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Structural damage identification of plates based on modal data using 2D discrete wavelet transform

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Abstract. An effective method for detection linear flaws in plate structures via two-dimensional discrete wavelet transform is proposed in this study. The proposed method was applied to a four-fixed supported rectangular plate containing damage with arbitrary length, depth and location. Numerical results identifying the damage location are compared with the actual results to demonstrate the effectiveness of the proposed method. Also, a wavelet-based method presented for de-noising of mode shape of plate. Finally, the performance of the proposed method for de-noising and damage identification was verified using experimental data. Comparison between the location detected by the proposed method, and the plate's actual damage location revealed that the methodology can be used as an accessible and effective technique for damage identification of actual plate structures.

Keywords: damage detection; de-noising; plate; wavelet transform; damage location

1. Introduction

In the last years, detection of damage during the service life of plate structures such as slab, tunnel and shear wall has increased the researcher's attenuation. Structural damage can be identified as the weakening of a plate causing negative changes in its performance. Damage may also be considered as any change in the property of a material and its original geometry of plate resulting in undesirable stress or displacement and vibration of the plate. Structural damage progressively impacts on the dynamic properties of the plate such as stiffness and damping at damage location (Doebling *et al.* 1996). Therefore, these changes cause alteration in the dynamic response behavior of plate structures. Identification of alteration in the mode shape and the natural frequency at the damaged element in comparison with the pre-damaged state of element is one of the popular methods in damage detection of plate structures. These changes are often small and measurements are polluted by noise making this method an inefficient method for detecting the proper location of damage. For this reason, the method needs an effective methodology to identify the damage location and so forth.

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The wavelet transform is a new method for precise signal analysis, which overcomes the problems exhibited by other signal processing techniques. Applying wavelet transform for the analysis of the damaged shape mode of structures produces satisfying results in damage identification. Sharp changes in the wavelet coefficient near the damage exhibit the presence and location of damage. There are a number of studies on the one-dimensional wavelet transforms. The possibility of applying various wavelets for the detection of beam cracks has been studied by Sun and Chang (2004), Han *et al.* (2005), Poudel *et al.* (2007), Lakshmanan *et al.* (2007), Gokdag (2008) and Gokdag *et al.* (2010). Frame structures have been analyzed by Ovanesova and Suarez (2004).

The damage detection in plate structures were addressed by many researchers. Cornwell *et al.* (1999) extended the modal energy method originally developed for damage detection in one-dimensional structures to plate structures. Yam and Yam (2002) identified damage utilizing wavelet analysis in composite plates to decompose the dynamic responses. Chang and Chen (2004) studied damage detection of a rectangular plate by spatial wavelet based approach. Loutridis *et al.* (2005) presented the method that a two-dimensional wavelet transform is used for identification of cracks in plate structures. A Nondestructive damage evaluation of plates using the multi-resolution analysis of two-dimensional Haar wavelet was suggested by Kim *et al.* (2006). Rucka and Wilde (2006) proposed a method for estimating damage localization in a plate by applying continuous wavelet transform. Bayissa and Haritos (2007) proposed a new damage identification technique based on the statistical moments of the energy density function of vibration responses in the time-scale domain. Hadjileontiadis and Douka (2007a) presented an effective method for detecting cracks in plate structures based on kurtosis analysis. Also, Hadjileontiadis and Douka (2007b) used fractal dimension for crack detection in plates. Wang and Yuan (2007) presented an active damage localization method based on energy propagation of Lamb waves. Bayissa *et al.* (2008) offered a new damage detection technique using wavelet transform based on the vibration responses of plate. Fan and Qiao (2009) developed a two-dimensional continuous wavelet transform-based damage detection algorithm using Dergauss2d wavelet for plate-type structures. Also, a distributed two-dimensional continuous wavelet transform algorithm was developed by Huang *et al.* (2009). They used data from discrete sets of nodes and provide spatially continuous variation in the structural response parameters to monitor structural degradation.

Recently, Bagheri *et al.* (2009) presented a novel method based on the discrete curvelet transform using unequally-spaced fast Fourier transforms to identify damage location in plate structures. Also, a method was proposed for detection of rectangular damage in plate structures using discrete wavelet transform by Ghodrati Amiri *et al.* (2009). In the current paper, a new method is proposed for the detection of linear damages in plate structure based on two-dimensional discrete wavelet transform. Especially, the two-dimensional discrete wavelet transform for de-noising of the vibration mode shapes of plate is applied. The damage is represented by elements with reduced thickness and then the mode shapes are analyzed by the two-dimensional discrate wavelet transform. The practicability of the proposed method is validated with a four-fixed supported rectangular plate containing damages in two states of numerical and experimental data. The proposed method is attractive because of the simplicity of the computational performance and accuracy of the results.

2. Discrete wavelet transform

The discrete wavelet transform is an implementation of the wavelet transform using a discrete set of the wavelet scales and translations obeying some defined rules. In discrete wavelet transform the signals can be represented by approximations and details. The detail at level j is defined as (Mallat 2001)

$$D_j(t) = \sum_{k \in Z} cD_{j,k}\,\psi_{j,k}(t) \tag{1}$$

where $\psi_{j,k}$ is wavelet functions and $cD_{j,k}$ is wavelet coefficients at level j.

The approximation at level j is defined as

$$A_j(t) = \sum_{k=-\infty}^{+\infty} cA_j(k)\,\phi_{j,k}(t) \tag{2}$$

where $\phi_{j,k}$ is scaling functions and $cA_{j,k}$ is scaling coefficients at level j.

The signal $f(t)$ can be represented by

$$f(t) = A_J + \sum_{j<J} D_j \tag{3}$$

In detection of singularities of signals the vanishing moments play an important role. A wavelet has n vanishing moments if the following equation is satisfied (Mallat 2001)

$$\int_{-\infty}^{+\infty} t^i \psi(t)\,dt = 0\,, \quad i = 1, 2, \ldots, n-1 \tag{4}$$

Hence the wavelet having n vanishing moments is orthogonal to polynomials up to degree $n-1$. Mallat (2001) proved that for wavelets with n vanishing moments and a fast decay there exist function $\theta(t)$ with a fast decay dened as follows

$$\psi(t) = \frac{d^n\theta(t)}{dt^n}\,, \quad \int_{-\infty}^{+\infty} \theta(t)\,dt \neq 0 \tag{5}$$

The one-dimensional wavelet transform can be extended to any dimensions. In this section, a two-dimensional wavelet transform is studied to obtain the damage localization of plate structures.

