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Vibration analysis of wave motion in micropolar thermoviscoelastic plate

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Abstract. The aim of the present article is to study the micropolar thermoelastic interactions in an infinite Kelvin-Voigt type viscoelastic thermally conducting plate. The coupled dynamic thermoelasticity and generalized theories of thermoelasticity, namely, Lord and Shulman's and Green and Lindsay's are employed by assuming the mechanical behaviour as dynamic to study the problem. The model has been simplified by using Helmholtz decomposition technique and the resulting equations have been solved by using variable separable method to obtain the secular equations in isolated mathematical conditions for homogeneous isotropic micropolar thermo-viscoelastic plate for symmetric and skew-symmetric wave modes. The dispersion curves, attenuation coefficients, amplitudes of stresses and temperature distribution for symmetric and skew-symmetric modes are computed numerically and presented graphically for a magnesium crystal.

Keywords: micropolar thermoviscoelastic plate; secular equations; dispersion curves; attenuation coefficients

1. Introduction

The micropolar elasticity theory takes into consideration the granular character of the medium which describes deformations by a micro-rotation and a micro-displacement. In recent years the micropolar thermoelasticity and micropolar thermoviscoelasticity gained great importance due to the large- scale development and utilization of composite, reinforced, and coarse-grained materials. Analyzing the problems of high-frequency short-wavelength vibrations and ultrasonic waves in elastic media, the classical theory is inadequate to describe the real phenomena, hence the necessity of recurring to the micropolar theory. The general theory of linear and nonlinear micropolar continuum mechanics was given by Eringen (1966). It was exended to include thermal effects by Nowacki (1966).

The linear viscoelasticity remains an important area of research not only due to the advent and use of polymer, but also because most solids when subjected to dynamic loading exhibit viscous effects.

Eringen (1967) extended the theory of micropolar elasticity to obtain linear constitutive theory for

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micropolar material possessing internal friction. A problem on micropolar viscoelastic waves has been discussed by McCarthy and Eringen (1969). Biswas *et al*. (1996) studied the axisymmetric problems of wave propagation under the influence of gravity in a micropolar viscoelastic semi-infinite medium when a time varying axisymmetric loading has been applied on the surface of the medium. Kumar (2000) discussed wave propagation in micropolar viscoelastic generalized thermoelastic solid. EI-Karamany (2003) studied uniqueness and reciprocity theorems in a generalized linear micropolar thermoviscoelasticity.

Simonetti (2004) studied Lamb wave propagation in elastic plates coated with viscoelastic materials. Sharma (2005) carried out some considerations on the Rayleigh-Lamb wave propagation in visco-thermoelastic plate. Sharma and Othman (2007) studied effect of rotation on generalized thermo-viscoelastic Rayleigh-Lamb waves. Kumar and Sharma (2008) discussed propagation of waves in micropolar viscoelastic generalized themoelastic solids having interficial imperfections. Kumar and Partap (2008) did analysis of free vibrations for Rayleigh-Lamb waves in a micropolar viscoelastic plate. Baksi *et al.* (2008) studied two dimensional visco-elastic problems in generalized thermoelastic medium with heat source. Sharma *et al.* (2009) studied propagation of Lamb waves in viscothermoelastic plates under fluid loadings. Kožar and Ožbolt (2010) discussed some aspects of load-rate sensitivity in visco-elastic microplane material model.

The present paper is aimed at studying the vibration analysis of wave motion in micropolar thermo viscoelastic plate of thickness 2d.

2. Basic equations

The equations of motion for micropolar elastic solid given by Eringen (1999) are

$$t_{kl,k} + \rho(f_l - \ddot{u}_l) = 0 \tag{1}$$

$$m_{kl,k} + \varepsilon_{lmn} t_{mn} = \rho j_{lm} \ddot{\phi}_m \tag{2}$$

The constitutive relations in a homogeneous isotropic micropolar thermoviscoelastic solid without body forces, body couples and heat sources are

$$t_{kl} = \lambda_I u_{r,r}\delta_{kl} + \mu_I(u_{k,l} + u_{l,k}) + K_I(u_{l,k} - \varepsilon_{klr}\phi_r) - \nu_I\left(T + \tau_1\frac{\partial T}{\partial t}\right)\delta_{kl} \tag{3}$$

$$m_{kl} = \alpha_I \phi_{r,r}\delta_{kl} + \beta_I \phi_{k,l} + \gamma_I \phi_{l,k} \tag{4}$$

The heat conduction equation of thermoelasticity in context of theories given by Lord and Shulman (1967) and Green and Lindsay (1972)

$$K^*\nabla^2 T = \rho C^*\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + \nu_I T_0\left(\frac{\partial}{\partial t} + \eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla\cdot\vec{u}) \tag{5}$$

Using the constitutive relations (3)-(4) in Eqs. (1)-(2), we get the following equations in vectorial form

$$(\lambda_I + 2\mu_I + K_I)\nabla(\nabla\cdot\vec{u}) - (\mu_I + K_I)\nabla\times\nabla\times\vec{u} + K_I\nabla\times\vec{\phi} - \nu_I\left(1 + \tau_1\frac{\partial}{\partial t}\right) = \rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{6}$$

$$(\alpha_I + \beta_I + \gamma_I)\nabla(\nabla \cdot \vec{\phi}) - \gamma_I \nabla \times (\nabla \times \vec{\phi}) + K_I \nabla \times \vec{u} - 2K_I \vec{\phi} = \rho j \frac{\partial^2 \vec{\phi}}{\partial t^2} \tag{7}$$

where

$$\lambda_I = \lambda + \lambda_v \frac{\partial}{\partial t}, \quad \mu_I = \mu + \mu_v \frac{\partial}{\partial t}, \quad K_I = K + K_v \frac{\partial}{\partial t}, \quad \alpha_I = \alpha + \alpha_v \frac{\partial}{\partial t}, \quad \beta_I = \beta + \beta_v \frac{\partial}{\partial t}$$

$$\gamma_I = \gamma + \gamma_v \frac{\partial}{\partial t}, \quad \nu_I = \nu + \nu_v \frac{\partial}{\partial t}$$

$\lambda_v, \mu_v, K_v, \alpha_v, \beta_v, \gamma_v, \nu_v$ are viscosity constants, $\lambda, \mu, \alpha, \beta, \gamma, K$ are material constants, ρ is the density, j is the microinertia, t_{kl} and m_{kl} are components of stress and couple stress tensors respectively. $\vec{u} = (u_r, u_\theta, u_z)$ is the displacement vector, $\vec{\phi} = (\phi_r, \phi_\theta, \phi_z)$ is the microrotation vector, δ_{ij} is the Kronecker delta, T is the temperature change, T_0 is uniform temperature, $\nu_I = (3\lambda_I + 2\mu_I + K_I)\,\alpha_t$, α_t is the coefficient of linear thermal expansion and K^* is the coefficient of thermal conductivity, C^* is specific heat at constant strain, τ_0, τ_1 are the thermal relaxation times,

$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ is the vector operator.

