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General evolutionary path for fundamental natural frequencies of structural vibration problems: towards optimum from below

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Abstract. In this paper, both an approximate expression and an exact expression for the contribution factor of an element to the natural frequency of the finite element discretized system of a structure in general and a membrane in particular have been derived from the energy conservation principle and the finite element formulation of structural eigenvalue problems. The approximate expression for the contribution factor of an element is used to predict and determine the elements to be removed in an iteration since it depends only on the quantities associated with the old system in the iteration. The exact expression for the contribution factor of an element makes it possible to check whether the element is correctly removed at the end of an iteration because it depends on both the old system and the new system in the iteration. Thus, the combined use of the approximate expression and the exact expression allows a considerable number of elements to be removed in a single iteration so that the efficiency of the evolutionary structural optimization method can be greatly improved for solving the natural frequency optimization problem of a structure. A square membrane with different boundary supports has been chosen to investigate the general evolutionary path for the fundamental natural frequency of the structure. The related results indicated that if the objective of a structural optimization is to raise the fundamental natural frequency of the structure to an optimal value, the general evolutionary path during its optimization is that the elements are gradually removed along the direction from the area surrounded by the contour of the highest value to that surrounded by the contour of the lowest value.

Key words: evolutionary criterion; natural frequency optimization; membrane vibration; general evolutionary path.

1. Introduction

The fundamental natural frequency of a structure under free vibration conditions is an impor-

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tant parameter for the dynamic design of the structure if the structure is subjected to the load of a low predominant frequency. The loads induced either by an earthquake or by a wind are typical examples of such dynamic loads for the dynamic design of civil engineering structures. Since the dynamic response of a structure depends mainly on the dynamic characteristics of the structure and the dynamic load exerted on the structure, the maximum dynamic response of the structure will occur if its fundamental natural frequency is in coincidence with the predominant frequency of the dynamic load. On the other hand, changing the fundamental natural frequency of a structure away from the predominant frequency of the dynamic load can lead to a significant reduction of the dynamic response of the structure so that a much improved structure is obtained for this particular dynamic load. If the difference between the fundamental natural frequency of a structure and the predominant frequency of the dynamic load is adequately selected, and optimal structure may be produced. Therefore, changing the fundamental natural frequency of a structure to a desired value, which is usually an optimal value, is a very important topic in the structural optimization design under dynamic loadings. This is referred to as the fundamental natural frequency optimization of a structure in this paper because an optimal fundamental natural frequency is related to an optimal structure in the sense that the smallest dynamic response of a structure can lead to the lightest weight of the structure for the structural design.

In the conventional structural optimization method (e.g., Barthelemy and Haftka 1993, Díaz and Bendsφe 1992, Gallagher and Zienkiewicz 1973, Grandhi ad Venkayya 1988, Kohli and Carey 1993, Masur 1984, Miura and Schmit 1978, Seyranian, Lund and Olhoff 1994, Venkayya 1978), the weight of structure was taken to be the objective function, while the natural frequency of the structure was considered as a constraint. The objective of a structural optimization is to design the structure of the minimum weight and to satisfy all the constraints imposed on the structural design. Many developments and refinements over the last several decades indicated that the structural optimization algorithm is iterative because of the highly nonlinear nature of the problem. The algorithm consists of an analysis of the structure and the modification of design variables at each iteration. The analysis of a structure is generally performed using the finite element method or other numerical methods due to the complicated feature of the structure in engineering practice. The modification of design variables is achieved using several methods such as the nonlinear mathematical programming method, the optimality criterion approach and so forth. According to the design variables used, the structure optimization encountered commonly in engineering practice can be classified into the following three basic categories. The first is the "shape" optimization problem in which the cross-sections and/or thickness of structural members such as beams, frames, plates and so forth are selected as basic design variables. The second is the topology optimization problem in which the space coordinates used to describe the geometry of a structure are selected as basic design variables. The third is the material optimization problem in which the structural material properties such as the elastic modulus and Poisson's ratio are selected as basic design variables.

Realizing the fact that almost all the conventional methods for structural optimization usually involved some complicated mathematical operations, Xie and Steven (1993, 1994a, 1994b) have recently presented an evolutionary structural optimization method to circumvent the use of any complicated mathematical operation. In the evolutionary structural optimization method, the "shape" optimization, topology optimization and material optimization can be implemented at the same time. Since the evolutionary criterion and the finite element method are the basis of the evolutionary structural optimization method, an original structure of any shape under certain requirements can evolve into an optimal one. Apart from its simplicity and versatility, another significant advantage of using the evolutionary structural optimization method is that for a sturctural optimization problem, the optimal shape of the structure can be found without the specification of more geometric details than the material used, the location of supports and the application points of loads. This makes the initial data preparation very simple for solving a structural optimization problem. Although the evolutionary structural optimization method has been successfully used to optimize the shape of a structure for both static and dynamic problems under plane stress conditions, some fundamental questions associated with the method have not been answered so far, especially for structural natural frequency optimization problems. For example, by simply using the related evolutionary criterion to remove elements from a structural system in the process of its natural frequency optimization, can the resulted structural shape be guaranted as an optimal shape of the structure under the given condition? At the previous stage, only a few elements are allowed to be removed in a single iteration for the natural frequency optimization of a structure, since the previous evolutionary criterion was derived under the assumption that the concerned modal shape of the structure before removing elements is approximately the same as that after removing elements in a single iteration. This makes the evolutionary structural optimization method of low efficiency for dealing with structural natural frequency optimization problems. Thus, another fundamental question is how to improve the efficiency of the evolutionary optimization method when it is used to deal with structural natural frequency optimization problems.

