Structural Engineering and Mechanics, Vol. 4, No. 4 (1996) 425-436
DOI: http://dx.doi.org/10.12989/sem.1996.4.4.425

Application of the first-order perturbation method to optimal structural design

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Abstract. An application of the perturbation method to optimum structural design with random parameters is presented. It is formulated on the basis of the first-order stochastic finite element perturbation method. It also takes into full account the stress, displacement and eigenvalue constraints, together with the rates of change of the random variables. A method for calculating the sensitivity coefficients in regard to the governing equation and the first-order perturbed equation has been derived, by using a direct differentiation approach. A gradient-based nonlinear programming technique is used to solve the problem. The numerical results are specifically noted, where the stiffness parameter and external load are treated as random variables.

Key words: stochastic finite element method; first-order perturbation; optimal design; sensitivity analysis.

1. Introduction

In structural system, a certain amount of uncertainty always persists, for instance, in material properties, geometric parameters and applied loads. Hence the structure must be designed to withstand the uncertainties which either measurement inaccuracies or system complexities cause. Random structures may be mathematically modelled using the stochastic finite element method. In the perturbation approach, the random functions are expressed as the sum of the deterministic component and a random component (Benaroya and Rehak 1988). This sum substitutes either for the variational functional or for the discretized equations. A considerable research has been made in the field of the structural analysis with random parameters using the stochastic finite element method. Among many researchers, Contreras (1980), Liu et al. (1986) and Hisada and Nakagiri (1981) applied the method to the analysis of static and dynamic problems.

The application of perturbation methods in optimal structural design was initiated by Wu (1968). Further developments of the method were made by Banichuk (1984) for solving static and eigenvalue problems. He applied the perturbation method to the distributed parameters, for which state variables were described by the differential equations. As a result, the methods with the distributed parameter are not convenient for the application to large structures.

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This paper offers an alternate and simplified procedure for the optimal structural design with the random parameter. In this procedure, the first-order perturbation method is employed in order to construct the stress, displacement and eigenvalue constraints. The constraints are taken into full consideration together with the rates of change of the random variables. The object function is formulated as expected values of function of the random variable. In the gradient-based optimization algorithms, there is a fundamental requirement for information in relation to the changes in the structural response to changes in the design variables. The static response design sensitivity for the random structure is derived by differentiating both the governing equation and the first-order perturbed equations. The eigenvalue and eigenvector derivatives can calculate the characteristic equation and the first-order perturbed equation. This is the key result of the research presented here.

The numerical results are presented for a ten-member cantilever truss (Haug and Arora 1979) and two-dimensional plate where the stiffness parameter and load vector are treated as random variables. To verify the design sensitivity analysis, the results are compared with the probabilistic finite difference method. The optimum results were obtained by using the recursive quadratic programming method which uses the active set strategy (Lim and Arora 1986).

2. Stochastic finite element analysis

2.1. Static problems

The stochastic finite element method is summarized in this section (Hisada and Nakagiri 1981). The linear finite element equations are

$$Kz=f \tag{1}$$

where f is the external load vector. The nodal displacement vector is denoted by z. The stiffness matrix K depends on the strain-displacement relation matrix B and the stress-strain relation matrix D,

$$K=\int_V B^T D\, B\, dV \tag{2}$$

The transpose is designated by a superscript T. V is the domain of the element. Suppose that the elastic constants and sizing design variables such as bar cross-sectional areas, moment of inertia and plate thickness have the uncertainties. Then the stiffness matrix involves uncertainty.

We assume the random variables α fluctuated as the lower and the upper bound of the coefficient of the variation. The coefficient of variation (COV) is defined as the ratio of the standard deviation to the mean for a given random variable,

$$|\alpha| \leq \mathrm{COV} \tag{3}$$

where α is small but zero-mean random variables.

The variation of the stiffness matrix, external load vector and nodal displacement can be expressed in the form of a first-order Taylor series expansion in relation to the random variables.

$$K = K^0 + \sum_{k=1}^{n} K_k^I \alpha_k + \cdots \tag{4}$$

$$f = f^0 + \sum_{k=1}^{n} f_k^I \alpha_k + \cdots \tag{5}$$

$$z = z^0 + \sum_{k=1}^{n} z_k^I \alpha_k + \cdots \tag{6}$$

where n denotes the total number of variables. The superscripts 0 and I denote zeroth and the first-order rates of change respectively.

