must be considered; also, the terms associated with ψ as compared to other terms in Eqs. (27)-(32) are negligible. Axial deformations which are developed using Eqs. (27)-(32) are dependent on the unknown parameter $d\psi/dy$ which is determined by substituting stresses that are created from applying concentrated moments, in the moment equilibrium equation of framed tube. Consequently, the following equation is obtained

$$M = EI\frac{d\psi}{dy} = \int_{-c_w}^{c_w} 2t_w(\sigma_y)_w z\,dz + \int_{-c_f}^{c_f} 2t_f(\sigma_y)_f c_w\,dx + 4A_r\sigma_r c_w \tag{33}$$

where σ_r is the axial stress in each corner column of the framed tube structure. Substituting $d\psi/dy$ with M/EI in moment equilibrium equation, the amount of equivalent bending stiffness (EI) for the

three types of loading noted earlier is obtained as follows

$$EI = \frac{4}{3}E_w t_w c_w^3\left(1-\frac{2}{5}\xi\right) + 4E_f t_f c_w^2 c_f\left(1-\frac{2}{3}\eta\right) + 4E_r A_r c_w^2 \tag{34}$$

Also the equivalent stiffness of concentrated moments placed at the outrigger-belt truss locations can be calculated using the following equations

$$EI_1 = \frac{4}{3}E_w t_w c_w^3\left(1-\frac{2}{5}\xi_{M_1}\right) + 4E_f t_f c_w^2 c_f\left(1-\frac{2}{3}\eta_{M_1}\right) + 4E_r A_r c_w^2 \tag{35}$$

$$EI_2 = \frac{4}{3}E_w t_w c_w^3\left(1-\frac{2}{5}\xi_{M_2}\right) + 4E_f t_f c_w^2 c_f\left(1-\frac{2}{3}\eta_{M_2}\right) + 4E_r A_r c_w^2 \tag{36}$$

where EI_1 and EI_2 are equivalent stiffness of concentrated moments placed at L_1 and L_2 locations.

Axial stresses in web and flange of the structure can be derive by calculating the amount of equivalent rotational stiffness in outrigger-belt truss and structure's rotation at the outrigger-belt truss location.

4. Lateral displacement of the combined system

Lateral displacement of a structure subjected to different types of lateral loading, i.e., the concentrated load at the top, uniformly or triangularly distributed loads along structure's height can be evaluated by first substituting EI, EI_1 and EI_2 into Eq. (16) to obtain ψ and then substituting it into Eq. (18). For obtaining lateral displacement of the combined system, it has been assumed that EI, EI_1 and EI_2 are constant along structure's height and are equal to their value at the base of structure (Kwan 1994). Behavior of this structure is then simulated using the equivalent cantilevered beam with box section model. Finally the lateral displacement of the combined system subjected to each type of lateral loadings is obtained and are as follows

Concentrated loading

for $y > L_2$

$$w = \frac{C}{EI}\left(\frac{1}{2}Hy^2 - \frac{1}{6}y^3\right) + \frac{C}{4G_w t_w c_w}y - \left(\frac{K_1\theta_{C_1}}{EI_1}\left(L_1 y - \frac{L_1^2}{2}\right) + \frac{K_2\theta_{C_2}}{EI_2}\left(L_2 y - \frac{L_2^2}{2}\right)\right) \tag{37}$$

for $L_1 < y \le L_2$

$$w = \frac{C}{EI}\left(\frac{1}{2}Hy^2 - \frac{1}{6}y^3\right) + \frac{C}{4G_w t_w c_w}y - \left(\frac{K_1\theta_{C_1}}{EI_1}\left(L_1 y - \frac{L_1^2}{2}\right) + \frac{K_2\theta_{C_2}}{2EI_2}y^2\right) \tag{38}$$

for $y < L_1$

$$w = \frac{C}{EI}\left(\frac{1}{2}Hy^2 - \frac{1}{6}y^3\right) + \frac{C}{4G_w t_w c_w}y - \frac{1}{2}y^2\left(\frac{K_1\theta_{C_1}}{EI_1} + \frac{K_2\theta_{C_2}}{EI_2}\right) \tag{39}$$

Uniformly distributed loading

for $y > L_2$

$$w = \frac{U_o}{EI}\left(\frac{1}{4}H^2y^2 - \frac{1}{6}Hy^3 + \frac{1}{24}y^4\right) + \frac{U_o}{4G_w t_w c_w}\left(Hy - \frac{1}{2}y^2\right) - \left(\frac{K_1\theta_{D_1}}{EI_1}\left(L_1 y - \frac{L_1^2}{2}\right) + \frac{K_2\theta_{D_2}}{EI_2}\left(L_2 y - \frac{L_2^2}{2}\right)\right) \quad (40)$$

for $L_1 < y \le L_2$

$$w = \frac{U_o}{EI}\left(\frac{1}{4}H^2y^2 - \frac{1}{6}Hy^3 + \frac{1}{24}y^4\right) + \frac{U_o}{4G_w t_w c_w}\left(Hy - \frac{1}{2}y^2\right) - \left(\frac{K_1\theta_{D_1}}{EI_1}\left(L_1 y - \frac{L_1^2}{2}\right) + \frac{K_2\theta_{D_2}}{2EI_2}y^2\right) \quad (41)$$

for $y < L_1$

$$w = \frac{U_o}{EI}\left(\frac{1}{4}H^2y^2 - \frac{1}{6}Hy^3 + \frac{1}{24}y^4\right) + \frac{U_o}{4G_w t_w c_w}\left(Hy - \frac{1}{2}y^2\right) - \frac{1}{2}y^2\left(\frac{K_1\theta_{D_1}}{EI_1} + \frac{K_2\theta_{D_2}}{EI_2}\right) \quad (42)$$