For the purpose of answering two basic questions raised above, the fundamental natural frequency optimization of a membrane with different boundary supports is carried out in this paper to investigate the general evolutionary path for the fundamental natural frequency of the membrane by using the evolutionary structural optimization method. From the relevant conclusions obtained by studying the general evolutionary path for the fundamental natural frequency of a membrane vibration problem, not only have the concerned fundamental questions about the evolutionary structural optimization method been answered, but also some interesting suggestions have been made for the dynamic design of a structure when the fundamental natural frequency of the structure plays an important role in determining the optimal shape of the structure.

2. Basic algorithms related to the evolutionary structural optimization method

Like other conventional structural optimization methods, the evolutionary structural optimization method is iterative because of the highly nonlinear nature of the structural optimization problem itself. The basic algorithms related to the evolutionary structural optimization method contains an algorithm for the analysis of a complicated structure and an algorithm for the modification of structural shape and/or material at each iteration. Since the finite element method has provided a very powerful tool for both the dynamic and the static response of almost all the structures in engineering practice, it is reasonable to include the finite element method into the evolutionary structural optimization method as an effective analytical tool for determining the response of a complicated structure. The algorithm related to the finite element method is well known and does not need to be repeated here. However, the algorithm related to the modification of structural shape and/or material at each iteration depends mainly on the corresponding evolutionary criterion used in the evolutionary structural optimization method, because it is the evolutionary criterion that bridges the gap between the finite element method and the structural optimization in the evolutionary structural optimization method. For this reason, the evolutionary criterion for the natural frequency optimization of a structure in general and a membrane in particular is established in this section.

In order to establish an evolutionary criterion, it is essential to evaluate the individual contribution of an element to the overall behaviour of the structure since finite elements are basic cells of the structure in the sense that the finite element method is used to model the structure. Although the contribution factor of an element to the natural frequency of the structure is available, it was derived under the assumption that the modal shape in relation to the concerned natural frequency of the structure does not change much before and after an element is removed from the finite element discretized system of the structure. This assumption seriously restricts the total number of elements to be removed at each iteration and therefore greatly reduces the efficiency of the evolutionary structural optimization method. To solve this problem, the above-mentioned assumption should be given up in the process of deriving the contribution factor of an element to the natural frequency of the structure so that removal of an element can be extended to the removal of a patch of elements at each iteration. Since the natural frequency of a structure is usually obtained from the solution to an eigenvalue problem of the structure under the undamped free vibration condition, it is possible to derive the contribution factor of an element to the natural frequency of the structure from the energy conservation principle because ther is no energy dissipation in such a structural system.

For the purpose of obtaining a finite element solution to the natural frequency of a membrane under undamped free vibration conditions, it is a common practice to assume that the membrane is subjected to a harmonic motion as follows:

$$w = w_A \sin\omega t \tag{1}$$

where w is the out-of-plane displacement of the membrane; w_A is the corresponding amplitude of the displacement and ω is the circular frequency at which the membrane vibrates.

Under the condition of harmonic motion, the strain energy and kinetic energy of an element in the finite element discretized system of the membrane can be expressed as

$$E_p^e = \frac{1}{2}\{w_A^e\}^T[k^e]\{w_A^e\}\sin^2\omega t$$

$$E_k^e = \frac{1}{2}\{w_A^e\}^T[m^e]\{w_A^e\}\,\omega^2\cos^2\omega t \tag{2}$$

where $\{w_A^e\}$ is the displacement amplitude vector of the element; E_p^e and E_k^e are the strain energy and kinetic energy of the element respectively; $[k^e\}$ and $[m^e]$ are the stiffness matrix and mass matrix of the element. These matrices can be obtained using the conventional finite element method.

The summation of the strain energy and kinetic energy from all elements in the finite element discretized system yields the total strain energy and total kinetic energy of the system.

$$E_p=\frac{1}{2}\{w_A\}^T[K]\{w_A\}\sin^2\omega t$$

$$E_k=\frac{1}{2}\{w_A\}^T[M]\{w_A\}\,\omega^2\cos^2\omega t \tag{3}$$

where $\{w_A\}$ is the displacement amplitude vector of the system; E_p and E_k are the total strain energy and total kinetic energy of the system; $[K]$ and $[M]$ are the global stiffness matrix and mass matrix of the system respectively.

From Eq. (3), the total energy of the system can be obtained as

$$E=E_p+E_k=\frac{1}{2}\{w_A\}^T([K]\sin^2\omega t+\omega^2[M]\cos^2\omega t)\{w_A\} \tag{4}$$

where E is the total energy of the system.

Note that for an undamped free vibration system, the total energy is constant. When the system passes through the static equilibrium position, the strain energy is zero and the kinetic energy has a maximum value which is equal to the total energy. On the other hand, when the system reaches the maximum displacement, the kinetic energy is zero and the strain energy has a maximum value which is also equal to the total energy of the system. This leads to the following expression:

$$E=\frac{1}{2}\{w_A\}^T[K]\{w_A\}=\frac{1}{2}\omega^2\{w_A\}^T[M]\{w_A\} \tag{5}$$

As shown in Fig. 1, the total energy of an element in the finite element discretized system can be expressed at the element level as

$$E^e=\frac{1}{2}\{w_A^e\}^T[k^e]\{w_A^e\}-\{w_A^e\}^T\{p^e\} \tag{6}$$

where E^e is the total energy of a element in the finite element discretized system of a membrane; $\{p^e\}$ is the nodal force vector of the element in the case of the system reaching its maximum strain energy. Since $\{p^e\}$ is an internal force, it will be balanced out when the element is assembled into the structure.