In two dimensions, a two-dimensional scaling function, $\phi(x, y)$, and three two-dimensional wavelets $\psi^H(x, y)$, $\psi^V(x, y)$ and $\psi^D(x, y)$, are required. While each one is the product of a one-dimensional scaling function ϕ and its corresponding wavelet ψ (Daubechies 1992)

$$\phi(x,y) = \phi(x)\phi(y) \tag{6}$$

$$\psi^H(x,y) = \psi(x)\phi(y) \tag{7}$$

$$\psi^V(x,y) = \phi(y)\,\psi(x) \tag{8}$$

$$\psi^D(x,y) = \psi(x)\,\psi(y) \tag{9}$$

where ψ^H measures variations along columns (like horizontal edges), ψ^V responds to variations along rows (like vertical edges), and ψ^D corresponds to variations along diagonals.

Like the one-dimensional discrete wavelet transform, the two-dimensional discrete wavelet transform can be implemented using digital filters and downsamplers. With separable two-dimensional scaling and wavelet functions, we simply take the one-dimensional fast wavelet

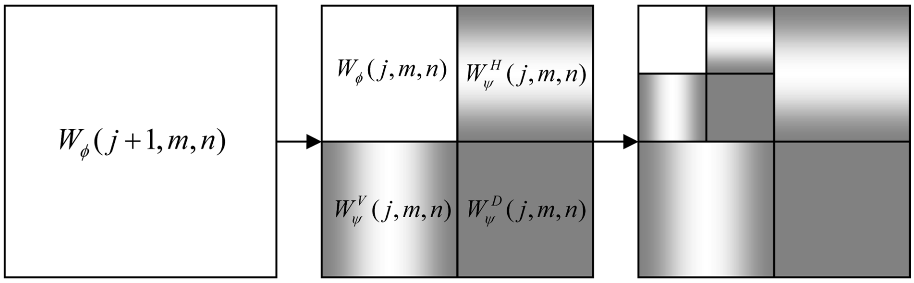


Fig. 1 Two-scale of tow-dimensional decomposition in wavelet transform

transform of the rows of $f(x, y)$, followed by the one-dimensional fast wavelet transform of the resulting columns.

As in the two-dimensional case, image $f(x, y)$ is used as the first scale input, which outputs four quarter-sized subimages W_ϕ, $W_\psi{}^H$, $W_\psi{}^V$, and $W_\psi{}^D$. These subimages are shown in the middle of Fig. 1.

In this paper, the reverse biorthogonal wavelet with four vanishing moments (rbio5.5) is used. This wavelet provided the best effectiveness in detecting damage position.

3. Damage detection method

This paper suggests the use of a two-dimensional discrete wavelet transform for the detection of linear damages in a plate. We have used the decomposing shape mode of the plate by applying wavelet multiresolution analysis. The mode shapes of the damaged rectangular plate is derived by finite element method. The damage is represented by elements with reduced thickness and then the mode shapes are analyzed by the two-dimensional discrate wavelet transform.

The mode shapes of the rectangular plate with and without damage are derived using finite element method. The geometry of a four-fixed supported damaged rectangular plate is shown in Fig. 2. The plate dimension is $L \times B \times t$ while the damage is represented as the elements with reduced thickness. The dimension of the damaged region is $W \times D$ whereas the location coordinates of the damage are $L1$, $B1$ with a directional angle of α. Also the damage region is represented as the

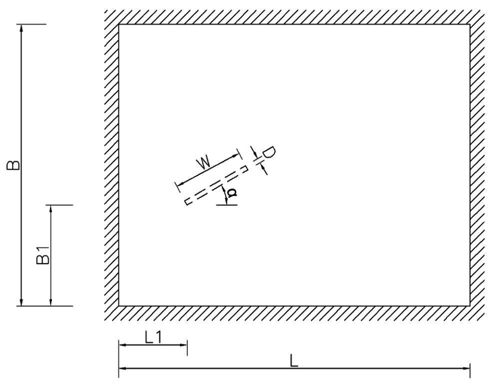


Fig. 2 Geometry of the plate with damage

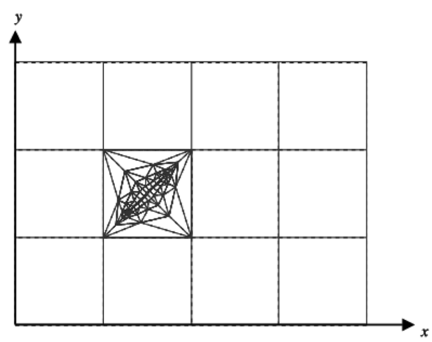


Fig. 3 Finite element model of the plate with damage

region with reduced thickness Δt. The finite element mesh of the damage is modeled using ABAQUS software. Fig. 3 shows the finite element model of the plate with damage.

The equation of the plate's free vibration is as follows

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0 \tag{10}$$

where **M**, **C** and **K** are mass, damping and stiffness matrices, respectively. By solving Eq. (10), the mode shape Φ_i, $i = 1, 2, ..., n$, is obtained where n is the number of structural modes. In this study, we used a fundamental shape mode of the damaged plate, denoted with Φ_d, whereas for the undamaged plate we used Φ_{ud}.

The two-dimensional discrete wavelet transform is applied to the mode shape of the plate to determine the coefficients of the two-dimensional discrete wavelet transform. The modules of the wavelet transform for the fundamental shape mode of the damaged plate Φ_d is given by

$$M^d = \sqrt{(W_\psi^V)^2 + (W_\psi^H)^2} \tag{11}$$

where M^d is the module of the wavelet transform of the damaged plate. M^{ud} is the modules of the wavelet transform for the fundamental mode shape of the undamaged plate.

The results of wavelet transform for Φ_d are defined as follows

$$R^d = \sqrt{(W_\psi^V)^2 + (W_\psi^H)^2 + (W_\psi^D)^2} \tag{12}$$

where R^d is the result of the wavelet transform for the damaged plate and R^{ud} is the result of the wavelet transform for the fundamental mode shape of the undamaged plate.

Finally, damage identification indices like relative root mean square error in modules of the wavelet transform (RRMSEM), the results of the wavelet transform (RRMSER), difference error in modules of the wavelet transform (DEM) and also the results of wavelet transform (DER) were used for locating the damage of a plate structure (Bagheri *et al*. 2009).

