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A method of global-local analyses of structures involving local heterogeneities and propagating cracks

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Abstract. This paper presents the global-local finite cover method (GL-FCM) that is capable of analyzing structures involving local heterogeneities and propagating cracks. The suggested method is composed of two techniques. One of them is the FCM, which is one of the PU-based generalized finite element methods, for the analysis of local cohesive crack growth. The mechanical behavior evaluated in local heterogeneous structures by the FCM is transferred to the overall (global) structure by the so-called mortar method. The other is a method of mesh superposition for hierarchical modeling, which enables us to evaluate the average stiffness by the analysis of local heterogeneous structures not subjected to crack propagation. Several numerical experiments are conducted to validate the accuracy of the proposed method. The capability and applicability of the proposed method is demonstrated in an illustrative numerical example, in which we predict the mechanical deterioration of a reinforced concrete (RC) structure, whose local regions are subjected to propagating cracks induced by reinforcement corrosion.

Keywords: global-local analysis; finite cover method; hierarchical analysis method; multiple cohesive crack growth; reinforcement corrosion.

1. Introduction

It has recently been well known that existing concrete and RC structures have serious risks for material or structural damage with crack formations attributed to the chloride attack, the alkali-aggregate reaction, the frost damage, etc., although these structures have ever been regarded as maintenance-free. In particular, special attention must be paid to the chloride attack and the carbonation against RC structures, because the reinforcement corrosion induces not only the degradation of stiffness/strength of RC structures, but also the detachment (falling) of cover concretes due to crack formation near the surface. Since it takes long periods of time for such mechanical deterioration phenomena to progress, the development of an effective simulation tool for predicting the degree of deterioration is urgently needed.

Numerical analysis to predict the degree of deterioration caused by reinforcement corrosion is generally categorized into two types. One is diffusion analysis of chemical species such as chloride

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ions in concrete. The other is stress/deformation analysis of concrete structures subjected to crack formations due to the corrosion-induced expansion of reinforcement. The numerical and experiment-based analytical studies of the former type have been made so far in consideration of the influence of cracks and concrete's meso/micro-structures (Zhang and Gjorv 1996, Maekawa *et al.* 2003, Pivonkaa *et al*. 2004, Zhang and Lounis 2006, Marsavina *et al*. 2007, Zuo *et al*. 2010). However, numerical studies of the latter type have less advanced, because most of them utilize an analytical or the classical smeared crack model to represent the formations of cohesive cracks and do not take into account the influence of the overall mechanical behavior of RC structures on the crack growth in local regions (Bhargava and Ghosh 2003, Du *et al*. 2006, Chen and Mahadevan 2008). As a result, numerical analysis methods to evaluate such mechanical behavior are still developing. In this context, the intrinsic difficulty arises from

- arbitrarily evolving cracks, which requires special and cumbersome treatments in finite element based numerical analysis methods, and
- mechanical coupling between the overall RC structure and local heterogeneous structures possibly involving crack propagations.

A class of partition-of-unity (PU)-based finite element methods has recently become popular as an efficient tool to represent arbitrary discontinuities explicitly with finite elements. The FCM (Terada *et al*. 2003), the X-FEM (Daux *et al*. 2000), the PUFEM (Strouboulis *et al*. 2001), the GFEM (Melenk and Babuška 1996) and the Hansbo method (Hansbo and Hansbo 2004) are known to be representatives of such methods and have been applied to various problems of weak/strong discontinuities (Duarte *et al*. 2001, Gasser and Holzapfel 2005, Asferg *et al*. 2007, Dumstorff and Meschke 2007, Kurumatani and Terada 2009). The most notable characteristic of these methods is that finite elements, namely sub-domains for approximation, can be constructed independently of the physical domains owing to the PU property of weight or shape functions, even if discontinuities evolve arbitrarily. Thus, the first issue pointed out above can be resolved by the introduction of the PU-based methods.

To overcome the second issue for deformation/stress analyses of mechanically deteriorating RC structures, we need some techniques to bridge between the local and global mechanical responses. Especially when reinforcements in RC structures are main sources of local heterogeneity, crack propagation must be analyzed in local regions, but the explicit modeling of the heterogeneity in other healthy regions without cohesive crack growth makes numerical analyses expensive in computational resources and time. Therefore, some sort of averaging schemes has to be employed to incorporate the effect of local heterogeneity in the healthy regions into the global stiffness, while the local regions involving propagating cracks can be glued to adjacent healthy ones in some fashion. In this regard, the method of mesh superposition (Fish 1992) seems meet the needs, since it enables us to perform global analysis with a coarse mesh, whose elements embed a fine mesh prepared for local analysis of local heterogeneous structures; see e.g., (Lee *et al.* 2004, Loehnert and Belytschko 2007, Duarte and Kim 2008). If a fine mesh for the local heterogeneity without cohesive crack growth is superimposed on the corresponding global element, its average stiffness can be evaluated a priori by performing the local analysis. On the other hand, when a fine mesh is used for crack propagation analysis of local structures by a PU-based method, it is directly connected to adjacent healthy global mesh instead of being used for averaging.

In this paper, we propose a global-local finite cover method (GL-FCM), which is capable of analyzing structures involving local heterogeneities and propagating cracks. The suggested method is composed of the above-mentioned techniques. One of them is the FCM, which is one of the PU-based methods, for the analysis of local cohesive crack growth. Then the so-called mortar method is utilized to transfer the mechanical responses thus evaluated within local regions to the global structure. The other technique is the method of mesh superposition for hierarchical modeling, which enables us to evaluate a priori the average stiffness by the numerical analysis of local heterogeneous structures not subjected to crack propagation. After the governing equation of the problem under consideration is presented in the next section, the GL-FCM is proposed in the third section. That is, after summarizing the modeling procedures for the global-local analysis along with the definitions of levels of heterogeneities, we explain in order the hierarchical modeling to evaluate the average stiffness of global structures and the FCM for the representation of crack propagations in local heterogeneous structures. In the fourth section, the basic performance of the present method is assessed by conducting some numerical experiments. Finally, an illustrative numerical example is presented to demonstrate the capability and applicability of the proposed method, in which we predict the mechanical deterioration of a RC structure, whose local structures are subjected to propagating cracks induced by reinforcement corrosion.

2. Governing equations

We provide the governing equations defining the global-local equilibrium problem for a structure composed of quasi-brittle materials, whose local heterogeneous structures are subjected to cohesive crack growth and reinforced by other stiffer elastic material. Also, the so-called cohesive crack model is introduced for modeling the softening behavior of the local structures due to the crack growth.