Triangularly distributed loading

for $y > L_2$

$$w = \frac{T_o}{EI}\left(\frac{1}{6}H^2y^2 - \frac{1}{12}Hy^3 + \frac{1}{120}\frac{y^5}{H}\right) + \frac{T_o}{4G_w t_w c_w}\left(\frac{Hy}{2} - \frac{1}{6}\frac{y^3}{H}\right) - \left(\frac{K_1\theta_{T_1}}{EI_1}\left(L_1 y - \frac{L_1^2}{2}\right) + \frac{K_2\theta_{T_2}}{EI_2}\left(L_2 y - \frac{L_2^2}{2}\right)\right) \quad (43)$$

for $L_1 < y \le L_2$

$$w = \frac{T_o}{EI}\left(\frac{1}{6}H^2y^2 - \frac{1}{12}Hy^3 + \frac{1}{120}\frac{y^5}{H}\right) + \frac{T_o}{4G_w t_w c_w}\left(\frac{Hy}{2} - \frac{1}{6}\frac{y^3}{H}\right) - \left(\frac{K_1\theta_{T_1}}{EI_1}\left(L_1 y - \frac{L_1^2}{2}\right) + \frac{K_2\theta_{T_2}}{2EI_2}y^2\right) \quad (44)$$

for $y < L_1$

$$w = \frac{T_o}{EI}\left(\frac{1}{6}H^2y^2 - \frac{1}{12}Hy^3 + \frac{1}{120}\frac{y^5}{H}\right) + \frac{T_o}{4G_w t_w c_w}\left(\frac{Hy}{2} - \frac{1}{6}\frac{y^3}{H}\right) - \frac{1}{2}y^2\left(\frac{K_1\theta_{T_1}}{EI_1} + \frac{K_2\theta_{T_2}}{EI_2}\right) \quad (45)$$

Analogously the rotation of combined system (θ) at outrigger-belt truss locations L_1 and L_2, for each load type are as follows

Concentrated loading

$$\theta_{C_1} = \frac{EI_1}{S_1L_1S_2L_2 - K_1K_2L_1^2}\left[\frac{C}{EI}\left(S_2L_2\left(HL_1 - \frac{L_1^2}{2}\right) - K_2L_1\left(HL_2 - \frac{L_2^2}{2}\right)\right) + \frac{C}{4G_w t_w c_w}(S_2L_2 - K_2L_1)\right] \quad (46)$$

$$\theta_{C_2} = \frac{EI_2}{S_1L_1S_2L_2 - K_1K_2L_1^2}\left[\frac{C}{EI}\left(S_1L_1\left(HL_2 - \frac{L_2^2}{2}\right) - K_1L_1\left(HL_1 - \frac{L_1^2}{2}\right)\right) + \frac{C}{4G_w t_w c_w}(S_1L_1 - K_1L_1)\right] \quad (47)$$

Uniformly distributed loading

$$\theta_{D_1} = \frac{EI_1}{S_1L_1S_2L_2 - K_1K_2L_1^2}\left[\frac{U_o}{EI}\left(S_2L_2\left(\frac{1}{2}H^2L_1 - \frac{1}{2}HL_1^2 + \frac{1}{6}L_1^3\right) - K_2L_1\left(\frac{1}{2}H^2L_2 - \frac{1}{2}HL_2^2 + \frac{1}{6}L_2^3\right)\right)\right.$$
$$\left. + \frac{U_o}{4G_wt_wc_w}(S_2L_2(H-L_1) - K_2L_1(H-L_2))\right] \tag{48}$$

$$\theta_{D_2} = \frac{EI_2}{S_1L_1S_2L_2 - K_1K_2L_1^2}\left[\frac{U_o}{EI}\left(S_1L_1\left(\frac{1}{2}H^2L_2 - \frac{1}{2}HL_2^2 + \frac{1}{6}L_2^3\right) - K_1L_1\left(\frac{1}{2}H^2L_1 - \frac{1}{2}HL_1^2 + \frac{1}{6}L_1^3\right)\right)\right.$$
$$\left. + \frac{U_o}{4G_wt_wc_w}(S_1L_1(H-L_1) - K_1L_1(H-L_1))\right] \tag{49}$$

Triangularly distributed loading

$$\theta_{T_1} = \frac{EI_1}{S_1L_1S_2L_2 - K_1K_2L_1^2}\left[\frac{T_o}{EI}\left(S_2L_2\left(\frac{1}{3}H^2L_1 - \frac{1}{4}HL_1^2 + \frac{1}{24}\frac{L_1^4}{H}\right) - K_2L_1\left(\frac{1}{3}H^2L_2 - \frac{1}{4}HL_2^2 + \frac{1}{24}\frac{L_2^4}{H}\right)\right)\right.$$
$$\left. + \frac{T_o}{4G_wt_wc_w}\left(S_2L_2\left(\frac{H}{2} - \frac{L_1^2}{2H}\right) - K_2L_1\left(\frac{H}{2} - \frac{L_2^2}{2H}\right)\right)\right] \tag{50}$$

$$\theta_{T_2} = \frac{EI_2}{S_1L_1S_2L_2 - K_1K_2L_1^2}\left[\frac{T_o}{EI}\left(S_1L_1\left(\frac{1}{3}H^2L_2 - \frac{1}{4}HL_2^2 + \frac{1}{24}\frac{L_2^4}{H}\right) - K_1L_1\left(\frac{1}{3}H^2L_1 - \frac{1}{4}HL_1^2 + \frac{1}{24}\frac{L_1^4}{H}\right)\right)\right.$$
$$\left. + \frac{T_o}{4G_wt_wc_w}\left(S_1L_1\left(\frac{H}{2} - \frac{L_2^2}{2H}\right) - K_1L_1\left(\frac{H}{2} - \frac{L_1^2}{2H}\right)\right)\right] \tag{51}$$

where S_1 and S_2 are

$$S_1 = K_1 + \frac{EI_1}{L_1} \tag{52}$$

$$S_2 = K_2 + \frac{EI_2}{L_2} \tag{53}$$

5. Equivalent stiffness of outrigger-belt truss system

In this combined system, the outrigger-belt truss structure is attached to the shear core and to the peripheral frames; furthermore, it connects the upper and lower stories (Fig. 4). In this paper each outrigger-belt truss structure is modeled as an equivalent rotational spring with equivalent stiffness K, as given by (Lee *et al.* 2008, Malekinejad and Rahgozar 2011)

$$K = \left(\frac{L}{2c_w^2AE_f} + \frac{c_w^2}{6(EI)_e} + \frac{1}{kGA\ l}\right)^{-1} \tag{54}$$

where in above equation, L, c_w, A and E_f are location of outrigger-belt truss from the base of structure, half width of frame's web, sum of columns cross sectional areas of a flange frame and