3.1 Damage indexes using RRMSEM and RRMSER

The damage indices are calculated using the relative root mean square error for an element node-based module and the results of the discrete wavelet transform are used for determining the damaged and the undamaged state of each element node. The RRMSEM damage index is defined as follows

$$DI^n\ (RRMSEM) = \sqrt{\frac{\frac{1}{m}\sum_{j=1}^{m}(M_j^d - M_j^{ud})^2}{\frac{1}{m}\sum_{j=1}^{m}(M_j^{ud})^2}} \tag{13}$$

where $DI^n(RRMSEM)$ is the element node-based damage index using RRMSEM; while m is the number of nodes leading into node n. M_j^d and M_j^{ud} are the modules of discrete wavelet transform for the damaged and undamaged plate element node n, respectively.

Also, the RRMSER damage index is defined as follows

$$DI^n\ (RRMSER) = \sqrt{\frac{\frac{1}{m}\sum_{j=1}^{m}(R_j^d - R_j^{ud})^2}{\frac{1}{m}\sum_{j=1}^{m}(R_j^{ud})^2}} \tag{14}$$

where $DI^n(RRMSER)$ is the element node-based damage index using RRMSER; R_j^d and R_j^{ud} are the results of discrete wavelet transform for the damaged and undamaged plate element node n, respectively.

3.2 Damage indexes using DEM and DER

The damage indices are calculated using the difference of an element node-based module and the results of a discrete wavelet transform for the damaged state and the undamaged state determined for each element node using the following equation

$$DI^n\ (DEM) = \left(\frac{\frac{1}{m}\sum_{j=1}^{m} M_j^d}{\frac{1}{n_n}\sum_{n=1}^{n_n} M_n^d} - \frac{\frac{1}{m}\sum_{j=1}^{m} M_j^{ud}}{\frac{1}{n_n}\sum_{n=1}^{n_n} M_n^{ud}}\right) \tag{15}$$

and

$$DI^n\ (DER) = \left(\frac{\frac{1}{m}\sum_{j=1}^{m} R_j^d}{\frac{1}{n_n}\sum_{n=1}^{n_n} R_n^d} - \frac{\frac{1}{m}\sum_{j=1}^{m} R_j^{ud}}{\frac{1}{n_n}\sum_{n=1}^{n_n} R_n^{ud}}\right) \tag{16}$$

where $DI^n(DEM)$ and $DI^n(DER)$ are the element node-based damage index using DEM and DER, respectively; n_n is the total number of nodes used in the model.

4. De-noising method of mode shape

The measured mode shapes of structure always contain noise. The existence of noise may have an appreciable influence on the accuracy of damage detection. A systematic investigation of the effects of noise on the performance of damage identification method is yet to be done.

The de-noising procedure using the wavelet transform involves three steps. The basic version of the procedure follows the steps described below:

1. Decompose: Choose a wavelet, choose a level J. Compute the wavelet decomposition of the measured mode shape at level J.
2. Threshold detail coefficients: For each level from 1 to J, select a threshold and apply soft thresholding to the detail coefficients.
3. Reconstruct: Compute wavelet reconstruction using the original approximation coefficients of level J and the modified detail coefficients of levels from 1 to J.

5. Numerical examples

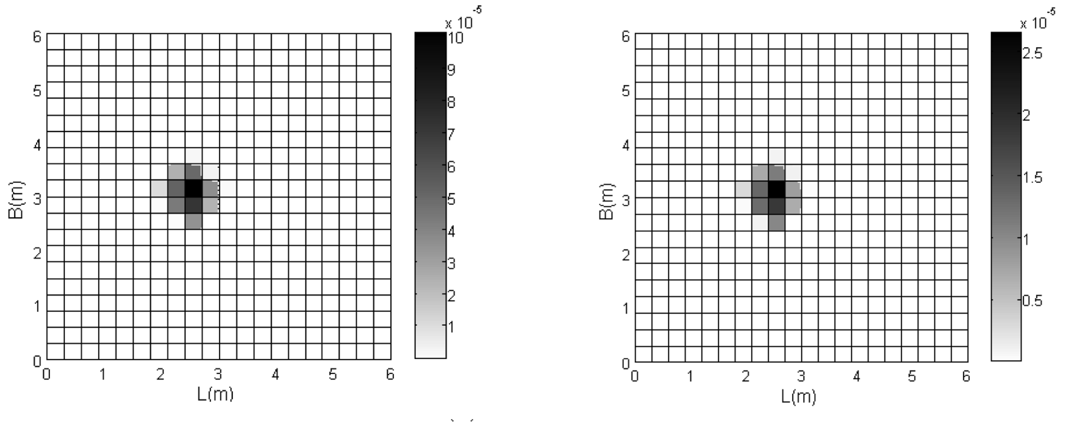
In this paper, the proposed method for damage detection in plates using 2D discrete wavelet transform has been applied to a four-fixed supported rectangular plate containing one or two linear damages with arbitrary length, depth and location. For damage identification studies a fixed supported rectangular plate with dimensions of 6 m × 6 m × 0.2 m was considered. The material properties for the plate include a Young's modulus of $E = 20$ GPa, mass density of $\rho = 2500$ kg/m^3, and Poisson's ratio of $\nu = 0.2$. In the finite element model, the damage was represented by elements with reduced thickness. In order to detect the damage, three different cases based on the number and location of the damage were considered.

In the first case of damage identification studies, a single damage induced in a plate with the damaged dimension region of 0.19 m × 0.01 m and damage location coordinates of is $L1 = 2.14$ m, $B1 = 2.71$ m with a directional angle of $\alpha = 90°$ was used. For different levels of damage in a plate with 10 and 5% reduced thickness was considered. Afterwards the finite element model analysis of the plate determined the mode shapes of the plate and the fundamental mode shape of the plate used for a proposed damage detection method. It is noted that the mode shapes have been decomposed with *rbio5.5* wavelet.

Using the proposed method for damage identification through the wavelet transform can determine the damage indices. Figs. 4 to 7 show the capability of the proposed method to identify the damage of a plate using the proposed damage indexes. The single damage in a plate is detected and localized using the damage index technique for different levels of damage. The results show that the peak values of the damage indices are observed at the exact damage locations of the model.

