Structural Engineering and Mechanics, Vol. 38, No. 2 (2011) 171-193
DOI: http://dx.doi.org/10.12989/sem.2011.38.2.171

Time-dependent analysis of cable trusses Part II. Simulation-based reliability assessment

S. Kmet*, M. Tomko and J. Brda

Faculty of Civil Engineering, Technical University of Kosice, Vysokoskolska 4, 042 00 Kosice, Slovak Republic

(Received December 3, 2009, Accepted December 8, 2010)

Abstract. One of the possible alternatives of simulation-based time-dependent reliability assessment of pre-stressed biconcave and biconvex cable trusses, the Monte Carlo method, is applied in this paper. The influence of an excessive deflection of cable truss (caused by creep of cables and rheologic changes) on its time-dependent serviceability is investigated. Attention is given to the definition of the basic random variables and their statistical functions (basic, mutually dependent random variables such as the pre-stressing forces of the bottom and top cable, structural geometry, the Young's modulus of elasticity of the cables, and the independent variables, such as permanent load, wind, snow and thermal actions). Then, the determination of the response of the cable truss to the loading effects, and the definition of the limiting values considering serviceability of the structure are performed. The potential of the method, using direct Monte Carlo technique for simulation-based time-dependent reliability assessment as a powerful tool, is emphasized. Results obtained by the First order reliability method (FORM) are compared with those obtained by the Monte Carlo simulation technique.

Keywords: cable truss; closed-form computational model; creep of cable; yielding support; random variables; reliability; serviceability; Monte Carlo method; simulation-based time-dependent reliability assessment.

1. Introduction

At present, stochastic approaches to the reliability assessment of structures are developed. The structural behaviour is described as depending on factors of random nature. Construction materials possess properties of statistical variability and the loads on structures represent random processes unfolding during a certain time period. Structural reliability theory has received considerable attention in the literature (Ditlevsen and Madsen 1996, Melchers 1999, Nowak and Collins 2001, Raizer 1998). The general principles for a probabilistic design of bearing structures were published by the Joint Committee on Structural Safety and can be found in JCSS (2001).

Modern codes of design of structures take into account the probabilistic nature of loads and of carrying capacity and serviceability of constructions only on the level of initial data treatment. Limit states design method used in the codes of design is semi-probabilistic and structural reliability is ensured by partial coefficients, i.e., partial reliability factors for different loads and materials, model

*Corresponding author, Professor, E-mail: stanislav.kmet@tuke.sk

coefficients and another coefficients of reliability. Design of structures reflecting their realistic behaviour during their construction and during their entire service life should be totally based on the theory of reliability, taking into account probabilistic methods, which give a more unbiased assessment of the structure during its expected service life. The development of computer techniques allow a change in approach regarding the probabilistic design. Modern computers give the possibility to automate complex calculations, which allows the use of modern numerical statistic simulation methods in the course of reliability assessment. Their most valuable characteristics are simplicity, absence of any limitations concerning the nature of initial statistical information, as well as the possibility of using real data (experimentally obtained physical, geometrical and load parameters of the structure, etc.) without making preliminary theoretical probabilistic models using stochastic factors that would be subsequently taken into account during the calculations. This opens large opportunities to the use of probabilistic simulation methods for design and for reliability assessment of structures. New developments in the use of various structures and elements include also the Monte Carlo simulation method and its modifications (Marek *et al*. 1995, 2001, Fishman 1996). Although reliability analysis can be carried out using Monte Carlo simulation, a large number of finite element executions for structural analysis can make the computational cost (CPU time) prohibitively high (Guan and Melchers 2001). This is especially true for large and complex cable structures requiring high reliability. In order to reduce the CPU time to an acceptable level, in some cases, the closed-form models could be applied.

Monte Carlo simulations have been used successfully in many cases of structural reliability assessments. Benedetti and Ceccoli (1987) used the Monte Carlo method to obtain information on the sensitivity of cable structures to initial imperfections. The normally distributed random variables such as nodal coordinates of the structure, mechanical properties and pre-stress of the cables and external loads were assumed. The simulation analysis of large-span cable structures needs very large computational efforts. This follows from the need of acquisition of sufficient amount of statistical data, and from the iterative character of non-linear solution of each system in the sample. For this reason the mentioned authors used the model analysis which means that the simulation analysis was performed with equivalent cable structures, i.e., models, having fewer variables than the original structures. A suitable extension of the Buckingham theorem extrapolates the results of the simulations to the analysis of more complex real structures. Imai and Frangopol (2000) evaluated response of geometrically non-linear long-span and slender structures such as suspension bridges, by first-order approximation and Monte Carlo simulation. They applied total Lagrangian formulation for finite-element discretization. Numerical examples are presented to illustrate the computational process and to study the effects of various parameters such as type of analysis (linear or non-linear), magnitude of loads and load effects, and type of approximation (first order or simulation) on the main descriptors of the response of geometrically non-linear structures. For the cases studied, the results show that first-order approximation and Monte Carlo simulation are in close agreement. This indicates that, at least for the numerical examples presented, first-order approximation can be used in place of Monte Carlo simulation. In this manner, computational time is drastically reduced without a significant loss in accuracy.

Ahammed and Melchers (2009) formulated a procedure in the space of the load processes, for estimating the reliability of structures subject to multi-parameter time-varying loading. Faber *et al*. (2003) developed models for the assessment of the strength and fatigue life of cables. Mahadevan and Raghothamachar (2000) proposed an efficient simulation-based methodology to estimate the system reliability of large structures. Melcher *et al.* (2004) presented the results of the experimental research of material and geometrical characteristics of steel products and the problems discussed also in a connection with the application of stochastic computations. Onoufriou and Frangopol (2002) presented a brief retrospective of the development and application of reliability-based techniques for assessment of complex structures with emphasis on inspection optimization of offshore and bridge structures. Shi *et al*. (2007) introduced a methodology for the strength estimation of suspension bridge cables using results of tensile strength tests performed on wire samples extracted from the bridge's main cables.

The possibilities of computational time reduction during the reliability analysis and of simplifying criteria introduction, that satisfy the needs of accuracy, are as follows: analysis of an appropriate model with the required accuracy instead of real structure, use of approximate analytical reliability assessment methods (e.g., distribution-free first-order second-moment method (FOSM), 1^{st} and/or 2^{nd}-order reliability methods (FORM, SORM)) instead of simulation techniques, the use of closed-form analysis instead of discrete one, etc. With respect to the fact that the number of simulations (separate steps which differ from each other by applying random input variables), depending on the problem of structural reliability solved, varies from ten thousands to millions, the use of FEM can be in some problems too cumbersome. In comparison with the numerical model, the proposed closed-form analytical model has some specific advantages. It does not need such a powerful computer and the response of cable structure is obtained during one simulation by direct substitution into the explicit formulae and expressions.

