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Buckling of cylindrical shells under external pressure proposition of a new shape of self-stiffened shell

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Abstract. We propose a new shape of cylindrical shell formed by multiples vaults which gives a self-stiffening against buckling. By an experimental and numerical study of cylindrical shells with a repeated defect, on the circumferential direction made only of outside oriented wave-defects, we show that multiple vault cylindrical shells can have a good behaviour in buckling. An optimal behaviour is obtained by optimization of the vaults number, with conduces to a special multiple vault cylindrical shell named "ASTER shell".

Key words: cylindrical shell; external pressure; buckling; self-stiffening; boundary conditions; geometric imperfection.

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1. Introduction

Thin shells are frequently used in industrial construction. Examples are numerous: nuclear reactors vessels, supporting structures for marine platforms, under sea projects, reservoirs, etc. These structures are built in different materials, metal, concrete, composites, the choice depending on the dimensions, the loading, the objectives and the contents.

Cylindrical shells with or without stiffeners are used to carry a variety of loads such as external pressure, axial compression or various combinations of loading.

Under certain types of loading, the failure of thin shells is caused by buckling, which can occur before the limit of strength of the structure material is reached. Further more, under these loadings, the magnitude of the safety margin is important in order to allow for the imperfections of the shell. The design of circular cylindrical shells under external pressure embraces shells both with and without stiffeners. The sizing of the structure is, in general, simple and calls upon simplified methods or numerical methods of varying complexity.

Stiffening is a satisfactory solution for increasing the critical load of a shell. The cost depends on the construction material, the level of loading and the acceptable margins of safety.

A number of investigations into the buckling of circular cylindrical shells subject to external

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pressure have established a knowledge of the physical phenomenological behaviour which occurs before the onset of instability and they have brought an understanding of the way in which instability is initiated.

This state of knowledge now suggests that the replacement of the circular cylindrical shape of the shell by a multivaulted shape, can opposite to the precritical geometry developing observed for the first shape, and thus, the strength of cylindrical shells can be increased. The optimal strength for the new shape depends on the number of vaults, for which we propose an optimal value.

The work presented here is supported by experimental results and numerical analysis relating to the different phases of behaviour of various shells.

2. The effect of external pressure on circular cylindrical shells

The different theories and experiences concerning the behaviour of thin circular cylindrical shells, subject to external pressure and having small or large displacements, predict an axisymmetrical precritical deformation (Donnell 1956, Montague 1969, Yamaki 1970, 1984, etc.). Instability is then characterised by the sudden change of this axisymmetric geometry by the appearence of a multimodal deformation in the circumferential and axial directions.

Geometrical defects are inevitable in the construction of thin shells both in practice and in the laboratory. Even with the numerous precautions under taken during fabrication these imperfections appear and modify the effect of the boundary conditions of the shell. The imperfections create very small radial displacements at the point where they occur, the magnitude of this effect is comparable to that induced by an external pressure applied to the surface of the shell. This is why the shell often shows the initial state of circumferential multi-modal imperfection near or equal to the critical mode under external pressure as well as the nul axial mode. an example of the initial geometric imperfection distribution is shown in Fig. 1.

Analysis of laboratory tests show that precritical deformation of the shell is not axisymmetrical but corresponds to the amplification of the imperfections on the critical mode with the creation of a geometry approximately that of the critical geometry (Araar 1990) Fig. 2.

At the time of loading, the variation in the amplitude of the imperfections is small but their

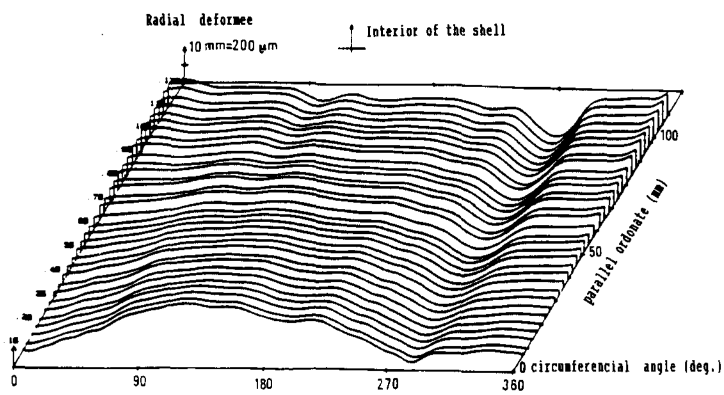


Fig. 1 Example of a development of an initial imperfect shell geometry.