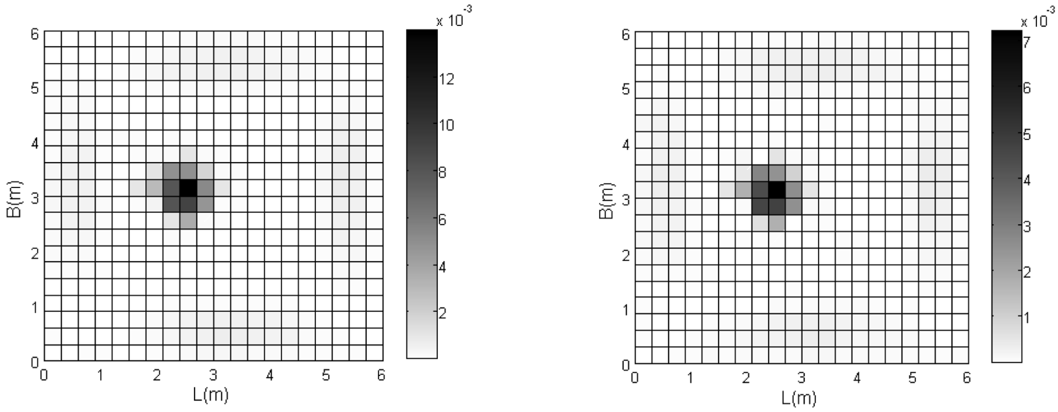
In the second case of the damage identification studies a single damage induced in the plate with the location coordinates of $L1 = 4.06$ m, $B1 = 4.45$ m and a directional angle of $\alpha = 135°$ was used while the dimensions of the damaged region were 0.27 m × 0.01 m. This has been considered for different levels of damage in the plate. The result of the damage indexes for this case at different levels of damage are shown in Figs. 8 to 11. Results show that the maximum values of the damage indices occur at the damages in the plate.

In the third case, two damages are induced in the plate. The dimensions of the damaged region



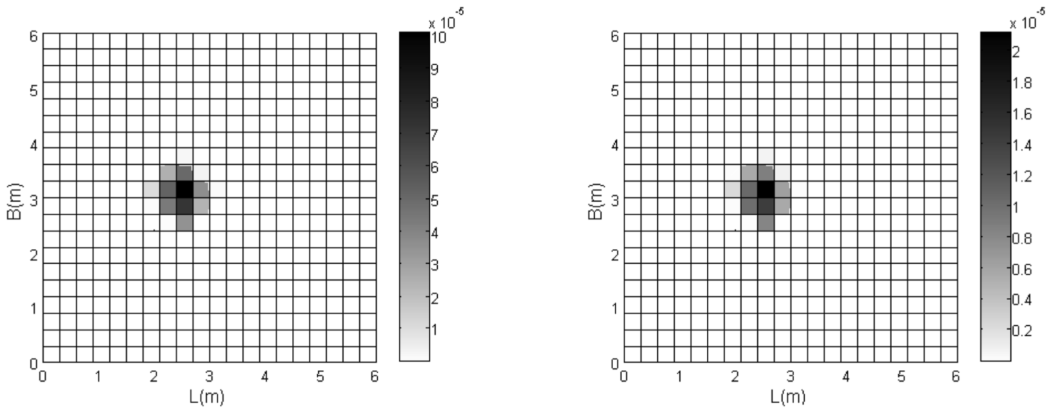
(a) (b)

Fig. 4 Single damage detection in the first case using RRMSER (a) 10% damage and (b) 5% damage



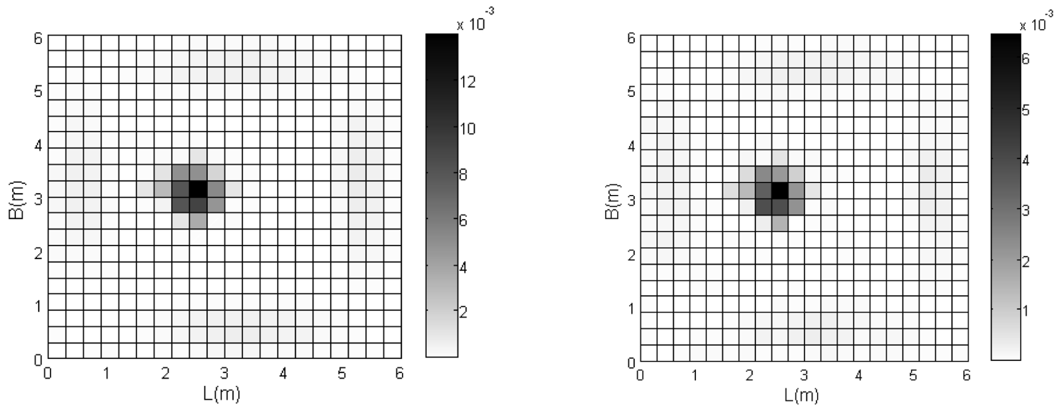
(a) (b)

Fig. 5 Single damage detection in the first case using DER (a) 10% damage and (b) 5% damage



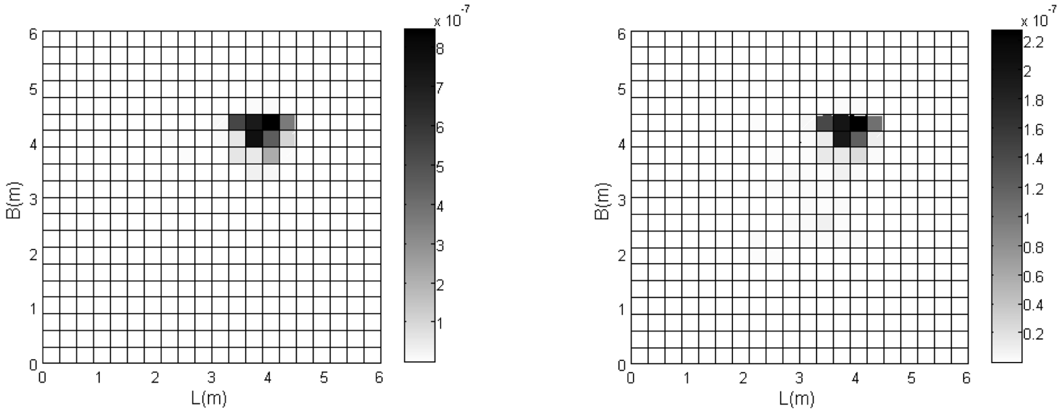
(a) (b)