In this paper, one of the possible alternatives of simulation-based time-dependent reliability assessment of the biconcave and biconvex pre-stressed cable trusses, the Monte Carlo method, is applied. The influence of an excessive deflection of cable truss (caused by creep and rheologic changes) on its serviceability in required time is investigated. Attention is given to the definition of basic random variables and their statistical functions (basic, mutually dependent random variables such as the pre-stressing forces of the bottom and top cable, structural geometry, the Young's modulus of elasticity of the cables, and the independent variables, like permanent load, wind, snow and thermal actions). Then, the determination of the response of the structure to the loading effects, and the definition of the limiting values considering serviceability of the structure is performed. The potential of the method using direct Monte Carlo technique for simulation-based time-dependent reliability assessment as a powerful tool is presented.

The derived closed-form model (see the first part of this paper Kmet and Tomko 2011) and its alternatives are used in the probabilistic serviceability assessment of cable truss by the Monte Carlo method. First, the subject of assessment is the structure without rheological factors and with homogeneous boundary conditions; then the model is applied to cable truss with rheological properties (the effect of cable creep is accounted for) and non-homogeneous boundary conditions (elastic flexible supports are considered). Results obtained by the First order reliability method (FORM) are compared with those obtained by the Monte Carlo simulation technique.

2. Probabilistic time-dependent reliability assessment of cable truss

The developed time-dependent analytical model and the fully probabilistic simulation-based time-dependent reliability assessment are applied on the pre-stressed biconcave cable truss suspended on hinge columns anchored by elastic anchoring cables showed in Fig. 1.

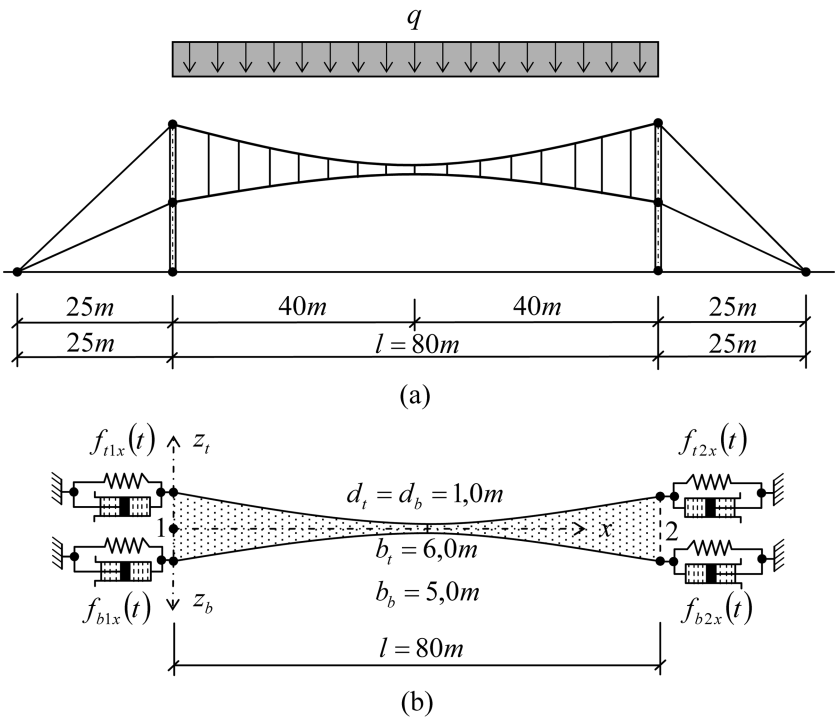


Fig. 1 Geometry, (a) load and (b) scheme of the investigated biconcave cable truss

2.1 Characteristics of cable truss and input data

For purpose of quantifying the uncertainties during the design stage of unconventional structures needed for a probabilistic reliability assessment, it is necessary to define a set of basic variables and to decide which quantities should be modelled by random variables and which by deterministic parameters.

The following deterministic input data of the investigated cable truss were specified: span length of $l = 80$ m, the geometrical quantities $b_b = 5.0$ m and/or $b_t = 6.0$ m and the cross-sectional areas of the bottom and top cable $A_b = 1.802.10^{-3}\text{m}^2$ as well as $A_t = 2.802.10^{-3}$ m^2, respectively.

Coefficients a, b and c of the creep constitutive equations in the forms of a logarithmic-exponential function given by Eqs. (9), see first part of this paper (Kmet and Tomko 2011), were obtained for the corresponding initial pre-stress levels. The resulting values were as follows: $a = 0.526774$, $b = 1065.143$ and $c = 0.026767$ for both bottom and top cable (due to the assumed pre-stressing forces and cross-sectional areas of the bottom and top cable, the resulting stresses after load application will be similar). In this example the expected average value of the creep is considered as deterministic. Generally, a random creep as a function of the actual stress level and time would be used.

The elastic flexibility at the structural supports (elastic flexibility of the anchored hinge columns), in the horizontal direction, at the bottom support $f_{xb}(t) = 0.00022\ \text{mkN}^{-1}$ and at the top support $f_{xt}(t) = 0.00015\ \text{mkN}^{-1}$ is considered. The axial tension stiffness of the cables is modified according to Eqs. (19), defined in the first part of this paper (Kmet and Tomko 2011).

A set of basic random variables, which affect the structural behaviour are defined as follows: the initial horizontal components of the pretensions in the bottom and top chords $H_b(t_0)$ and $H_t(t_0)$, respectively, arise from geometrical properties of the cable truss (characterized by a camber $c = b_b - d_b$ of the stabilizing cable and a sag $s = b_t - d_t$ of the carrying cable) and also from mechanical properties of the cables (characterized by the Young's modulus of the bottom $E_b(t)$ and the top cable $E_t(t)$). Therefore, these three groups of random variables are considered to be mutually dependent in this study.

The statistical evaluation of the random variable material characteristics of steel cables is based on the experimental research.

One of the most significant variables affecting the reliability of cable trusses is loading (action). The basic random variables will be the intensity of the permanent actions $q(t)$, wind $w(t) = q_w(t)$ and snow $s(t) = q_s(t)$ actions (climatic actions) and a change of the atmospheric external (outdoor) temperature $T_e(t)$ (climatic thermal action).

In the current specifications the case of independent loads – i.e., loads that do not depend on one another - usually applies and our examples given in this paper fall into this category. Loads that cannot occur on a cable truss at the same time (e.g., high temperature together with snow) are eliminated and are not considered.