Substituting Eqs. (4)-(6) into (1) and equating equal order terms, the mean-centered first-order perturbation to, the results are:

Zeroth-order

$$K^0 z^0 = f^0 \tag{7}$$

First-order

$$K^0 z_k^I = f_k^I - K_k^I z^0 \tag{8}$$

2.2. Eigenvalue problems

Consider the eigenvalue formulation for natural frequency or buckling (Collins and Thomson 1969).

$$(K - \lambda M)\phi = 0 \tag{9}$$

where λ and ϕ are eigenvalue and the corresponding nontrivial eigenvector, respectively. λ are assumed to be distinct. M is the mass matrix in case of the vibration analysis and the the geometric stiffness matrix in case of the buckling analysis. For the vibration problems λ is the square of the frequency of free vibration and for the buckling problem it is the buckling load factor. K and M are assumed to be the symmetric, positive definite matrices. The eigenvector ϕ is normalized by the condition

$$\phi^T M \phi = 1 \tag{10}$$

Suppose the stiffness and mass matrices have random variables α, then the variation of mass matrix can be expressed as a first-order Taylor series expansion.

$$M = M^0 + \sum_{k} M_k^I \alpha_k + \cdots \tag{11}$$

The corresponding variations of λ and ϕ are assumed to be the following form.

$$\lambda = \lambda^0 + \sum_{k} \lambda_k^I \alpha_k + \cdots \tag{12}$$

$$\phi = \phi^0 + \sum_{k} \phi_k^I \alpha_k + \cdots \tag{13}$$

Substituting Eqs. (11)-(13) and (4) into characteristic Eq. (9) and equating equal order terms, the result give rise to the following equations.

Zeroth-order

$$(K^0 - \lambda^0 M^0)\,\phi^0 = 0 \tag{14}$$

First-order

$$(K^0 - \lambda^0 M^0)\,\phi_k{}^I = -(K_k{}^I - \lambda_k{}^I M^0 - \lambda^0 M_k{}^I)\,\phi^0 \tag{15}$$

The first-order perturbation of the eigenvalues can be obtained by premultiplying Eq. (15) by ϕ^{0T}.

$$\lambda_k{}^{Ii} = \frac{\phi^{0iT}(K_k{}^I - \lambda^{0i} M_k{}^I)\phi^{0i}}{\phi^{0iT} M^0 \phi^{0i}} \tag{16}$$

The perturbation of the eigenvectors can be computed from the M-orthonormal Eq. (11). Substituting the two Eqs. (11) and (13) into the characteristic Eq. (9) and equating equal order terms, the zeroth- and the first-order equations corresponding to Eq. (10) are

Zeroth-order

$$\phi^{0iT} M^0 \phi^{0i} = 1 \tag{17}$$

First-order

$$2\phi^{0iT} M^0 \phi_k{}^{Ii} + \phi^{0iT} M_k{}^I \phi^{0i} = 0 \tag{18}$$

Taking Eqs. (15) and (18) together, we obtain a consistent but overdetermined system, which can be written in partitioned form as

$$\underset{(n+1)\times n}{\begin{bmatrix} K^0 - \lambda^{0i} M^0 \\ 2\phi^{0iT} M^0 \end{bmatrix}} \underset{n\times 1}{\phi_k{}^{Ii}} = -\underset{(n+1)\times n}{\begin{bmatrix} K_k{}^I - \lambda_k{}^{Ii} M^0 - \lambda^{0i} M_k{}^I \\ \phi^{0iT} M_k{}^I \end{bmatrix}} \underset{n\times 1}{\phi^{0i}} \tag{19}$$

where n is the number of degrees of freedom. Denoting the term on the left side of $(n+1)\times n$ matrix denote P and the right side of $(n+1)\times n$ matrix Q, then the first-order perturbed eigenvectors can be obtained in the next equation.

