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Structural design using topology and shape optimization

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Abstract. A topology optimization and shape optimization method are widely used in the design area of engineering field. In this paper, a unified procedure to combine both topology and shape optimization method is used. A material distribution method is used first to extract necessary design parameters of the structure and a shape optimization scheme using genetic algorithm and satisfying all the condition follows. As an example, a GFRP bridge deck is designed and compared with other commercial products. The performance of the designed deck shows that the used design procedure is very efficient and safe. This procedure can be generalized for using in other areas of engineering.

Keywords: topology optimization; shape optimization; genetic algorithm; minimum compliance design; GFRP deck.

1. Introduction

An efficient use of material is important and it is one of key features of good design. Due to this fact, optimization has been performed for almost every engineering area. If we focus on the structural elements, a structural element is the result of individual or systematic optimization.

When we design a structural element, structural optimization is, in an ordinary sense, to make the structure deform or resist stress up to some limit point without fail using minimal material. The method of topology and shape optimization has been developed by many researchers to fulfill these requirements. The theory of shape design sensitivity analysis was performed by Zolesio (1981) and Haug *et al.* (1986). Bendsoe and Kikuchi (1988) developed the homogenization method for structural topology optimization and many researchers performed related researches (Suzuki and Kikuchi 1991, Diaz and Bendsoe 1992, Allaire and Kohn 1993, Bendsoe and Haber 1993, Allaire 1997). Kim and Kwak (2002) proposed design space optimization, where design parameters and layout varies in the course of optimization. Wang *et al.* (2003) developed a level set method, where the boundary is represented by a level set model that is embedded in a scalar function of a higher dimension. Kutylowski (2009) utilized minimum compliance method for topology optimization, where the variation approach of the problem is formulated. However, these researches generally utilized only one optimization method for each design.

One of active application areas of optimal design is the design of FRP decks. Cassidy *et al.*

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(2002), Keller and Gurtler (2006), and Keller and Schollmayer (2004, 2006) reported on the composite action developed between pultruded FRP bridge deck system and supporting main girders. Qiao *et al.* (2000) presented a systematic approach for analysis and design of all FRP deck/stringer bridges. Lee *et al.* (2004) and Ahn (2007) used shape optimization procedures to produce pultruded FRP bridge deck designs. Recently, Lee (2009) designed a GFRP deck using a topology optimization followed by a shape optimization.

In this study, we develop a systematic procedure for the design of a structure, which was partially developed in Lee (2009). A topology optimization is used to select necessary design parameters and a shape optimization is used to determine values of design parameters.

2. Theory

2.1 Topology optimization

Topology Optimization is to make the topological geometry of a structure be most efficient to resist against external loads. One of the methods is to adopt minimum compliance design using the set up of a material distribution problem (Bendsoe and Sigmund 2004).

2.2 Problem statement

For real displacement **u** and arbitrary virtual displacement **v**, we introduce the energy bilinear form

$$a(\mathbf{u},\mathbf{v}) = \int_{\Omega} E_{ijkl}\varepsilon_{ij}(\mathbf{u})\varepsilon_{kl}(\mathbf{v})d\Omega \tag{1}$$

with linearized strain $\varepsilon_{ij}(\mathbf{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ and the linear load form

$$l(\mathbf{u}) = \int_{\Omega} \mathbf{f}\cdot\mathbf{u}d\Omega + \int_{\Gamma} \mathbf{t}\cdot\mathbf{u}ds \tag{2}$$

The elastic modulus tensor E_{ijkl}, body force **f**, and surface traction vector **t** are introduced in Eq. (2). Main functions **u**, and **v** corresponds to real displacements and virtual displacements, respectively. The minimum compliance problem takes the following form:

Minimize $l(\mathbf{u})$ such that

$$a(\mathbf{u},\mathbf{v}) = l(\mathbf{v}) \text{ for all } \mathbf{v} \tag{3}$$

which means that we try to find optimal E_{ijkl} which minimizes the work done by external force such that the structure has maximum elastic stiffness.