2.2 Density and distribution functions of the random variables

Cable trusses are characterized by a strong interaction between pretensions and geometry. Consequently, the initial components of the pretensions in the bottom and top chords $H_b(t_0)$ and $H_t(t_0)$ are in correlation with the camber $c = b_b - d_b$ of the stabilizing cable and the sag $s = b_t - d_t$ of the carrying cable. Young's modulus affects the patterning of cable trusses, which determines the shape, forces and displacements of the structure at the initial conditions and naturally also during its entire expected service-life (Lewis 2003). Therefore the actual values of the Young's modulus (tangential modulus of elasticity calculated from the stress-strain function of the cables) corresponding to the stresses and/or forces in the bottom and top cables will be determined. In previous finite elements based computer studies, various algebraic expressions have been derived by the first author (Kmet 1994) to reflect the observed nature of stress-strain diagrams. Such constitutive relations can be made to accurately reflect the behaviour of cable structures. Three different types of steel cable are common in structural applications: structural strand, rope and parallel wire strands. Due to the varying constructional methods the Young's modulus of elasticity varies. Values of 140 kNmm^{-2} for rope, 170 kNmm^{-2} for strand and 190 kNmm^{-2} for parallel wire strand appear to be typical. Generally the Young's modulus of elasticity can depend on the stress level and whether the cable is subject to first loading or repeated loading. The Young's modulus used for structural analysis should be obtained for each cable type and diameter, by measuring the secant modulus after a sufficient number of load cycles (pre-stretching process) between the minimum and the maximum cable force under the characteristic permanent and variable actions, to get stable values. However, this is not necessary in the present study and experimental data, i.e., the stress-strain diagrams of the bottom and top cable of the truss are all that are required. At all stress levels, depending on the actual initial forces in the bottom $N_b(t_0)$ and top $N_t(t_0)$ cables at time t_0, and on the forces $N_b(t)$ and $N_t(t)$ due to the loading at time t, the corresponding values of the Young's modulus reflecting the stress-strain curves of the cables were determined. Therefore, finally three groups of the dependent random variables were considered at time t_0: $H_b(t_0)$ and $H_t(t_0)$, $E_b(t_0)$ and

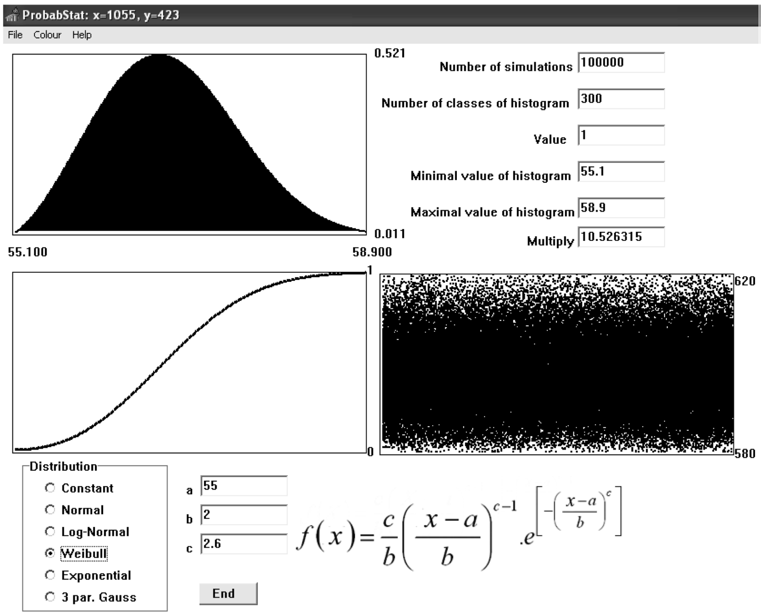


Fig. 2 Density function of the horizontal component of the initial pre-stressing force $H_b(t_0)$ of cable and the corresponding distribution function. The bottom part of the figure shows the graphic description, i.e., the time record of a selection of random variable values from the corresponding set. All data are outputs from the *ProbabStat* software developed by the authors

$E_t(t_0)$, and $N_b(t_0)$ and $N_t(t_0)$.

A probability density function in the form of the Weibull distribution, see Fig. 2

$$f(x) = \frac{2.6}{2}\left(\frac{x-55}{2}\right)^{1.6} e^{\left[-\left(\frac{x-55}{2}\right)^{2.6}\right]} \tag{1}$$

is used to represent $H_b(t_0)$ in the interval <580,620> kN.

The density functions of the other dependent random variables such as $H_t(t_0)$, the camber $c = b_b - d_b$ and sag $s = b_t - d_t$, $E_b(t)$ and $E_t(t)$, are similar. Their distributions are influenced by the corresponding correlations as described below.

Correlation between initial pre-stressing forces of the bottom and top cables $H_b(t_0)$ and $H_t(t_0)$, respectively, and the actual geometry is given by

$$H_t(t_0) = H_b(t_0)\frac{b_b - d_b}{b_t - d_t} = H_b(t_0)\frac{c(H_b(t_0))}{s} \tag{2}$$

Assuming, that the cables are perfectly flexible, i.e., bending moments in all cable cross sections are equal to zero

$$H_b(t_0)c(H_b(t_0)) = \frac{q_{b,p}l^2}{8} = 0 \tag{3}$$

The initial value of uniformly distributed load $q_{b,p}$ corresponding to the known value of $H_b(t_0)$ can be obtained as

$$q_{b,p} = \frac{8c(H_b(t_0))H_b(t_0)}{l^2} \tag{4}$$

and the camber $c(H_b(t_0))$ as a function of the pretension in the bottom chord $H_b(t_0)$ from

$$c(H_b(t_0)) = \frac{q_{b,p}l^2}{8H_b(t_0)} \tag{5}$$

Since we assume that the ties are inextensible and in the analysis they are replaced by a continuous diaphragm, then adjacent vertical elements may slide freely with respect to each other, and the dependence between sag and camber of the cables at mid-span of the truss is considered as below. Given the geometrical quantities $d_b(c)$ and d_t at mid-span of the truss, then

$$d_b(c) + d_t = const$$
$$d_t = const - d_b(c) \tag{6}$$

Substituting expressions $d_b(c) = b_b - c(H_b(t_0))$ and $d_t = b_t - s$ into $d_t = const - d_b(c)$ gives, $-s = const - b_b + c(H_b(t_0)) - b_t$, and thus

$$s(c) = b_b + b_t - c(H_b(t_0)) - const \tag{7}$$

In the present example, constant $d_b(c) + d_t = const = 2,0$ m is considered. As an initial value the camber $c = b_b - d_b = 5,0 - 1,0 = 4,0$ m for the stabilizing cable is considered.