$$\phi_k{}^{Ii} = (P^T P)^{-1}(-P^T Q \phi^{0i}) \tag{20}$$

3. Design sensitivity analysis for random structural system

3.1. Displacement and stress derivatives

Taking the derivative of zeroth-order Eq. (7) with respect to design variable x^0 and rearranging yields (Haug, Choi and Komkov 1984)

$$K^0 \frac{\partial z^0}{\partial x^0} = \left(\frac{\partial f^0}{\partial x^0} - \frac{\partial K^0}{\partial x^0} \tilde{z}^0 \right) \tag{21}$$

where the tilde indicates a variable that is to be held constant for the partial differentiation. In many cases, the applied forces are independent of the design variables and these terms are zero. This assumption will be made for the remainder of the development. In a similar manner, one may also differentiate the first-order Eq. (8) with respect to x^0 to become

$$K^0 \frac{\partial z_k^I}{\partial x^0} = \frac{f_k^I}{\partial x^0} - \left(\frac{\partial K_k^I}{\partial x^0} z^0 + K_k^I \frac{\partial z^0}{\partial x^0} + \frac{\partial K^0}{\partial x^0} z_k^I \right) \tag{22}$$

Therefore the displacement derivatives for the structural system with the random parameter can be obtained by differentiating Eq. (6).

$$\frac{\partial z}{\partial x^0} = \frac{\partial z^0}{\partial x^0} + \sum_{k=1}^{n} \frac{\partial z_k^I}{\partial x^0} \alpha_k \tag{23}$$

Thus, the displacement derivatives are expressed as the sum of the deterministic component and the random component.

Consider the general expression for the stresses.

$$\sigma = DBz \tag{24}$$

One may differentiate Eq. (24) with respect to x^0 to obtain

$$\frac{\partial \sigma}{\partial x^0} = \frac{\partial DB}{\partial x^0} z + DB \frac{\partial z}{\partial x^0} \tag{25}$$

3.2. Eigenvalue and eigenvector derivatives

In order to evaluate the derivative of first-order perturbed eigenvalue, the derivatives of the zeroth-order eigenvectors are required. The symmetric, positive definite matrices K and M are continuously differentiable with respect to design variables. Differentiating Eqs. (14) and (17) with respect to x^0, we obtain the eigenvalue and eigenvector derivatives for deterministic system (Haftka, Gürdal and Kamat 1990).

$$(K^0 - \lambda^0 M^0) \frac{d\phi}{dx^0} - \frac{d\lambda^0}{dx^0} M\phi = -\left(\frac{dK^0}{dx^0} - \lambda^0 \frac{dM^0}{dx^0} \right) \phi^0 \tag{26}$$

$$\phi^{0T} M^0 \frac{d\phi^0}{dx^0} = -\frac{1}{2} \phi^{0T} \frac{dM^0}{dx^0} \phi^0 \tag{27}$$

To obtain the derivatives of the eigenvalue and the eigenvector, we can use the direct approach and Combine Eqs. (27) and (28) as

$$\underset{(n+1)\times(n+1)}{\begin{bmatrix} (K^0 - \lambda^{0i} M^0) & -M^0 \phi^0 \\ -\phi^{0iT} M^0 & 0 \end{bmatrix}} \underset{(n+1)\times 1}{\begin{bmatrix} \dfrac{d\phi^0}{dx^0} \\ \dfrac{d\lambda^0}{dx^0} \end{bmatrix}} = \underset{(n+1)\times 1}{\begin{bmatrix} -\left(\dfrac{dK^0}{dx^0} - \lambda^0 \dfrac{dM^0}{dx^0} \right) \phi^0 \\ \dfrac{1}{2} \phi^{0T} \dfrac{dM^0}{dx^0} \phi^0 \end{bmatrix}} \tag{28}$$

The first-order perturbed derivatives of the eigenvalue and the eigenvector may be obtained differentiating Eqs. (15) and (18) and rearranging as

$$\underset{(n+1)\times(n+1)}{\begin{bmatrix} (K^0 - \lambda^{0i} M^0) & -M^0 \phi^0 \\ -\phi^{0iT} & 0 \end{bmatrix}} \underset{(n+1)\times 1}{\begin{bmatrix} \dfrac{d\phi_k^{Ii}}{dx^0} \\ \dfrac{d\lambda_k^{Ii}}{dx^0} \end{bmatrix}} = \underset{(n+1)\times 1}{\begin{bmatrix} E \\ F \end{bmatrix}} \tag{29}$$