2.1 Global-local equilibrium problem

Let us consider the quasi-static equilibrium problem of a structure involving both local heterogeneities and local regions with propagating cracks as shown in Fig. 1. The materials used in the structure are assumed to be linearly elastic, but possibly quasi-brittle. The whole domain, denoted by Ω, consists of the global domain $\Omega^{[H]}$ and the local domain $\Omega^{[FC]}$ with heterogeneities or cracks, and is defined as follows

$$\Omega = \overline{\Omega^{[H]} \cup \Omega^{[FC]}}, \quad \Omega^{[FC]} = \overline{\Omega^{[1]} \cup \Omega^{[2]}} \tag{1}$$

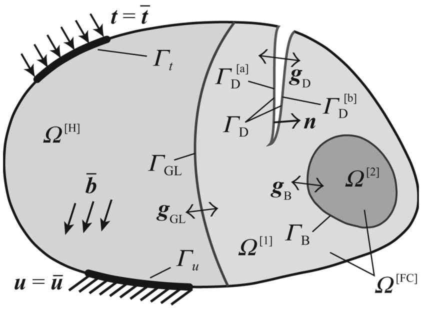


Fig. 1 Quasi-static equilibrium problem of a quasi-brittle solid with local heterogeneity

Here, we assume that $\Omega^{[FC]}$ is composed of two kinds of materials and superscripts $[i]$ are identifiers for materials i. The whole boundary Γ surrounding Ω is described as

$$\Gamma = \overline{\Gamma_u \cup \Gamma_t}, \quad \Gamma_D = \overline{\Gamma_D^{[a]} \cup \Gamma_D^{[b]}} \tag{2}$$

$$\Gamma_u \cap \Gamma_t \cap \Gamma_{GL} \cap \Gamma_B \cap \Gamma_D = \emptyset \tag{3}$$

where Γ_{GL} is the interface between the global and local domains, and Γ_B and Γ_D are the material interface and the cracked boundary in a local domain, respectively.

The equilibrium equation for stress $\boldsymbol{\sigma}$, the relationship between strain $\boldsymbol{\varepsilon}$ and displacement $\mathbf{u}$, and the constitutive equation are given as follows

$$\nabla \cdot \boldsymbol{\sigma} + \bar{\mathbf{b}} = \mathbf{0} \quad \text{in} \quad \Omega \tag{4}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}\{\nabla \mathbf{u} + (\nabla \mathbf{u})^T\} \quad \text{in} \quad \Omega \tag{5}$$

$$\boldsymbol{\sigma} = \mathbf{c} : \boldsymbol{\varepsilon} \quad \text{in} \quad \Omega \tag{6}$$

where $\mathbf{c}$ is the fourth-order elasticity tensor, ∇ is the nabla operator and $\bar{\mathbf{b}}$ is the body force vector. Here and in what follows, variables such as $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ and $\mathbf{u}$ without superscripts [1], $[H]$, etc. are commonly defined in the whole domain.

The following two types of boundary conditions are considered

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on} \quad \Gamma_u \tag{7}$$

$$\mathbf{t} := \boldsymbol{\sigma}\mathbf{n} = \bar{\mathbf{t}} \quad \text{in} \quad \Gamma_t \tag{8}$$

where $\mathbf{n}$ is the outward unit normal vector, $\bar{\mathbf{u}}$ and $\bar{\mathbf{t}}$ are the prescribed displacement and surface traction vectors, respectively. At the same time, the following interfacial conditions are imposed at Γ_{GL} and Γ_B

$$\mathbf{u}^{[H]} = \mathbf{u}^{[FC]} \quad \text{and} \quad \mathbf{t}^{[H]} = -\mathbf{t}^{[FC]} \quad \text{in} \quad \Gamma_{GL} \tag{9}$$

$$\mathbf{u}^{[1]} = \mathbf{u}^{[2]} \quad \text{and} \quad \mathbf{t}^{[1]} = -\mathbf{t}^{[2]} \quad \text{in} \quad \Gamma_B \tag{10}$$

Here, the relative displacement defined as

$$\mathbf{g}_D := \mathbf{u}^{[a]} - \mathbf{u}^{[b]} \quad \text{on} \quad \Gamma_D \tag{11}$$

is often referred to as the crack opening displacement vector, which characterizes the crack growth.

2.2 Cohesive crack model

Quasi-brittle materials such as concrete often reveal softening behavior during the process of fracture, which is characterized by the formation of fracture process zone (FPZ) as shown in Fig. 2.

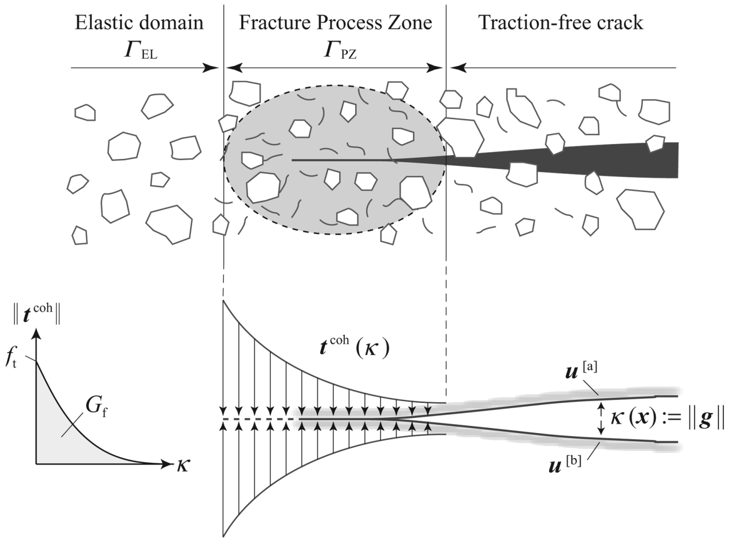


Fig. 2 Fracture process zone and fracture mechanics model for quasi-brittle material

The FPZ is formed between the unbroken (or healthy) elastic domain and the fully separated stress-free fractured domain, and allows the stress to be transmitted over the interlocking of aggregates and micro-crack interactions. Then, the decrease of the stress transfer causes softening behavior as crack opening displacement (11) is increased.

The cohesive crack model (Hillerborg *et al*. 1976) is one of the major fracture mechanics models formulated to represent such softening behavior of concrete. In this model, the stress transfer in the FPZ is substituted for the cohesive traction force between two fictitious fractured surfaces facing with each other as shown in Fig. 2. The relationship between the cohesive traction and the crack opening displacement (named as the traction-separation law) provides a tension softening curve, and the area under the curve is called the fracture energy, which represents the energy necessary to create unit crack area.

In this study, we employ the following traction-separation law (Wells and Sluys 2001, Mergheim *et al*. 2005)

$$\|\mathbf{t}^{coh}(w)\| = f_t \exp\left(-\frac{f_t}{G_f}w\right) \quad \text{on} \quad \Gamma_{PZ} \tag{12}$$

where the tensile strength f_t and the fracture energy G_f are introduced as material parameters. Here, $\|\mathbf{t}^{coh}(w)\|$ is the norm of the cohesive traction vector, and w is a history parameter defined as the maximum value of the norm of the crack opening displacement $\|\mathbf{g}_D\|$ ever experienced during the loading process.