Fig. 6 Single damage detection in the first case using RRMSEM (a) 10% damage and (b) 5% damage



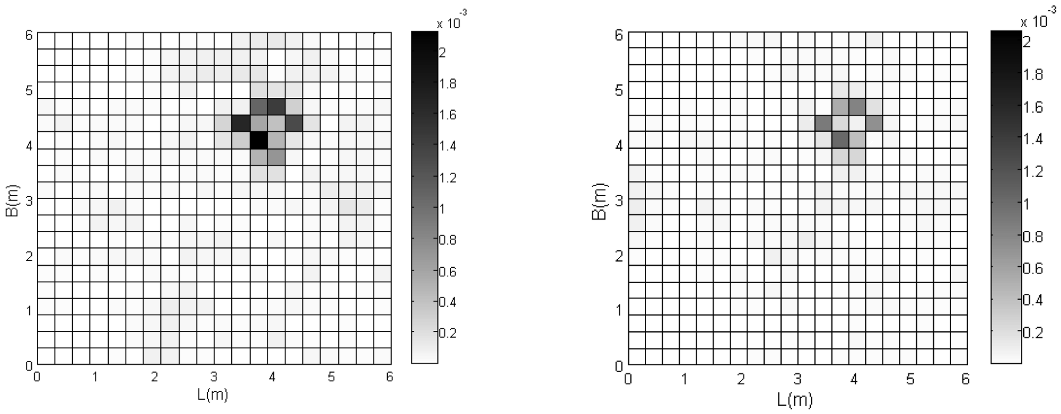
(a) (b)

Fig. 7 Single damage detection in the first case using DEM (a) 10% damage and (b) 5% damage



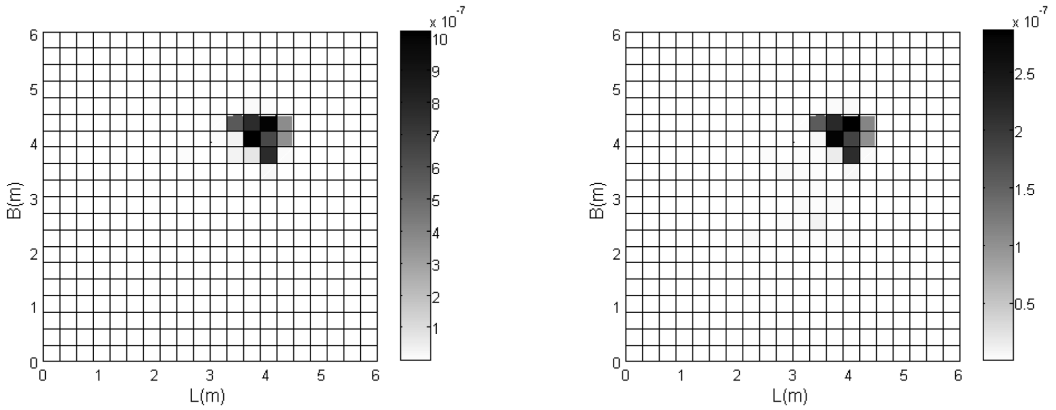
(a) (b)

Fig. 8 Single damage detection in the second case using RRMSER (a) 10% damage and (b) 5% damage



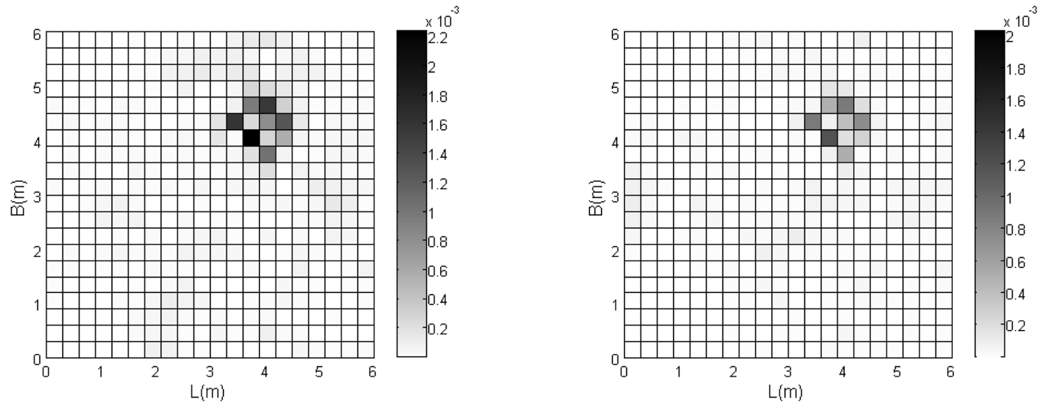
(a) (b)

Fig. 9 Single damage detection in the second case using DER (a) 10% damage and (b) 5% damage



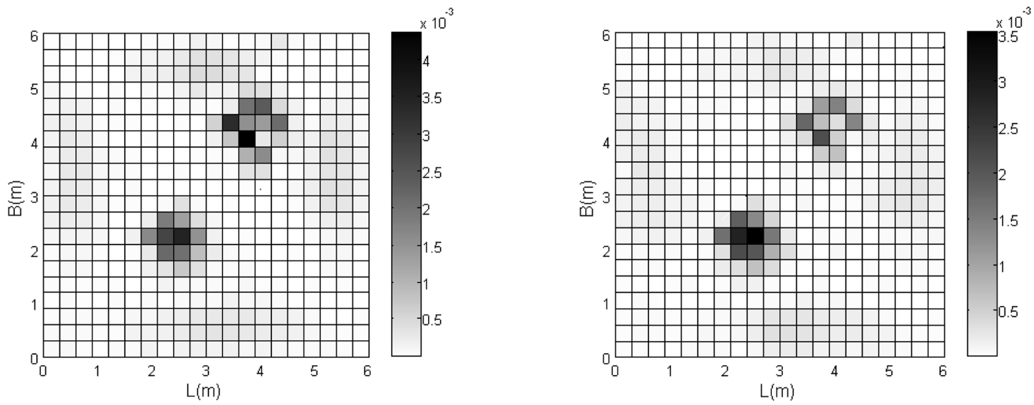
(a) (b)

Fig. 10 Single damage detection in the second case using RRMSEM: (a) 10% damage and (b) 5% damage