3. Formulation of the problem

We consider an infinite, homogeneous isotropic thermally conducting micropolar viscoelastic plate bounded by two parallel surfaces free of tractions at $z = \pm d$, $z = 0$ being the midplane of the plate. The circular cylindrical co-ordinates (r, θ, z) have been used to describe the response of the plate as shown in Fig. 1. The plate is axisymmetric with the z-axis as the axis of the symmetry. The plate of thickness $2d$ initially undisturbed and at uniform temperature T_0. The origin of the co-ordinate system (r, θ, z) is taken on the middle surface of the plate and z-axis normal to it along the thickness. We take $r - z$ plane as the plane of incidence.

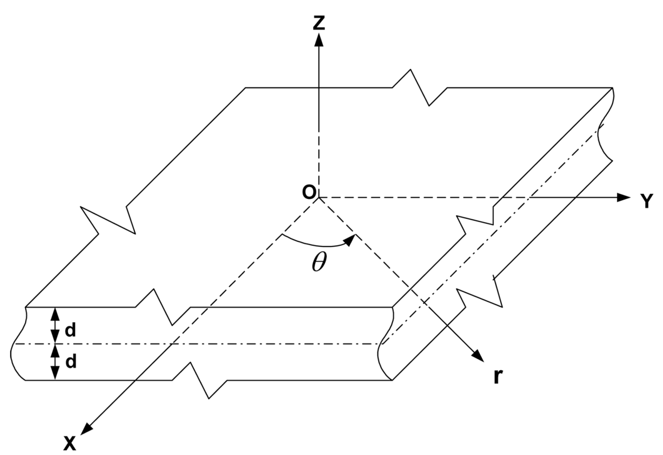

Fig. 1 Geometry of the problem

For two dimensional problem, we take $\vec{u} = (u_r, 0, u_z)$ and $\vec{\phi} = (0, \phi_\theta, 0)$. (8)

We define the non-dimensional quantities

$$r' = \frac{\omega^*}{c_1}r,\quad z' = \frac{\omega^*}{c_1}z,\quad u_r' = \frac{\rho\omega^* c_1}{v_I T_0}u_r,\quad u_z' = \frac{\rho\omega^* c_1}{v_I T_0}u_z,\quad t' = \omega^* t,\quad \phi_\theta' = \frac{\rho c_1^2}{v_I T_0}\phi_\theta,\quad T' = \frac{T}{T_0}$$

$$\tau_1' = \omega^*\tau_1,\quad \tau_0' = \omega^*\tau_0,\quad h' = \frac{c_1 h}{\omega^*},\quad d' = \frac{\omega^* d}{c_1},\quad t_{ij}' = \frac{1}{v_I T_0}t_{ij},\quad m_{ij}' = \frac{\omega^*}{v_I T_0 c_1}m_{ij},\quad p = \frac{K_I}{\rho c_1^2}$$

$$\delta^2 = \frac{c_2^2}{c_1^2},\quad \delta_1^2 = \frac{c_4^2}{c_1^2},\quad \delta^* = \frac{K_I c_1^2}{\gamma_I \omega^*} \tag{9}$$

where $c_1^2 = \dfrac{\lambda_I + 2\mu_I + K_I}{\rho}$, $c_2^2 = \dfrac{\mu_I + K_I}{\rho}$, $c_4^2 = \dfrac{\gamma_I}{\rho j}$, $\in = \dfrac{v_I^2 T_0}{\rho^2 c_1^2 C^*}$, $\omega^* = \dfrac{\rho C^* c_1^2}{K^*}$, ω^* is the characteristic frequency of the medium, $\in$ is the thermoelastic coupling constant.

Introducing the potential functions ϕ and ψ through the relations

$$u_r = \frac{\partial\phi}{\partial r} + \frac{\partial\psi}{\partial z},\quad u_z = \frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial r} - \frac{\psi}{r} \tag{10}$$

Using Eqs. (8)-(10) in Eqs. (6)-(7) and (5) and after suppressing the primes for convenience, we obtain

$$\left(\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\phi - \left(T + \tau_1\frac{\partial T}{\partial t}\right) = 0 \tag{11}$$

$$\left(\nabla^2\psi - \frac{\psi}{r^2}\right) - \frac{p\phi_\theta}{\delta^2} - \frac{1}{\delta^2}\frac{\partial^2\psi}{\partial t^2} = 0 \tag{12}$$

$$\left(\nabla^2 - \frac{1}{r^2}\right)\phi_\theta + \delta^*\left(\nabla^2 - \frac{1}{r^2}\right)\psi - 2\delta^*\phi_\theta - \frac{1}{\delta_1^2}\frac{\partial^2\phi_\theta}{\partial t^2} = 0 \tag{13}$$

$$\nabla^2 T - (\dot{T} + \tau_0\ddot{T}) = \in\left(\frac{\partial}{\partial t} + \eta_0\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\dot{\phi} \tag{14}$$

where $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial z^2}$ is the Laplacian operator.

3.1 Boundary conditions

The boundaries of the plate are assumed to be stress-free thermally insulated/isothermal. Therefore, we consider following types of boundary conditions.

3.2 Mechanical conditions

The non-dimensional mechanical boundary conditions at $z = \pm d$ are given by

$$t_{zz} = 0\,, \quad t_{zr} = 0\,, \quad m_{z\theta} = 0 \tag{15}$$

where

$$t_{zz} = (\lambda_I + 2\mu_I + K_I)\frac{\partial u_z}{\partial z} + \lambda_I\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) - \nu_I\left(T + \tau_1\frac{\partial T}{\partial t}\right)$$

$$t_{zr} = (\mu_I + K_I)\frac{\partial u_r}{\partial z} + \mu_I\frac{\partial u_z}{\partial r} - K_I\phi_\theta, \quad m_{z\theta} = \gamma_I\frac{\partial \phi_\theta}{\partial z}$$

3.3 Thermal conditions

The thermal boundary condition is given by

$$T_{,z} + hT = 0 \tag{16}$$

where h is the surface heat transfer coefficient. Here $h \to 0$ corresponds to thermally insulated boundaries and $h \to \infty$ refers to isothermal one.