2.3 Weak form of the global-local problem

In order to account for the coupling between global and local domains, we employ the Lagrange multiplier method, by which the interfacial constraints are imposed in a weak sense. The corresponding weak forms for the set of solutions ($\mathbf{u}$, λ) that govern the quasi-static boundary value problem defined above are given by

$$\int_{\Omega} \nabla \delta \mathbf{u} : \mathbf{c} : \nabla \mathbf{u} d\Omega + \int_{\Gamma_{GL}} \delta \mathbf{g}_{GL} \cdot \boldsymbol{\lambda} d\Gamma + \int_{\Gamma_B} \delta \mathbf{g}_B \cdot \boldsymbol{\lambda} d\Gamma$$
$$= \int_{\Omega} \delta \mathbf{u} \cdot \bar{\mathbf{b}} d\Omega + \int_{\Gamma_t} \delta \mathbf{u} \cdot \bar{\mathbf{t}} d\Gamma + \int_{\Gamma_D} \delta \mathbf{g}_D \cdot \mathbf{t}_D d\Gamma \tag{13}$$

$$\int_{\Gamma_{GL}} \delta \boldsymbol{\lambda} \cdot \mathbf{g}_{GL} d\Gamma + \int_{\Gamma_B} \delta \boldsymbol{\lambda} \cdot \mathbf{g}_B d\Gamma = 0 \tag{14}$$

where $\delta(\bullet)$ represents the variation of $\bullet$, and the relative displacement vectors have been defined as

$$\mathbf{g}_{GL} := \mathbf{u}^{[H]} - \mathbf{u}^{[FC]} \quad \text{on} \quad \Gamma_{GL} \tag{15}$$

$$\mathbf{g}_B := \mathbf{u}^{[1]} - \mathbf{u}^{[2]} \quad \text{on} \quad \Gamma_B \tag{16}$$

along with (11). Here, assuming that the cohesive traction vector is directed to the normal to Γ_{PZ}, the surface traction vector $\mathbf{t}_D$ on Γ_D is determined according to the cohesive crack model (12) as

$$\mathbf{t}_D = \boldsymbol{\sigma}\mathbf{n} = \begin{cases} \mathbf{t}^{\mathrm{coh}} & \text{on } \Gamma_{PZ} \\ \mathbf{0} & \text{on } \Gamma_D / \Gamma_{PZ} \end{cases} \tag{17}$$

Although the solutions of the present weak forms can be solved by the standard finite element method with the help of the mortar elements for the interfacial constraints as in (Kurumatani and Terada 2005), the nonlinearity involved in the cohesive crack model makes the implementation complicated. To avoid the complexity, we here employ the iterative solution method suggested in (Kurumatani and Terada 2009), in which a semi-implicit time integration is applied to the cohesive term in conjunction with the penalty method.

3. Global-local finite cover method

In this section, the global-local finite cover method (GL-FCM) is proposed. After summarizing the modeling procedures as well as the definitions of three distinct levels of local heterogeneities, we present two computational techniques, both of which are the key ingredients in the proposed method. One of them is used for the hierarchical modeling, based on the mesh superposition technique, to evaluate the average stiffness for the coarsest mesh, whereas the other is the FCM for crack propagation analysis in the finest mesh.

3.1 Modeling procedure with definitions of levels of heterogeneities

The schematic of the global-local problem assumed in this study is illustrated in Fig. 3, where the whole structure involves a local region with a circular inclusion subjected to crack generations. To realize the global-local analysis for this type of structures, we define three levels of heterogeneities depicted as this figure. The modeling procedure of each level as well as its definition is explained below.

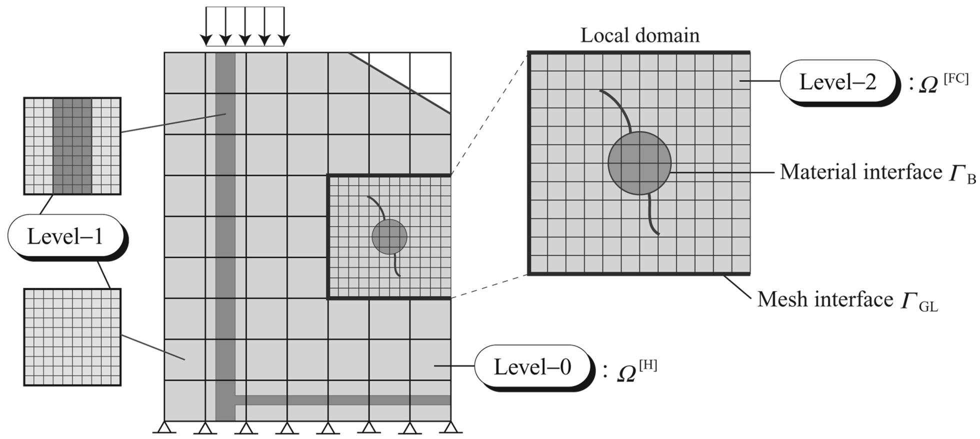


Fig. 3 Hierarchy of heterogeneities for the global-local finite cover method

First, we define the finest mesh to represent the finest level of heterogeneity, called here Level-2, and discretize the whole domain by a regularly structured mesh of pixel-or voxel-type, which can easily be generated by using its digital image data. In an RC structure, for instance, the dimension of reinforcements corresponds to this level.

Second, the local heterogeneous structures that are subjected to crack propagation are assumed and are analyzed by the FCM with the finest level of heterogeneity, namely Level-2. That is, crack propagation analyses in the present GL-FCM are always made by the FCM with Level-2 mesh. The modeling and analysis procedures in this level can be carried out by the method proposed in (Kurumatani and Terada 2009), which will be summarized in the next section.

Third, the coarsest mesh, which defines Level-0, is prepared for the overall domain to evaluate the average behavior by the global analysis. The element size of Level-0 is set much larger than that of Level-2 so that one element of Level-0 covers Level-2 heterogeneity. The local heterogeneous structure associated with one element is defined as Level-1 structure, but its level of heterogeneity is in effect the same as that of Level-2. However, the Level-1 mesh is assumed to be healthy or not subjected to crack propagation, and is superimposed on one element of Level-0 to account for the effect of the finest heterogeneity on the stiffness of the Level-0 mesh.

Thus, the fourth procedure is to evaluate the average element stiffness matrix in the Level-0 mesh by using the Level-1 mesh. That is, each pixel or voxel finite element mesh is superimposed on a single coarse element to characterize the stiffness of each Level-0 element. This hierarchical modeling can be realized by the combination of the method of superposition, known as s-FEM (Fish 1992), and the well-known static condensation scheme. Its formulation is explained after the next subsection.

In summary, the global analysis is performed with the Level-0 mesh, each of whose element is covered by a Level-1 mesh to evaluate its element stiffness matrix a priori by the mesh-superposition technique. At the same time, the global model of the Level-0 mesh is connected to the local model of the Level-2 mesh for the local heterogeneous structure, in which the propagations of cohesive cracks are analyzed by the FCM. The mortar elements are utilized to glue the Level-2 and Level-0 meshes together as mentioned in the previous section.

3.2 Hierarchical modeling of global stiffness

We here present the hierarchical modeling to evaluate the average stiffness for the global structure. The method can be realized by the combination of the method of mesh superposition of (Fish 1992) with the static condensation procedure as mentioned above.