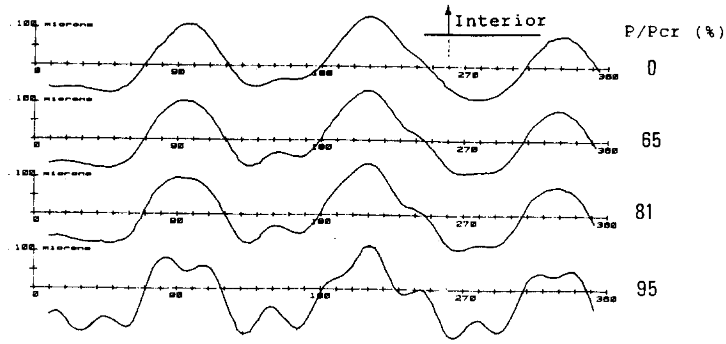


Fig. 2 Precritical development of the radial geometry, under external pressure, of the mid-height section of near perfect cylindrical shell.

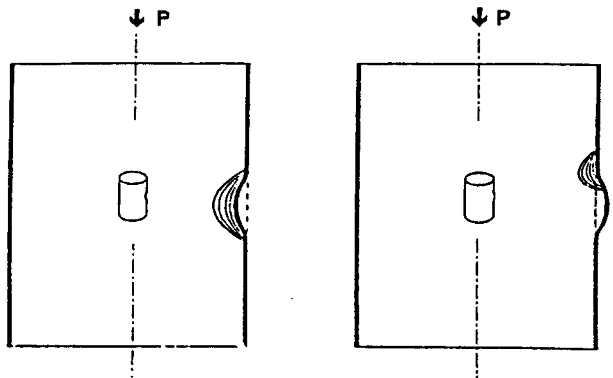


Fig. 3 Precritical development of a localized geometrical imperfections in axially loaded cylindrical shell.

magnitude grows asymptotically to that value which occurs at the critical load, there their size depends on the geometric parameters and the boundary conditions.

Similar behaviour has been observed (Waeckel and Jullien 1984) under axial compression Fig. 3, where the sign of the curvature of a local imperfection determines its amplification. An inward imperfection is amplified and forms a deflection wave pointing towards the interior, whereas an outward imperfection remains constant.

The experimental results and the calculated value of the critical loads under external pressure are very close for the dimensions investigated. These results confirm earlier work and the recorded differences are dependent principally on the quality of the boundary conditions, which have an important effect (Debbaneh 1988, Galletly and Bart 1956).

3. The effect of external pressure on cylindrical shells of optimal shape

3.1. Basic principle

The concept of multi-vaulted cylindrical shells is based on the knowledge that the waves

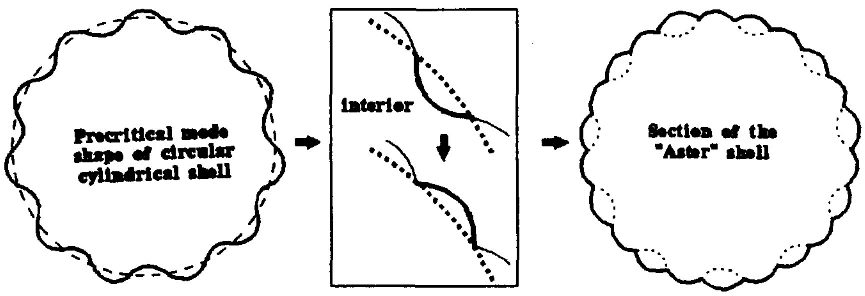


Fig. 4 Conception of the multi-vaulted cylindrical shell "ASTER".

in the circumferential precritical geometry must be continuous and the observation that the precritical geometry is initiated by inward pointing defects.