4. Formal solution of the problem

We assume the solutions of Eqs. (11)-(14) of the form

$$[\phi, \psi, \phi_\theta, T] = [f(z)J_0(\xi r), g(z)J_1(\xi r), w(z)J_1(\xi r), h(z)J_0(\xi r)]e^{-i\omega t} \tag{17}$$

where ω is the circular frequency, ξ is the wave number and $c = \omega/\xi$ is the phase velocity.

Substituting the values of ϕ, ψ, ϕ_θ and T from Eq. (17) in Eqs. (11)-(14) and solving the resulting differential equations, the expressions for ϕ, ψ, ϕ_θ and T are obtained as

$$\phi = (A\cos m_1 z + B\sin m_1 z + C\cos m_2 z + D\sin m_2 z)J_0(\xi r)e^{-i\omega t} \tag{18}$$

$$\psi = (A'\cos m_3 z + B'\sin m_3 z + C'\cos m_4 z + D'\sin m_4 z)J_1(\xi r)e^{-i\omega t} \tag{19}$$

$$\phi_\theta = \frac{\delta^2}{p}[(b^2 - m_3^2)(A'\cos m_3 z + B'\sin m_3 z) + (b^2 - m_4^2)(C'\cos m_4 z + D'\sin m_4 z)]J_1(\xi r)e^{-i\omega t} \tag{20}$$

$$T = ik_1^{-1}\omega^{-1}[(a^2 - m_1^2)(A\cos m_1 z + B\sin m_1 z) + (a^2 - m_2^2)(C\cos m_2 z + D\sin m_2 z)]J_0(\xi r)e^{-i\omega t} \tag{21}$$

where

$$m_i^2 = \xi^2(c^2 a_i^2 - 1)\,, \qquad i = 1,2,3,4\,; \quad a^2 = \xi^2(c^2 - 1)\,, \quad b^2 = \xi^2\left(\frac{c^2}{\delta^2} - 1\right)$$

$$k_0 = \tau_0 + i\omega^{-1}\,, \quad k_0' = n_0\tau_0 + i\omega^{-1}\,, \quad k_1 = \tau_1 + i\omega^{-1}$$

$$(a_1^2, a_2^2) = \frac{1}{2}\{[1 + k_0 - i\omega \in k_0'k_1] \pm [\{(1 - k_0 - i\omega \in k_0'k_1)\}^2 - 4i\omega \in k_0'k_1k_0]^{1/2}\}$$

$$(a_3^2, a_4^2) = \frac{1}{2}\left\{\begin{array}{c}\left[\frac{1}{\delta_1^2}+\frac{1}{\delta^2}+\frac{\delta^{*2}}{\omega^2\delta^2}(p-2\delta^2)\right] \pm \\ \left[\left\{\frac{1}{\delta^2}-\frac{1}{\delta_1^2}+\frac{\delta^{*2}}{\omega^2\delta^2}(p-2\delta^2)\right\}^2+\frac{4\delta^{*2}}{\omega^2\delta_1^2\delta^2}\{p-2(\delta^2-\delta_1^2)\}\right]^{1/2}\end{array}\right\}$$

With the help of Eqs. (10), (18) and (19) we obtain the displacement components u_r and u_z as

$$u_r = \begin{bmatrix} -\xi(A\cos m_1 z + B\sin m_1 z + C\cos m_2 z + D\sin m_2 z) + \\ (-A' m_3 \sin m_3 z + B' m_3 \cos m_3 z - C' m_4 \sin m_4 z + D' m_4 \cos m_4 z) \end{bmatrix} J_1(\xi r) e^{-i\omega t} \tag{22}$$

$$u_z = \begin{bmatrix} (-m_1 A\sin m_1 z + m_1 B\cos m_1 z - m_2 C\sin m_2 z + m_2 D\cos m_2 z) - \\ \xi(A'\cos m_3 z + B'\sin m_3 z + C'\cos m_4 z + D'\sin m_4 z) \end{bmatrix} J_0(\xi r) e^{-i\omega t} \tag{23}$$

5. Derivation of the secular equations

Using the boundary conditions (15) and (16) on the surfaces $z = \pm d$ of the plate and with the help of Eqs. (18)-(21), we obtain a system of eight linear equations.

$$P(Ac_1 + Bs_1 + Cc_2 + Ds_2) + Q\{m_3(A's_3 - B'c_3) + m_4(C's_4 - D'c_4)\} = 0 \tag{24}$$

$$P(Ac_1 - Bs_1 + Cc_2 - Ds_2) + Q\{m_3(-A's_3 - B'c_3) + m_4(-C's_4 - D'c_4)\} = 0 \tag{25}$$

$$Q\{(-As_1 + Bc_1)m_1 + (-Cs_2 + Dc_2)m_2\} + P(A'c_3 + B's_3 + C'c_4 + D's_4) = 0 \tag{26}$$

$$Q\{(As_1 + Bc_1)m_1 + (Cs_2 + Dc_2)m_2\} + P(A'c_3 - B's_3 + C'c_4 - D's_4) = 0 \tag{27}$$

$$f_3(-A's_3 + B'c_3)m_3 + f_4(-C's_4 + D'c_4)m_4 = 0 \tag{28}$$

$$f_3(A's_3 + B'c_3)m_3 + f_4(C's_4 + D'c_4)m_4 = 0 \tag{29}$$

$$g_1[(Hc_1 - m_1 s_1)A + (Hs_1 + m_1 c_1)B] + g_2[(Hc_2 - m_2 s_2)C + (Hs_2 + m_2 c_2)D] = 0 \tag{30}$$

$$g_1[(Hc_1 + m_1 s_1)A + (-Hs_1 + m_1 c_1)B] + g_2[(Hc_2 + m_2 s_2)C + (-Hs_2 + m_2 c_2)D] = 0 \tag{31}$$

where

$$P = b^2 - \xi^2 + \frac{p\xi^2}{\delta^2}, \quad Q = -2\xi\left(1 - \frac{p}{2\delta^2}\right), \quad s_i = \sin m_i d, \quad c_i = \cos m_i d, \quad i = 1, 2, 3, 4.$$

$$f_i = b^2 - m_i^2,\ i = 3, 4; \quad g_i = a^2 - m_i^2, \quad i = 1, 2$$

This system of eight linear equations has a non-trivial solution if the determinant of the coefficients of amplitudes $[A, B, C, D, A', B', C', D']^T$ vanishes.