Correlation between $H_b(t_0)$ and $H_t(t_0)$, and the corresponding actual values of $E_b(t_0)$ and $E_t(t_0)$ is given by the following procedure. We start with

$$N_b(t_0) = \frac{H_b(t_0)}{\varphi_b(t_0)} \quad \text{and} \quad N_t(t_0) = \frac{H_t(t_0)}{\varphi_t(t_0)} \tag{8}$$

where $N_b(t_0)$ and $N_t(t_0)$ are the initial normal tensile forces in the bottom and top chords, respectively, at initial time t_0; $\varphi_b(t_0)$ and $\varphi_t(t_0)$ are the known initial inclinations of the tangent to the cable profile curve in the bottom and top chords, respectively, at the initial time t_0 and analogously at time t

$$N_b(t) = \frac{H_b(t)}{\varphi_b(t)} = \frac{H_b(t_0) - \Delta H_b(t)}{\varphi_b(t)}$$
$$N_t(t) = \frac{H_t(t)}{\varphi_t(t)} = \frac{H_t(t_0) + \Delta H_t(t)}{\varphi_t(t)} \tag{9}$$

Thus one can obtain the normal stresses from

$$\sigma_b(t_0) = \frac{N_b(t_0)}{A_b}, \quad \sigma_t(t_0) = \frac{N_t(t_0)}{A_t}, \quad \sigma_b(t) = \frac{N_b(t)}{A_b}, \quad \sigma_t(t) = \frac{N_t(t)}{A_t} \tag{10}$$

and using the relevant stress-strain constitutive functions $\sigma_b(\varepsilon_b)$ and $\sigma_t(\varepsilon_t)$, (appropriate approximation stress-strain functions, see Kmet (1994) for more information) the corresponding values of Young's modulus of elasticity can be determined from

$$E_b(t_0) = \frac{d\sigma_b(\varepsilon_b, t_0)}{d\varepsilon_b(t_0)}, \quad E_t(t_0) = \frac{d\sigma_t(\varepsilon_t, t_0)}{d\varepsilon_t(t_0)}, \quad E_b(t) = \frac{d\sigma_b(\varepsilon_b, t)}{d\varepsilon_b(t)}, \quad E_t(t) = \frac{d\sigma_t(\varepsilon_t, t)}{d\varepsilon_t(t)} \tag{11}$$

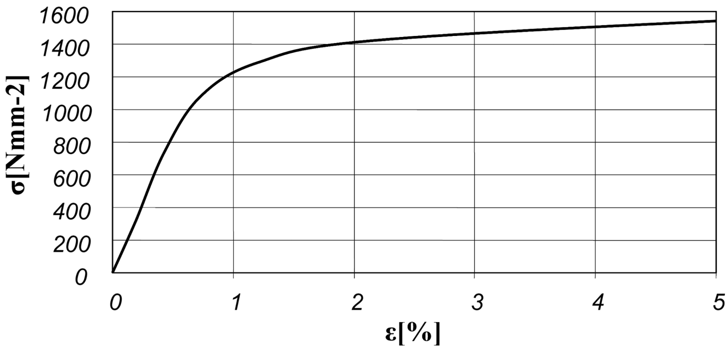


Fig. 3 The stress-strain diagram of the bottom and top cable

The described procedure is repeated and applied in each simulation. In this study the stress-strain diagram shown in Fig. 3 will be considered for the bottom and top cable.

A pre-stress error in cables is caused either through an uncertainty of pressure indicated by jack (accuracy of technology) or by the variations of the cable length due to the thermal cycling, creep and deformation of external supports. An uncertainty in the position of cable truss is mainly due to two factors: first, the finite precision of the measuring devices used for assembling the cable structure, second, a subsequent sag or camber depending on the total cable lengths. The subsequent value of Young's modulus can change with time due to creep, stress and thermal cycling and due to a deformation of the external restraints. Notional values of the Young's modulus of cables, when the test results are not available, are given in EN 1993-1-11 (2006).

In this study, the intensity of permanent action, wind and snow actions represent the basic random load variables. These are described as a uniformly distributed load over the entire span of the cable truss and considered as the independent quantities.

To describe the time-dependent loads, one needs the probability distribution values for an arbitrary point in time and a description of the variations in time. In our examples, a FBC model (Ferry-Borges-Castanheta model) based on the rectangular wave process with equidistant time intervals is used. The load intensities in subsequent time intervals of this model are independent.

The distribution of the wind load acting perpendicularly to the area of the cylindrical biconcave cable truss roof is shown in Fig. 4(a) (Kadlcak 1995). The closed-form analysis of a biconcave cable truss under variable wind load acting perpendicularly to the roof area is complicated. Approximately and safely, it is possible to calculate the equivalent vertically acting wind load uniformly distributed over the entire span of the cable truss. Consequently, the equivalent wind suction of 70% intensity (acting vertically upwards) and the equivalent wind pressure of 40% intensity (acting vertically downwards) of the characteristic value of the basic wind pressure will be considered (Fig. 4(b)).

A probability density function in the form of lognormal distribution, see Fig. 5

$$f(x) = \frac{1}{0.03x\sqrt{2\pi}} e^{\left[-\frac{(\ln x - 4.26)^2}{2.0.03^2}\right]} \tag{12}$$

is used to represent the permanent load $q(t)$ in the interval <0.9, 1,1> kNm^{-1}.

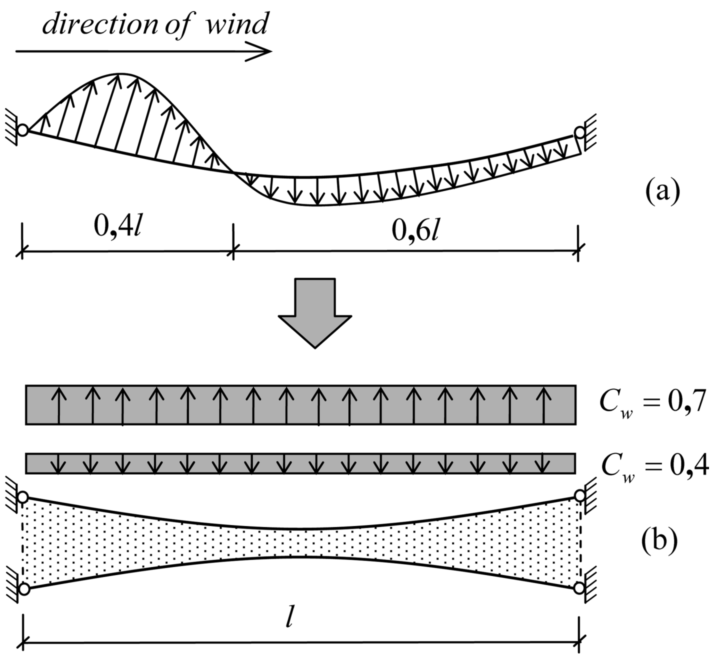


Fig. 4 Distribution of (a) suction and pressure of the wind on a cylindrical suspension cable roof and (b) the equivalent approximate wind loading