We have sought to oppose the formation of the circumferential mode by creating a surface geometry of multiple meridian vaults whose curvature is such that the vaults always point outwards. The optimal number of vaults proposed is equal to twice the number of the critical mode. The section form of the resulting shell with multiple vaults is shown in Fig. 4, and is called an "ASTER" shell (Araar and Debanneh 1991, Jullien and Araar 1990). The depth of the vaults is another parameter to be considered.

3.2. experimental and numerical validation of the concept

The behaviour of this novel form of "ASTER" shell is investigated experimentally and results are compared with those of a circular cylindrical shell (Araar 1990) and with another multiple vault shell whose number of vaults is different from the Aster shell.

3.2.1. Details of specimens

The overall geometry of the three specimens is common: $R=75$ mm, $L=120$ mm, $h=150$ to 160 microns. The three specimens are studied with encastrate conditions at each end. These boundary conditions were obtained by fusing the ends of the shell into a base plate.

The number of vaults in the Aster shell was deduced from a linear analysis of the reference shell buckling under external pressure, the mode of this buckling is dependent on the quality of the boundary conditions. It is 12 when the shell is perfectly encastre at the two ends and decreases to about 10 when the local axial displacements around the circumference at the assumed point of fixity are free, Fig. 5.

The experimental conditions are intermediate between these two and the observed mode is 11, Fig. 6. This later value is used in the following studies, hence the Aster shell has 22 vaults. The number of vaults chosen for the multi-vault shell used for comparison is arbitrary; the number chosen was 14, and the shell was designated by VM14.

The form of the vaults is deduced from the sinusoidal precritical geometry which developes on the circumference. For simplicity, and in order to facilate fabrication, the vaults are defined as an arc of that circle which approximates the half sine wave, Fig. 7, ($r=20$ mm for the Aster

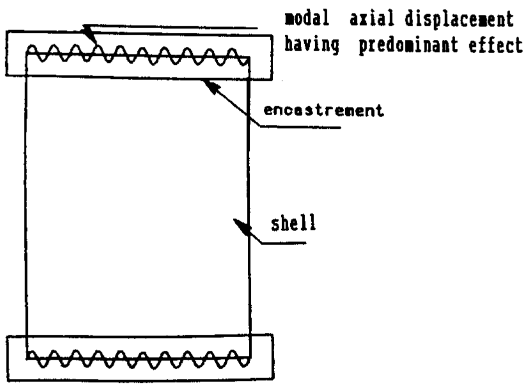


Fig. 5 Bounadry conditions-axial mode having a predominant effect.

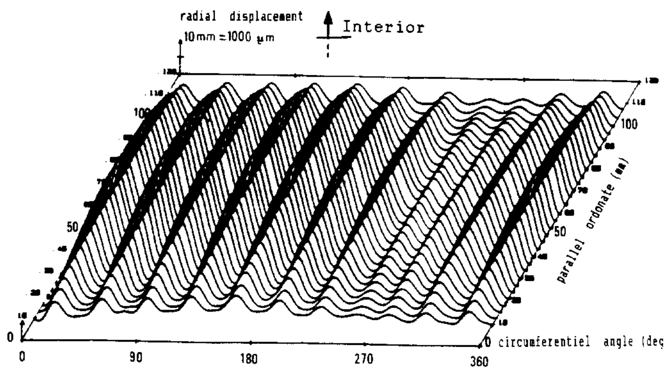


Fig. 6 Experimental buckling mode of the reference circular cylindrical shell.

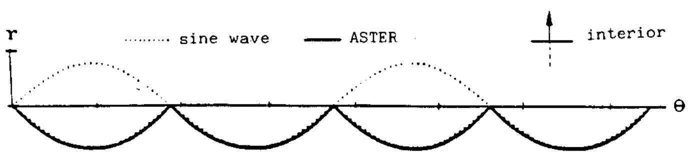


Fig. 7 Approximation of half sine wave to an arc of a circle.