We obtain the following secular equations after applying lengthy algebraic reductions and manipulations

$$\left[\frac{\tan m_1 d}{\tan m_3 d}\right]^{\pm 1} - \frac{m_1(m_1^2-a^2)}{m_2(m_2^2-a^2)}\left[\frac{\tan m_2 d}{\tan m_3 d}\right]^{\pm 1} - \frac{m_3(m_3^2-b^2)}{m_4(m_4^2-b^2)}\left[\frac{\tan m_1 d}{\tan m_4 d}\right]^{\pm 1}$$

$$+\frac{m_1 m_3(m_1^2-a^2)(m_3^2-b^2)}{m_2 m_4(m_2^2-a^2)(m_4^2-b^2)}\left[\frac{\tan m_2 d}{\tan m_4 d}\right]^{\pm 1} = \frac{-4\xi^2\left(1-\frac{p}{2\delta^2}\right)^2 m_1 m_3(m_2^2-m_1^2)(m_4^2-m_3^2)}{\left(b^2-\xi^2+\frac{p\xi^2}{\delta^2}\right)^2(m_4^2-b^2)(m_2^2-a^2)} \tag{32}$$

for stress free thermally insulated boundaries $(h\to 0)$ of the plate.

$$\left[\frac{\tan m_1 d}{\tan m_3 d}\right]^{\pm 1} - \frac{m_2(m_1^2-a^2)}{m_1(m_2^2-a^2)}\left[\frac{\tan m_2 d}{\tan m_3 d}\right]^{\pm 1} + \frac{\left(b^2-\xi^2+\frac{p\xi^2}{\delta^2}\right)^2(m_2^2-m_1^2)(m_3^2-b^2)}{4\xi^2\left(1-\frac{p}{2\delta^2}\right)^2 m_1 m_4(m_2^2-a^2)(m_4^2-m_3^2)}\left[\frac{\tan m_4 d}{\tan m_3 d}\right]^{\pm 1}$$

$$= \frac{\left(b^2-\xi^2+\frac{p\xi^2}{\delta^2}\right)^2(m_2^2-m_1^2)(m_4^2-b^2)}{4\xi^2\left(1-\frac{p}{2\delta^2}\right)^2 m_1 m_3(m_4^2-m_3^2)(m_2^2-a^2)} \tag{33}$$

for stress free isothermal boundaries $(h\to\infty)$ of the plate.

Here the exponent +1 refers to skew-symmetric and −1 refers to symmetric modes. Eqs. (32) and (33) are the most general dispersion relations involving wave number and phase velocity of various modes of propagation in a micropolar thermoviscoelastic plates under the considered situations.

5.1 Particular cases

(a) Micropolar thermoviscoelastic plate with one relaxation time (Lord and Shulman 1967)

In this case, $\tau_1 = 0$, $\tau_0 > 0$ and $\eta_0 = 1$.

(b) Micropolar thermoviscoelastic plate with two relaxation time (Green and Lindsay 1972)

In this case, $\tau_1 \ge \tau_0 > 0$ and $\eta_0 = 0$.

6. Amplitudes of stresses, couple stresses and temperature

The amplitudes of stresses, couple stresses and temperature distribution for symmetric and skew-symmetric modes of plate waves, have been computed using Eqs. (15) and (18)-(21) as

$$(t_{zz})_{sym} = \{G_1\cos m_1 z + G_2 L\cos m_2 z - G_5(M m_3\cos m_3 z + N m_4\cos m_4 z)\}AJ_0(\xi r)e^{-i\omega t}$$

$$(t_{zz})_{asym} = \{G_1\sin m_1 z + G_2 L'\sin m_2 z - G_5(M' m_3\sin m_3 z + N' m_4\sin m_4 z)\}BJ_0(\xi r)e^{-i\omega t}$$

$$(t_{zr})_{sym} = \{-G_5(m_1\sin m_1 z + L' m_2 \sin m_2 z) - (G_3 M' \sin m_3 z + G_4 N' \sin m_4 z)\} B J_1(\xi r) e^{-i\omega t}$$

$$(t_{zr})_{asym} = \{(-G_5(m_1\cos m_1 z + L m_2 \cos m_2 z)) + (G_3 M \cos m_3 z + G_4 N \cos m_4 z)\} A J_1(\xi r) e^{-i\omega t}$$

$$(m_{z\theta})_{sym} = \frac{m_3\delta^2\gamma}{p}\left\{(b^2 - m_3^2)\cos m_3 z - (b^2 - m_4^2)\frac{f_3 c_3}{f_4 c_4}\cos m_4 z\right\} B' J_1(\xi r) e^{-i\omega t}$$

$$(m_{z\theta})_{asym} = \frac{m_3\delta^2\gamma}{p}\left\{(b^2 - m_3^2)\sin m_3 z - (b^2 - m_4^2)\frac{f_3 c_3}{f_4 c_4}\sin m_4 z\right\} A' J_1(\xi r) e^{-i\omega t}$$

$$(T)_{sym} = i k_1^{-1}\omega^{-1}\{(a^2 - m_1^2)\cos m_1 z + (a^2 - m_2^2) L \cos m_2 z\} A J_0(\xi r) e^{-i\omega t}$$

$$(T)_{asym} = i k_1^{-1}\omega^{-1}\{(a^2 - m_1^2)\sin m_1 z + (a^2 - m_2^2) L' \sin m_2 z\} B J_0(\xi r) e^{-i\omega t}$$

where

$$L = -\frac{g_1 m_1 s_1}{g_2 m_2 s_2}, \quad M = \frac{P(g_2 m_2 T_1^{-1} - g_1 m_1 T_2^{-1}) f_4}{Q g_2 m_2 m_3 (f_4 - f_3) T_3^{-1}}\frac{s_1}{s_3}, \quad N = \frac{P(g_1 m_1 T_2^{-1} - g_2 m_2 T_1^{-1}) f_3}{Q g_2 m_2 m_4 (f_4 - f_3) T_4^{-1}}\frac{s_1}{s_4}$$

$$L' = -\frac{g_1 m_1 c_1}{g_2 m_2 c_2}, \quad M' = \frac{P(g_2 m_2 T_1 - g_1 m_1 T_2) f_4}{Q g_2 m_2 m_3 (f_4 - f_3) T_3}\frac{c_1}{c_3}, \quad N' = \frac{P(g_1 m_1 T_2 - g_2 m_2 T_1) f_3}{Q g_2 m_2 m_4 (f_4 - f_3) T_4}\frac{c_1}{c_4}$$

$$T_i = \tan m_i d, \quad i = 1,2,3,4$$

$$G_i = -\{(\lambda_I + 2\mu_I + K_I) m_i^2 + \lambda_I \xi^2 + \nu_I (i k_1^{-1}\omega^{-1} + \tau_1 k_1^{-1})(a^2 - m_1^2)\}, \quad i = 1,2$$

$$G_j = \left\{(\mu_I + K_I) m_j^2 - \mu_I \xi - \frac{K_I \delta^2}{p}(b^2 - m_j^2)\right\}, \quad j = 3,4; \; G_5 = (2\mu_I + K_I)\xi$$

7. Numerical results and discussion

With the view of illustrating theoretical results obtained in the preceding sections, we now present some numerical results.