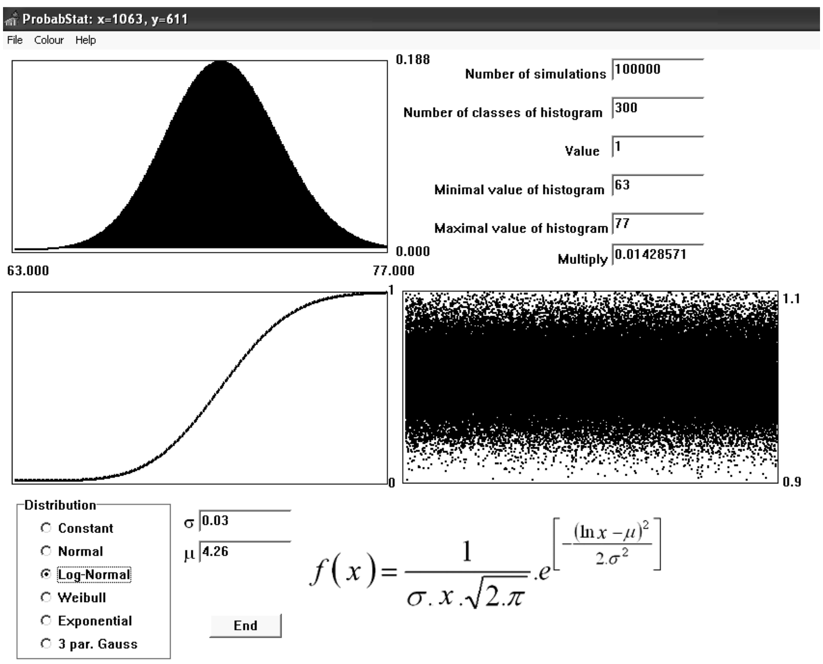
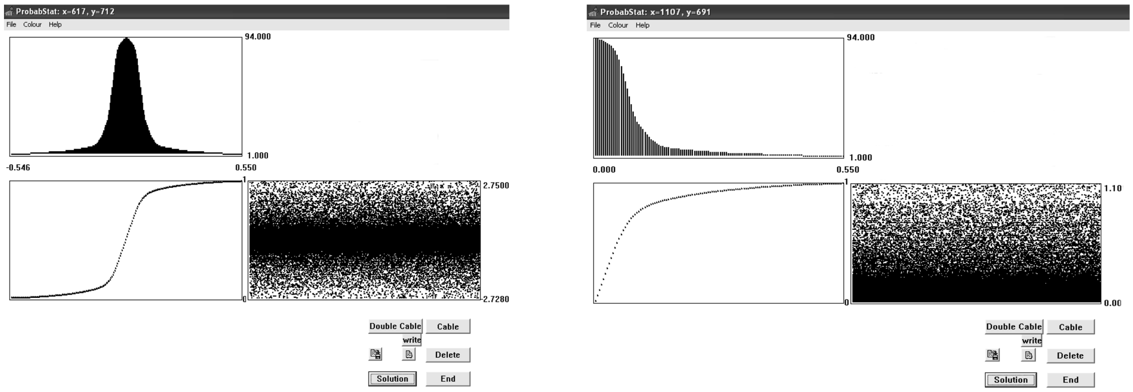


Fig. 5 Density function of uniformly distributed permanent load $q(t)$ of the cable truss and the corresponding distribution function



(a) (b)

Fig. 6 Density function of (a) wind action $q_w(t)$ and (b) wind pressure acting on the cable truss and the corresponding distribution functions

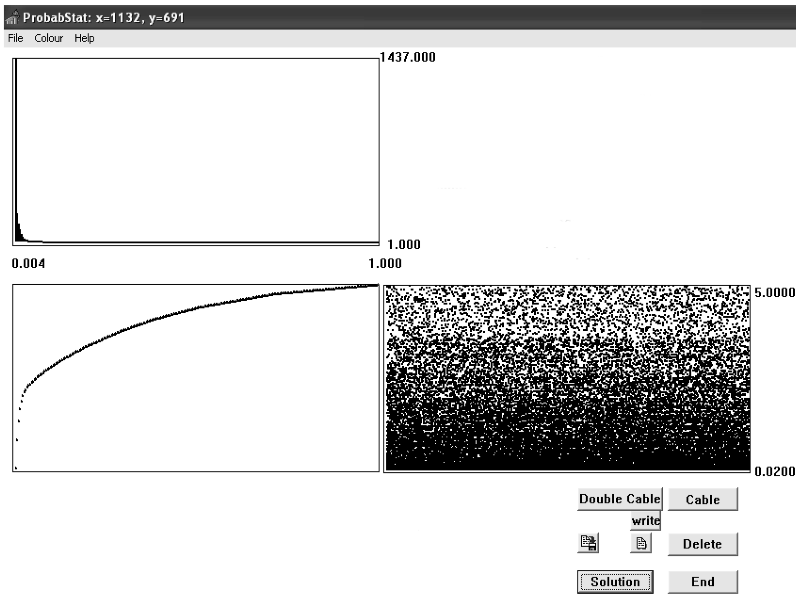


Fig. 7 Density function of snow action $q_s(t)$ of the cable truss and the corresponding distribution function

The real values and distributions for the wind and snow were obtained from the fifty years measurements at the Czech Hydro-meteorological Institute. Daily average values of one year for the wind and of the four months for the snow were used for a construction of the non-parametric distributions as shown in Fig. 6 and Fig. 7. The wind $q_w(t)$ and snow $q_s(t)$ actions in the intervals $<-2{,}728{,}2{,}75>$ kNm^{-1} (Fig. 6(a)) and $<0.0{,}5.0>$ kNm^{-1} (Fig. 7), respectively, are considered. In this example, only a combination with the wind pressure defined by the interval $<0.0{,}2.75>$. $C_w = <0.0{,}2.75> . 0.4 = <0.0{,}1.1>$ kNm^{-1} is performed. The corresponding distribution is shown in Fig. 6(b). A loading width of the cable truss of 5.0 m was considered.