In the numerical analysis, the whole domain of interest is divided into small elements. The problem of best topological geometry is now changed to the problem of determination of material element or empty element such that it is called *material distribution*. This approach implies that admissible stiffness tensor can be represented by

$$E_{ijkl} = 1_{\Omega}^{mat} E_{ijkl}^{0} \tag{4}$$

where $1_{\Omega}^{mat} = 1$ if the element include material point and $1_{\Omega}^{mat} = 0$ if the element is void. The reference elastic modulus tensor E_{ijkl}^{0} is the one when every point in one element is a material point. In the calculation, the penalized proportional stiffness model is used

$$E_{ijkl}(\mathbf{x}) = \rho(\mathbf{x})^{p} E_{ijkl}^{0}, \ \ p > 1 \tag{5}$$

with a material density condition

$$\int_{\Omega} \rho(\mathbf{x}) d\Omega = V, \quad 0 \le \rho(\mathbf{x}) < 1 \tag{6}$$

where the volume of entire body is denoted as V.

Introduction of three Lagrange multipliers $\Lambda, \lambda^{+}, \lambda^{-}$ leads to another constrained problem definition as

$$\begin{aligned} L = {} & l(\mathbf{u}) - (a(\mathbf{u}, \mathbf{v}) - l(\mathbf{v})) + \Lambda\left(\int_{\Omega} \rho(\mathbf{x}) d\Omega - V\right) \\ & + \int_{\Omega} \lambda^{+}(\mathbf{x})(\rho(\mathbf{x}) - 1) d\Omega + \int_{\Omega} \lambda^{-}(\mathbf{x})(\rho_{\min} - \rho(\mathbf{x})) d\Omega \end{aligned} \tag{7}$$

Under the assumption that $\rho \ge \rho_{\min} \ge 0$, the optimal status occurs when the condition for ρ becomes

$$p\rho(\mathbf{x})^{p-1} E_{ijkl}^{0} \varepsilon_{ij}(\mathbf{u}) \varepsilon_{kl}(\mathbf{u}) = \Lambda \tag{8}$$

such that we try to make the following B_K be 1

$$B_K = \Lambda_K^{-1} p\rho(\mathbf{x})^{p-1} E_{ijkl}^{0} \varepsilon_{ij}(\mathbf{u}_K) \varepsilon_{kl}(\mathbf{u}_K) \tag{9}$$

where K is the step number.

A fixed point type update scheme for $\rho(\mathbf{x})_{K+1}$ is as follows

$$\rho_{K+1} = \begin{cases} \max\{(1-\xi)\rho_K, \rho_{\min}\} & \text{if } \rho_K B_K^{\eta} \le \max\{(1-\xi)\rho_K, \rho_{\min}\} \\ \min\{(1+\xi)\rho_K, 1\} & \text{if } \min\{(1+\xi)\rho_K, 1\} \le \rho_K B_K^{\eta} \\ \rho_K B_K^{\eta} & \text{otherwise} \end{cases} \tag{10}$$

where η is a tuning parameter and ξ is a move limit. A typical value for η and ξ is 0.5 and 0.2, respectively.

When the above optimization is done, a topological geometry of the structure for a given density level is decided. The next step is to extract geometrical parameters which can define the exact shape and dimension of the structure. An expert can choose necessary parameters for each density level.

2.2 A method of shape optimization

Shape optimization is to find an optimal shape which minimizes a certain cost functional while keeping its topological geometry. In structural design area, it is often to minimize the total mass of the structure while satisfying several design requirements.

In case the cost functional consists of continuous functions of their parameters, the optimal design is to find the point where the derivative with respect to each parameter is zero. However, in many cases those functions are not continuous/regular such that a more general optimization scheme is appropriate.

In this study, we utilize a typical genetic algorithm (GA) scheme for the shape optimization in the following way:

1) l the design parameters are chosen already.
2) A reasonable range can be set for each design parameter.
3) Each range is discretised with equal intervals. To get more precise solution, the size of an interval should be smaller.
4) Each discrete value of one parameter corresponds to one binary digit in GA.
5) Repeat step 4 to make Parent DNAs which include all the initial parameter information.
6) Generate child DNAs which are checked to satisfy several design conditions.
7) Select best DNAs among those who satisfy design conditions to make next Parents.
8) Repeat from step 6 to step 7 until the DNA values are converged.
9) Convert the DNA into real specifications of the structure.

In step 1, we assume design parameters of a structure are previously chosen. In step 4, the initial discrete value of a parameter is transformed into a binary digit. In step 6, the GA program calls external program which calculates design conditions using generated DNAs. If a child DNA is not satisfactory, it is discarded.