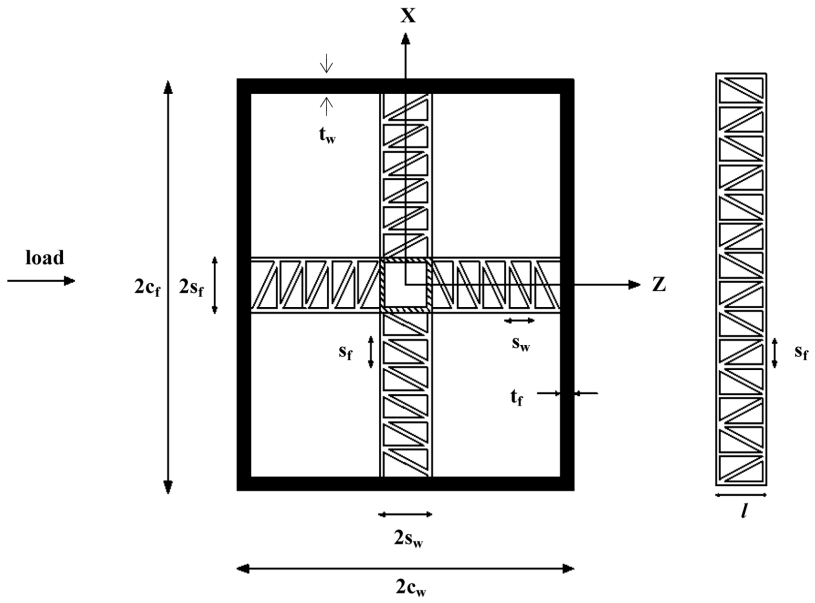

Fig. 4 Schematic plan of combined system at the location of outrigger-belt truss (Malekinejad and Rahgozar 2011)

equivalent elastic modulus of frame's flange respectively. $(EI)_e$ is the effective flexural stiffness of the outrigger, modeled as though its length extended from the column to core's centroid. $(EI)_e$ can be obtained from outrigger's actual flexural rigidity EI by converting the flexural rigidity of a wide-column beam, to that of an equivalent full-span beam (Smith and Salim 1981, Smith and Coull 1996, Lee *et al.* 2008, Malekinejad and Rahgozar 2011). kGA and l are effective shear rigidity (k is shear coefficient, G is shear modulus and A is sectional area of an outrigger-belt truss) and height of outrigger-belt truss respectively.

6. Comparison between results of proposed method and finite element analyses

In this section a numerical example is utilized to verify the accuracy and efficiency of the proposed method. A high-rise 50 storey reinforced concrete structure with the combined system has been analyzed. All the beam, column and outrigger-belt truss members are of sizes 0.8 m × 0.8 m. The specifications that are used in this numerical example have been presented in Table 1.

Table 1 Specifications of the 50 storey building for the test case

Plan dimensions		Height of storey	Dimension of core			Space of spans	
Web	Flange					Web	Flange
$2c_w$ (m)	$2c_f$ (m)	h (m)	b_c (m)	t_c (m)	Area (m^2)	s_w (m)	s_f (m)
30	35	3	5	0.25	4.75	2.5	2.5

Table 2 Properties of actual and equivalent structure

Actual structure		Equivalent membranes			Equivalent membranes		
Material		Web			Flange		
E (GPa)	G (GPa)	E_w (GPa)	G_w (GPa)	t_w (m)	E_f (GPa)	G_f (GPa)	t_f (m)
20	8	20	1.48	0.256	20	1.48	0.256

Table 3 Comparison of the results between proposed method and SAP2000 for 50 storey building subjected to concentrated loading at the top level of the structure

Locations of outrigger-belt truss from the base of building	Stresses in corner columns of flange panel (MPa)		Stresses in middle columns of flange panel (MPa)		Displacement at the top of the building (cm)	
	Proposed method	SAP2000	Proposed method	SAP2000	SAP2000	Proposed method
$H/6$ & $H/4$	9.24	10.18	6.99	6.61	45.27	48.25
$H/6$ & $H/2$	9.31	10.40	6.77	6.44	44.51	47.77
$H/6$ & $3H/4$	9.33	10.53	6.66	6.28	44.45	47.43
$H/6$ & H	9.34	10.57	6.64	6.22	45.39	47.40
$H/4$ & $H/2$	9.41	10.81	6.62	6.23	44.34	47.50
$H/4$ & $3H/4$	9.42	10.92	6.51	6.09	44.17	47.16
$H/4$ & H	9.44	10.96	6.49	6.03	45.09	47.12
$H/2$ & $3H/4$	9.49	11.43	6.27	5.64	44.19	46.65
$H/2$ & H	9.50	11.46	6.25	5.59	44.97	46.61
$3H/4$ & H	9.51	11.65	6.12	5.35	45.32	46.25

Table 4 Comparison of the results between proposed method and SAP2000 for 50 storey building subjected to uniformly distributed loading