The input data for thermal action were defined according to the European standards ISO (1996). Let us consider a lightweight roof cladding made of corrugated steel sheets on which a thermal insulation with a protective water-proof layer as shown in Fig. 8 is placed. A random variable internal surface temperature of the top carrying cable $\Theta_{t,s,i}(t) = T_{s,i}(t)$ at time t is obtained from

$$T_{s,i}(t) = T_i(t) - \frac{T_i(t) - T_e(t)}{R_0} R_{s,i} \tag{13}$$

where $T_i(t)$ and $T_e(t)$ are the characteristic values of the internal (indoor) and external (outdoor) temperatures at time t, respectively. The heat transfer resistance R_0 of the roof covering with the three layers is defined as

$$R_0 = R_{s,i} + \sum_{j=1}^{n} R_j + R_{s,e} = R_{s,i} + \sum_{j=1}^{n} \frac{d_j}{\lambda_j} + R_{s,e} \tag{14}$$

where $R_{s,i}$ and $R_{s,e}$ are the characteristic values of the internal and external surface resistances, respectively, d_j is the thickness and λ_j is the thermal conductivity of the material for the corresponding layer. The following input data are considered in this example: $R_{s,i} = 0.1$ m^{2}KW^{-1}, $R_{s,e} = 0.04$ m^{2}KW^{-1} for a winter period and $R_{s,i} = 0.17$ m^{2}KW^{-1}, $R_{s,e} = 0.07$ m^{2}KW^{-1} for a summer period, $d = 0.16$ m and $\lambda = 0.038$ Wm^{-1}K^{-1} (hence, only one layer of the thermal insulation

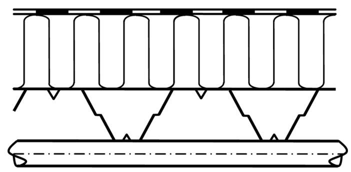


Fig. 8 Scheme of a lightweight roof cladding

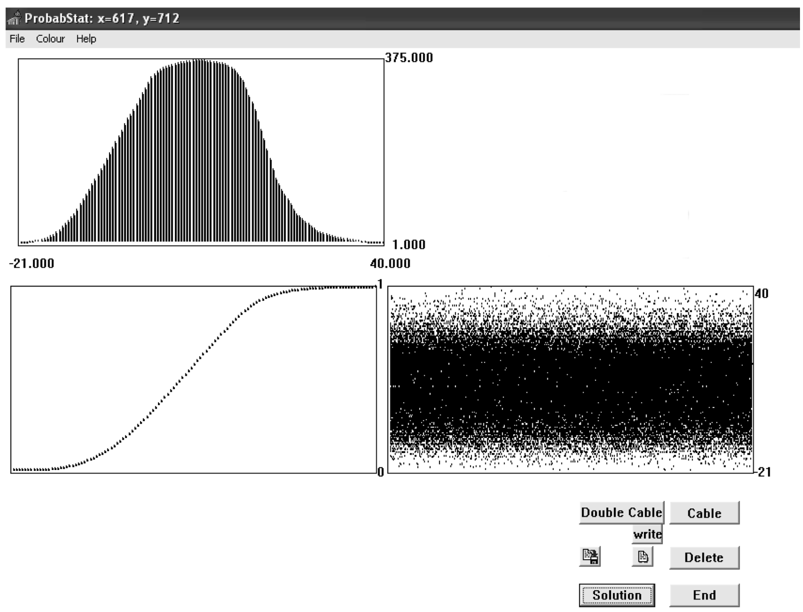


Fig. 9 Density function of the external temperature $T_e(t)$ and the corresponding distribution function

is considered). $T_i(t) = T_i = 18\,^{\circ}\mathrm{C}$ for the entire service-life of the structure. A probability density function in the form of the non-parametric distribution, as shown in Fig. 9, is used to represent the random variable $T_e(t)$ in the interval $<-21^{\circ}\mathrm{C} \div 40^{\circ}\mathrm{C}>$. Corresponding values of the random variable temperature $T_{s,i}(t)$ at time t is calculated by Eq. (13).

Finally, the random variable design (computational) temperature difference $\Delta T_t(t)$ of the top carrying cable is defined as

$$\Delta T_t(t) = T_{s,i}(t) - T(t_0) \tag{15}$$

where $T(t_0) = 15\,^{\circ}\mathrm{C}$ is the initial, i.e., the assembling temperature at time t_0.

2.3 Simulation-based time-dependent reliability assessment of cable truss

The fully probabilistic simulation-based method using direct Monte Carlo technique is applied to the time-dependent reliability assessment of the cable truss shown in Fig. 1. Experience with a number of cable structures has shown that their failure is characterised by the time-dependent rheologic changes in the tension stiffness of the cables. This is the reason why the serviceability limit states are assessed in the present study. Probabilistic time-dependent reliability assessment of the analysed cable truss is based on its serviceability and not on ultimate limit states.

When the creep data of a cable component are available, we can find the reliability $P_s(t)$ or failure probability $P_f(t)$ of the cable truss at an arbitrary time t from the required time interval during its service life. The time interval is divided into the required time domains of the lengths $\Delta t_1, \Delta t_2, \ldots, \Delta t_i, \ldots, \Delta t_N$. Then the Monte Carlo method is used to estimate the reliability or failure probability of the cable truss, with the corresponding random variables and creep strain increments, at the considered time t.

Three cable trusses with the different properties used as the examples were selected for the investigation of their reliability and for the illustration of the mentioned procedure and capability of the analytical model. The first cable truss analysed was the structure with fixed, i.e., un-flexible supports, without creep effect. As the second case, the truss with fixed supports and with creep effect was analysed. Finally, the cable structure with elastic flexible supports and with creep effect was assessed.

Serviceability assessment refers to the vertical deflection limit value $w_{lim}(t) = 0.341$ m at the mid-span of the cable truss in case of the fixed supports and to the value $w_{lim}(t) = 0.445$ m for the elastic flexible supports (for image: $l/200 = 80/200 = 0.400$ m and/or $l/150 = 80/150 = 0.533$ m). These limit deflections were calculated for the design failure probability $P_{fd} = 7.10^{-2}$ (design index of reliability $\beta_d = 1.48$), with the corresponding design reliability $P_{sd} = 1 - P_{fd} = 1 - 7.10^{-2} = 0.93$, for the normal reliability level from the point of view serviceability limit states and for the design technical service life of the structure $t_d = 80$ years (JCSS 2001).