3. A structural design procedure

A design procedure in this study includes both topology and shape optimization explained in the previous section. It can be seen as a general procedure for optimal design of a structure, which consists of eleven steps as follows:

1) The material properties of the target structure are chosen.
2) The maximum outer bound of the structure is chosen. (If the mass ratio is 1.0, the shape of the designed structure is the same as that of the outer bound which is filled with the one material)
3) Design loads and boundary conditions are chosen and applied to the structure.
4) Set the mass ratio be the low limit.
5) Perform the topology optimization of section 2.1
6) Extract design parameters from the result of step 5.
7) Increase small amount of mass ratio and go to step 5.
8) If enough number of geometric parameters are extracted, stop the loop (step 5-7).
9) Perform the shape optimization of section 2.2 using the extracted parameters of step 6.
10) All the values of those parameters are decided upon the convergence of the shape optimization process.
11) Final verification of performance is done.

In step 1, basic material properties are the elastic modulus and poisson's ratio in general. Each material model has proper modulus. In step 5, the topological geometry of the structure is decided at the lowest material ratio status in the beginning. In step 6, the shape of the whole structure can be determined exactly by those parameters. As the mass ratio increases in step 7, the topological geometry of the structure changes after step 5 and an additional parameter, which is essential to describe the new geometry, might show up. In step 11, the material ratio of step 5 should match that of step 9 approximately.

As a summary, the topology optimization step is first used to determine design parameters and the shape optimization step follows to set values of design parameters extracted in topology optimization step.

4. Design example

As an example application of the introduced procedure, a GFRP (Glass Fiber Reinforced Polymer) bridge deck cross section is optimized (Lee 2009)

A typical material property of GFRP is listed in Table 1. To decide the out bound of the GFRP deck, sizes of two commercial products are investigated (Fig. 1). In these figures, the heights of both decks are equal to 200 mm. Based on this information, a rectangular box with 200 mm height is chosen as the maximum out bound of the cross section.

A GFRP deck is generally optimized to have the maximum second moment of inertia to resist the bending moment. However, this kind of optimization does not guarantee the best performance in the orthogonal direction. The topological optimization is performed under the condition in Fig. 2.

Initially, the mass ratio is set to be 0.25 and the topological optimization leads to a truss-like structure with rigid joints (Fig. 3).

Table 1 Material properties of typical GFRP (Keller 2003)

Elastic Modulus - Fiber Direction	30 GPa
Elastic Modulus - Orthogonal Direction	7 GPa
Shear Modulus	3.5 GPa
Tensile/Compressive Strength	200 MPa