After an element is removed from the finite element discretized system in Fig. 1, the total energy of the new system can be expressed as

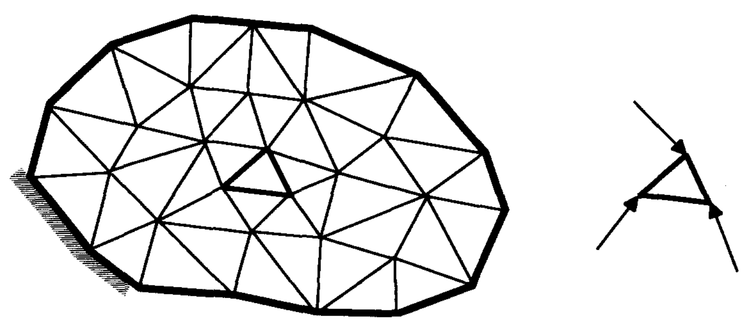


Fig. 1 Removal of an element from the system.

$$E_n=\frac{1}{2}\{w_A\}_n^T([K]-[K^e])\{w_A\}_n \tag{7a}$$

$$E_n=\frac{1}{2}\omega_n^2\{w_A\}_n^T([M]-[M^e])\{w_A\}_n \tag{7b}$$

where $[K^e]$ and $[M^e]$ are enlarged stiffness matrix and mass matrix of the element. These enlarged matrices of the element have the same order as the global matrices of the system. ω_n and $\{w_A\}_n$ are the circular frequency and the displacement amplitude vector of the new system. They can be defined as

$$\begin{aligned}\omega_n&=\omega_0+\Delta\omega\\ \{w_A\}_n&=\{w_A\}_0+\{\Delta w_A\}\end{aligned} \tag{8}$$

where ω_0 and $\{w_A\}_0$ are the circular frequency and the displacement amplitude vector of the old system in which the element is not removed. $\Delta\omega$ expresses the circular frequency difference between the new and old systems so that it can be called as the contribution factor of an element to the circular frequency of the system. $\{\Delta w_A\}$ represents the displacement amplitude difference between the new and old systems.

The total energy of the old system can be written as

$$E_0=\frac{1}{2}\{w_A\}_0^T[K]\{w_A\}_0 \tag{9a}$$

$$E_0=\frac{1}{2}\omega_0^2\{w_A\}_0^T[M]\{w_A\}_0 \tag{9b}$$

Using Eqs. (7a) and (9a), the total energy difference between the old and new systems can be evaluated as

$$\Delta E_1=E_0-E_n=\frac{1}{2}\{w_A\}_0^T[K]\{w_A\}_0-\frac{1}{2}\{w_A\}_n^T([K]-[K^e])\{w_A\}_n \tag{10}$$

Similarly, using Eq. (7b) and (9b), the total energy difference between the old and new systems can be expressed as

$$\Delta E_2=E_0-E_n=\frac{1}{2}\omega_0^2\{w_A\}_0^T[M]\{w_A\}_0-\frac{1}{2}(\omega_0+\Delta\omega)^2\{w_A\}_n^T([M]-[M^e])\{w_A\}_n \tag{11}$$

Under the energy conservation condition, the following equation exists.

$$\Delta E_1=\Delta E_2 \tag{12}$$

Substituting Eqs. (10) and (11) into Eq. (12) yields the following equation:

$$\begin{aligned}&\frac{1}{2}\{w_A\}_0^T[K]\{w_A\}_0-\frac{1}{2}(\{w_A\}_0^T+\{\Delta w_A\}^T)([K]-[K^e])(\{w_A\}_0+\{\Delta w_A\})\\ &=\frac{1}{2}\omega_0^2\{w_A\}_0^T[M]\{w_A\}_0-\frac{1}{2}(\omega+\Delta\omega)^2(\{w_A\}_0^T+\{\Delta w_A\}^T)([M]-[M^e])(\{w_A\}_0+\{\Delta w_A\})\end{aligned} \tag{13}$$

The governing equation of motion for the new system under the undamped free vibration condition can be expressed as

$$[(\omega+\Delta\omega)^2([M]-[M^e])-([K]-[K^e])]\,(\{w_A\}_0+\{\Delta w_A\})=\{0\} \tag{14}$$

Substituting Eq. (14) into Eq. (13) yields the following equation:

$$\omega_0^2\{w_A^e\}_0^T[m^e]\{w_A^e\}_0-\{w_A^e\}_0^T[k^e]\{w_A^e\}_0+\omega_0^2\{\Delta w_A\}^T([M]-[M^e])\{\Delta w_A\}-\{\Delta w_A\}^T([K]-[K^e])\{\Delta w_A\}$$
$$=[2\omega_0\Delta\omega+(\Delta\omega)^2](\{w_A\}_0^T([M]-[M^e])\{w_A\}_0-\{\Delta w_A\}^T([M]-[M^e])\{\Delta w_A\}) \tag{15}$$