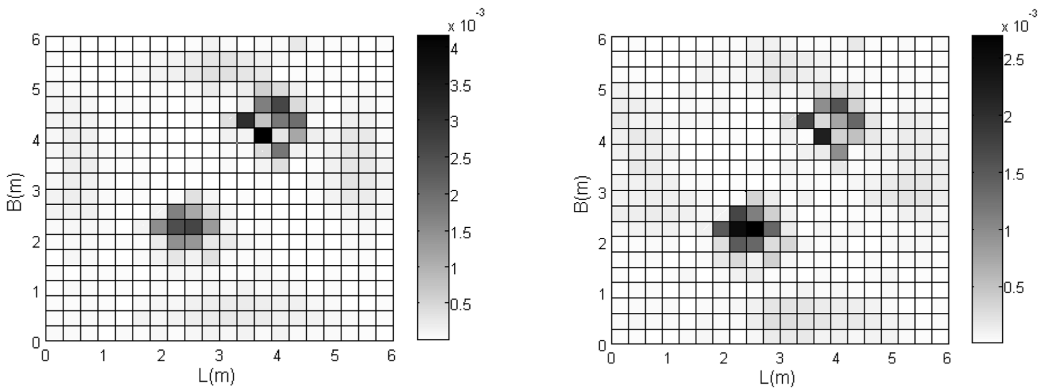
(a) (b)

Fig. 11 Single damage detection in the second case using DEM (a) 10% damage and (b) 5% damage



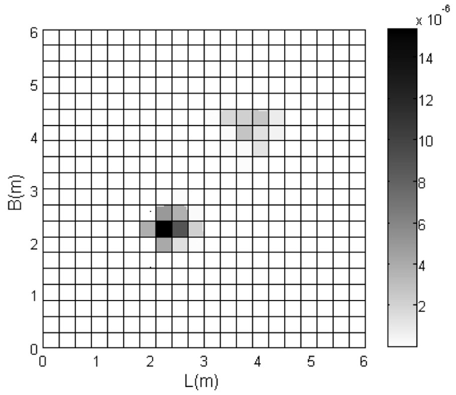
(a) (b)

Fig. 12 Two damage identification in the plate using DER (a) 20% damage and 5% damage and (b) 10% damage and 5% damage

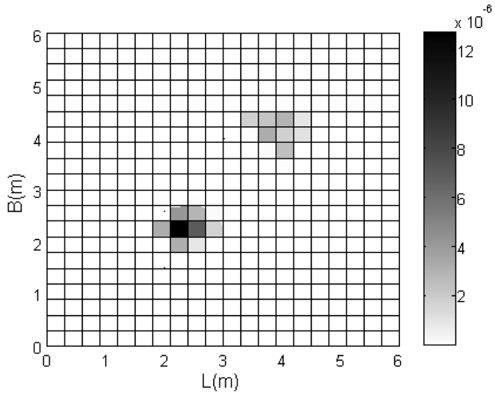


(a) (b)

Fig. 13 Two damage identification in the plate using DEM (a) 20% damage and 5% damage and (b) 10% damage and 5% damage



(a)



(b)

Fig. 14 Two damage identification in the plate using: (a) RRMSER and (b) RRMSEM for 20% damage and 5% damage

are 0.19 m × 0.01 m and 0.27 m × 0.01 m. The location coordinates of the damages are $L1 = 2.14$ m, $B1 = 2.71$m and $L2 = 4.06$ m, $B2 = 4.45$ m with directional angles of $\alpha = 90°$ and $\alpha = 135°$, respectively. For different levels of damage in the plate with reduced thickness of 20% and 5%, and also 10% and 5% are considered.

For this case, the result of damage indices in various levels of damage in the plate is shown in Figs. 12 to 14. Furthermore, the locations of the plate damages are identified using the results of the proposed damage indexes.

Finally, it can be concluded that the proposed damage identification method is much more efficient to detect damage. This is due to the use of 2D discrete wavelet transform in the data processing than other presented methods.

6. Experimental example

In the previous section, the damage identification method was demonstrated using various numerical examples with realistic levels of plate damage. However, it is still useful to examine the empirical performance of the proposed method by applying the measurement data from an experimental study. Therefore, in this section, the performance of the damage detection using a curvelet transform is validated by the use of vibration response data measured from a steel plate tested by Rucka and Wilde (2006).

6.1 Properties of plate

The steel plate used had the following dimensions: length $L = 560$ mm, width $B = 480$ mm and thickness $t = 2$ mm. The material properties for the plate includes Young's modulus of $E = 192$ GPa, mass density of $\rho = 7430$ kg/m^3, and Poisson's ratio of $\nu = 0.25$. The plate contains a rectangular defect of length $W = 80$ mm, width $D = 80$ mm and thickness of $\Delta t = 0.5$ mm. While the location coordinates of the damage is $L1 = 200$ mm, $B1 = 200$ mm. Experimental set up of plate is shown in Fig. 15.

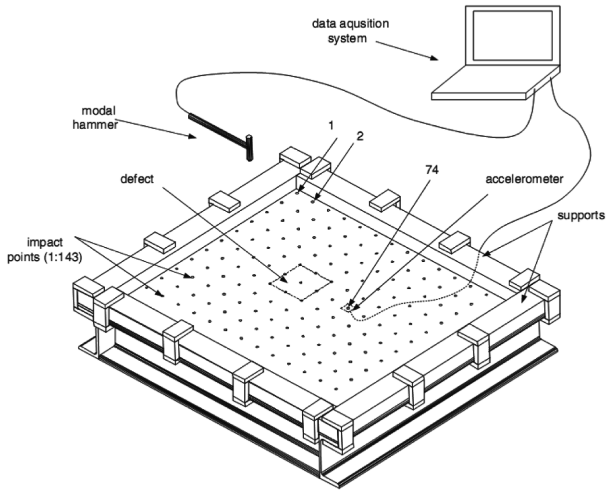


Fig. 15 Experimental set up (Rucka and Wilde 2006)

The plate was subjected to the pulse load applied at 143 points situated on its surface. The measurements were made using one accelerometer to record the response of the structure. Dynamic pulse load was induced by the modal hammer PCB086C03. The data were collected by the data acquisition system Pulse type3650C (Rucka and Wilde 2006).