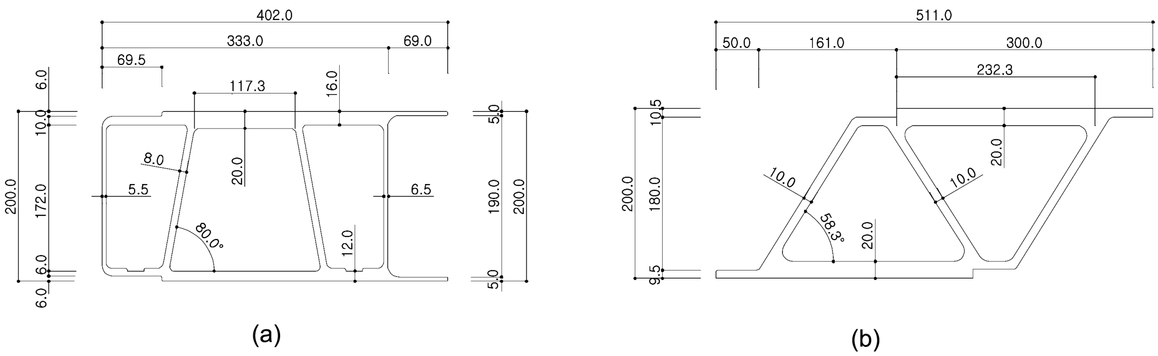


Fig. 1 Cross section (a) Delta deck, (b) ASSET

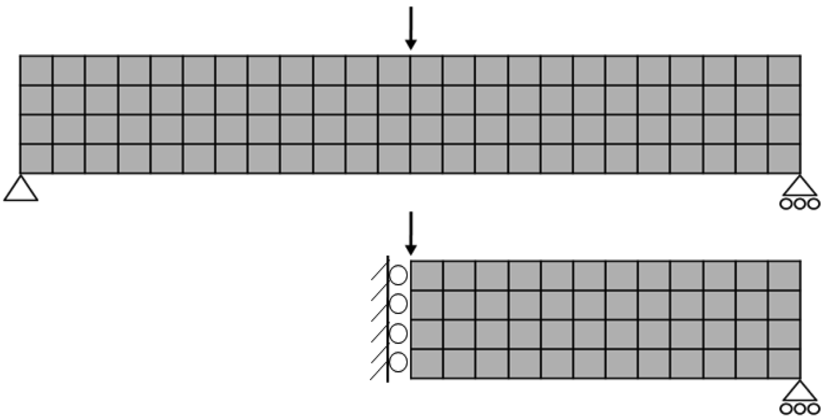


Fig. 2 Load and Boundary Conditions for the Cross section of GFRP deck

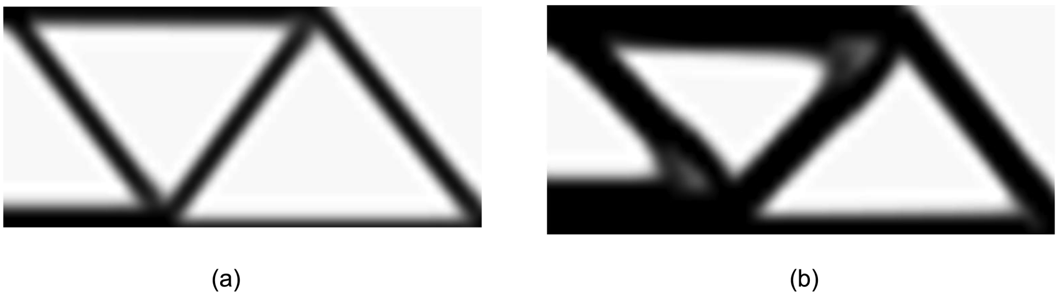


Fig. 3 Cross Section (a) Mass ratio = 0.25, (b) Mass ratio = 0.5

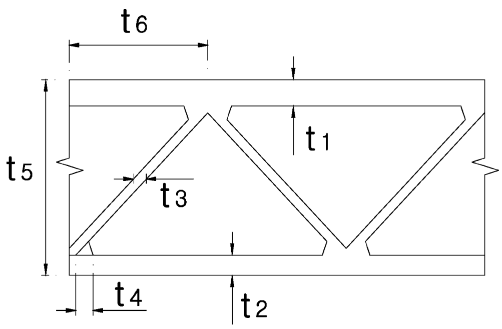


Fig. 4 Final design parameters

In Fig. 3(a), parameters t_1, t_2, t_3, and t_6 (Fig. 4) are extracted. As the mass ratio increases, another parameter t_4 can be found in Fig. 3(b) and is indicated in Fig. 4. The value of $t_5 = 200$ mm is already selected.

All the necessary design parameters are extracted and the shape optimization phase begins. The design limitations are defined first. The major resistance factor of a bridge deck is the bending stress due to the design load, i.e., the standard traffic load (DB24). Assuming a span length of 35 m bridge, the maximum bending stress f is calculated by the beam theory

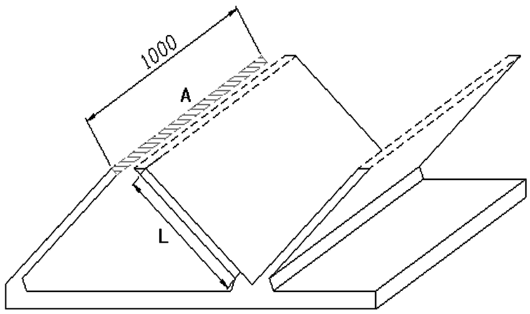


Fig. 5 Local buckling of web

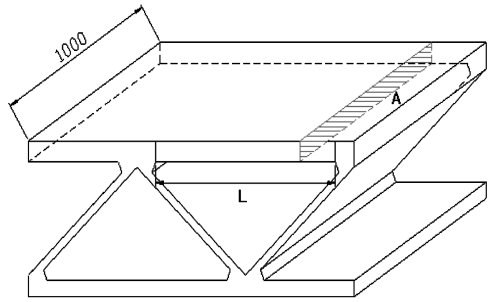


Fig. 6 Local buckling of flange

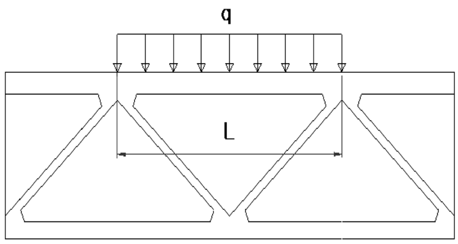


Fig. 7 Local deflection of flange

$$f = \frac{Mh}{2I} \tag{11}$$

where M is the maximum bending moment along the bridge, h is the height of the deck, and I is the second moment of inertia of the cross section. Using the strength of GFRP (= 200 MPa), safety factor 2.5 for tensile stress, and safety factor 3.0 for compressive stress, the allowable tensile stress is 80 MPa and the allowable compressive stress is 66.7 MPa.