Locations of outrigger-belt truss from the base of building	Stresses in corner columns of flange panel (MPa)		Stresses in middle columns of flange panel (MPa)		Displacement at the top of the building (cm)	
	Proposed method	SAP2000	Proposed method	SAP2000	Proposed method	SAP2000
$H/6$ & $H/4$	5.30	5.88	2.84	3.05	21.73	18.24
$H/6$ & $H/2$	5.41	6.23	2.76	2.99	21.69	19.06
$H/6$ & $3H/4$	5.46	6.15	2.76	2.79	21.76	19.10
$H/6$ & H	5.50	6.16	2.79	2.77	22.10	19.71
$H/4$ & $H/2$	5.77	6.48	2.79	2.66	21.59	18.44
$H/4$ & $3H/4$	5.83	6.54	2.78	2.58	21.65	19.11
$H/4$ & H	5.62	6.56	2.69	2.56	21.79	19.71
$H/2$ & $3H/4$	5.68	6.95	2.57	2.21	21.56	19.72
$H/2$ & H	5.45	6.97	2.48	2.19	21.69	20.27
$3H/4$ & H	5.24	6.37	2.35	2.05	21.72	21.10

Table 5 Comparison of the results between proposed method and SAP2000 for 50 storey building subjected to triangularly distributed loading

Locations of outrigger-belt truss from the base of building	Stresses in corner columns of flange panel (MPa)		Stresses in middle columns of flange panel (MPa)		Displacement at the top of the building (cm)	
	Proposed method	SAP2000	Proposed method	SAP2000	Proposed method	SAP2000
H/6 & *H*/4	6.82	7.41	4.02	4.21	30.20	26.12
H/6 & *H*/2	6.93	7.64	3.88	4.02	30.20	25.98
H/6 & 3*H*/4	7.01	7.45	3.87	3.88	30.08	26.88
H/6 & *H*	7.06	7.72	3.93	3.82	30.32	27.71
H/4 & *H*/2	7.36	8.08	3.93	3.79	30.20	25.90
H/4 & 3*H*/4	7.43	8.17	3.92	3.67	29.90	26.73
H/4 & *H*	6.93	8.19	3.88	3.63	30.20	27.74
H/2 & 3*H*/4	7.22	8.67	3.61	3.23	29.69	27.21
H/2 & *H*	7.27	8.69	3.65	3.19	29.89	28.13
3*H*/4 & *H*	6.67	7.96	3.30	2.99	29.90	29.21

The structure is subjected to three different types of lateral loadings in turn:
concentrated loading at the top level of structure: $C = 18 \times 10^4$ kN
uniformly distributed loading: $U_o = 120$ kN/m
triangularly distributed loading: $T_o = 240$ kN/m

Equivalent elastic properties for orthotropic membranes that have been presented by Takabatake *et al.* (1993) and Kwan (1994) and the material properties are shown in Table 2.

After determining equivalent elastic properties for orthotropic membranes and calculating shear lag coefficients for different types of loading, the values of axial stresses and lateral displacements can be obtained from Eqs. (27)-(32) and (37)-(45), respectively. Outrigger-belt truss systems are located at different heights of the structure (*H*/6, *H*/4; *H*/6, *H*/2; *H*/6, 3*H*/4; *H*/6, *H*; *H*/4, *H*/2; *H*/4, 3*H*/4; *H*/4, *H*; *H*/2, 3*H*/4; *H*/2, *H*; 3*H*/4, *H*) and the results from proposed method and finite element analyses

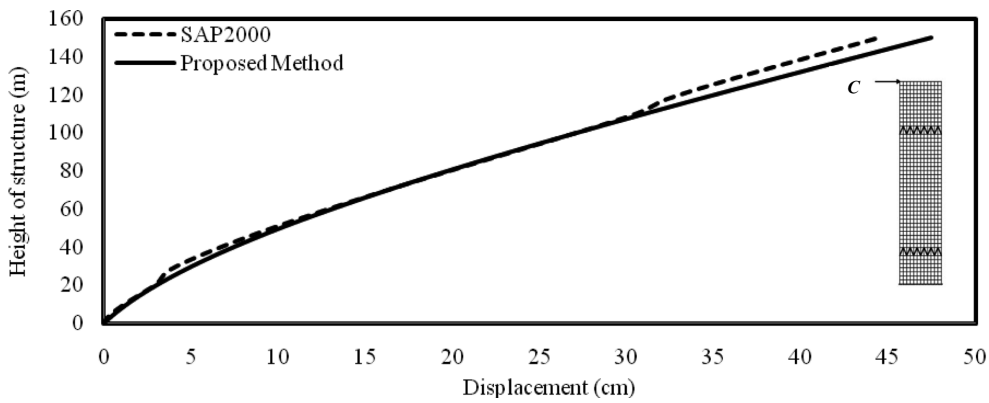

Fig. 5 Displacement distribution of the 50 storey combined system subjected to concentrated loading at the top of the structure (O.B. = H/6 & 3H/4)

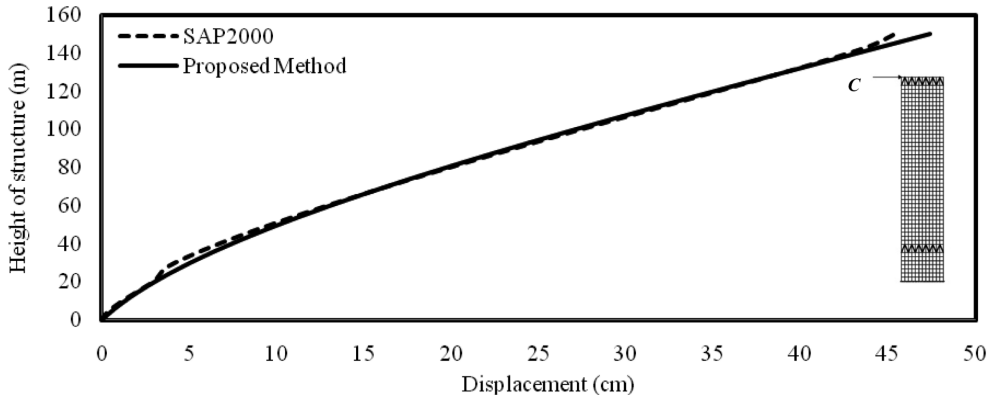

Fig. 6 Displacement of the 50 storey combined system subjected to concentrated loading at the top of the structure (O.B. = H/6 & H)