Let us start by considering the following weak form for a single, displacement-based standard finite element e

$$\int_{\Omega_e} \nabla \delta\mathbf{u} : \mathbf{c} : \nabla \mathbf{u} d\Omega = \int_{\partial\Omega_e} \delta\mathbf{u} \cdot \mathbf{t} d\Gamma \qquad \forall \delta\mathbf{u} \tag{18}$$

where Ω_e and $\partial\Omega_e$ respectively indicate the domain and the boundary of element e. Here, the displacement $\mathbf{u}$ is defined as the sum of the global displacement $\mathbf{u}^0$ and local displacement $\mathbf{u}^1$, and is approximated as follows

$$\mathbf{u} = \mathbf{u}^0 + \mathbf{u}^1 = \mathbf{N}^0\mathbf{d}^0 + \mathbf{N}^1\mathbf{d}^1 \tag{19}$$

where $\mathbf{N}^0$ and $\mathbf{d}^0$ are the matrix of the standard element shape functions and the element nodal displacement vector, respectively, and are defined over the single element. On the other hand, $\mathbf{N}^1$ and $\mathbf{d}^1$ are the matrix containing global shape functions and the global nodal displacement vector for the Level-1 mesh, respectively, and thus have the degrees of freedom (DOFs) of the Level-1 mesh prepared for the corresponding local structure. That is, superscripts 0 and 1 correspond to the levels of meshes (domains) defined above and depicted in Fig. 3. It is to be noted that the Level-1 mesh, which involves relevant number of elements to represent the local heterogeneity, is embedded in a single Level-0 element.

Substitution of the relation (19) into the weak form (18) yields the following system of equations involving both the Level-0 element stiffness matrix and the global stiffness matrix of the Level-1 mesh

$$\begin{bmatrix} \mathbf{K}_e^{00} & \mathbf{K}_e^{01} \\ \mathbf{K}_e^{10} & \mathbf{K}_e^{11} \end{bmatrix} \begin{Bmatrix} \mathbf{d}_e^0 \\ \mathbf{d}_e^1 \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_e^0 \\ \mathbf{0} \end{Bmatrix} \tag{20}$$

Here, the block matrices in the left-hand side are defined as

$$\mathbf{K}_e^{00} = \int_{\Omega_e} (\mathbf{B}_e^0)^T \mathbf{C} \mathbf{B}_e^0 d\Omega \tag{21}$$

$$\mathbf{K}_e^{01} = \int_{\Omega_e} (\mathbf{B}_e^0)^T \mathbf{C} \mathbf{B}_e^1 d\Omega = (\mathbf{K}_e^{10})^T \tag{22}$$

$$\mathbf{K}_e^{11} = \int_{\Omega_e} (\mathbf{B}_e^1)^T \mathbf{C} \mathbf{B}_e^1 d\Omega \tag{23}$$

$$\mathbf{F}_e^0 = \int_{\partial\Omega_e} (\mathbf{N}_e^0)^T \bar{\mathbf{t}} d\Gamma \tag{24}$$

where $\mathbf{C}$ is the elasticity matrix and $\mathbf{B}$ is the displacement-strain matrix. The actual size of each matrix is consistent with the DOFs of the single element of the Level-0 mesh as well as all the elements in the Level-1 mesh, and can be recognized by the superscripts 0 and 1 as indicated above.

Assuming the displacement $\mathbf{u}^1$ is independent of the adjacent global element, we apply the static condensation to the above matrix equation. That is, from the second matrix equation in (20), $\mathbf{d}_e^1$ can be written as

$$\mathbf{d}_e^1 = -(\mathbf{K}_e^{11})^{-1}\mathbf{K}_e^{10}\mathbf{d}_e^0 \tag{25}$$

which can be set back to the first matrix equation in (20) to have the following equation

$$\mathbf{K}_e^H\mathbf{d}_e^0 = \mathbf{F}_e^0 \tag{26}$$

Here,

$$\mathbf{K}_e^H = \mathbf{K}_e^{00} - \underbrace{\mathbf{K}_e^{01}(\mathbf{K}_e^{11})^{-1}\mathbf{K}_e^{10}}_{\text{Local heterogeneity}} \tag{27}$$

represents the averaged stiffness of each element in the Level-0 mesh for the overall or global structure in consideration of the Level-1 local heterogeneity. Note that $-\mathbf{K}_e^{01}(\mathbf{K}_e^{11})^{-1}\mathbf{K}_e^{10}$ can be interpreted as a correction term due to local heterogeneity to the standard element stiffness matrix $\mathbf{K}_e^{00}$, and the static condensation possibly increases the computational cost in a static analysis because of the pre-solving step in (25).

The evaluation of the correction term $-\mathbf{K}_e^{01}(\mathbf{K}_e^{11})^{-1}\mathbf{K}_e^{10}$ requires us to preliminarily solve the system of linear equations as in (25), which is obtained by the discretization of the boundary value problem for the Level-1 domain. To this end, some sort of boundary conditions must be imposed on the local displacement $\mathbf{u}^1$ at the boundary of the Level-1 mesh to eliminate the singularity of $\mathbf{K}_e^{11}$. In this context, periodic or homogeneous boundary conditions seem to be natural. Because of its simplicity in implementation, we introduce the latter condition in this study as

$$\mathbf{u}^1 = \mathbf{0} \quad \text{on} \quad \partial\Omega_e \tag{28}$$

It has been confirmed that the hierarchical element thus defined pass the patch tests.

3.3 Finite cover method for Level-2 local domain

We employ the finite cover method (FCM), which is known as one of the PU-based analysis methods as typified in (Terada *et al*. 2003), for the crack propagation analyses in the Level-2 domain. The applicability of the FCM has been demonstrated in dealing with complicated meso-/micro-scale structures of heterogeneities solids (Kurumatani and Terada 2005) and arbitrary crack propagations with spatially fixed meshes (Kurumatani and Terada 2009). In this section, the finite cover approximations is summarized with a view to the numerical simulation of crack formations in the Level-2 mesh involving local heterogeneities with material interfaces.

3.3.1 Representation of discontinuous deformation

In the FCM, the finite number of mathematical sub-domains called mathematical covers, each of which is endowed with the partition-of-unity (PU), is introduced as shown in Fig. 4. The totality of mathematical covers, corresponding to a mathematical mesh, is constructed so that it keeps the PU