where E and F are force derivatives defined as

$$\{E\}=-\left(\frac{dK^0}{dx^0}-\lambda^0\frac{dM^0}{dx^0}-\frac{d\lambda^0}{dx^0}M^0\right)\phi_k^{li}$$
$$-\left(\frac{dK_k^I}{dx^0}-\lambda_k^I\frac{dM^0}{dx^0}-\frac{d\lambda^0}{dx^0}M_k^I-\lambda^0\frac{dM_k^I}{dx^0}\right)\phi^{0i}$$
$$-(K_k^{li}-\lambda_k^{li}M^0-\lambda^0 M_k^I)\frac{d\phi^0}{dx^0} \tag{30}$$

$$\{F\}=\phi_k^{liT}M^0\frac{d\phi^{0i}}{dx^0}+\phi^{0iT}\frac{dM^0}{dx^0}\phi_k^{li}+\phi^{0iT}M_k^I\frac{d\phi^{0i}}{dx^0}$$
$$+\frac{1}{2}\phi^{0iT}\frac{dM_k^I}{dx^0}\phi^{0i} \tag{31}$$

Therefore total derivatives of the eigenvalue and the eigenvector for the random structural system can be obtained by differentiating Eq. (12) and (13), respectively.

$$\frac{d\lambda}{dx^0}=\frac{d\lambda^0}{dx^0}+\sum_{i=1}^{n}\frac{d\lambda_k^I}{dx^0}\alpha_k \tag{32}$$

$$\frac{d\phi}{db^0}=\frac{d\phi^0}{dx^0}+\sum_{i=1}^{n}\frac{d\phi_k^I}{dx^0}\alpha_k \tag{33}$$

4. Formulation of optimal design problem extended to perturbation treatment

The optimum design problem with the random parameter can be formulated as the nonlinear programming method.

Minimize

$$F(x^0) \tag{34}$$

subject to the equilibrium equations

$$K^0 z^0=f^0 \tag{35}$$

and

$$(K^0-\lambda^0 M^0)\phi^0=0 \tag{36}$$

the first-order perturbed equations

$$K_k^I z^0+K^0 z_k^I=f_k^I \tag{37}$$

and

$$(K^0-\lambda^0 M^0)\phi_k^I=-(K_k^I-\lambda_k^I M^0-\lambda^0 M_k^I)\phi^0 \tag{38}$$

and the constraints

$$G_i \leq 0 \qquad i=1, 2, \cdots, m \tag{39}$$

and the bounds

$$x^{0L} \geq x^0 \geq x^{0U} \tag{40}$$

In Eq. (34), $F(x^0)$ is taken as the expected cost function that is to be minimized. The x^0 are treated as design variables such as the member cross-sectional areas and plate thickness. G_i represents limits on stresses, displacements, and frequencies of the structure. In Eq. (40) x^{0L} and x^{0U} are lower and upper bounds on x^0, respectively. It is important to note that formulation Eqs. (34)-(40) are different from the deterministic one since the constraints will include the rate of changes of the random variables.

The optimal design problem with the random parameter stated in Eqs. (34)-(40) are solved using PLBA(Pshenichny-Lim-Belegundu-Arora 1986) program which is based on the quadratic programming method. The algorithm uses the second order information in the direction finding problem and the active set strategy.

5. Numerical examples

5.1. Ten-member cantilever truss

5.1.1. Problem definition

The optimal design of a ten-member cantilever truss will be represented below. The problem has been formulated using the first-order stochastic finite element method. The design sensitivity analysis has been carried out using the method given previously and the result was compared with the finite difference method. Computing times reported are for a HWS-S200K workstation.

A ten-member cantilever truss is considered as a typical example (Haug and Arora 1979). The geometry of the truss and the loading are shown in Fig. 1. The material data are: the expected value of the modulus of the elasticity=6,894.76 MPa, material density(γ_i)=2,867.99 kg/m^3, and the initial expected value of the area (x_i^0)=6.45$\times 10^{-4}$ m^2. The imposed constraints include:

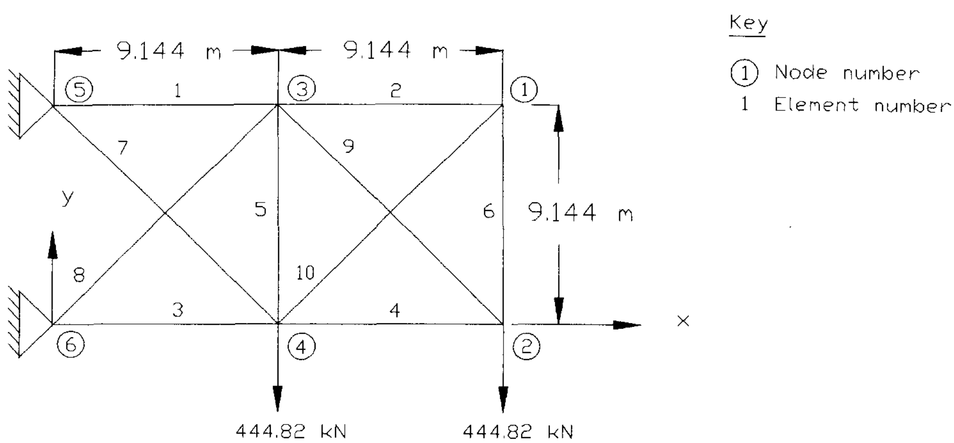

Fig. 1 Ten-member cantilever truss.

the lower limit on expected area (x_i^{0L})=6.45×10^{-6} m^2, the upper limit on expected area (x_i^{0U}) =0.645 m^2, the displacement limit (z_i^a)=$\pm$0.058 m, the stress limit (σ_j^a)=$\pm$172.37 MPa, and the lower limit of the frequency constraint (λ^a)=18 Hz. The member cross-sectional areas (x_i), the modulus of elasticity (E) and the external force (f) are treated as the random variables. The random variables are assumed to be the random parameters and modeled as following

$$E_i=E^0(1+\alpha_i),\qquad x_i=x_i^0(1+\alpha_i),\qquad f=f^0(1+\alpha_i) \tag{41}$$

The mean values of the member cross-sectional areas are considered as design variables. The material density is assumed to be deterministic. In this problem, the COV are taken as 0.01 and 0.1, respectively. The truss is designed for the load of 444.822 kN in the negative y-direction each at nodes 2 and 4. Design variable linking is not used in this example.

First, the optimum design of the ten-member cantilever truss can be expressed as
Minimize

$$F(x^0)=\sum_{i=1}^{10}\gamma_i l_i x_i^0 \tag{42}$$

subject to the displacement constraints

$$\frac{|z_i|}{z_i^a}-1.0\leq 0,\ i=1,\ 2,\ \cdots,\ 8 \tag{43}$$

stress constraints

$$\frac{|\sigma_j|}{\sigma_j^a}-1.0\leq 0,\ j=1,\ 2,\ \cdots,\ 10 \tag{44}$$

frequency constraint

$$\frac{\lambda^a}{\lambda}-1.0\leq 0 \tag{45}$$

and the bounds

$$x_i^{0L}\leq x_i^0\leq x_i^{0U} \tag{46}$$

The objective is to minimize the expected weight of the truss where l_i represents the member lengths while satisfying constraints on natural frequency, stress, displacement, and member sizes.

5.1.2. Sensitivity analysis results

The comparisons of the displacement, and the stress derivatives for the first-order perturbed

Table 1 Comparison of the truss sensitivity calculation with COV=0.1

Const.	ψ_i^1	ψ_i^2	$\Delta\psi_i$	ψ_i'	$(\psi_i'/\Delta\psi_i\times100)$%
ψ_1	17.288	17.107	−0.181	−1.6992E-01	93.88
ψ_2	17.984	17.796	−0.188	−1.7622E-01	93.73
ψ_3	0.5615	0.5461	−0.0154	−1.3386E-02	86.92
ψ_4	0.2249	0.2249	0.0	1.0336E-02	—

system are given in Table 1. Define ψ_i^1 and ψ_i^2 as constraint function values for the design x^0 and modified design $x^0+\delta x^0$, respectively. Let $\Delta\psi_i$ be the difference $\psi_i^2-\psi_i^1$, and let ψ_i' be the difference predicted by design sensitivity calculations. The ratio of ψ_i' and $\Delta\psi_i$ times 100 is used as a measure of accuracy of the derivative. 100 percent means that the predicted change is exactly the same as the finite difference.

In Table 1, ψ_1 and ψ_2 are displacement constraints in the negative y-direction each at nodes 2 and 4. ψ_3 is the stress constraints for the 5th member and ψ_4 is the natural frequency constraint. The numerical results were obtained with 1% of the design change, $\Delta x^0=0.01x^0$.