It is noted that Eq. (15) is called as the energy equation for the old and new systems. From this energy equation, the following expression can be derived.

$$\Delta\omega=\sqrt{(\omega_0^2+\alpha)}-\omega_0 \tag{16}$$

where α can be expressed as:

$$\alpha=\frac{\omega_0^2\{w_A^e\}_0^T[m^e]\{w_A^e\}_0-\{w_A^e\}^T[k^e]\{w_A^e\}+\omega_0^2\{\Delta w_A\}^T([M]-[M^e])\{\Delta w_A\}-\{\Delta w_A\}^T([K]-[K^e])\{\Delta w_A\}}{\{w_A\}_0^T([M]-[M^e])\{w_A\}_0-\{\Delta w_A\}^T([M]-[M^e])\{\Delta w_A\}} \tag{17}$$

It is worth noting that Eq. (16) is an exact expression for the contribution factor of an element to any natural frequency of the undamped free vibration system. However, if the first order variations are only considered in Eq. (15), the contribution factor of an element to the natural frequency of an undamped free vibrations system can be approximately expressed as:

$$\Delta\omega^*=\frac{\omega_0^2\{w_A^e\}_0^T[m^e]\{w_A^e\}_0-\{w_A^e\}_0^T[k^e]\{w_A^e\}_0}{2\omega_0\{w_A\}_0^T[M]\{w_A\}_0} \tag{18}$$

If the ith natural frequency and its corresponding vibration mode of a system are expressed as ω_i and $\{d_i\}$, Eqs. (16) and (18) can be further simplified using the following equation in structural dynamics.

$$M_i=\{d_i\}^T[M]\{d_i\} \tag{19}$$

where M_i is the modal mass in correspondence with the ith natural frequency of the system.

Considering Eqs. (16), (17) and (19) and simultaneously yields the following equation:

$$\Delta\omega_i=\sqrt{(\omega_i^2+\alpha)}-\omega_i \tag{20}$$

where

$$\alpha=\frac{\omega_i^2\{d_i^e\}^T[m^e]\{d_i^e\}-\{d_i^e\}^T[k^e]d_i^e\}+\omega_i^2\{\Delta d_i\}^T([M]-[M^e])\{\Delta d_i\}-\{\Delta d_i\}^T([K]-[K^e])\{\Delta d_i\}}{M_i-\{d_i\}^T[M^e]\{d_i\}-\{\Delta d_i\}^T([M]-[M^e])\{\Delta d_i\}} \tag{21}$$

where $\Delta\omega_i$ is the exact contribution factor of an element to the ith natural frequency of the system; $\{d_i^e\}$ is the modal shape vector of the element to be removed. The other symbols in Eq. (21) are of the same meaning as mentioned before.

Similarly, considering Eqs. (18) and (19) simultaneously yields the following equation:

$$\Delta\omega_i^*=\frac{\omega_i^2\{d_i^e\}^T[m^e]\{d_i^e\}-\{d_i^e\}^T[k^e]\{d_i^e\}}{2\omega_i M_i} \tag{22}$$

where $\Delta\omega_i^*$ is the approximate contribution factor of an element to the ith natural frequency of the system in the sense that only the first order variations are considered during considering the energy conservation before and after an element is removed from the system.

It is noted that $\Delta\omega_i^*$, which is the same as the previously derived contribution factor of an element to the ith natural frequency of the system, depends only on the quantities associated with the old system, whereas the exact $\Delta\omega_i$ depends on the quantities associated with both the old and the new systems in an iteration. Thus, a new enhanced evolutionary criterion can be established by using the combination of $\Delta\omega_i^*$ and $\Delta\omega_i$. That is to say, $\Delta\omega_i^*$ is used to predict and determine which element should be removed from the old system in an iteration, while $\Delta\omega_i$ is used to check whether the element is correctly removed at the end of the iteration. If the check indicates that an element is wrongly removed in the iteration, the element can be returned to the system before starting next iteration. This new enhanced evolutionary criterion allows a patch of elements to be removed from the system in a single iteration because the use of the exact $\Delta\omega_i$ eliminates the assumption employed in the process of deriving the previous contribution factor of an element to the natural frequency of the system. Since the main purpose of this paper is to investigate the general evolutionary path when the fundamental natural frequency of a membrane is raised to a given value during its shape optimization, the most efficient way is to remove the elements, the contribution factors of which are of or close to the maximum positive value to the concerned natural frequency, from the system in an iteration. This forms the evolutionary criterion used in this study.

3. General evolutionary path for the fundamental natural frequency optimization of a square membrane

Using the new enhanced evolutionary criterion presented in the last section, the fundamental