The material chosen for this purpose is Magnesium crystal (micropolar thermoviscoelastic solid), the physical data for which is given below

Micropolar parameters given by Eringen (1984)

$$\rho = 1.74 \times 10^3 \text{ kg/m}^3, \quad \lambda = 9.4 \times 10^{10} \text{ N/m}^2, \; \mu = 4.0 \times 10^{10} \text{ N/m}^2$$

$$K = 10 \times 10^{10} \text{ N/m}^2, \quad \gamma = 0.779 \times 10^{-9} \text{ N}, \quad j = 0.2 \times 10^{-19} \text{ m}^2$$

Thermal parameters given by Dhaliwal and Singh (1980)

$$\tau_0 = 6.131\times10^{-13}\ \text{sec},\quad \tau_1 = 8.765\times10^{-13}\ \text{sec},\quad \in\ = 0.028,\quad T_0 = 298^{\circ}\text{K},$$

$$C^* = 1.04\times10^{3}\ \text{J/kg deg},\quad K^* = 1.7\times10^{6}\ \text{J/m sec deg},\quad \nu = 2.68\times10^{6}\ \text{N/m}^2\ \text{deg},\quad d = 0.01\ \text{m}$$

For a particular model of a micropolar thermoviscoelastic solid, the relevant parameters are expressed as $\chi_I = \chi(1 - iQ_k^{-1})$, k = 1, 2, 3, 4, 5 for $\chi = \lambda, \mu, K, \gamma, \nu$, respectively where Q_1^{-1} = 0.05, $Q_2^{-1} = 0.01$, Q_3^{-1} = 0.015, $Q_4^{-1} = 0.1$, $Q_5^{-1} = 0.15$.

A FORTRAN program has been developed for the solution of Eq. (32) to compute phase velocity c for different values of n by using the relations $\tan\theta = \tan(n\pi + \theta)$ and $m_i^2 = \xi^2(c^2 a_i^2 - 1)$.

The phase velocity and attenuation coefficient of symmetric and skew-symmetric modes of wave propagation in the context of L-S theory of thermoelasticity have been computed for various values of wave number from dispersion Eq. (32) for stress free thermally insulated boundaries in case of

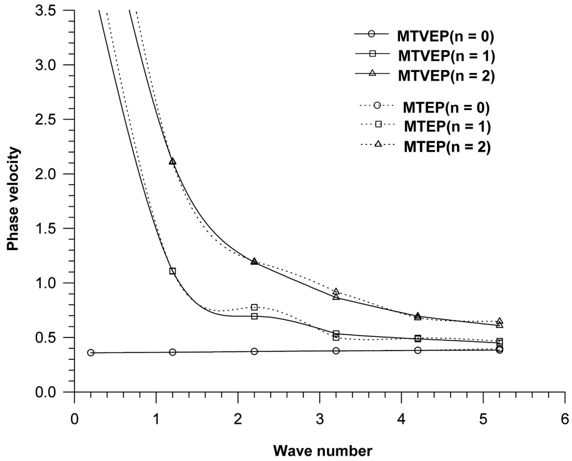

Fig. 2 Variation of phase velocity of symmetric modes of wave propagation

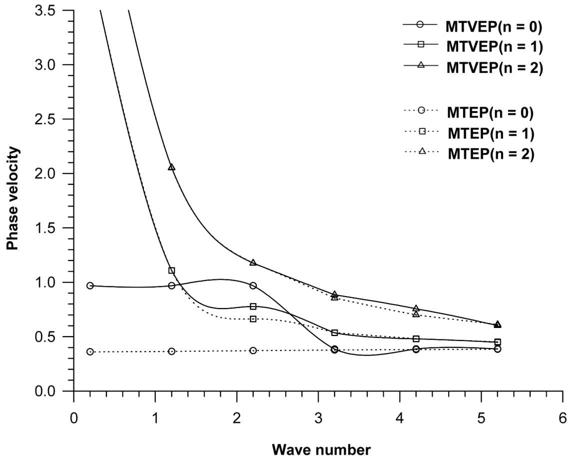

Fig. 3 Variation of phase velocity of skew-symmetric modes of wave propagation

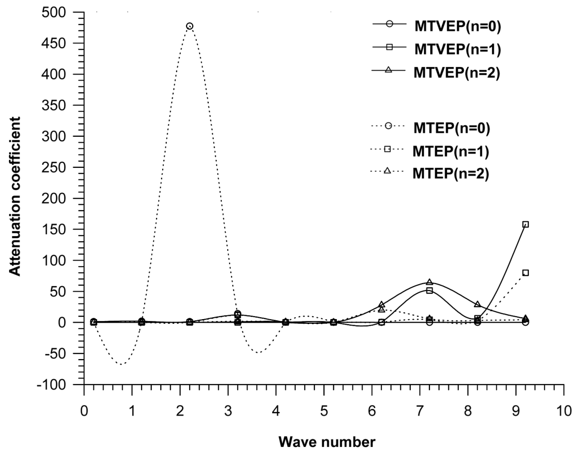

Fig. 4 Variation of attenuation coefficient of symmetric modes of wave propagation

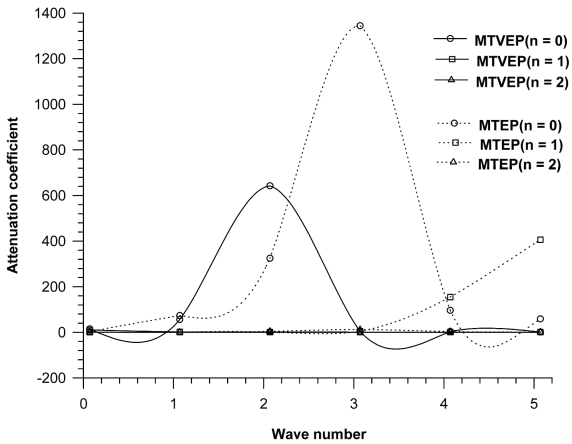

Fig. 5 Variation of attenuation coefficient of skew-symmetric modes of wave propagation