5.1.3. Optimal design result

Starting with a initial design having the expected value of the area (x^0) 6.45×10^{-4} m^2 for all design variables, the optimum design was achieved for the three different cases which the deterministic case and the COV are 0.01 and 0.1, respectively. In the PLBA algorithm, the penalty parameter is selected as 1.0. Thickness for determining active constraints is chosen as 0.1. The convergence criterion and tolerance value for line search are set to 0.0001, respectively.

Table 2 contains a comparison of solutions for the three cases. For the deterministic case, the optimum weight is 2008 kg and 6 constraints are active at optimum. These are the stress constraint on member 5, the downward displacements constraints at node 1 and 2 and minimum-size constraints on members 2, 5 and 10. For the first-order perturbed case with COV=0.01,

Table 2 Results for 10-Member cantilever truss

Member		Optimum cross-section area, m^2		
Number		Deterministic	COV=0.01	COV=0.1
1		1.7178E-02	1.8008E-02	1.9532E-02
2		6.4500E-05	6.4500E-05	6.4500E-05
3		1.3231E-02	1.3672E-02	1.4524E-02
4		8.4819E-03	7.9775E-03	9.3264E-03
5		6.4500E-05	6.4500E-05	6.4500E-05
6		1.7865E-04	6.4500E-05	2.4436E-04
7		4.5728E-03	4.7910E-03	5.0464E-03
8		1.1748E-02	1.2756E-02	1.2685E-02
9		1.1986E-02	1.0938E-02	1.3205E-02
10		6.4500E-05	6.4500E-05	6.4500E-05
At optimum	Wt, kg	2,008.0	2,031.0	2,217.0
	MCV	5.804×10^{-3}	7.192×10^{-3}	1.317×10^{-3}
	NI	25	25	27
	NFE	87	87	88
	NGE	25	25	27
	CPUs	9.52	71.9	79.2

MCV: Maximum Constraint Violation
NI: No. of Iteration
NFE: No. of Calls for Function Evaluation
NGE: No. of Calls for Gradient Evaluation
CPUs: CPU time in seconds

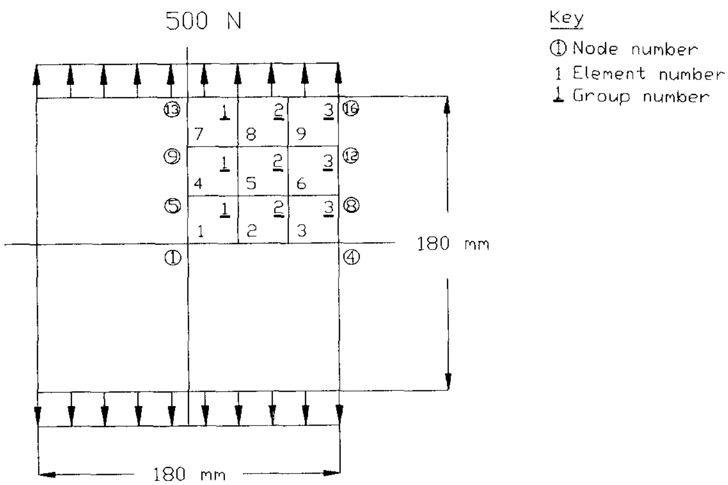

Fig. 2 Quadrilateral plate.

the optimum weight is 2031 kg and 7 constraints are active at optimum. These are the downward displacement constraints at node 1 and 2 and the minimum-size constraints on members 2, 5, 6 and 10. For the first-order perturbed case with COV=0.1, the optimum weight is 2217 kg and 7 constraints are active at optimum. These are the same as the coefficient of variation 0.01.

Therefore the results for the ten-member cantilever truss with the random parameters have 1% variation in case of COV=0.01 and 10% variation in COV=0.1 in respect to the deterministic solution.

5.2. Two-dimensional plate

5.2.1. Problem definition

This example consists of a 180×180 mm rectangular plate, supported at four points. The geometry of the plate and loading are shown in Fig. 2. The external loading of 500 N is applied to the upper side and the lower side. Fig. 2 shows the symmetric model of one quarter of the plate. The finite element model shown in Fig. 2 contains 9 rectangular elements linked in 3 groups (underline is the group number). The material properties are: the expected value of modulus of elasticity=7280 kgf/mm^2, Poisson's ratio=0.3, and material density $(\gamma_i)=78.57\times10^{-9}$ kg·s^2/mm^4. The imposed constraints include: the lower limit on the expected plate thickness $(t^{0L})=1.0$ mm, the upper limit on the expected plate thickness $(t^{0U})=300.0$ mm, the displacement limits $(z^a)=$ 0.05 mm, the allowable stresses $(\sigma^a)=10.0$ kgf/mm^2, the lower bound on natural frequency $(\lambda^a)=$ 763.3 Hz.