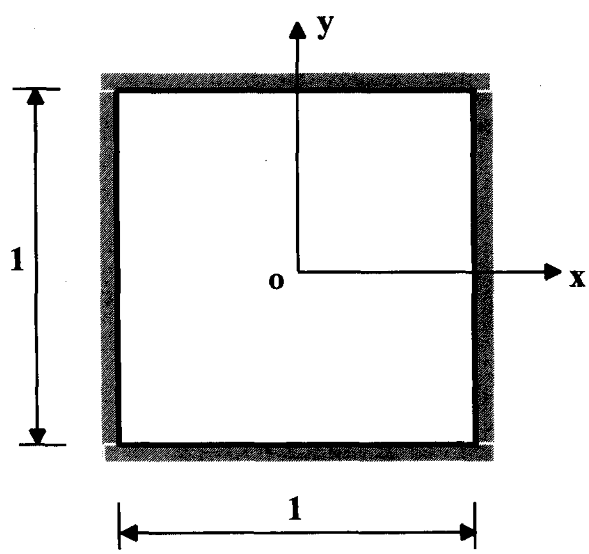
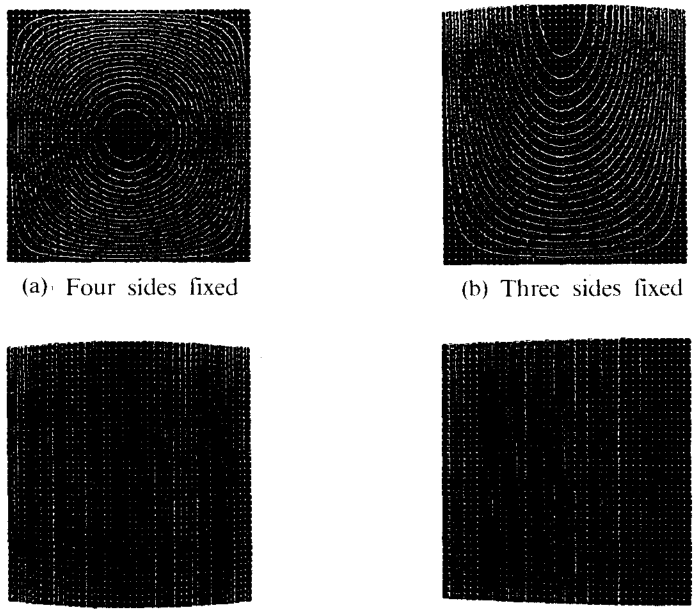


Fig. 2 A square membrane with four sides fixed.



(a) Four sides fixed (b) Three sides fixed

(c) Two sides fixed (d) One side fixed

Fig. 3 First vibration mode variation of a membrane due to different boundary supports.

natural frequency of a square membrane is raised to an optimal value in this section to investigate the general evolutionary path during its optimization. As shown in Fig. 2, a square membrane with the side length of one meter is supported along its four sides. The membrane is initially discretized into 1600 square finite elements at the beginning of its shape optimization. For the purpose of investigating the general evolutionary path of the fundamental natural frequency of the membrane, the following parameters have been used in the calculation. The thickness of the membrane is 0.002 m and the wave velocity of the membrane is 1 m/s. Since the fundamental natural frequency and its corresponding vibration mode of the initial square membrane are dependent on its boundary supports, four different boundary supports are taken into account in the calculation. In order to provide some necessary references for later analysis, the contours of the first vibration mode of the membrane due to four different boundary supports have been shown in Fig. 3. As expected, the general distribution patterns of the contours are different for the membrane with different boundary supports. However, the distribution pattern of the contour for the membrane with one side fixed is similar as that for the left half or the right half of the membrane with two sides fixed. For this reason, the attention will be given to the study on the membrane with the first three boundary supports in Fig. 3.

Figs. 4, 6 and 8 show the shape variations of the membrane at different stages in the first natural frequency optimization process of the membrane due to different boundary supports. It is noted form these figures that the greater the stage number, the more the elements removed from the initial square membrane. This is due to the fact that the main purpose of the membrane optimization in this section is to find out its optimal shape, in which the first natural frequency is increased to a desired value but the material usage is decreased to a maximum extent. This leads to the removal of the elements, whose contribution factors to the first natural frequency are equal or close to a maximum value, from the finite element discretized system of the memb-

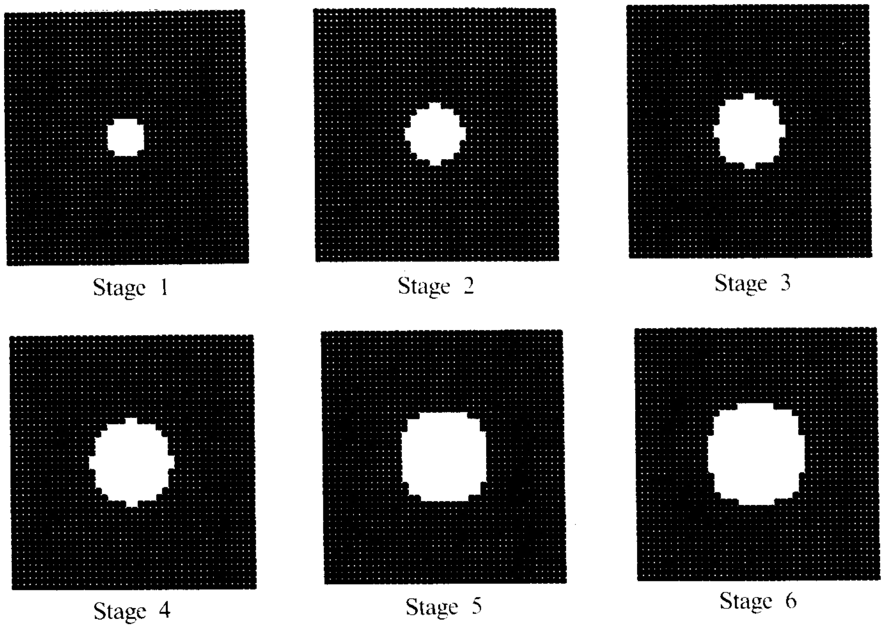


Fig. 4 First natural frequency optimization process of a membrane (Four sides fixed).