6.2 Estimation of mode shape

Laplace transform of equation of motion any discrete system can be described by

$$(\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K})\mathbf{X}(s) = \mathbf{F}(s) \tag{17}$$

where s is a Laplace variable. Eq. (17) can be rewritten as

$$\mathbf{B}(s)\mathbf{X}(s) = \mathbf{F}(s) \tag{18}$$

where $\mathbf{B}(s)$ known as a system matrix. Transfer function matrix is defined as follows

$$\mathbf{H}(s) = \mathbf{B}(s)^{-1} \tag{19}$$

Evaluating the transfer function matrix along the frequency axis results in frequency response function (FRF) matrix given as

$$\mathbf{H}(\omega) = \mathbf{X}(\omega)[\mathbf{F}(\omega)]^{-1} \tag{20}$$

To determine one row of the FRF matrix $\mathbf{H}(\omega)$ the modal hammer signal is measured in all points whereas the acceleration is measured still in the same one point. Mode shape is contained in each row or columns of FRF matrix. The obtained FRFs allowed precise identification of the structures lowest frequencies. The fundamental frequency of the plate was $f = 65.0$ Hz. More details can be found in Rucka and Wilde (2006).

Based on the measured data, the mode shape of the plate is determined and the result of the experimental fundamental mode shape is shown in Fig. 16.

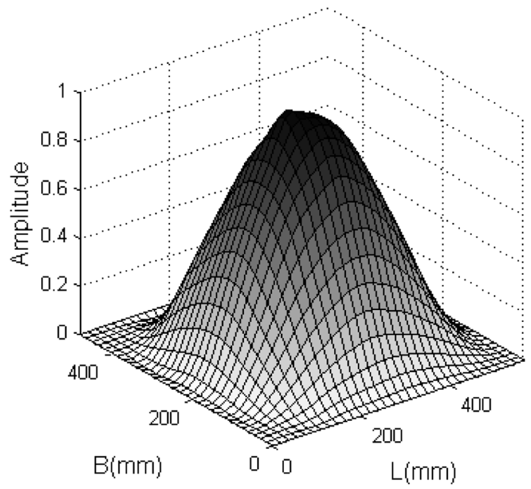


Fig. 16 Experimental fundamental mode shape for the plate with damage

6.3 Results of de-noising and damage detection

In practice, experimentally collected data includes measurement noise. Therefore, it seems necessary to perform a noise reduction process. To achieve this, the presented method for de-noising of mode shape was employed. Fig. 17 indicates the obtained results of de-noising for experimental fundamental mode shape of the plate.

Finally, using the proposed method for damage detection by applying the 2D wavelet transform can determine the proposed damage indexes. Figs. 18 and 19 show the capability of the proposed method for damage detection in the experimented plate using the proposed damage indexes. The damage in plate is detected and localized by using damage index techniques. Therefore, it can be concluded that the proposed method is strong and reliable to detect the damage of actual structures.

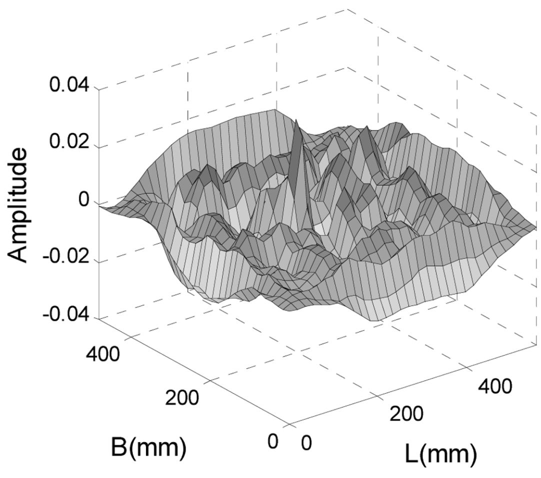
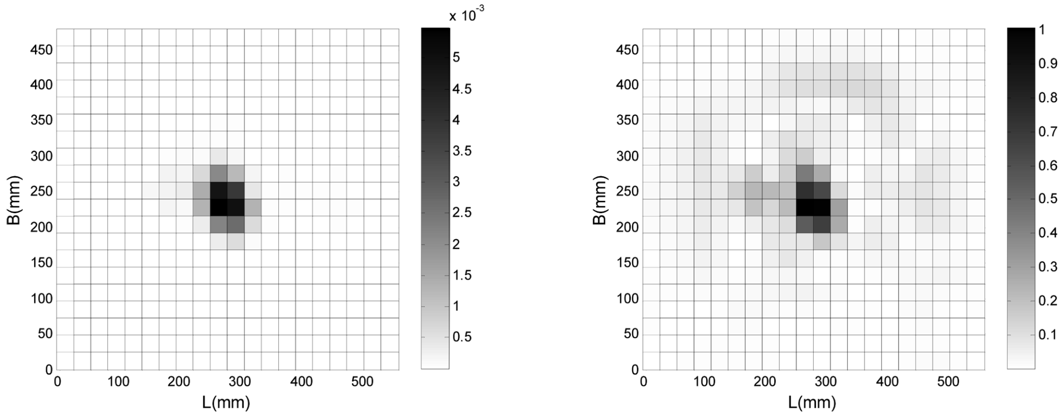
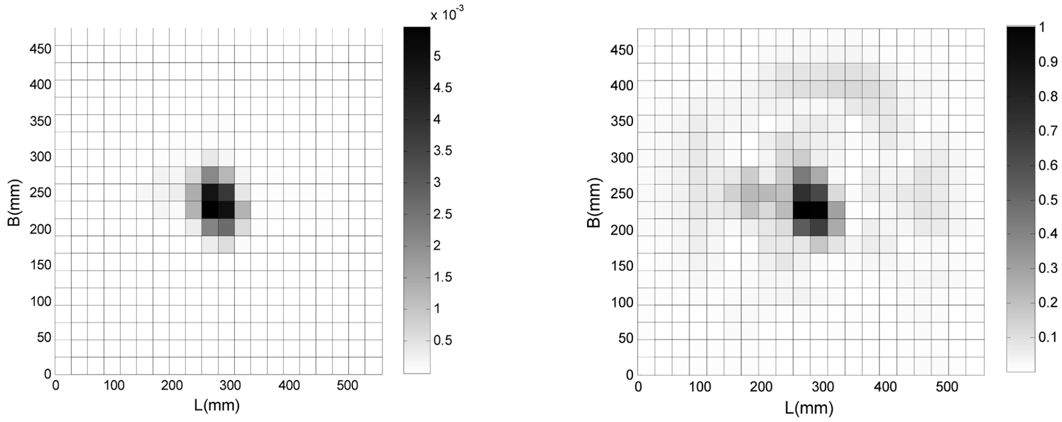


Fig. 17 Obtained results of de-noising for experimental fundamental made shape of the plate



(a) (b)

Fig. 18 Damage identification in the experimented plate after de-noising using (a) RRMSEM and (b) DEM



(a) (b)

Fig. 19 Damage identification in the experimented plate after de-noising using (a) RRMSER and (b) DER

7. Conclusions

In this study, a vibration-based structural damage detection technique is proposed for the localization of the damage in plate structures based on a two-dimensional discrete wavelet transform. The damage is represented by elements with reduced thickness. The effectiveness of the proposed method is demonstrated by conducting a detailed damage identification study on the plate using simulated damage conditions.