In this method the pseudo-random numbers generator is used to select a simulation value of each of the random parameters of the structure for the corresponding distribution function (see section 2.2). The number of selections is equal to the number of one million simulations. The simulation systems response, i.e., the resulting vertical deflections, at the investigated times ($t = 0$ initial time); 0.003; 0.04; 1; 5; 10; 25; 50; 80 and 100 years, are compared with the specified limit deflections. After generating the predetermined number of simulations, the overall reliability (or failure probability) of the cable truss, at the investigated time t, is computed. *ProbabStat* program output, i.e., serviceability reliability function of the pre-stressed biconcave cable truss is shown, as an

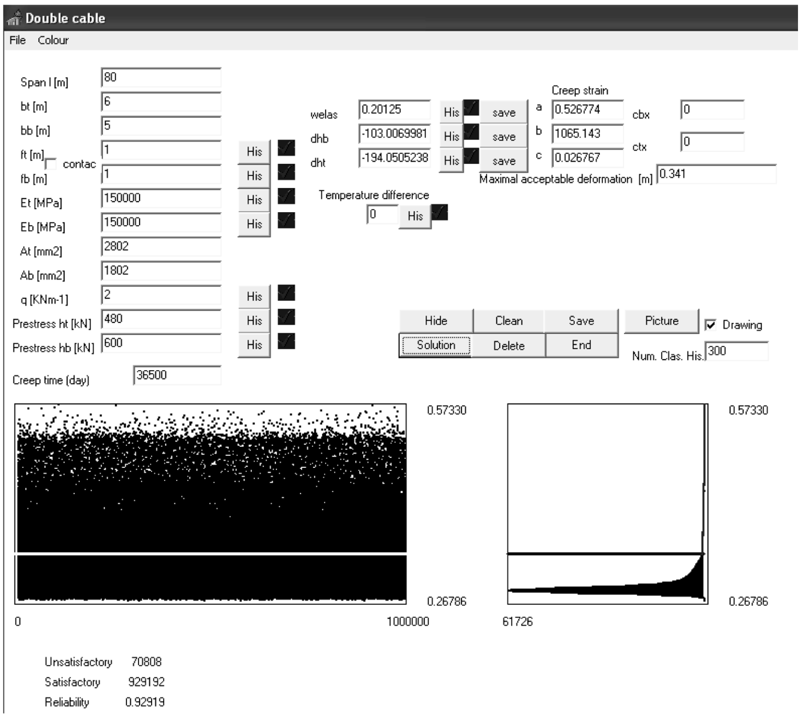


Fig. 10 *ProbabStat* program output, i.e., the serviceability function of pre-stressed biconcave cable truss

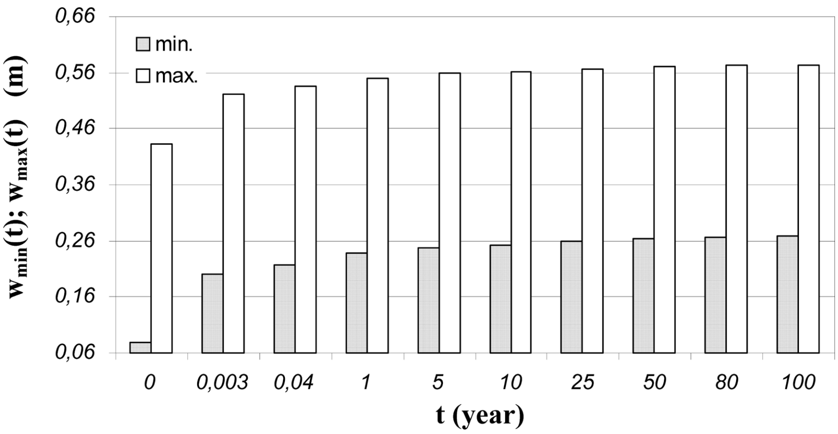


Fig. 11 Minimum $w_{\min}(t)$ and maximum $w_{\max}(t)$ deflections at the mid-span of the cable truss with fixed supports at the investigated times t, under 1 000 000 simulations

illustration, in Fig. 10. The boundary between successful and unsuccessful simulations is marked by the line.

The minimum $w_{\min}(t)$ and maximum $w_{\max}(t)$ mid-span deflections of the cable truss (obtained from one set of simulations at time t), at the investigated times $t = 0$ (initial time); 0.003; 0.04; 1; 5; 10; 25; 50; 80 and 100 years, are shown, for the corresponding type of the supports, in Fig. 11 and Fig. 13. The corresponding time-dependent reliabilities $P_s(t)$ are shown in Fig. 12 and Fig. 14. The

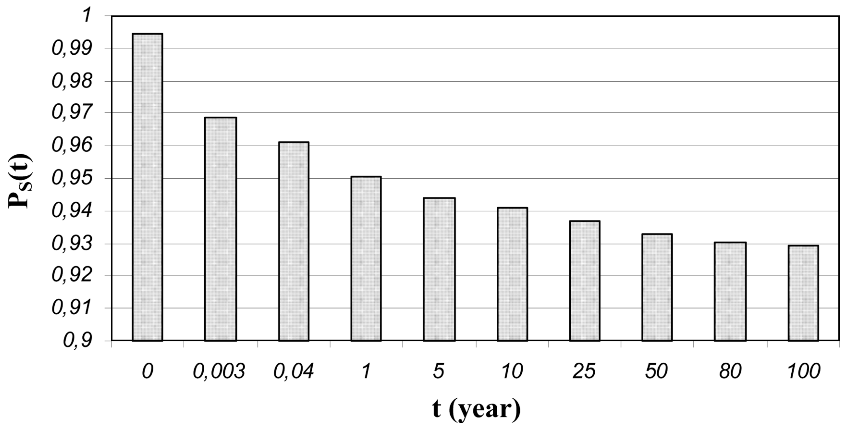


Fig. 12 Time-dependent reliability $P_s(t)$ of the cable truss with fixed supports at the investigated times t, under 1 000 000 simulations

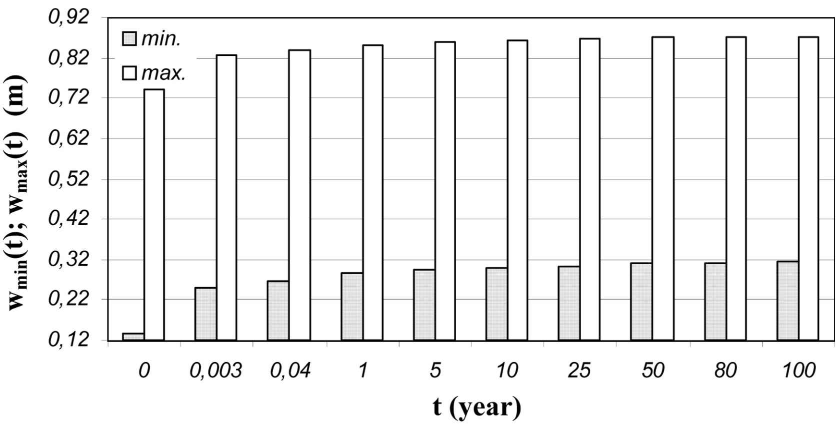


Fig. 13 Minimum $w_{\min}(t)$ and maximum $w_{\max}(t)$ deflections at the mid-span of the cable truss with elastic yielding supports at the investigated times t, under 1 000 000 simulations