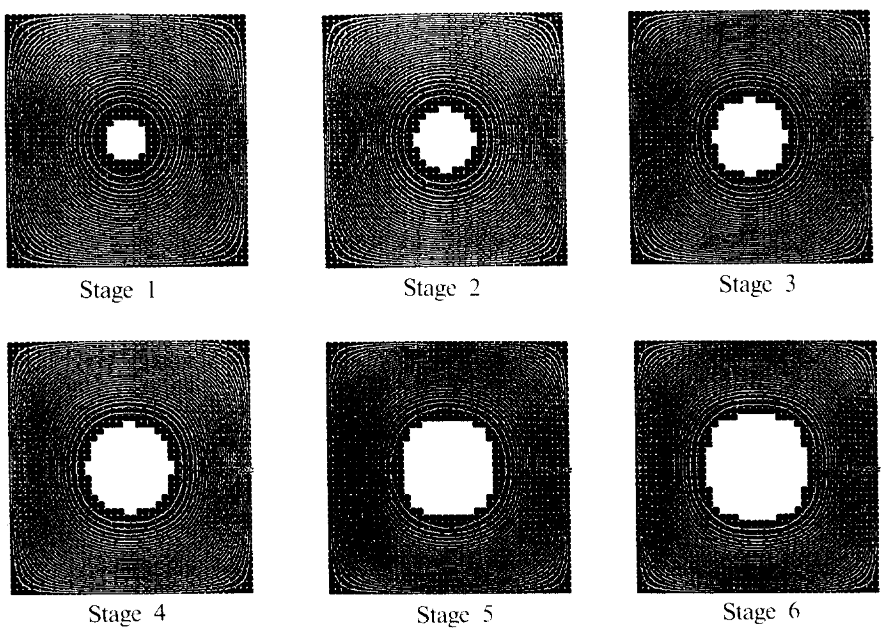


Fig. 5 First vibration mode variation during optimization (Four sides fixed).

rane in each iteration. Tables 1 to 3 show some detailed information about the number of elements removed in a stage, the number of total elements removed from the beginning of optimization to the stage, the value of the first natural frequency increase in the stage, the total value of the first natural frequency increase from the beginning of optimization to the stage

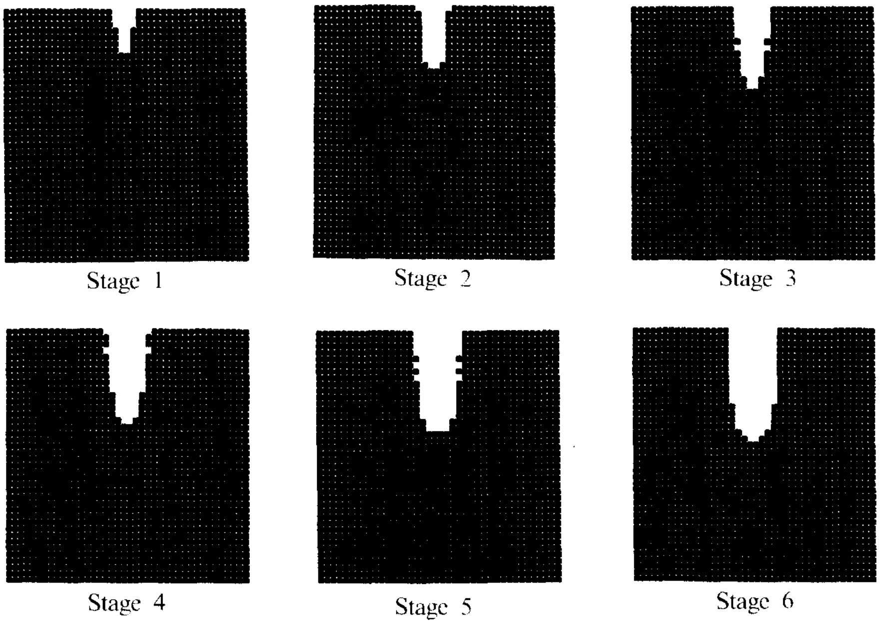


Fig. 6 First natural frequency optimization process of a membrane (Three sides fixed).

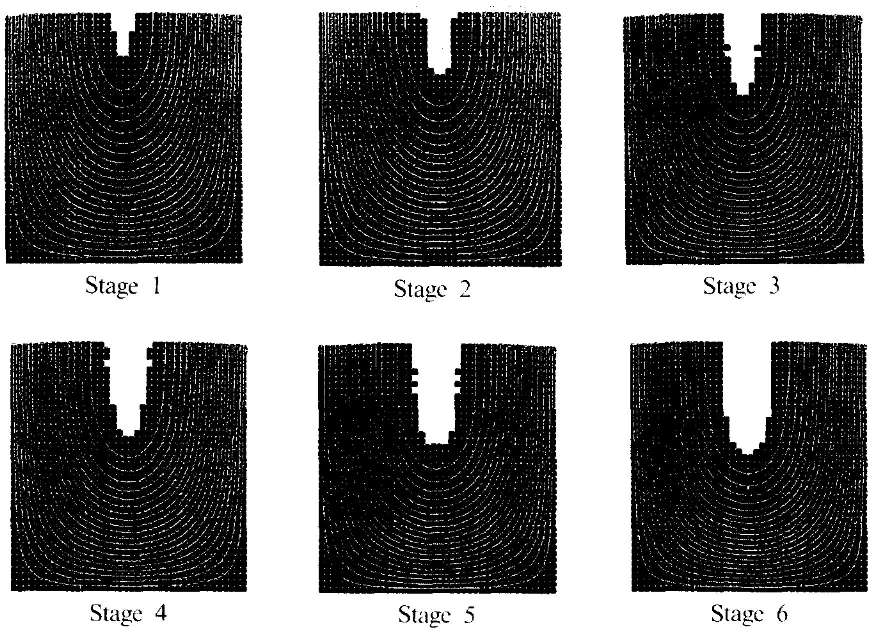


Fig. 7 First vibration mode variation during optimization (Three sides fixed).