The next condition is the critical stress σ_{cr} of the web due to local buckling and it is calculated based on the theory of Euler column (Fig. 5)

$$\sigma_{cr} = \frac{\pi^2 EI}{L^2 A} \tag{12}$$

The third condition is the critical stress σ_{cr} of the flange due to local buckling and it is calculated by the Eq. (11) and Fig. 6.

The last condition deals with the local deflection of flange between webs, which limits the local deflection δ in Eq. (12) to be less than $L/300$ in Fig. 7

$$\delta = \frac{qL^4}{384EI} \tag{13}$$

The above design conditions and factor of safety are determined according to GFRP design specification (Park *et al.* 2008) and a KSCE (2008). All the procedure of shape optimization with genetic algorithm is summarized in the flow chart (Fig. 8). In this study, the objective function ϕ to be minimized is the area of the deck cross section

$$\phi = n \times (t_5 - t_1 - t_2) \times t_3 + (t_4 \times t_5 \times t_4 \div (\sqrt{t_5^2 + t_6^2})) + L(t_1 + t_2) \tag{14}$$

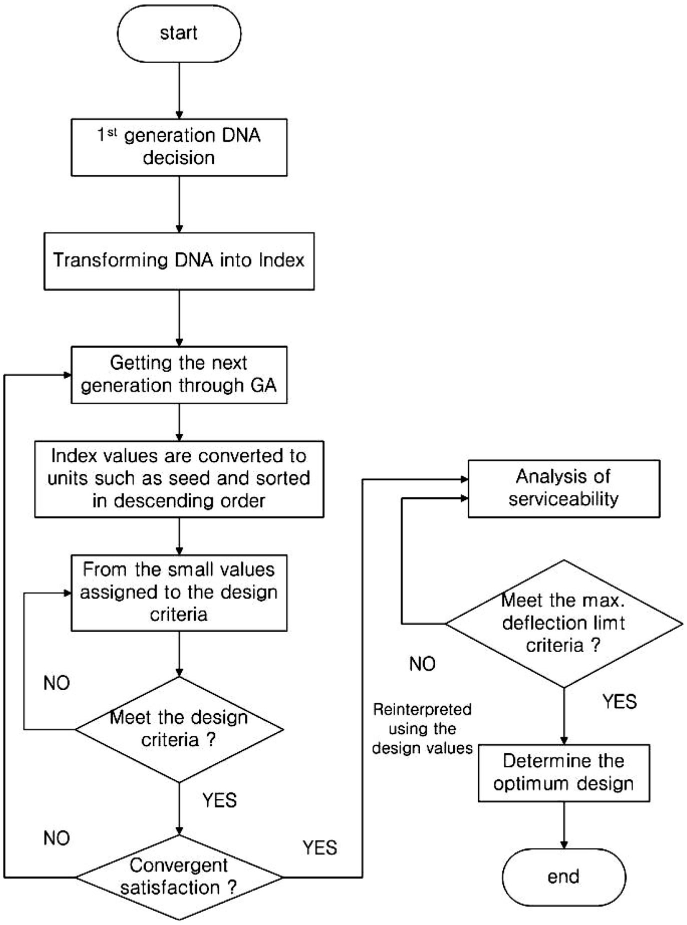


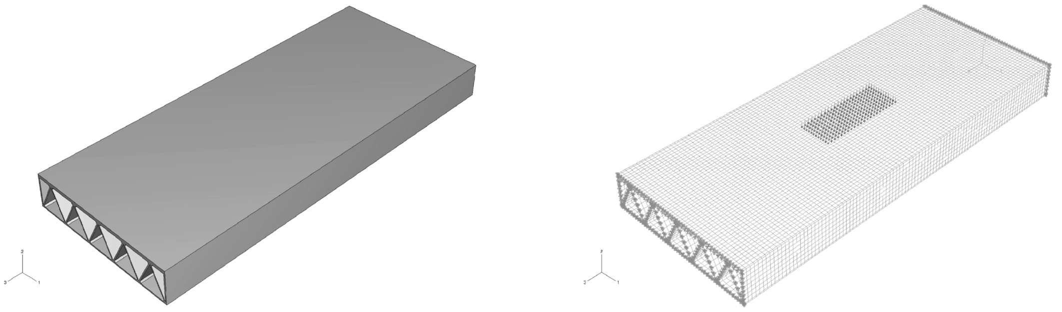
Fig. 8 flow chart of the GA in shape optimization

