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Simplified dynamic analysis of slender tapered thin-walled towers with additional mass and rigidity

Discussion by P.A.A. Laura†

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The authors are to be congratulated for their interesting and practical paper (Takabatake and Mizuki 1995).

It is also the purpose of this discussion to mention the possibility of using an alternate approximating function instead of the cantilever beam function employed by the authors, Eq. (39) in the paper under discussion

$$\phi_n = ch[k_n(x_B - x)] - \cos[k_n(x_B - x)] - \alpha_n\{sh[k_n(x_B - x)] - \sin[k_n(x_B - x)]\} \tag{1}$$

It has been shown by Laura and Gutierrez (1986) that a very convenient approximation for the fundamental mode of vibration of the structural systems depicted in Fig. 1 is the polynomial expression

$$W_a(x) = C(\alpha_4 x^\gamma + \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + 1);\ x = \bar{x}/L \tag{2}$$

where the α_i's are obtained substituting Eq. (2) in the essential boundary conditions

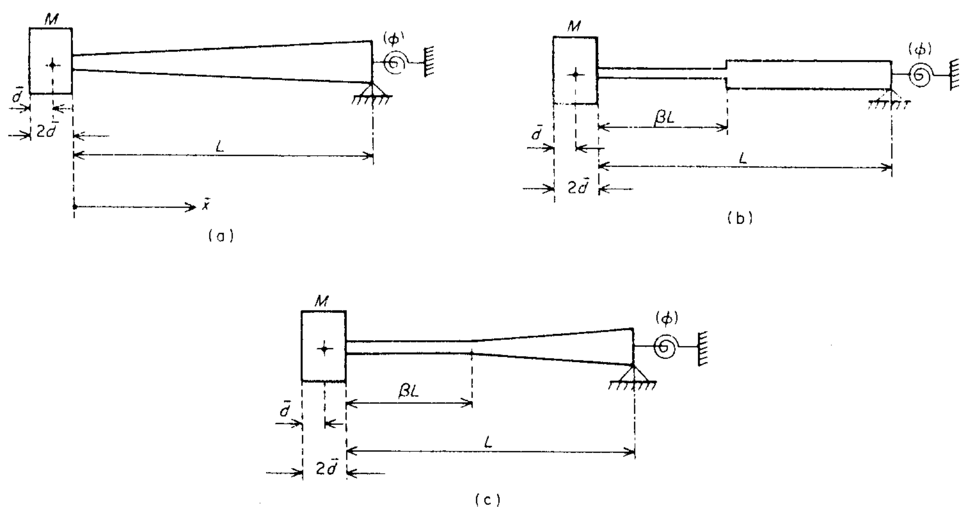

Fig. 1 Mechanical systems executing transverse vibrations (Laura and Gutierrez 1986).

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$$W_a''(0)=0;\ W_a'''(0)=0;\ W_a(1)=0;\ W_a'(1)=-\phi' W_a''(1) \quad \text{(3a, b, c and d)}$$

where $\phi'=EI(1)\phi/L$ and ϕ: flexibility coefficient defined by Eq. 3(d), see Fig. 1.

The exponential parameter γ contained in Eq. (2) allows for minimization of the fundamental eigenvalue since, when employing the Rayleigh-Ritz method, one obtains upper bounds.

The procedure can be extended using additional polynomials in order to increase the accuracy of the results and to deal with frequencies of excitation of higher order.

However for frequencies below the second natural frequency, expression Eq. (2) yields excellent engineering accuracy. Expression Eq. (2) is particularly advantageous in view of its simplicity and the fact that it is immediately applicable for any value of the flexibility coefficient. Eq. (1), on the other hand, posseses a more complicated mathematical structure.

Consideration of local variations in mass and/or rigidity does not introduce any formal difficulties when Eq. (2) is used.

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The authors would like to thank Professor Laura for his interest in Takabatake and Mizuki (1995) regarding the simplified dynamic analysis of slender tapered thin-walled towers with additional mass and/or rigidity.

There are many papers concerning uniform and variable cantilevers with or without tip mass, as shown in Laura, Pombs, and Susemihl (1974), Mabie and Rogers (1974), Kaiser, Shaker,

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and Nayfeh (1974), Goel (1976), Bhat and Wagner (1976), To (1982), and Lau (1984).

The motivation of current our paper (1995) is to solve simply the dynamic analysis of slender tapered thin-walled towers with additional mass and rigidity. Therefore, the discontinuous variation of mass and rigidity due to local additional mass and/or local structural rigidity is formulated rationally as a continuous function by means of a characteristic function. The characteristic function proposed here is defined as the Dirac function exists continuously in a prescribed region. The use of this function has the following merits on the formulation and computations:

(1) the variation of mass and rigidity is expressed exactly;
(2) the continuous condition of elastic deflections and rotations at discontinuous points in rigidity is satisfied because cutting at the discontinuous points is unnecessary;
(3) the treatment is very simple and general in the formulation and computation; and
(4) when the region of local and additional mass and rigidity is extremely smaller than the total height, the formulation and computation are simplified by means of an approximation proposed.

Thus, in current paper the Galerkin method is employed to utilize effectively the above-mentioned merits of characteristic function proposed here. The accuracy of results obtained from the Galerkin method depends on shape functions used. The shape functions of current member adapt well-known ones for a cantilevered uniform member without additional mass and rigidity from the following reasons: the tapering angle of the variable member considered here is small and the behavior is dominated by the original variable member without additional mass and rigidity.

Although the shape function used is simple, the numerical results obtained from the function show to be effective in practice. The shape functions depend on the magnitude and location of the additional mass and rigidity, on the tapering angle of variable members, and on the supporting conditions at the base. Laura and Gutierrez (1986) proposed an effective shape function by means of a polynomial. However, since this shape function includes optimization parameter, it is not effective to apply its function to the current Galerkin method. The effect of additional mass at the top is effective to employ the shape functions proposed by Laura, Pombo, and Susemihl (1974) and To (1982). Meanwhile, Goel (1976) proposed a shape function of transverse vibrations of tapered beams. Although this function is exact, the computation is very complicated because of including Bessel functions. A simplified shape function, which is usable to variable members with the arbitrary location and magnitude of additional mass and rigidity, is lacking. Current analysis is considered to provide engineering approach and effective approximate solutions in quasi-closed from solution for dynamic analysis of slender tapered thin-walled towers with additional mass and/or rigidity.

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