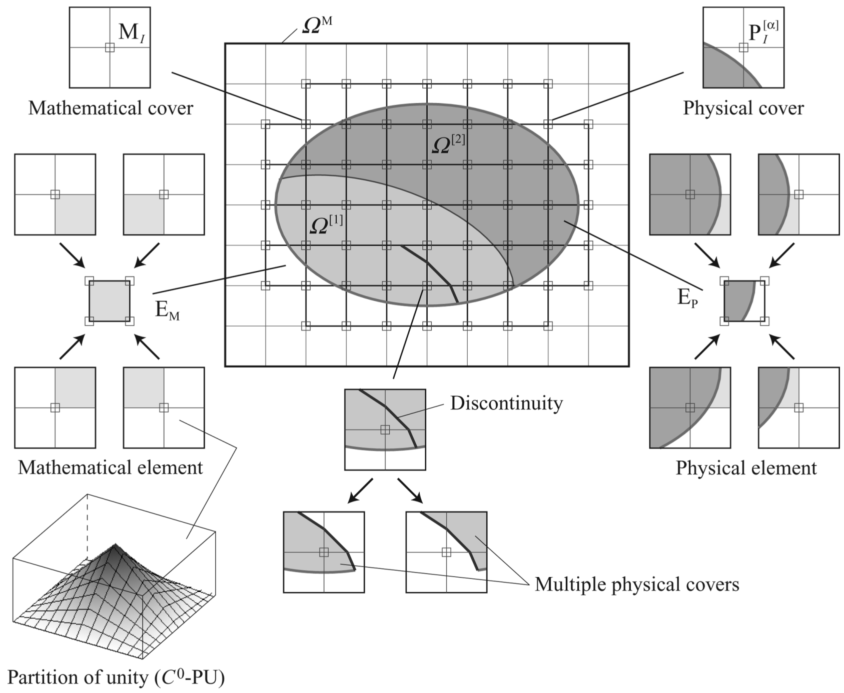
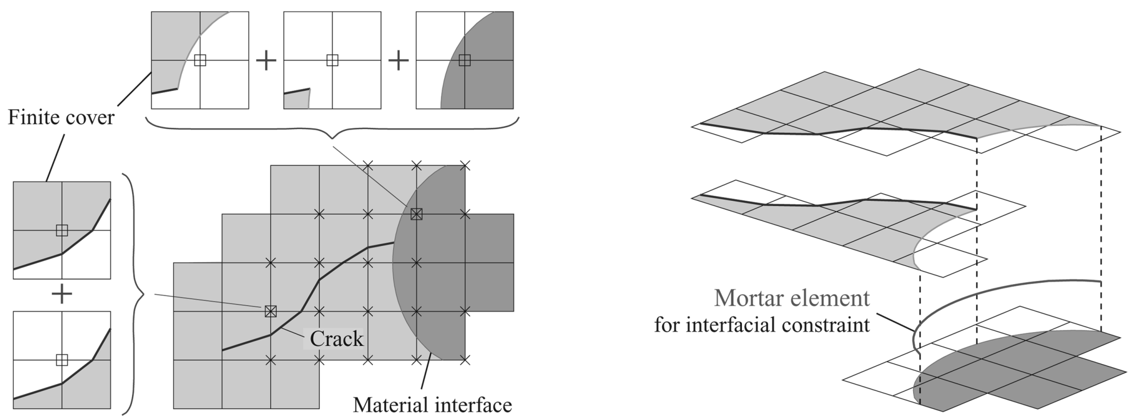


Fig. 4 Components for finite cover method



(a) Multiple set of finte covers for each discontinuity

(b) Cover layers for seperate physical domains

Fig. 5 Multi-cover-layers for finite cover approximations of strong and weak discontinuities

property and is placed to cover the structure independently of its topology. Since the mesh topology in the FCM does not need to conform to the physical boundaries of the structure, the mesh can be regular and structured.

An intersection of each mathematical cover with the domain of the structure, as shown in Fig. 4, is called a physical cover, in which physical quantities are defined. Also, the common regions of the physical covers are defined as elements, which are equivalent to the finite elements. Discontinuous deformations associated with crack formation and interfaces can be represented by the combination of multiple sets of physical covers in the FCM as shown in Fig. 5(a) so that the physical quantities divided by discontinuities are properly defined. Thus, the division of the physical domain by cracks or interfaces implies the increase of DOFs, or equivalently the increase of physical covers. Material discontinuities are also represented in the same way by the multiple sets of the physical covers with the interface elements (mortar elements) being attached to impose the continuity on the displacement and the traction at the interface (Kurumatani and Terada 2005). In summary, the weak and strong discontinuities, respectively associated with cracks and material interfaces, in heterogeneous solids can easily be represented by the introduction of layers of physical covers as schematized in Fig. 5(b).

3.3.2 Treatment of propagating cracks

When concrete materials are assumed for the crack simulations by the GL-FCM, it is known that mode I in cracking is dominant over the other modes since they are sensitive to tensile loading (Farid *et al*. 2004). Therefore, for the sake of simplicity, we employ the following failure criterion that is based on the maximum principal stress theory to judge the generation of a crack

$$F = \sigma_{\max} - f_t = 0 \tag{29}$$

where $\sigma_{\max}$ is the positive maximum principal stress, which can be computed in the finite cover analysis. The orientation of the crack is determined to be perpendicular to the major principal direction. Also, each discontinuity is introduced as a straight line and its crack-tip is assumed not to stay within a physical element, but always on a cover boundary.

When the discontinuity is generated, the cohesive traction is activated between the discontinuous surfaces according to the cohesive crack model in (12). The model employed to represent the strong discontinuities in this paper is equivalent to those used in the literature (Gasser and Holzapfel 2005, Wells and Sluys 2001, Mergheim *et al*. 2005), and its validity has already been verified. Also, as mentioned earlier in this paper, the replacement of the Lagrange multipliers by the penalized traction vectors, which is known to be a reasonable approximation, makes it easy to implement the model. Moreover, the nonlinearity due to the cohesive model is regularized by the semi-implicit time integration along with the iterative procedure as suggested in (Kurumatani and Terada 2009) in the context of the FCM.

Once a crack is generated in the finite cover analyses, physical covers with arbitrary physical geometry are generated. This implies that a special integration scheme is required to calculate the corresponding element stiffness matrices. In this study, we apply triangular Gaussian integration scheme with the area coordinate system as in (Kurumatani and Terada 2005), which ensures the numerical accuracy to some extent.

4. Numerical verification of GL-FCM

This section is devoted to two numerical experiments to assess the approximation properties of the proposed GL-FCM. One is to examine the performance of hierarchical elements in a Level-0 mesh, each of whose stiffness matrices reflects geometrical and material information defined in the Level-1 mesh. The other is to verify the validity of the treatment of a global structure (Level-0 mesh) accompanied by the Level-2 mesh, by which cracks propagation analyses are performed by the FCM. 2-D plane strain condition is assumed in both of the examples.

4.1 Overall behavior of RC structure

We first examine the performance of the hierarchical element presented in Subsection 3.2 for the global analysis. The numerical experiment is performed for the static equilibrium problem of the 2-D RC bridge pier as shown in Fig. 6, which is subjected to distributed dead load on top surface. It is assumed, for the sake of simplicity, that the concrete is assumed to be a homogeneous material so that the overall structure is composed of two materials; that is, concrete and steel. Those materials are assumed to be linearly elastic and thus subjected to neither cracking nor plastic deformation. The material parameters are given in this figure. Two distinct levels of heterogeneities for hierarchical elements are illustrated in the same figure, where the local mesh (Level-2) possesses 160×120 elements and the global mesh (Level-0) into 40×30. Thus, each global element of the Level-0 mesh is covered by the Level-1 mesh that has 4×4 finite elements to discretize the local region. It is to be noted that the element stiffness matrices of all the global elements are evaluated