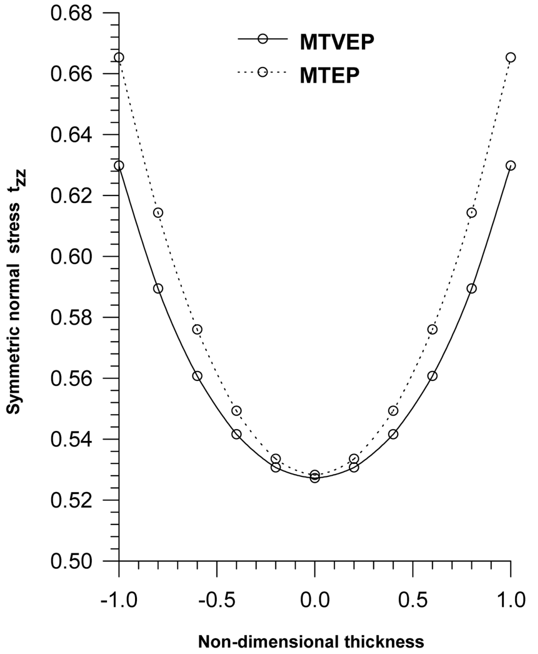

Fig. 6 Variation of amplitude of symmetric normal stress t_{zz}

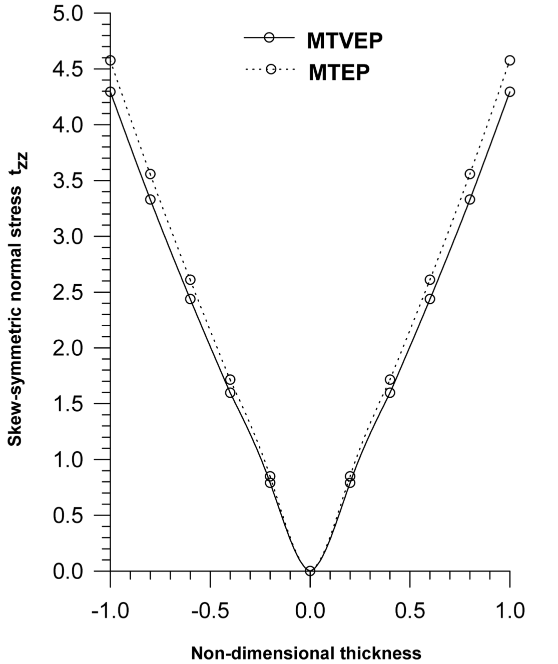

Fig. 7 Variation of amplitude of skew-symmetric normal stress t_{zz}

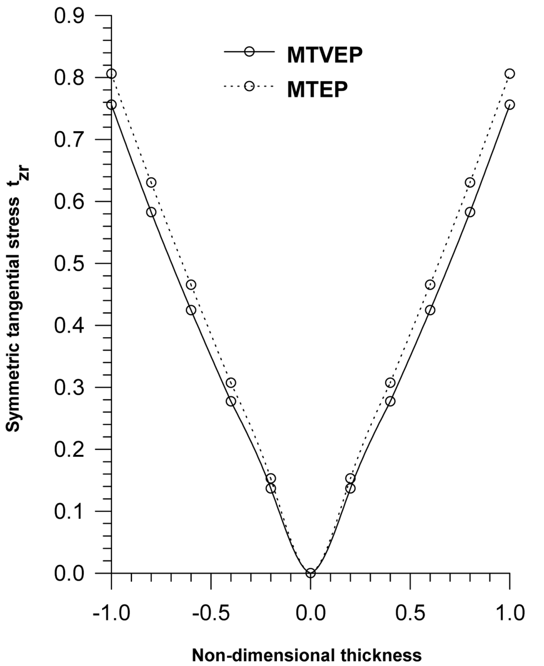

Fig. 8 Variation of amplitude of symmetric tangential stress t_{zr}

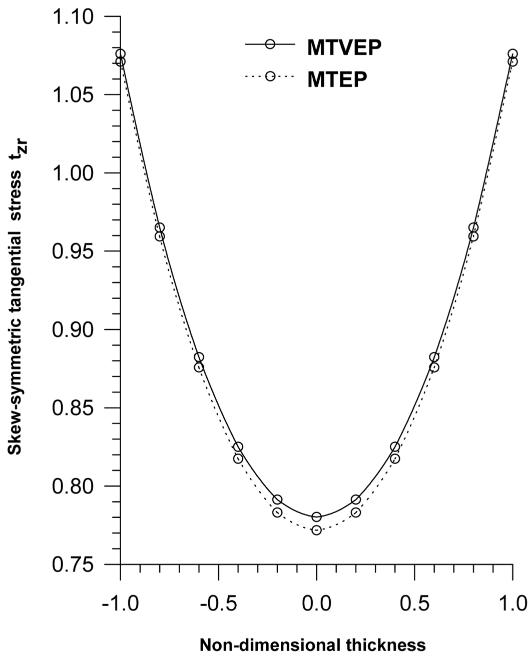

Fig. 9 Variation of amplitude of skew-symmetric tangential stress t_{zr}

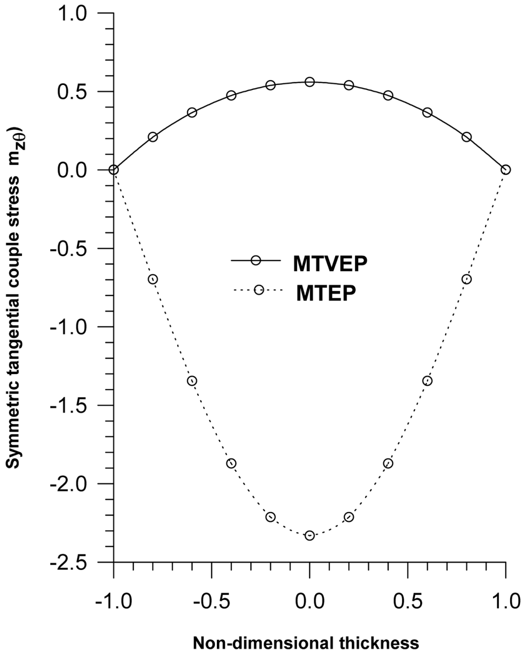

Fig. 10 Variation of amplitude of symmetric tangential couple stress $m_{z\theta}$

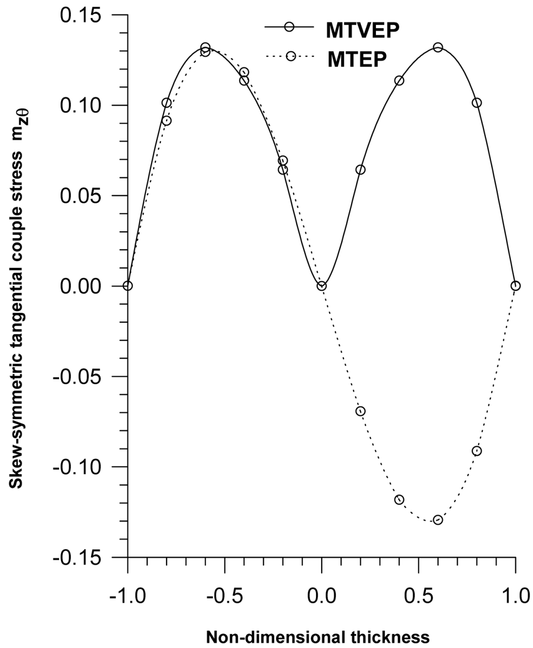

Fig. 11 Variation of amplitude of skew-symmetric tangential couple stress $m_{z\theta}$