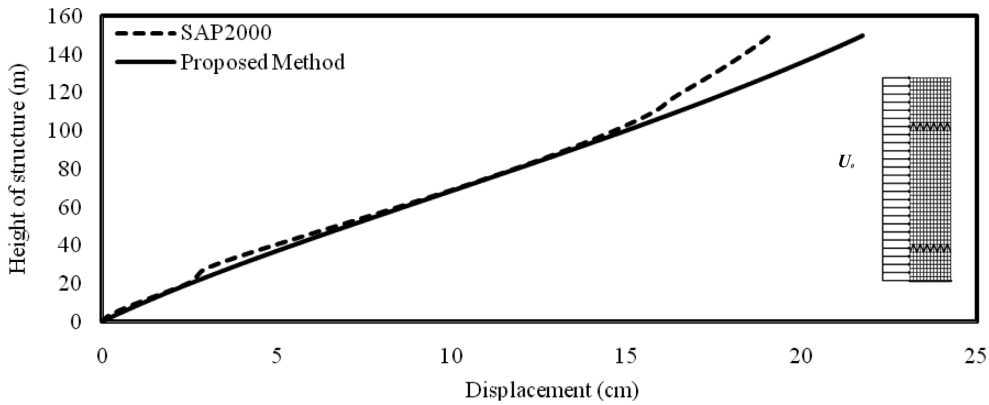

Fig. 7 Displacement of the 50 storey combined system subjected to uniformly distributed loading (O.B. = H/6 & 3H/4)

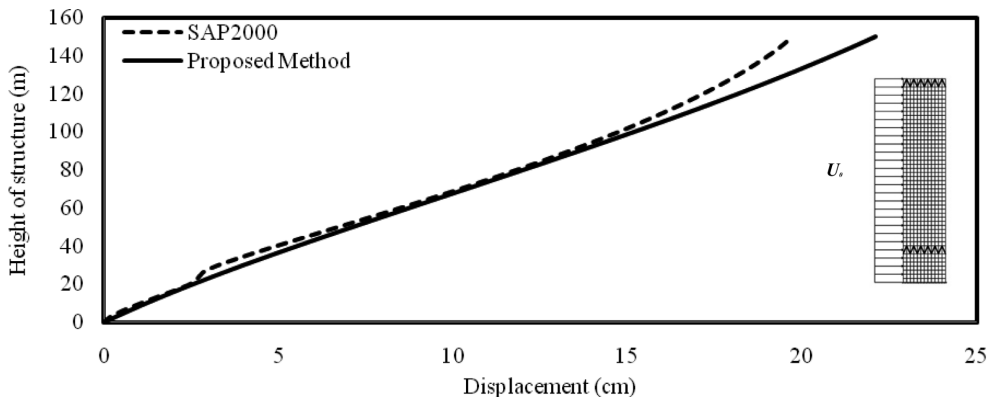

Fig. 8 Displacement of the 50 storey combined system subjected to uniformly distributed loading (O.B. = H/6 & H)

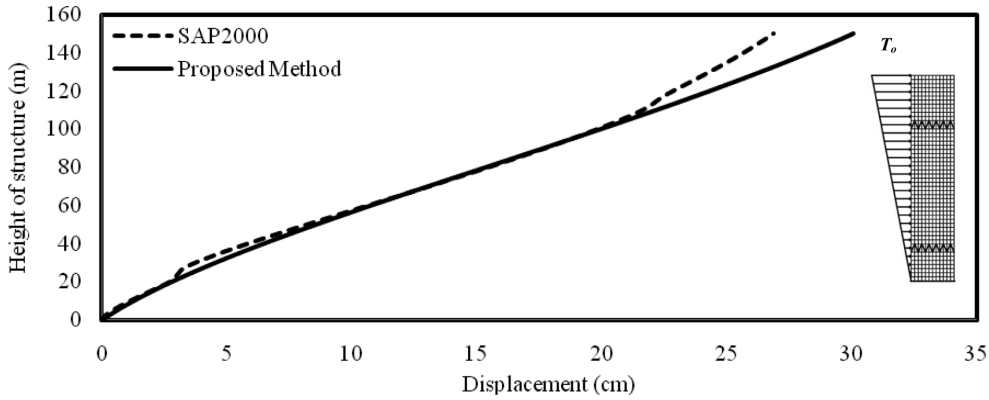

Fig. 9 Displacement of the 50 storey combined system subjected to triangularly distributed loading (O.B. = H/6 & 3H/4)

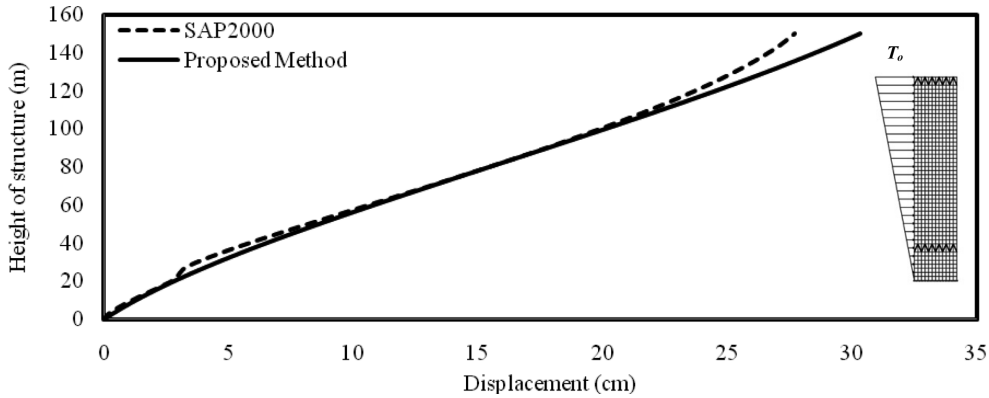

Fig. 10 Displacement of the storey combined system subjected to triangularly distributed loading (O.B. = H/6 & H)