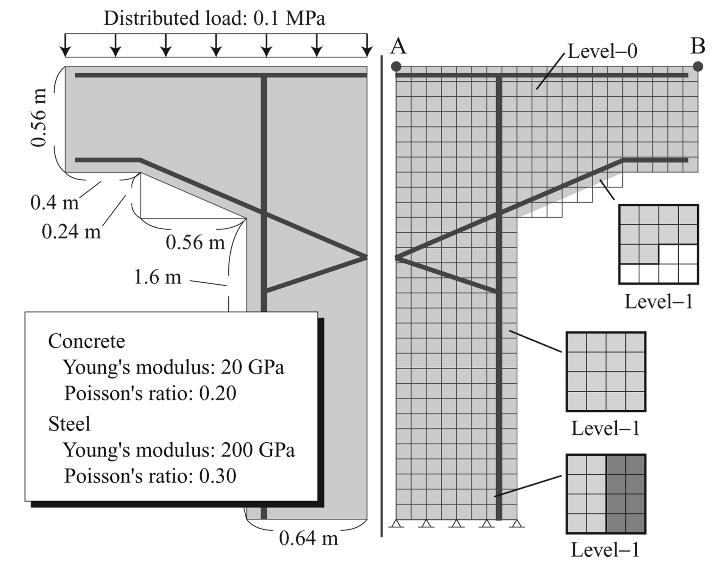


Fig. 6 RC bridge pier and its global and local meshes

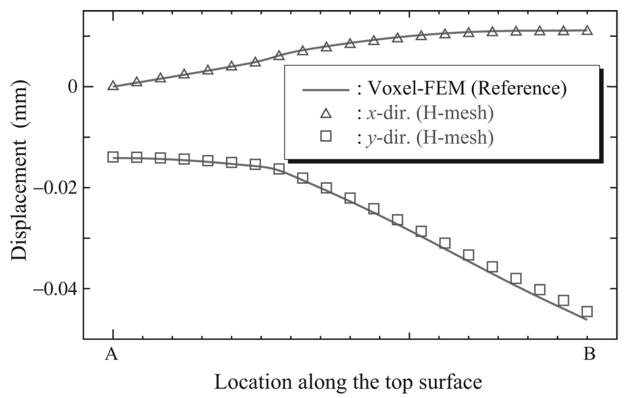


Fig. 7 Overall displacement profiles at the loading surface of RC bridge pier

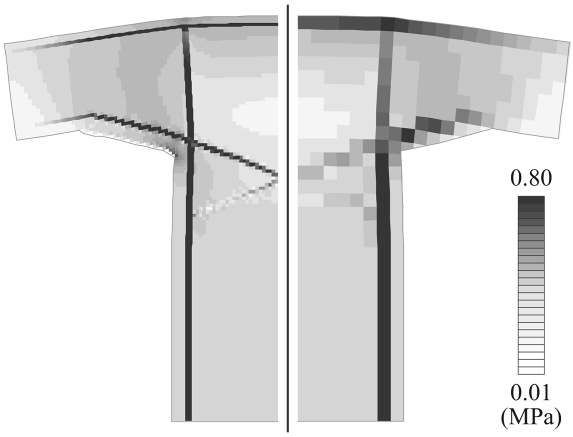


Fig. 8 Deformed configuration with stress distribution of RC bridge pier

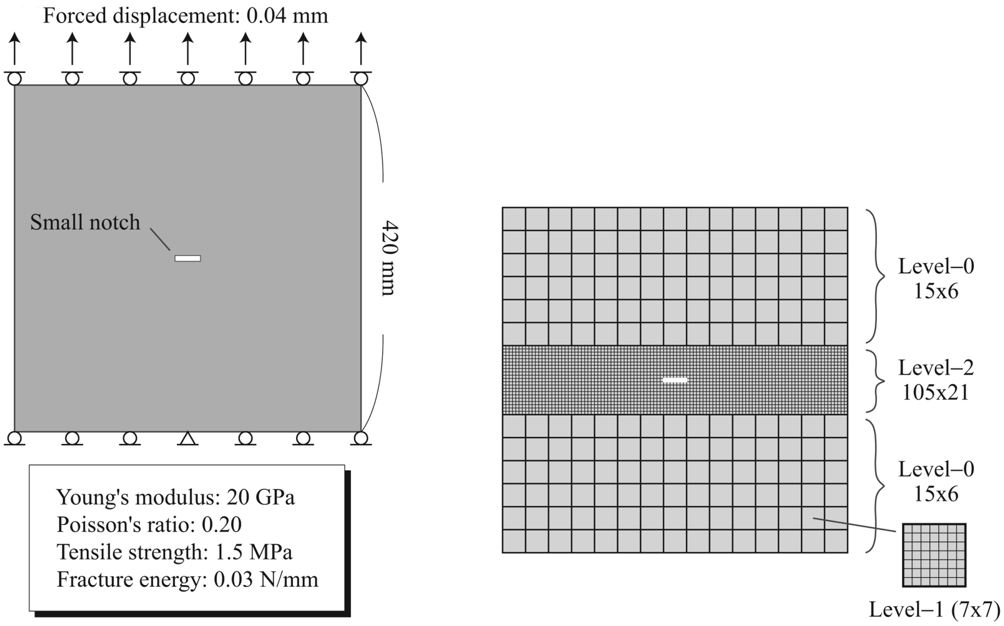
with the Level-1 mesh, even though there is no explicit heterogeneity. This brings the equivalent effect to the use of incompatible modes for finite elements and improves the overall accuracy of the coarse global elements.

After the average element stiffness matrices are a priori evaluated as explained in Subsection 3.2, the global analysis is performed. For comparison purpose, we also carried out the direct FE analysis using the mesh with Level-1 resolution. Fig. 7 shows the displacement profiles measured along the segment A-B in Fig. 6. Here, "H-mesh" indicates the result obtained by the global mesh with hierarchical elements, whereas the reference solution by the voxel-based FE analysis is indicated by "Voxel-FEM". As can be seen from this figure, the approximate solution by the hierarchical elements with the average stiffness is nearly identical with the reference solution obtained by the detailed voxel analysis.

The distributions of the von-Mises equivalent stress along with the deformed configurations are shown in Fig. 8, which compares the results obtained by the global analysis with the hierarchical elements and those by the direct analysis with the voxel FEM. It can be seen from the figure that the overall deformation and the stress distribution evaluated by the H-mesh seem to resemble the actual ones in an average sense, though the values are different. As demonstrated here, the hierarchical elements reflect the effects of local heterogeneities in the stiffness matrix for the global (Level-0) mesh. It is thus concluded that the present analysis method with the hierarchical elements enables us to simulate the average behavior of structures with local heterogeneities with desired accuracy.

4.2 Global-local behavior of a structure involving propagating cracks

We here verify the validity of the proposed global-local analysis of a structure involving propagating cracks generated from a small notch, as shown in Fig. 9, which is subjected to tensile loading in the vertical direction. The material parameters are given in the same figure. The element size of the Level-2 mesh is determined so as to represent the notch, whereas the element of the Level-0 consists of 7×7 Level-2 elements. At the same time, the element stiffness matrix of the Level-0 is again evaluated by using the Level-1 mesh (that is the same as the Level-2 mesh) for the



(a) Quasi-brittle structure with a small notch (b) Global and local meshes for GL-FCM

Fig. 9 Structure subjected to cracking from a small notch

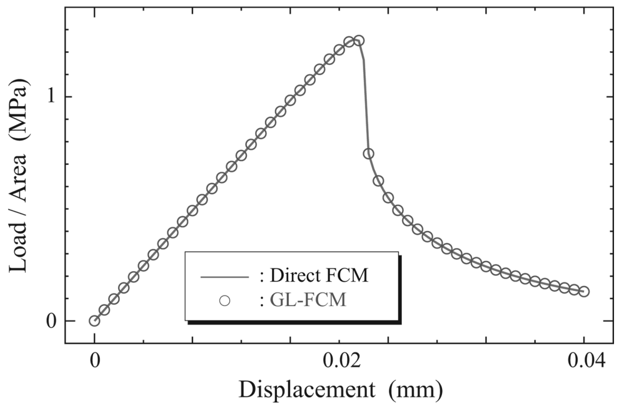


Fig. 10 Load-displacement curve of the structure with a small notch

healthy region, and is enhanced owing to the correction term in Eq. (27). Hence, the Level-0 mesh enhanced by the Level-1 mesh contributes to the improvement of accuracy and the reduction of computational cost even if the structure under consideration is homogeneous.