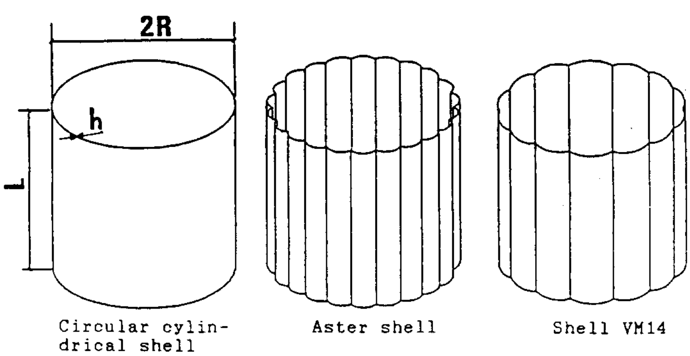


Fig. 8 Geometry of the three types of shell studied.

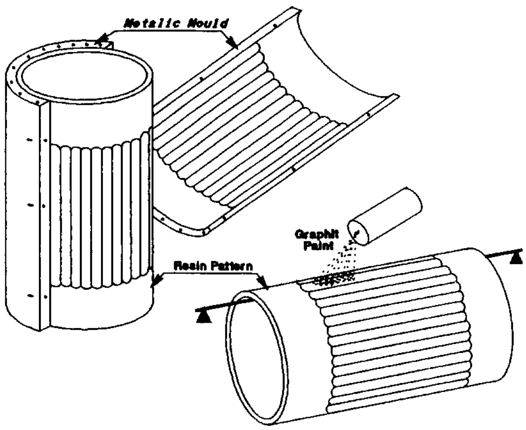


Fig. 9 Fabrication of multi-vaulted cylinders.

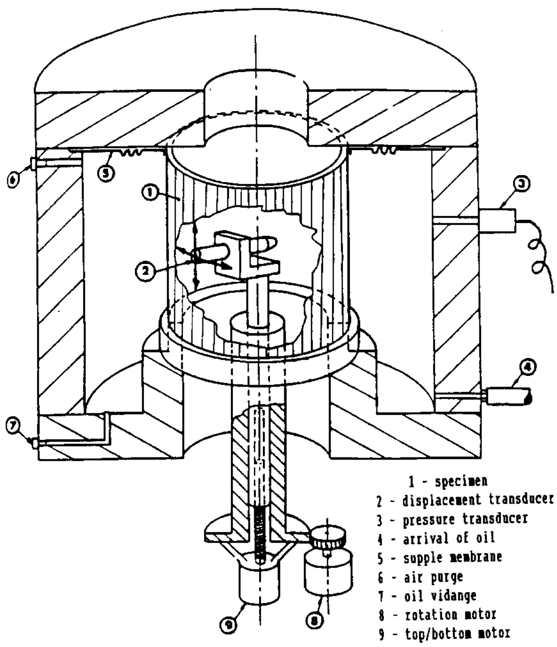


Fig. 10 Test device of the Aster shell.

shell and $r=35$ mm for the shell VM14). The Amplitude of the vaults is 2.3 mm for the Aster shell and 2.35 mm for the shell VM14. These values are a compromise between research requirme-nts and the method of fabrication. The geometrical caracteristics of the three shells are presented in the Fig. 8.

For the given geometry of the Aster shell, the internal volume is 4.2% greater and the weight of the constituent material is 5.3% greater than that of the reference circular cylindrical shell of identical thickness.

The shells were constructed in Nickel which was electro-deposited on a pattern having the geometrical form of the specimen, Fig. 9.

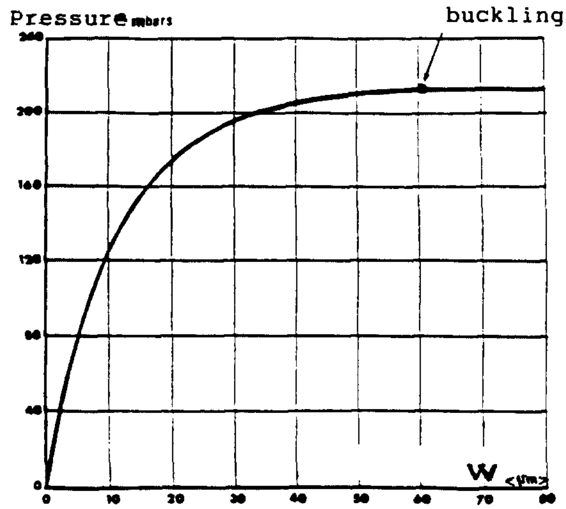


Fig. 11 Evolution of pressure-radial displacement on a caracteristic point of the circular cylindrical shell.