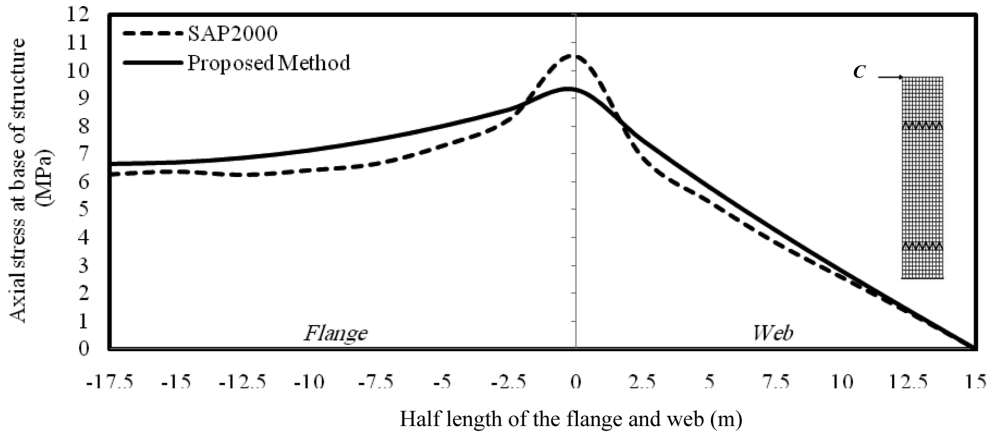

Fig. 11 Stress distribution of the 50 storey combined system subjected to concentrated loading at the top of the structure (O.B. = H/6 & 3H/4)

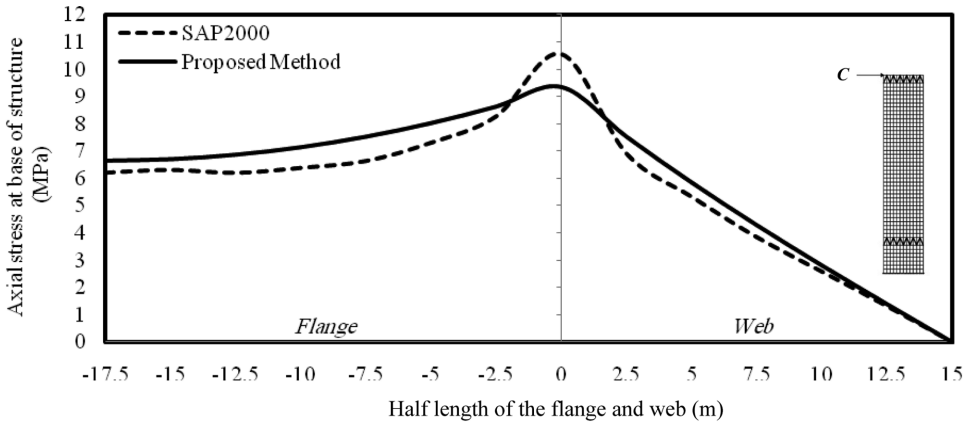

Fig. 12 Stress distribution of the 50 storey combined system subjected to concentrated loading at the top of the structure (O.B. = H/6 & H)

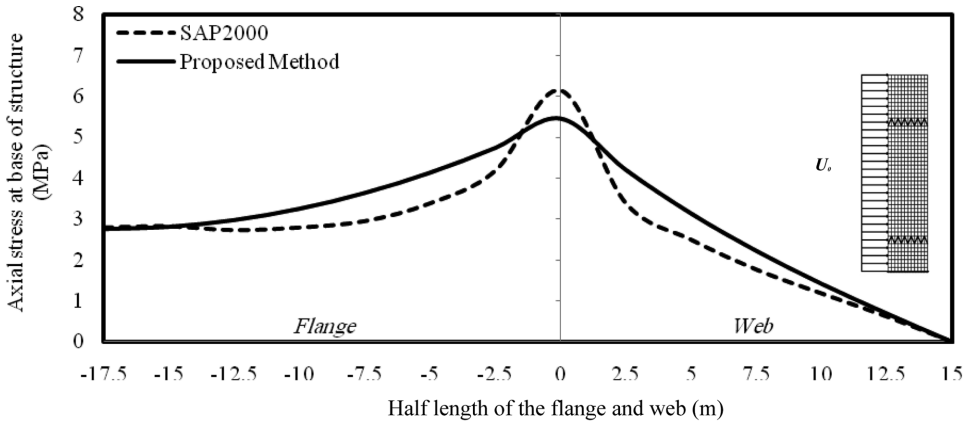

Fig. 13 Stress distribution of the 50 storey combined system subjected to uniformly distributed loading (O.B. = H/6 & 3H/4)

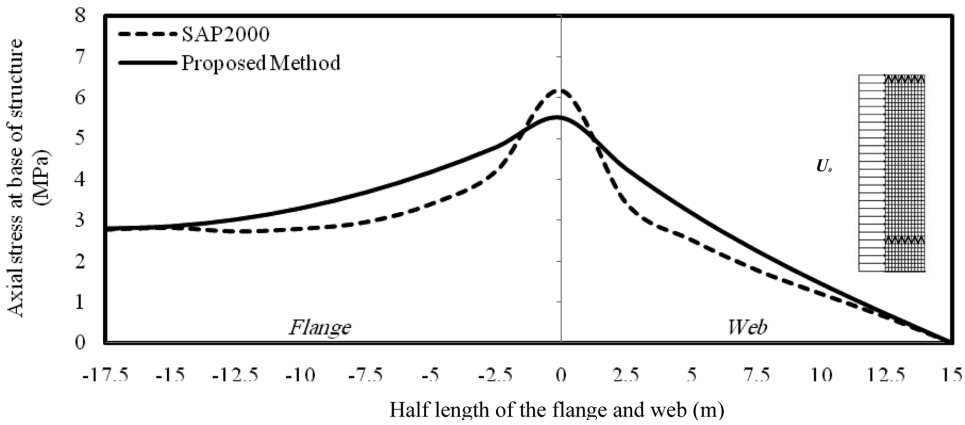

Fig. 14 Stress distribution of the 50 storey combined system subjected to uniformly distributed loading (O.B. = H/6 & H)