The plate thickness (t_i), modulus of elasticity (E) and external force (f) are treated as the random variables. The random variables are assumed to be the random parameters and modeled as a random vector field.

$$E_i=E^0(1+\alpha_i),\qquad t_i=t_i^0(1+\alpha_i),\qquad f=f^0(1+\alpha) \tag{47}$$

Table 3 Results of the quadrilateral plate

Case	Cost(kg)	Design Variables (mm) t_1^0	t_2^0	t_3^0	NFE	NGE	CPU
Deterministic	52.47	8.975	7.932	8.304	18	7	16.4
$\alpha=0.01$	52.97	9.056	8.012	8.385	21	8	48.1
$\alpha=0.1$	55.70	9.028	8.833	8.904	40	13	82.7

The mean values of the plate thickness (t_i^0) are chosen as the design variables. The material density is assumed to be deterministic.

The optimum design formulation of the quadrilateral plate can be expressed as
Minimize

$$F=\sum_{i=1}^{9} \gamma A\, t_i^0 \tag{48}$$

subject to displacement constraints

$$\frac{|z_i|}{z^a}-1.0\leq 0, \qquad i=1,\ 2,\ \cdots,\ 24 \tag{49}$$

stress constraints

$$\frac{|\sigma_j|}{\sigma^a}-1.0\leq 0, \qquad j=1,\ 2,\ \cdots,\ 9 \tag{50}$$

frequency constraint

$$\frac{\lambda^a}{\lambda}-1.0\geq 0 \tag{51}$$

and the bounds

$$t^{0L}\geq t^0\geq t^{0U} \tag{52}$$

The objective is to minimize the expected weight of the plate where A is the area, while satisfying constraints on natural frequency, stress, displacement, and member sises.

5.2.2. Optimal design result

Starting with a initial design having the expected value of the plate thickness (t^0) 10.0 mm for all design variable groups, the optimum design was achieved for the three cases which the deterministic case and the COV are 0.01 and 0.1. In the PLBA algorithm, the penalty parameter is selected as 1.0. Thickness for determining active constraints is chosen as 0.1. The convergence criterion and the tolerance value for the line search are set to 1.0×10^{-5} and 0.001, respectively.

Table 3 contains a comparison of solution for the three cases. For the deterministic case, the optimum weight is 52.47 kg and 5 constraints are active at optimum. These are displacement constraints at node 13, 14, 15, and 16 and the natural frequency constraint. For the first-order perturbed case with COV=0.01, the optimum weight is 52.97 kg and the active constraints are the same as the deterministic case. For the first-order perturbed case with COV=0.1, the optimum weight is 55.70 kg and the active constraints are the same as the deterministic case. Therefore the results for the two-dimensional plate have 1% variation in case of COV=0.01 and 6% variation in respect to the deterministic solution.

6. Conclusions

A general formulation of the design optimization problem with random parameters has been presented. The formulation is based on the stochastic finite element method. It takes into full account of the stress, displacement, and natural frequency together with the rates of change of the random variables. Static and dynamic response of the random system, including uncertainties for the random variable, is calculated with the first-order perturbation method to the original governing equation. In optimal design methods, there is a fundamental requirement for the design gradient. A method for calculating the first-order sensitivity coefficients is developed using the direct differentiation method for the governing equation and the first-order perturbed equation. The design sensitivity derivatives for the random structural system are expressed as the sum of its deterministic component and random component. A gradient nonlinear programming technique is used to solve the problem.

The numerical results show that the optimum design of the ten bar truss have 1% variation in case of COV=0.01 and 10% variation in COV=0.1 in respect to the deterministic solution. In case of the quadrilateral plate, it have 1% variation in case of COV=0.01 and 6% variation in COV=0.1. The comparison of the results show that the solution is sensitive to the variation of the load vector. Therefore the uncertainties must be taken into account in the design phase. The comparison of design sensitivity between the actual changes and predictions yields good results.

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