The study on wavelet analysis applied in damage detection leads to the following conclusions:

- The comparisons of the results obtained from the numerical studies and the experimental verification study indicate that the proposed damage identification methodology is strong and effective to detect structural damage in plate structures.
- The two-dimensional discrete wavelet transform has been adopted and applied for plate structure responses. Modulus and results of two-dimensional discrete wavelet transform are good parameters for generation of the damage indexes.

- Symmetric wavelets are appropriate for plate damage detection. The considered the reverse biorthogonal wavelet proved to be effective in the presented examples.
- The number of vanishing moments of the applied wavelet plays an important role in damage detection.
- The wavelet-based method localized the small aws in plate structures.

In addition, the results of the present study refer to a rectangular plate but they can be easily extended to consist of more complex structures.

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References

Bayissa, W.L. and Haritos, N. (2007), "Structural damage identification in plates using spectral strain energy analysis", *J. Sound Vib.*, **307**, 226-249.

Bayissa, W.L., Haritos, N. and Thelandersson, S. (2008), "Vibration-based structural damage identification using wavelet transform", *Mech. Syst. Signal Pr.*, **22**, 1194-1215.

Bagheri, A., Ghodrati Amiri, G. and Seyed Razzaghi, S.A. (2009), "Vibration-based damage identification of plate structures via curvelet transform", *J. Sound Vib.*, **327**, 593-603.

Chang, C.C. and Chen, L.W. (2004), "Damage detection of a rectangular plate by spatial wavelet based approach", *Appl. Acoust.*, **65**, 819-832.

Cornwell, P., Doebling, S.W. and Farrar, C.R. (1999), "Application of the strain energy damage detection method to plate-like structures", *J. Sound Vib.*, **224**, 359-374.

Daubechies, I. (1992), "Ten lectures on wavelets", *CBMS-NSF Conference Series in Applied Mathematics*, Montpelier, Vermont.

Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W. (1996), "Damage identification and health monitoring of structural and mechanical systems from change in their vibration characteristics: a literature review", Las Alamos National Laboratory Report LA-13070-MS, Las Alamos, New Mexico.

Fan, W. and Qiao, P. (2009), "A 2-D continuous wavelet transform of mode shape data for damage detection of plate structures", *Int. J. Solids Struct.*, **46**, 4379-4395.

Ghodrati Amiri, G., Bagheri, A., Seyed Razaghi, S.A. and Asadi, A. (2009), "Structural damage detection in plates using wavelet transform", *Proceedings of the Fifth International Structural Engineering and Construction Conference (ISCE5),* Les Vegas, NV, USA.

Gokdag, H. (2008), "Application of continuous wavelet transform to detect damage in thin-walled beams coupled in bending and torsion", *Struct. Eng. Mech.*, **29**(3), 351-354.

Gokdag, H. and Kopmaz, O. (2010), "A new structural damage detection index based on analyzing vibration modes by the wavelet transform", *Struct. Eng. Mech.*, **35**(2), 257-260.

Han, J., Ren, W. and Sun, Z. (2005), "Wavelet packet based damage identification of beam structures", *Int. J. Solids Struct.*, **42**, 6610-6627.

Hadjileontiadis, L.J. and Douka, E. (2007a), "Kurtosis analysis for crack detection in thin isotropic rectangular plates", *Eng. Struct.*, **29**, 2353-2364.

Hadjileontiadis, L.J. and Douka, E. (2007b), "Crack detection in plates using fractal dimension", *Eng. Struct.*, **29**, 1612-1625.

Huang, Y., Meyer, D. and Nemat-Nasser, S. (2009), "Damage detection with spatially distributed 2D Continuous

Wavelet Transform", *Mech. Mater.*, **41**, 1096-1107.

Kim, B.H., Kim, H. and Park, T. (2006), "Nondestructive damage evaluation of plates using the multi-resolution analysis of two-dimensional Haar wavelet", *J. Sound Vib.*, **292**, 82-104.

Lakshmanan, N., Raghuprasad, B.K., Muthumani, K., Gopalakrishnan, N. and Basu, D. (2007), "Wavelet analysis and enhanced damage indicators", *Smart Struct. Syst.*, **3**(1), 23-49.

Loutridis, S., Douka, E., Hadjileontiadis, L.J. and Trochidis, A. (2005), "A two-dimensional wavelet transform for detection of cracks in plates", *Eng. Struct.*, **27**, 1327-1338.

Mallat, S. (2001), *A Wavelet Tour on Signal Processing*, Academic Press, NewYork.

Ovanesova, A.V. and Suarez, L.E. (2004), "Application of wavelet transform to damage detection in frame structures", *Eng. Struct.*, **26**, 39-49.

Poudel, U.P., Fu, G. and Ye, J. (2007), "Wavelet transformation of mode shape difference function for structural damage location identification", *Earthq. Eng. Struct. D.*, **36**, 1089-1107.

Rucka, M. and Wilde, K. (2006), "Application of continuous wavelet transform in vibration based damage detection method for beams and plates", *J. Sound Vib.*, **297**, 536-550.

Sun, Z. and Chang, C.C. (2004), "Statistical Wavelet-Based Method for Structural Health Monitoring", *J. Struct. Eng.-ASCE*, **130**(7), 1055-1062.

Wang, L. and Yuan, F.G. (2007), "Active damage localization technique based on energy propagation of Lamb waves", *Smart Struct. Syst.*, **3**(2), 201-217.

Yam, Y.J. and Yam, L.H. (2002), "Online detection of crack damage in composite plates using embedded piezoelectric actuators/sensors and wavelet analysis", *Compos. Struct.*, **58**, 29-38.