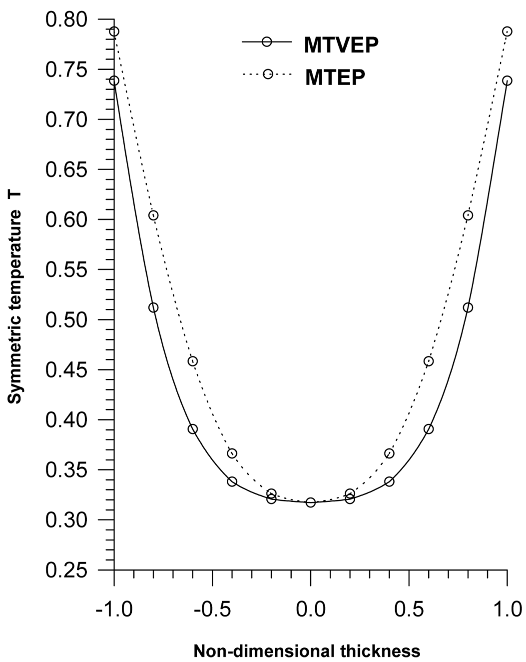

Fig. 12 Variation of amplitude of symmetric temperature T

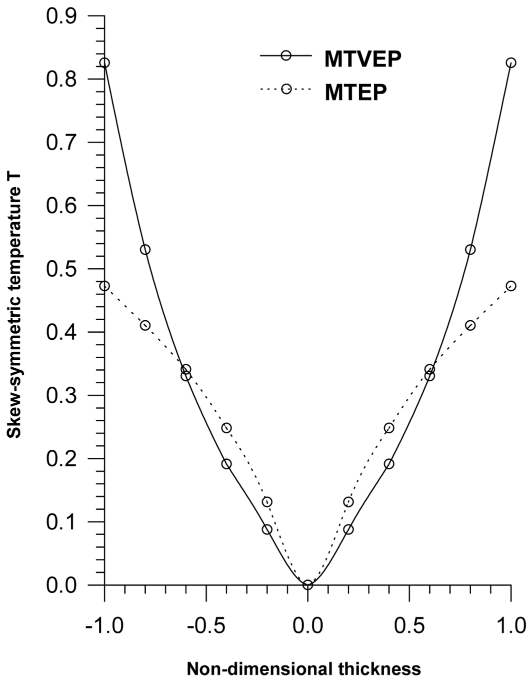

Fig. 13 Variation of amplitude of skew-symmetric temperature T

micropolar thermoviscoelastic plate (MTVEP) and micropolar thermoelastic plate (MTEP) and have been represented graphically for different modes (n = 0 to n = 2) in Figs. 2 to 5.

Figs. 6-13 depict the variations of symmetric and skew-symmetric amplitudes of normal stress t_{zz}, tangential stress t_{zr}, tangential couple stress $m_{z\theta}$ and temperature distribution T. The solid curves correspond to micropolar thermoviscoelastic plate (MTVEP) and dotted curves refer to micropolar thermoelastic plate (MTEP).

7.1 Phase velocities

The phase velocities of higher modes of propagation, symmetric and skew-symmetric attain quite large values at vanishing wave number which sharply slashes down to become steady and asymptotic with increasing wave number.

For symmetric modes of wave propagation, we notice the following from Fig. 2(a) for lowest mode $n = 0$, phase velocity profiles for MTVEP and MTEP coincide (b) for mode $n = 1$, phase velocity for MTVEP is less than in case of MTEP in the regions $0 \le \xi \le 1.1$, $1.4 \le \xi \le 2.8$ and $4.2 \le \xi \le 5.2$; phase velocity for MTVEP is more in comparison to MTEP for wave number lying between 2.8 and 4.2; phase velocity profiles for MTVEP and MTEP coincide for wave number $\xi \ge 1.1$ and $\xi \le 1.4$ (c) for mode $n = 2$, phase velocity for MTVEP is less than in case of MTEP in the regions $0 \le \xi \le 1.2$, $2.2 \le \xi \le 4.2$ and $4.7 \le \xi \le 5.2$; whereas phase velocity for MTVEP is more in comparison to MTEP in the regions $1.2 \le \xi \le 2.2$ and $4.2 \le \xi \le 4.7$.

For skew-symmetric modes of wave propagation, we observe the following from Fig. 3(a) for lowest mode $n = 0$, phase velocity for MTVEP is more than in case of MTEP for wave number $\xi \le 3.2$; phase velocity for MTVEP is less than in case of MTEP for wave number lying between 3.2 and 4.2; phase velocity for MTVEP is slightly more than in case of MTEP for wave number $\xi \ge 4.2$ (b) for mode $n = 1$, phase velocity profiles for MTVEP and MTEP coincide for wave number $\xi \le 1.3$ and $\xi \ge 4.2$; phase velocity for MTVEP is more than in case of MTEP in region $1.3 \le \xi \le 3.2$; phase velocity for MTVEP is slightly less than in case of MTEP for wave number lying between 3.2 and 4.2 (c) for mode $n = 2$, phase velocity profiles for MTVEP and MTEP coincide for wave number $\xi \le 2.5$; phase velocity for MTVEP is more than in case of MTEP for wave number $\xi \ge 2.5$.

7.2 Attenuation coefficients

In general, wave number and phase velocity of the waves are complex quantities, therefore, the waves are attenuated in space. If we write

$$c^{-1} = s^{-1} + i\omega^{-1}q \tag{34}$$

then $\xi = K_1 + iq$, where $K_1 = \omega/s$ and q are real numbers. This shows that s is the propagation speed and q is attenuation coefficient of waves. Upon using Eq. (34) in the FORTRAN program developed for the solution of Eq. (32) to compute phase velocity c, attenuation coefficient q for different modes of wave propagation can be obtained.

The variation of attenuation coefficient with wave number for symmetric and skew-symmetric modes is represented graphically in Figs. 4 and 5 respectively.