The global-local analysis is performed by the proposed GL-FCM. The relationship between the reaction force and the displacement at the loading surface is shown in Fig. 10 and is compared with that of the reference solution that has been obtained from the direct analysis by the FCM with the Level-2 mesh only. As can be seen, there is little difference in their overall responses. Fig. 11 shows the histories of the cracked configuration with von-Mises equivalent stress distribution. It can be found that the stress distributions and discontinuous deformations caused by the small notch in the

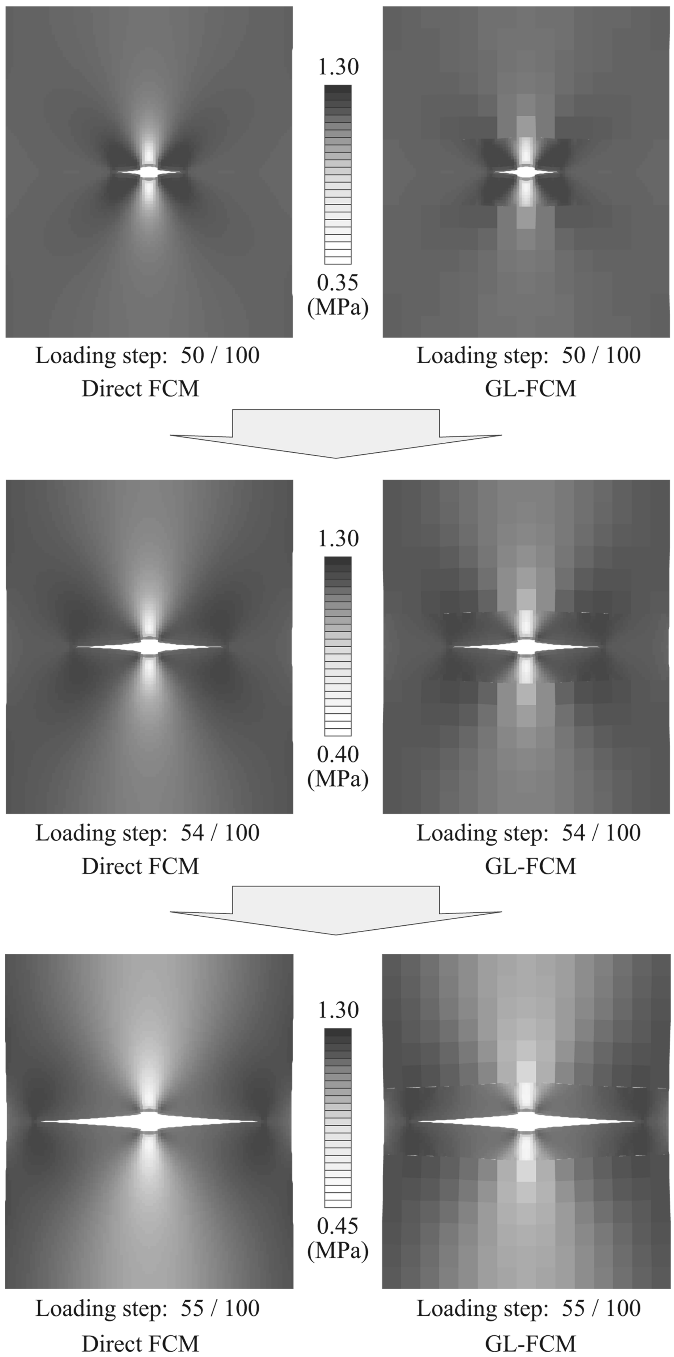


Fig. 11 Cracked configurations with stress distribution of the structure with a small notch

local domain (Level-2 mesh) are nearly identical. It is thus probably safe to conclude that the present GL-FCM is applicable for the analysis of structures involving both local heterogeneities and crack propagations.

5. Simulation of deteriorating RC structure subjected to crack growth

In order to demonstrate the capability and applicability of the present GL-FCM, we apply the method to the mechanical deterioration problem of the RC bridge pier, as shown in Fig. 12, which is subjected to the dead load distributed on the top surface. The meshes of global-local analysis and the material parameters used are given in the same figure. In this study, we assume that cracks are generated and propagate only around the reinforcements in regions P and Q, as depicted in this figure, and behave so as to cause the mechanical deterioration of the overall structure. Although such a deterioration phenomenon is commonly caused by the diffusion of chemical species such as chloride ions and the corresponding chemical reactions, only the mechanical response is considered in this study.

Assuming that these local regions P and Q are subjected to some amount of chemical species so that corrosion products are formed and inflate, we gradually apply the pseudo thermal (expansive) stress up to 10 MPa uniformly over the interface between the concrete and the reinforcement. The material parameters assumed for the corrosion product is taken from the literature (Suda *et al*. 1993).

The deformation histories of the global and local structures in regions P and Q are shown in Fig. 13. As can be seen from the figures, the cracks propagate from the surface of the reinforcement toward the external surface of the RC structure as the expansive stress is increased, and seem to reach the free surfaces, suggesting the detachment of cover concrete. The predicted results also include the mechanical behavior of the overall RC structure subjected to the dead load, owing to the introduction of the global-local analysis. It is thus demonstrated that the proposed method is capable of reproducing this type of deterioration phenomena that is possibly observed in existing RC structures.

It is again to be noted that the global-local analysis by the proposed GL-FCM is carried out on both the Level-0 and the Level-2 mesh at the same time, while the analysis for the healthy region with the Level-1 mesh is performed in advance to evaluate the average stiffness of the Level-0 element. Therefore, the crack propagations in the local regions (with the Level-2 mesh) are affected