and the final value of the first natural frequncy reached in the stage during the membrane optimization. It is obvious that the same number of elements removed in different stages does not cause the same amount of the natural frequency increase in these stages. Taking the results in Table 1 as an example, although 36 elements have been removed in both stage 2 and stage

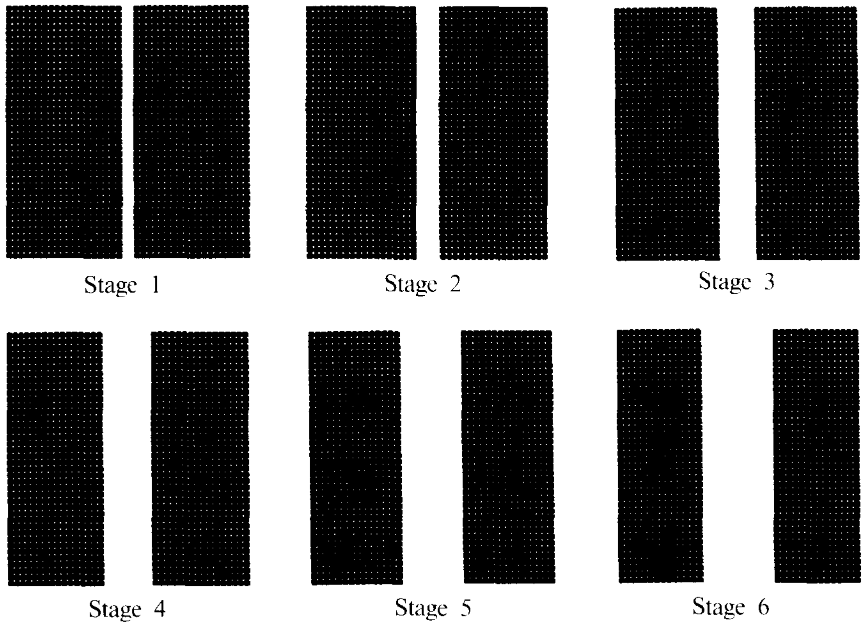


Fig. 8 First natural frequency optimization process of membrane (Two sides fixed).

Table 1 First natural frequency variation during optimization (Four sides fixed)

Stage number	Elements removed in the stage	Total elements removed	Frequency increase in the stage	Total frequency increase	Final frequency (rad/s)
0	0	0	0	0	4.4389
1	32	32 (2%)	0.1692	0.1692 (3.81%)	4.6081
2	36	68 (4.25%)	0.1856	0.3548 (7.99%)	4.7937
3	36	104 (6.5%)	0.1868	0.5416(12.20%)	4.9805
4	36	140 (8.75%)	0.1908	0.7324(16.50%)	5.1713
5	36	176(11%)	0.1965	0.9289(20.93%)	5.3678
6	40	216(13.5%)	0.2263	1.1552(26.02%)	5.5941

5, the increase values of the first natural frequency are 0.1856 rad/s and 0.1965 rad/s for stage 2 and stage 5 respectively. This means that the contribution factor of each removed element to the natural frequency of a structure is different in the process of the structural optimization, except for its symmetric counterparts if the optimization problem is a symmetric one. Also, it is noted that with an increase of stage number, the first natural frequency of the membrane increases, but the material used for the membrane decreases accordingly. In the case of stage 6 shown in Table 1, compared with the initial square membrane, the increase of the first natural frequency is 26.02% and the reduction of the material used for the membrane is 13.5%. This fact demonstrated the usefulness of the evolutionary structural optimization method to the natural frequency optimization of a structure.

Figs. 5, 7 and 9 show the first natural vibration mode contours of the membrane at different stages during the first natural frequency optimization of the membrane with three different boun-

Table 2 First natural frequency variation during optimization (Three sides fixed)

Stage number	Elements removed in the stage	Total elements removed	Frequency increase in the stage	Total frequency increase	Final frequency (rad/s)
0	0	0	0	0	3.5107
1	20	20(1.25%)	0.083	0.083 (2.36%)	3.5937
2	20	40(2.5%)	0.0792	0.1622 (4.62%)	3.6729
3	20	60(3.75%)	0.0777	0.2399 (6.83%)	3.7506
4	22	82(5.13%)	0.085	0.3249 (9.25%)	3.8356
5	22	104(6.5%)	0.0853	0.4102(11.68%)	3.9209
6	22	126(7.88%)	0.0861	0.4963(14.14%)	4.007

Table 3 First natural frequency variation during optimization (Two sides fixed)