We observe the following from Fig. 4 (i) for lowest symmetric mode, magnitude of attenuation coefficient for MTVEP has negligible variation with wave number in regions $0.2 \le \xi \le 2.2$ and $4.2 \le \xi \le 9.2$; attenuation coefficient attains value 12.04 at $\xi = 3.2$ in the region $2.2 \le \xi \le 4.2$ (ii) for first symmetric mode, magnitude of attenuation coefficient for MTVEP has negligible variation with wave number in region $0.2 \le \xi \le 6.2$; attenuation coefficient attains value 51.31 at $\xi = 7.2$ in the region $6.2 \le \xi \le 8.2$; attenuation coefficient increases sharply to 157.9 as wave number increases from 8.2 to 9.2. (iii) for second symmetric mode, the magnitude of attenuation coefficient for MTVEP has negligible variation with wave number in region $0.2 \le \xi \le 5.2$; attenuation coefficient attains peak value 63.91 at $\xi = 7.2$ in the region $5.2 \le \xi \le 9.2$ (iv) for lowest symmetric mode, magnitude of attenuation coefficient for MTEP has negligible variation with wave number in regions $0.2 \le \xi \le 1.2$ and $3.2 \le \xi \le 9.2$; attenuation coefficient attains highest peak value 477.3 at $\xi = 2.2$ in the region $1.2 \le \xi \le 3.2$. (v) for first symmetric mode, magnitude of attenuation coefficient for MTEP has negligible variation with wave number in regions $0.2 \le \xi \le 2.2$ and $4.2 \le \xi \le 9.2$; attenuation coefficient attains value 11.66 at $\xi = 3.2$ in the region $2.2 \le \xi \le 4.2$. (vi) for second symmetric mode, magnitude of attenuation coefficient for MTEP has negligible variation with wave number in regions $0.2 \le \xi \le 5.2$ and $7.2 \le \xi \le 9.2$; attenuation coefficient attains value 19.71 at $\xi = 6.2$ in the region $5.2 \le \xi \le 7.2$.

As observed from Fig. 5, the attenuation coefficient has negligible variation with wave number for first (n = 1) and second (n = 2) skew-symmetric modes in case of MTVEP, whereas attenuation coefficient has negligible variation with wave number for second skew-symmetric mode in case of MTEP. The attenuation coefficient for MTVEP attains high value 642.2 at $\xi = 2.2$ in the region $0.2 \le \xi \le 3.2$ and has negligible variation with wave number in the region $3.2 \le \xi \le 5.2$ for lowest skew-symmetric mode as noticed from Fig. 5.

For lowest skew-symmetric mode, attenuation coefficient attains highest peak value 1344 at $\xi = 3.2$ in the region $0.2 \le \xi \le 5.2$ in case of MTEP as seen from Fig. 5. The attenuation coefficient in case of MTEP for first skew-symmetric mode (n = 1) increases to 4.111 in the region $0.2 \le \xi \le 3.2$ at $\xi = 3.2$ and attains high values 154.1 and 406.0 at $\xi = 4.2$ and at $\xi = 5.2$ respectively in the $3.2 \le \xi \le 5.2$ as observed from Fig. 5.

7.3 Amplitudes

The amplitudes of symmetric normal stress t_{zz}, skew-symmetric tangential stress t_{zr} and symmetric temperature distribution T is minimum at the centre and maximum at the surfaces, whereas the amplitudes of skew-symmetric normal stress t_{zz}, symmetric tangential stress t_{zr} and skew-symmetric temperature distribution T is zero at the centre and maximum at the surfaces in case of micropolar thermoviscoelastic plate (MTVEP) and micropolar thermoelastic plate (MTEP).

The amplitudes of symmetric tangential couple stress $m_{z\theta}$ is maximum at the centre and minimum at the surfaces in case of MTVEP; is minimum at the centre and maximum at the surfaces in case of MTEP.

The amplitudes of skew-symmetric tangential couple stress $m_{z\theta}$ is zero at the centre and at the surfaces in case of MTVEP and MTEP; is maximum at the surfaces $z = \pm d/2$ in case of MTVEP, whereas it is maximum at the surface $z = -d/2$ and minimum at the surface $z = d/2$ in case of MTEP. $(t_{zz})_{sym}, (t_{zz})_{asym}, (t_{zr})_{sym}, (t_{zr})_{asym}, (m_{z\theta})_{sym}, (m_{z\theta})_{asym}, (T)_{sym}$ and $(T)_{asym}$ correspond to the values of $t_{zz}, t_{zq}, m_{z\theta}$ and T for symmetric and skew-symmetric modes respectively.

It is observed that behavior and trend of variations $(t_{zz})_{sym}, (t_{zr})_{asym}$ and $(T)_{sym}$ is similar and that of $(t_{zz})_{asym}$ and $(t_{zr})_{sym}$ resembles with each other.

The values of the symmetric normal stress t_{zz} of the plate in case of MTVEP are less in comparison to MTEP. The values of the skew-symmetric normal stress t_{zz} and symmetric tangential stress t_{zr} in case of MTVEP are less in comparison to MTEP except at the centre of the plate. The values of the skew-symmetric tangential stress t_{zr} of the plate in case of MTVEP are slightly more than in case of MTEP. The values of the symmetric tangential couple stress $m_{z\theta}$ of the plate in case of MTVEP are more in comparison to MTEP except at the surfaces. The values of the skew-symmetric tangential couple stress $m_{z\theta}$ of the plate in case of MTVEP are more in the region $0<z<d$, are slightly less in the region $0<z<-0.5d$ and are slightly more in the region $-0.5d \le z \le -d$ than in case of MTEP. The values of the temperature distribution T of the plate in case of MTVEP are less in comparison to MTEP for symmetric modes except at the centre. The values of the skew-symmetric temperature distribution T of the plate in case of MTVEP are less in the region $0<z\le \pm 0.6d$ and are more in region $\pm 0.6d \le z \le \pm d$ than in case of MTEP.

8. Conclusions

(i) The vibration analysis of wave motion in micropolar thermoviscoelastic plate subjected to stress free thermally insulated and isothermal boundary is carried out in the context of Lord and Shulman (L-S) theory of thermoelasticity (ii) The secular equations for symmetric and skew-symmetric wave modes in stress free thermally insulated and isothermal micropolar thermoviscoelastic plate are derived (iii) The phase velocities of higher modes of propagation, symmetric and skew-symmetric attain quite large values at vanishing wave number, which sharply slashes down to become steady and asymptotic with increasing wave number (iv) The variations of symmetric and skew-symmetric amplitudes of normal stress t_{zz}, tangential stress t_{zr}, tangential couple stress $m_{z\theta}$ and temperature distribution T has been studied in case of micropolar thermoviscoelastic plate (MTVEP) and micropolar thermoelastic plate (MTEP).

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