Table 2 Result of shape optimization

Design Variables	Value
Top flange thickness t_1	19.6 mm
Bottom flange thickness t_2	7.6 mm
Web thickness t_3	9.7 mm
Node reinforcement t_4	8.0 mm
Web interval t_5	102 mm

Those parameters in Eq. (13) are listed in Fig. 4. After the shape optimization, the final result is listed in Table 2.

Performance of the current design is compared with two commercial products. The span between girders L_G is 2.5 m and the length along the traffic direction is 1 m (Fig. 9). The standard DB24 wheel load is uniformly distributed on the inner rectangular area.



(a) (b)

Fig. 9 (a) 3D Model of the GFRP deck, (b) Finite elements and external load

Table 3 Analysis results

	Current Design	ASSET	Delta Deck
Flange deflection	2.262 mm	5.161 mm	4.908 mm

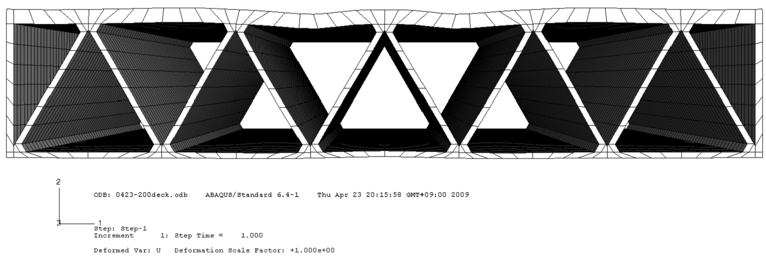


Fig. 10 Deformed shape of the designed deck

In this analysis, bottom flange deflection of each deck is calculated and presented in Table 3 and Fig. 10.

One of the most conservative design criteria's is AASHTO (2004) and it specifies the deflection limit to be $L_G/800$. For this problem, the deflection limit corresponds to 2500/800 = 3.125 mm and only the current design satisfies the limit condition.

GFRP structures are generally orthotropic and do not show yielding behavior easily. Therefore, Tsai-Wu failure analysis is appropriate to evaluate the factor of safety (KICT 2006). The Tsai-Wu Failure Index (F.I) is as follows

$$F.I. = (F_1\sigma_1 + F_2\sigma_2) + (F_1 1\sigma_1^2 + F_2 2\sigma_2^2 + 2F_1 2\sigma_1\sigma_2 + F_{66}\tau_{12}^2) \tag{15}$$

with $F_1 = \frac{1}{X_1^T} - \frac{1}{X_1^C}$, $F_2 = \frac{1}{X_2^T} - \frac{1}{X_2^C}$, $F_{11} = \frac{1}{X_1^T X_1^C}$, $F_{22} = \frac{1}{X_2^T X_2^C}$, $F_{66} = \frac{1}{S_{12}^2}$, and $F = -\frac{1}{2}\sqrt{\frac{1}{X_1^T 1 X_1^C 1 X_2^T X_2^C}}$.

The stress components $\sigma_1, \sigma_2, \tau_{12}$ and strength components S_{12}, X_1, X_2 are introduced in Eq. (14). Accordingly, the factor of safety (F.S.) is expressed in the following

Table 4 Factor of safety

Category	Design result	ASSET	Delta deck
Tsai-Wu F.I.	0.159	0.160	0.071
Factor of Safety	6.304	6.234	14.136

$$F.S. = \frac{1}{F.I.} \tag{16}$$

For this analysis, the result is given in Table 4 and three cross sections are verified to be safe enough.

5. Conclusions

A systematic way for structural optimization is presented. The original concept of this optimization is to use the topological and shape optimization together to get the most efficient design.

A material distribution method is used to extract design parameters, which is effectively used by increasing the mass ratio of cross section in the example problem. After acquisition of all necessary design parameters, a shape optimization using genetic algorithm followed to get the final result.