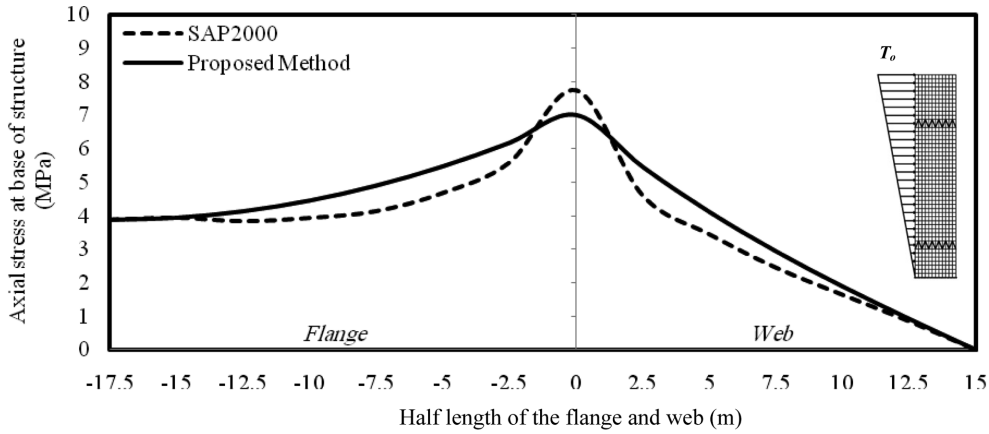

Fig. 15 Stress distribution of the 50 storey combined system subjected to triangularly distributed loading (O.B. = H/6 & 3H/4)

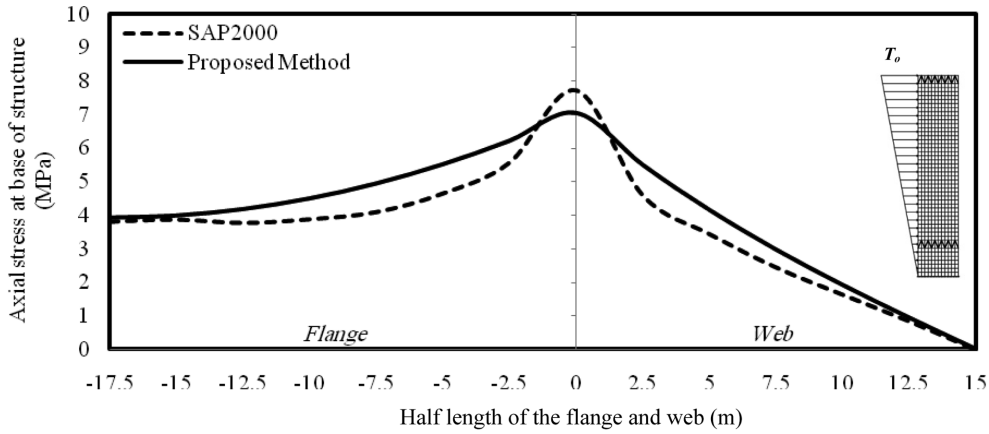

Fig. 16 Stress distribution of the 50 storey combined system subjected to triangularly distributed loading (O.B. = H/6 & H)

performed using SAP2000 for corner and middle columns of flange panel and lateral displacement at the top of the structure for three different types of loading are listed in Tables 3-5. Lateral displacement along the height of structure and stress distribution in half of flange and web at the base of the combined system of a 50 storey building are plotted and compared in Figs. 5-16 for two cases, note that because of practical consideration made in construction of a real tall building, only the graphs that represent practical locations such as ($H/6$, H) and ($H/6$, $3H/4$) are presented by these figures (subscript *O.B.* in figures demonstrates location of outrigger-belt trusses). For example in the case of practice, when the outrigger-belt truss systems located at ($H/6$, H) from the base of structure, the values of stresses in corner columns of flange panel is overestimated by 11%, 10% and 8% at the base; underestimate the stresses in middle columns of flange panel by 7%, 1% and 3% at the base; and underestimate the lateral displacements at the top level of the structure by 4%, 12% and 9% for concentrated load at the top, uniformly and triangularly distributed loads respectively.

It can be seen when the outrigger-belt truss systems are located at ($H/6$, H) and ($H/6$, $3H/4$) from the base of the structure, the approximate lateral displacements at top of the structure are in good agreement with those obtained using SAP2000 and stress distributions in web and flange panel at the base of the structure have acceptable differences which satisfied the objectives of decreasing displacement and smoothing stresses in framed tube. Therefore, the proposed approximate method can be used in preliminary stages of design for estimating the dimensions of beams and columns.

7. Conclusions

A simple mathematical model has been proposed that gives reasonably accurate results that is useful for the preliminary analysis and design of framed tube, shear core and outrigger-belt truss structures subjected to lateral loading. In the proposed method, the interaction between outrigger-belt truss with framed tube and shear core is considered as a concentrated moment applied at the outrigger-belt truss location which is modeled as torsional spring. The effects of shear lag in web and flange of the frames are considered in the proposed method. Based on the principle of minimum potential energy, closed form solution for lateral displacement and axial stress distributions are obtained. The results of numerical example for a 50 storey building subjected to three types of lateral loading are obtained from SAP2000 and compared to those of the proposed method; which are found to be in good agreement with those obtained from detailed analyses performed using SAP2000. It is revealed that the proposed approximate method overestimate the stresses in corner columns of flange panel at the base; underestimate the stresses in middle columns of flange panel at the base and the lateral displacement at the top level of the structure which satisfied the objectives of decreasing displacement and smoothing stresses in framed tube. Since only the global structural behavior is needed during the preliminary stages of design, the proposed approximate method offers a simple and inexpensive alternative to the full blown analysis of these combined systems.