Stage number	Elements removed in the stage	Total elements removed	Frequency increase in the stage	Total frequency increase	Final frequency (rad/s)
0	0	0	0	0	3.1408
1	80	80 (5%)	0.1652	0.1652 (5.26%)	3.306
2	80	160(10%)	0.1836	0.3488(11.11%)	3.4896
3	80	240(15%)	0.2051	0.5539(17.64%)	3.6947
4	80	320(20%)	0.2307	0.7846(24.98%)	3.9254
5	80	400(25%)	0.2615	1.0461(33.31%)	4.1869
6	80	480(30%)	0.2987	1.3448(42.82%)	4.4856

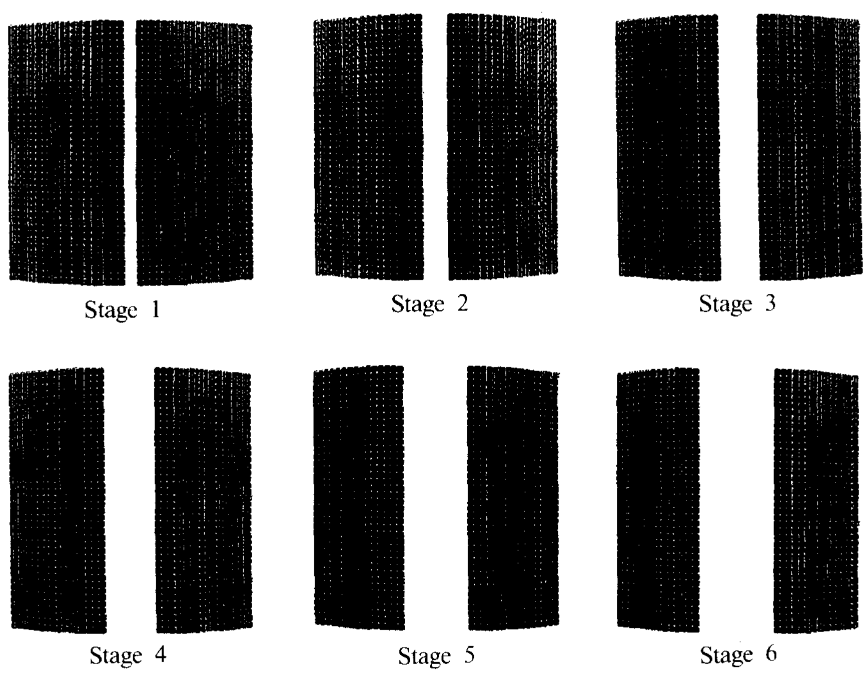


Fig. 9 First vibration mode variation during optimization (Two sides fixed).

dary supports respectively. As can be observed from these figures, the general evolutionary path for the fundamental natural frequency optimization of the membrane is that the elements are gradually removed along the direction from the area surrounded by the contour of the highest value to that surrounded by the contour of the lowest value. This fact indicates that the optimal shape of the membrane can be obtained by simply removing the elements, the contribution factors of which are of or close to the maximum value to the concerned natural frequency, from the system in each iteration. The reason for this is that in order to keep the natural frequency of a system increasing gradually, the elements, whose contribution factors are of or close to the smallest positive value to the concerned natural frequency, are located at the area surrounded by two specific contours of some middle values between the maximum and the minimum values of the all contours. If these elements are removed in an iteration, the elements located inside these two specific contours will automatically be removed because the connection between the inside and the outside of these two specific contours is broken due to removing the elements in the above-mentioned way in the iteration. On the other hand, it is also desirable to demonstrate that no matter how elements are removed in an iteration in the evolutionary structural optimization method, the same optimal shape of the system can be achieved when the concerned natural frequency is increased to a desired value. For this reason, each stage in Table 1 is divided into five substages. At each substage, four or eight elements, whose contribution factors are of the maximum value to the fundamental natural frequency of the membrane, are removed from the system due to the symmetric nature of the problem. It has been found that at the end of every five substages in a stage, the same results are obtained as using the previous one stage alone. This fact demonstrates that the evolutionary structural optimization method can result in an optimal shape for solving the natural frequency optimization problem of a structure.

4. Conclusions

There is an urgent need to answer two fundamental questions associated with the evolutionary structural optimization method when it is used to deal with the natural frequency optimization problem of a structure. The first is to confirm that the structural shape obtained by using the evolutionary structural optimization method is the optimal shape of the structure after its natural frequency optimization. The second is how to improve the efficiency of the evolutionary structural optimization method when used to solve structural natural frequency optimization problems.

For the purpose of answering the first question, the study on the natural frequency optimization of a square membrane with different boundary supports has been carried out to investigate the general evolutionary path for the fundamental natural frequency of the structure. It has been found that if the purpose of a structural optimization is to raise the fundamental natural frequency of the structure to an optimal value, the general evolutionary path during its optimization is that the elements are gradually removed along the direction from the area surrounded by the contour of the highest value to that surrounded by the contour of the lowest value. This finding demonstrates that the evolutionary structural optimization method can result in an optimal shape for solving the natural frequency optimization problem of a structure.

To answer the second question, both an approximate expression and an exact expression for the contribution factor of an element to the concerned natural frequency of the finite element discretized system of a structure in general and a membrane in particular have been derived in this paper from the energy conservation principle and the finite element formulation of structural eigenvalue problems. The combined use of the approximate expression and the exact expression allows a considerable number of elements to be removed in a single iteration so that the efficiency of the evolutionary structural optimization method can be greatly improved for solving the natural frequency optimization of a structure.

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