A performance verification analysis checked the deflection and the factor of safety of the structure and it is found that the current design is good enough in serviceability (deflection) and safety proving that this design scheme uses the material very efficiently. It would be possible to make this design procedure be more generalized and to apply it to different area of manufacturing.

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References

Ahn, I.C. (2007), "A study on the optimal FRP bridge deck using genetic algorithm", *M.S. Thesis*, Seoul National University, Korea.

Allaire, G. (1997), "The homogenization method for topology and shape optimization", *Topology Optimization in Structural Mechanics*, GISM, (Ed. Rozvany, G.I.N.), Springer Wien New York, New York.

Allaire, G. and Kohn, R.V. (1993), "Topology optimization and optimal shape design using homogenization", *Topology Design of Structures*, 207-218.

Bendsoe, M.P. and Harber, R. (1993), "The Michell layout problem as a low volume fraction limit of the homogenization method for topology design: An asymptotic study", *Struct. Opti.*, **6**, 263-267.

Bendsoe, M.P. and Kikuchi, N. (1988), "Generating optimal topologies in structural design using a homogenization methods", *Comput. Meth. Appl. Mech. Eng.*, **71**, 197-224.

Bendsoe, M.P. and Sigmund, O. (2004), *Topology Optimization*, Springer.

Cassidy, P., Richards, D. and Gillespie, J. (2002), "Composite acting FRP deck and girder system", *Struct. Eng. Int*., **12**(2), 71-75.
Diaz, A.R. and Bendsoe, M.P. (1992), "Shape optimization of structures for multiple loading conditions using a homogenization method", *Struct. Opti.*, **4**, 17-22.
Haug, E.J., Choi, K.K. and Komkov, V. (1986), *Design Sensitivity Analysis of Structural Systems,* Academic Press.
Keller, T. (2003), *Use of Fibre Reinforced Polymers in Bridge Construction*, IABSE-AIPC-IVBH.
Keller, T. and Gurtler, H. (2006), "In-plane compression and shear performance of FRP bridge decks acting as top chord of bridge girders", *Compos. Struct.*, **72**, 151-162.
Keller, T. and Schollmayer, M. (2004), "Plate bending behavior of a pultruded GFRP bridge deck system", *Compos. Struct.*, **64**, 285-295.
Keller, T. and Schollmayer, M. (2006), "In-plane tensile performance of a cellular FRP bridge deck acting as top chord of continuous bridge girders", *Compos. Struct.*, **72**, 130-140.
Kim, I.Y. and Kwak, B.M. (2002), "Design space optimization using a numerical design computation method", *Int. J. Numer. Meth. Eng.*, **53**(8), 1979-2002.
Korea Institute of Construction Technology (2006), *Development of Long-life Deck Systems for Bridges : FRP Deck*, KICT, Korea.
KSCE (2008), *Korea Highway Bridge Design Specification and Explanation*, Korea Society of Civil Engineers.
Kutylowski, R. (2009), "Topology optimization of the structure using multimaterial inclusions", *Struct. Eng. Mech.*, **33**(3), 258-306.
Lee, J.S., Cho, H.N., Choi, Y.M., Kim, H.S. and Kim, H.Y. (2004), "Shape optimum design of pultruded FRP bridge decks", *J. Comput. Struct. Eng. Inst. Korea*, **17**(3), 319-332.
Lee, E.H. (2009), "Design of a FRP deck using topology and shape optimization", M.E. Thesis, Dankook University, Korea.
Park, J., Choo, P. and Hong, K.J. (2008), "Design of a FRP bridge", *Technical Report of Korea Bridge Design & Engineering Research Center*, Seoul National University.
Qiao, P., Davalos, J.F. and Brown, B. (2000), "A systematic analysis and design approach for single-span FRP deck/stringer bridges", *Compos. Part B-Eng.*, **31**, 593-609.
Suzuki, K. and Kikuchi, N. (1991), "A homogenization method for shape and topology optimization", *Comput. Meth. Appl. Mech. Eng.*, **92**(3), 291-318.
Wang, M.Y., Wang, X. and Guo, D. (2003), "A level set method for structural topology optimization", *Comput. Meth. Appl. Mech. Eng*., **192**, 227-246.
Zolesio, J.P. (1981), "The material derivative method for shape optimization", *Optimization of Distributed Parameter Structures*, Sijthoff & Noordhof, Netherlands, 1089-1151.