References

Chang, S.T. and Zheng, F.Z. (1987), "Negative shear lag in cantilever box girders with constant depth", *J. Struct. Eng.-ASCE,* **113**(1), 20-35.

Connor, J.J. and Pouangare, C.C. (1991), "Simple model for design of framed tube structures", *J. Struct. Eng.-ASCE,* **117**, 3623-3644.

Coull, A. and Ahmed, K. (1978), "Deflection of framed-tube structures", *J. Struct. Div.*, **104**, 857-862.

Coull, A. and Bose, B. (1975), "Simplified analysis of frame tube structures", *J. Struct. Div.*, **101**, 2223-2240.

Coull, A. and Bose, B. (1976), "Torsion of frame-tube structures", *J. Struct. Div.-ASCE*, **102**(12), 2366-2370.

Coull, A. and Bose, B. (1977), "Discussion of simplified analysis of frame-tube structures", *J. Struct. Div.-ASCE,* **103**(1), 297-299.

Coull, A. and Subedi, N. (1971), "Framed-tube structures for high rise buildings", *J. Struct. Div.*, **97**, 2097-2105.

Ding, J.M. (1991), "Optimum belt truss location for high-rise structures and top level drift coefficient", *J. Build. Struct.*, **4**, 10-13.

Foutch, D.A. and Chang, P.C. (1982), "A shear lag anomaly", *J. Struct. Eng.-ASCE*, **108**(7), 1653-1658.

Fu, X.Y. (1999), "Design proposal for reinforced concrete high-rise building structure with outrigger belts", *J. Build. Struct.*, **10**, 11-19.

Gao, P. (1998), "Structural behavior of ultra high-rise building with outrigger belts", *J. Build. Struct.*, **10**, 8-12.

Gerasimidis, S., Efthymiou, E. and Baniotopoulos, C.C. (2009), "Optimum outrigger locations of high-rise steel buildings for wind loading", EACWE 5, Florence, Italy.

Gunel, H.M. and Ilgin, E.H. (2007), "A proposal for the classification of structural systems of tall buildings", *J. Build. Environ.,* **42**, 2667-2675.

Ha, A.H., Fazio, P. and Moselhi, O. (1978), "Orthotropic membrane for tall building analysis", *J. Struct. Div.-ASCE,* **104**(9), 1495-1505.

Hoenderkamp, J.C.D and Bakker, M.C.M. (2003), "Analysis of high-rise braced frames with outriggers", *J. Struct. Des. Tall Spec. Build.,* **12**, 335-350.

Kwan, A.K.H. (1994), "Simple method for approximate analysis of framed-tube structures", *J. Struct. Eng.-ASCE,* **120**, 1221-1239.

Kwan, A.K.H. (1996), "Shear lag in shear core walls", *J. Struct. Eng.-ASCE*, **122**, 1097-1104.

Lam, N., Wilson, J. and Lumantarna, E. (2010), "Modeling of seismically induced storey-drift in buildings", *Struct. Eng. Mech.*, **35**(4), 459-478.

Lee J., Bang, M. and Kim, J.Y. (2008), "An analytical model for high-rise wall-frame structures with outriggers", *J. Struct. Des. Tall Spec. Build.*, **17**, 839-851.

Malekinejad, M. and Rahgozar, R. (2011), "Free vibration analysis of tall buildings with outrigger-belt truss system", *J. Earthq. Struct.*, **2**(1), 89-107.

McNabb, J.W. and Muvdi, B.B. (1975), "Drift reduction factors for belt high-rise structures", *Eng. J.*, 3rd Quarter, 88-91.

Rahgozar, R. and Sharifi, Y. (2009), "An approximate analysis of combined system of framed tube, shear core and belt truss in high-rise buildings", *J. Struct. Des. Tall Spec. Build.*, **18**(6), 607-624.

Rahgozar, R., Ahmadi, A. and Sharifi, Y. (2010), "A simple mathematical model for approximate analysis of tall buildings", *J. Appl. Math. Model.*, **34**, 2437-2451.

Saffari, H., Rahgozar, R. and Mahjoob, R. (2003), "Simple method for analysis of tube frame", *Sixth International Conference on Civil Engineering, Isfahan University of Technology*, Isfahan, Iran, 198-204.

SAP2000 Advanced 12.0.0, Computers and Structures, Berkeley, California, USA.

Shushkewich, K.W. (1991), "Negative shear lag explained", *J. Struct. Eng.-ASCE*, **117**(11), 3543-3545.

Singh, Y. and Nagpal, A.K. (1993), "Secondary web-flange interaction in framed tube buildings", *J. Struct. Des. Tall Spec. Build.,* **2**, 325-331.

Smith, S. and Coull, A. (1996), *Tall Building Structures*, McGraw Hill Book Company, New York.

Smith, S.B. and Salim, I. (1981), "Parameter study of outrigger braced tall building structures", *J. Struct. Eng.*, **107**(10), 2001-2014.

Takabatake, H., Mukai, H. and Hirano, T. (1993), "Doubly symmetric tube structures, I: static analysis", *J. Struct. Eng.-ASCE*, **119**(7), 1981-2001.

Takewaki, I. and Tsujimoto, H. (2011), "Scaling of design earthquake ground motions for tall buildings based on drift and input energy demands", *Earthq. Struct.*, **2**(2), 171-187.

Taranath, B.S. (1975), "Optimum belt truss location for high-rise structures", *Struct. Eng.*, **53**(8), 18-21.

Taranath, B.S. (1988), *Structural Analysis and Design of Tall Buildings*, McGraw Hill Book Company, New York.

Zhang, Z.G., Fu, X.Y., Wang, J.J. and Wei, Y.N. (1996), "Studies on structural performance of ultra high-rise building with outrigger belts", *J. Build. Struct.*, **17**(4), 2-9.