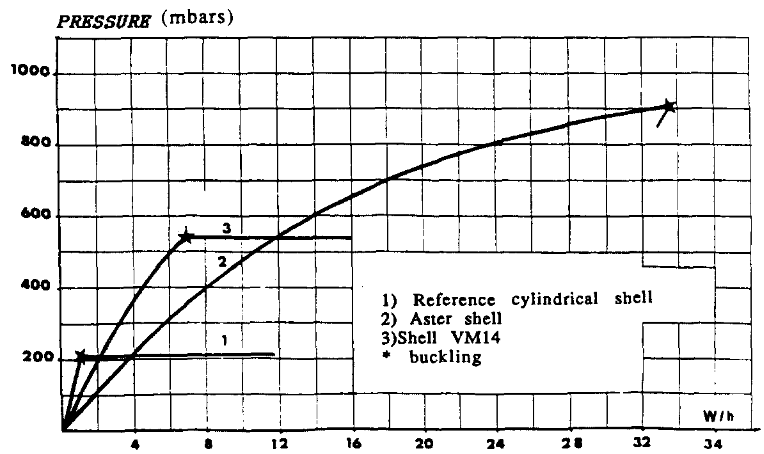


Fig. 12 Behaviour of the three types of shells under external pressure.

The production of the specimens was carefully controlled in all aspects in order to obtain the best experimental results. Dimensional accuracy, uniform mechanical characteristics, absence of residual stresses, minimization of geometric imperfections, etc., are qualities that have the same level of importance as good test equipment and test procedure.

3.2.2. Test method

The shells were subjected to an external pressure and the experimental behaviour was entirely monotorised by a computer which recorded and processed the desired readings. One of the test devices is presented in Fig. 10.

The load was kept constant at many levels to allow the radial and axial deformations to

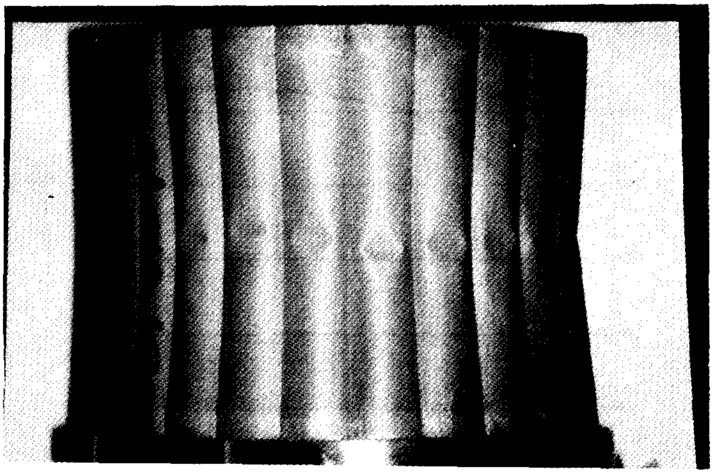


Fig. 13 Post-critical state of Aster shell.

be measured. This was done by means of an axial transducer without contact with the surface.

3.2.3. Analysis of behaviour

The global behaviour of a shell is characterised by the relationship between pressure and radial displacement (Fig. 11). For the shells analysed the axial mode was the same. The measurement of radial displacement was taken at mid-height.

The measurement points were defined after having observed that the precritical deformation of the Aster shell was totally axisymmetrical. Hence the characteristic point used was situated at the intersection of two vaults for this shell and at the summit of an internal wave for the circular shell.

The critical load under external pressure of the Aster shell was of the order of 4 times that of the reference circular cylindrical shell. The corresponding value for the shell VM14 was 2.5 times (Fig. 12).