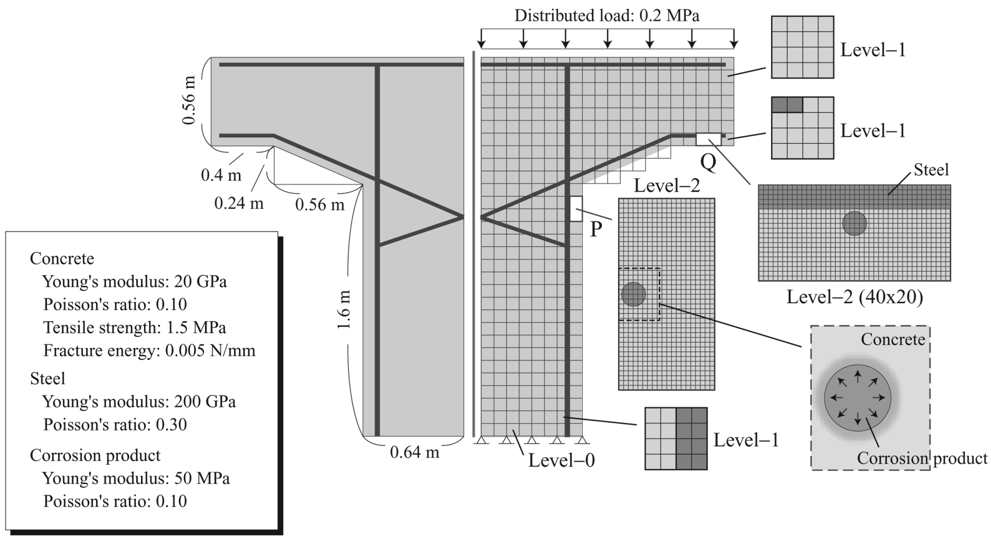
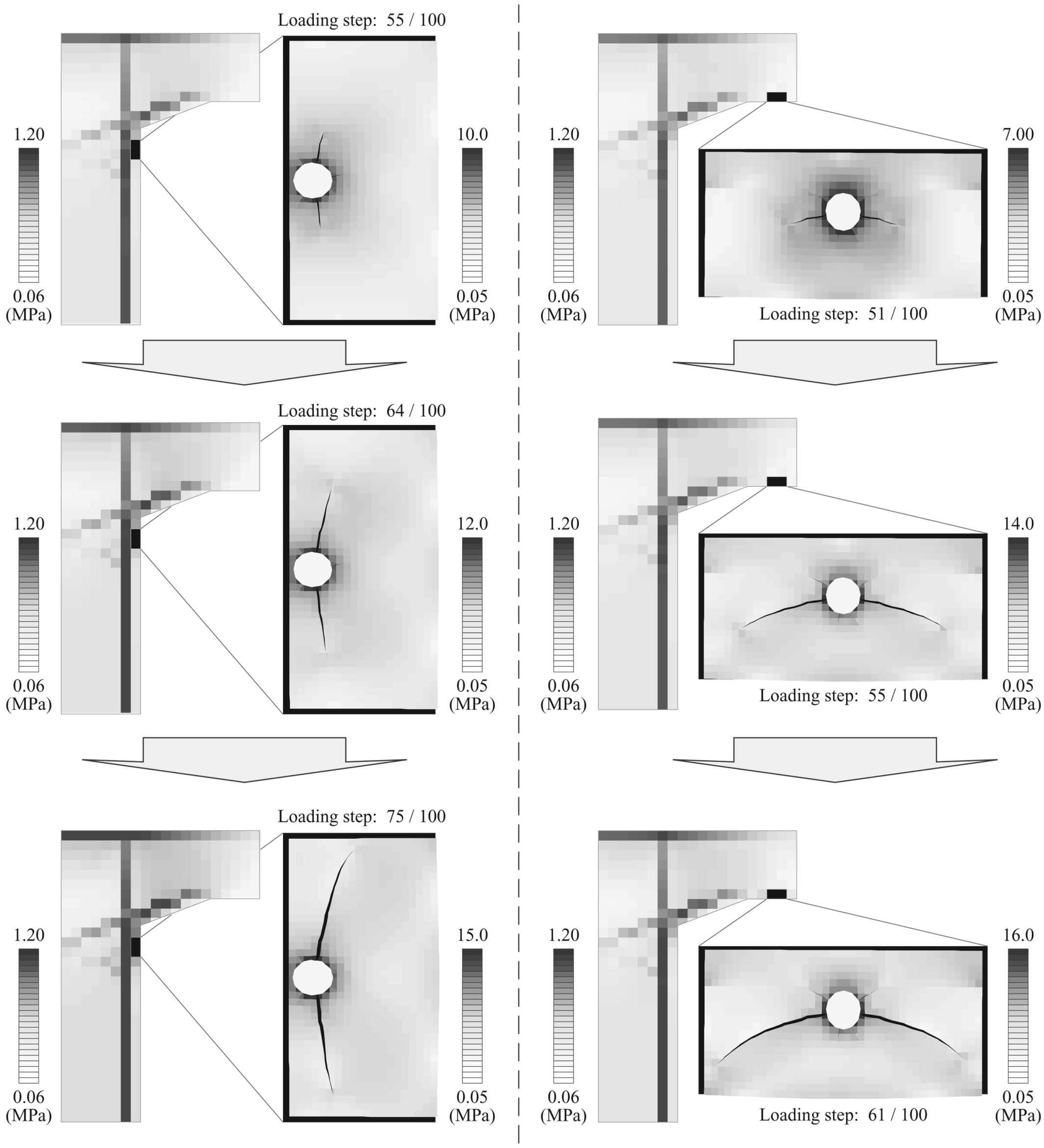


Fig. 12 RC bridge pier subjected to the reinforcement corrosion and its global and local meshes



(a) Numerical results of global-local FCM in local domain P (b) Numerical results of global-local FCM in local domain Q

Fig. 13 States of deterioration of the RC bridge pier due to the reinforcement corrosion in local regions P and Q

by the overall mechanical behavior evaluated by the Level-0 mesh. In this context, it is possible to consider a virtual rate of deterioration in terms of loading steps. For instance, it is pointed out that the rate of deterioration in region P is slower than that in region Q as can be seen in Fig. 13, because region P is subjected to the compressive stress due to the dead load. Thus, the GL-FCM suggested in this paper can be used for predicting the degree of deterioration in RC structures subjected to the reinforcement corrosion, if more sophisticated 3D modeling is made, and if more detailed material model and parameters are used.

6. Conclusions

We have developed the GL-FCM, which is capable of characterizing the coupling phenomena of the local structural damage due to cohesive crack growth and the stiffness/strength degradation of overall structures. The suggested method is composed of two distinct techniques: one is the hierarchical modeling based on the mesh superposition technique to evaluate a priori the average stiffness of the global or overall structure. The other is the FCM for crack simulations for local heterogeneous structures. The interface between the coarse global mesh and the finite cover mesh for crack propagation analyses is glued together with the mortar element based on the Lagrange multiplier formulation.

To examine the fundamental approximation properties of the present method, we performed two numerical experiments. First numerical experiments illustrated that the present hierarchical modeling to evaluate the stiffness matrices of coarsest elements enables us to approximate the average behavior of heterogeneous structures with desired accuracy and is feasible enough for the evaluation of global responses whose mechanical behavior is affected by the local heterogeneity. It was found from the second experiment that the coupling between the crack propagation phenomena analyzed in local regions and the overall average behavior characterized by the global analysis are reasonably simulated by the present GL-FCM. To demonstrate the capability and applicability of the proposed GL-FCM, we applied it to simulate the deterioration of the RC structure involving reinforcement corrosion. Although the analyzed examples were limited to 2-D problems, the result was reasonable enough in the sense that cracks generated at the interface between the concrete and the reinforcement propagate toward the external surface of the RC structure and that the behavior of propagating cracks in the local regions is affected by the overall state of stress. Thus, the proposed method would be an effective tool to predict the degree of deterioration of actual RC structures, if we make both the geometry modeling and the material modeling more sophisticated and if relevant data are available.

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