The results of the critical load and the precritical geometry show that it is possible to conceive forms of cylindrical shells having much improved loading capacity using a similar quantity of material. Certainly the radial stiffness of multi-vaulted shells in the precritical phase is greater than that of circular cylindrical shells; of the order of 4 to 1 for the Aster shells. This result is because of the nature of the precritical deformation which is constrained to be axisymmetrical in the case of the Aster shell and which has a sinusoidal mode for the circular shell. These two deformation forms are coupled in the shell VM14. Compared to the circular cylindrical shell, the higher strength and the stiffeness of multi-vaulted shell follow from the lack of circumferential membrane forces and the reduction in importance of the local radial bending curvature.

The 22 vaults in the Aster shell should perhaps be considered an optimum since no geometric model reproduces axisymmetric deformation. Only a variation in the curvature of the vaults is observed corresponding to the constraints of each. The post-critical state of the Aster shell under the external pressure load corresponds to a local instability of each vault at mid-height of the shell (Fig. 13). Shell VM14 shows a failure mode combining the form of the precritical mode with that of the failure of the vaults (Fig. 14).

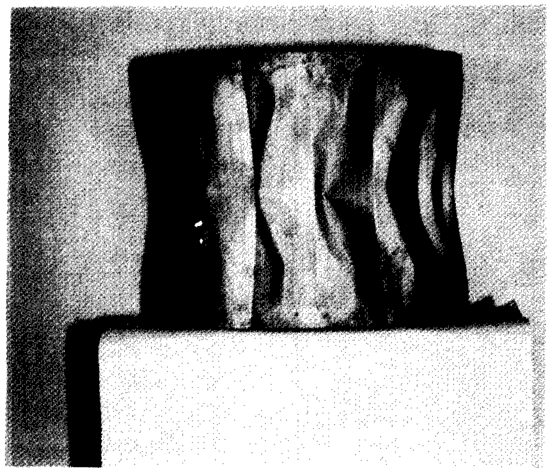


Fig. 14 Post-critical state of VM14 shells under external pressure.

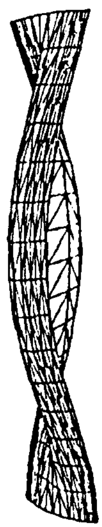
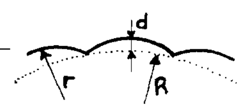


Fig. 15 Calculated buckling mode.

Table 1 Influence of the vaults rise on the critical pressue of the Aster shell

δ mm	δ/R %	$\frac{\delta}{h}$	r mm	Pcr mbars	$\frac{Pcr}{Pref}$
1.63	2.2	10	25	415	1.7
2.32	3.0	15	20	1002	4.1
3.70	5.0	25	15	2140	8.7



The numerical analysis of multi-vaulted shells is carried out using linear three dimensional elasticity and small displacements, program BILBO of the CASTEM system (CEA-France).

The load at which bifurcation takes place is correctly forecast for the two multi-vaulted shells, Aster and VM14, as compared to the experimental values. In contrast, the axisymmetric mode of instability for the Aster shell is not forecast. The calculated critical mode Fig. 15 corresponds favorably with the test only for the shell VM14.

These comparaisons have been made for a vault rise of 2.3 mm, which corresponds to 3% of the radius of the shell and is 15 times the thickness. The effect of the vaults rise was analysed numericaly in Table 1. The capacity of the shell is increased as the rise of the vaults is increased.

5. Conclusions

We have presented a study of a particular type of geometric imperfection of cylindrical shells which has a serious effect on their critical load. Automatic stiffening is created by inclusion of multiple vaults on the circumference which are uniform over the whole height and whose number is optimised. This optimal number is equal to double the buckling circumferential mode under external pressure of the corresponding circular cylinder of the same thickness. we call this shell by ASTER. The increase in the critical load is a function of the vaults rise and the radius of the shell. An Aster shell with a vaults rise of 3% of the radius of the shell has a self-stiffening giving an experimental external pressure failure load which is of the order of 4 times that supported by the corresponding circular cylindrical shell. These results have been drawn principally from the tests in the laboratory of the Aster shell and of the multi-vaulted shells. These shells were developed from the physical understanding of buckling and prebuckling behaviour of circular cylindrical shells and the analysis of geometrical imperfections, in particular, the average development of the geometry before and after buckling.

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