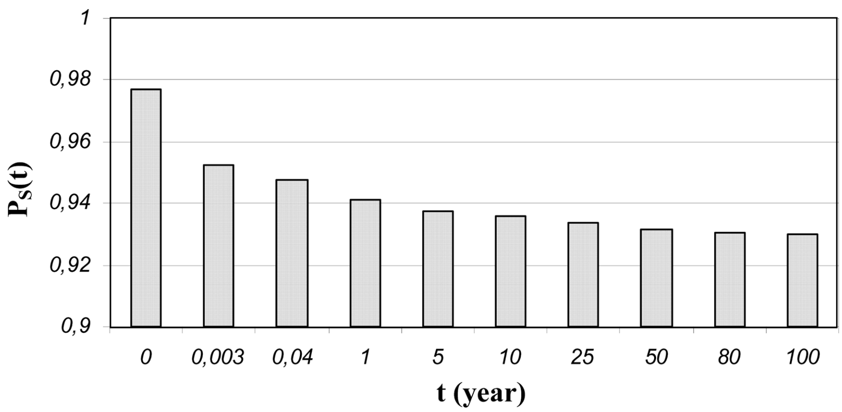


Fig. 14 Time-dependent reliability $P_s(t)$ of the cable truss with elastic yielding supports at the investigated times t, under 1 000 000 simulations

deflections of the cable truss increased with the time increment due to the creep influences and reliability of the cable truss is decreased, when serviceability limit states assessment is conducted.

3. Comparison of the FORM and the Monte Carlo simulation technique

The simulations based on Gaussian and independent assumptions give results that can be obtained faster by other methods that are much more efficient, such as the FORM and that of the response surface. The First Order Reliability method (FORM) for the assessment probability of structural failure is well developed and is generally robust in applications using random variables described by completely continuous probability density functions (Melchers 1999). Convergence problems may arise in some cases, when the standard FORM algorithm is applied to structural reliability problems involving random variables with high discontinuous, truncated or spiked probability distributions. The reasons for these problems were described and some simple modifications to the standard iterative FORM algorithm were proposed by Melchers *et al*. (2003).

The example illustrating the potential of the First Order Reliability method is given and the results are compared with ones obtained by Monte Carlo simulation technique. These methods are applied to the pre-stressed biconcave cable truss with fixed supports shown in Fig. 1.

3.1 Characteristics of cable truss and input data

The same deterministic input data of the investigated cable truss (for its geometrical and stiffness parameters as well as for the creep characteristic) as mentioned in Section 2.1 are specified for this example. The following mean - central values for the random variable input data are specified: the vertical uniformly distributed load $q = 2.0\ \mathrm{kNm^{-1}}$, $d_b = d_t = 1.0$ m, $E_b(t_0) = E_t(t_0) = 1.5\ .\ 10^8\ \mathrm{kNm^{-2}}$, $H_b(t_0) = 600$ kN and $H_t(t_0) = 480$ kN .

3.2 Results of the deterministic behaviour analysis at initial time t_0

In case of the deterministic behaviour analysis of the cable truss with fixed supports (without considering creep effects of the cables), when the mentioned input data are considered, the following partial and final results were obtained: $L_{eb} = 81.6$ m and $L_{et} = 82.5$ m (as Eqs. (15) from the first part of this papert (Kmet and Tomko 2011) are used), $\overline{M} = 1600$ kNm, $k_b(t_0) = -0.8179\ \mathrm{m^{-1}}$ and $k_t(t_0) = -1.5724\ \mathrm{m^{-1}}$ (Eqs. (18)), $\Delta H_b(t_0) = -107.8525$ kN and $\Delta H_t(t_0) = -207.3449$ kN (Eqs. (16)), $H_b(t=0) = 492.1475$ kN and $H_t(t=0) = 687.3449$ kN (Eqs. (21)), and $w(t_0) = 0.122$ m (at the midspan of the truss) (Eq. (5) and Table 1).

3.3 Density and distribution functions of the random variables

The following probability density functions in the forms of the Gaussian distribution, see Figs. 17-20, are used to represent the basic random variables:

• d_t in the interval <0.975,1.025> m (Fig. 15)

$$f(x) = 110.35e^{\left[\frac{-(x-0.9994)^2}{2(-0.004^2)}\right]} \qquad (16)$$

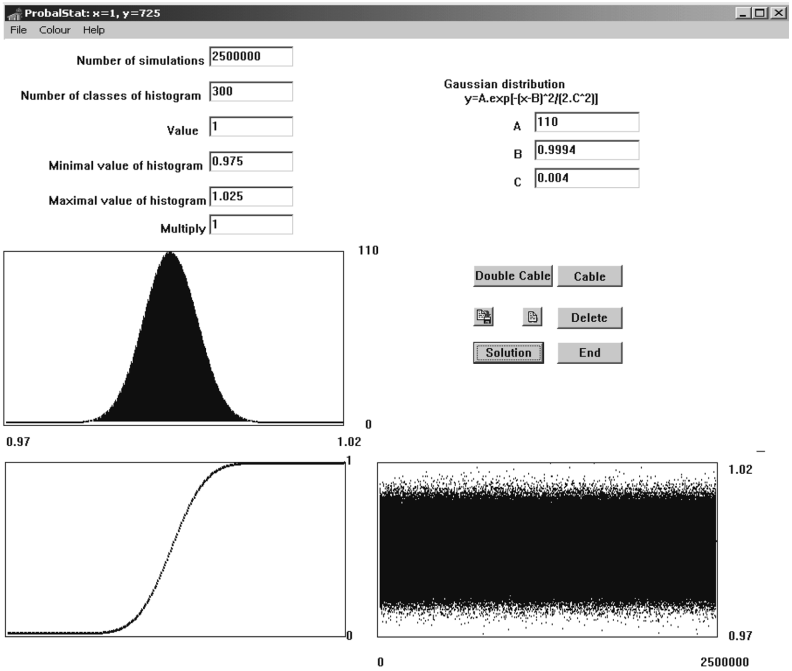


Fig. 15 Density function of the geometrical variable d_t at the mid-span of the top cable of the cable truss and the corresponding distribution function

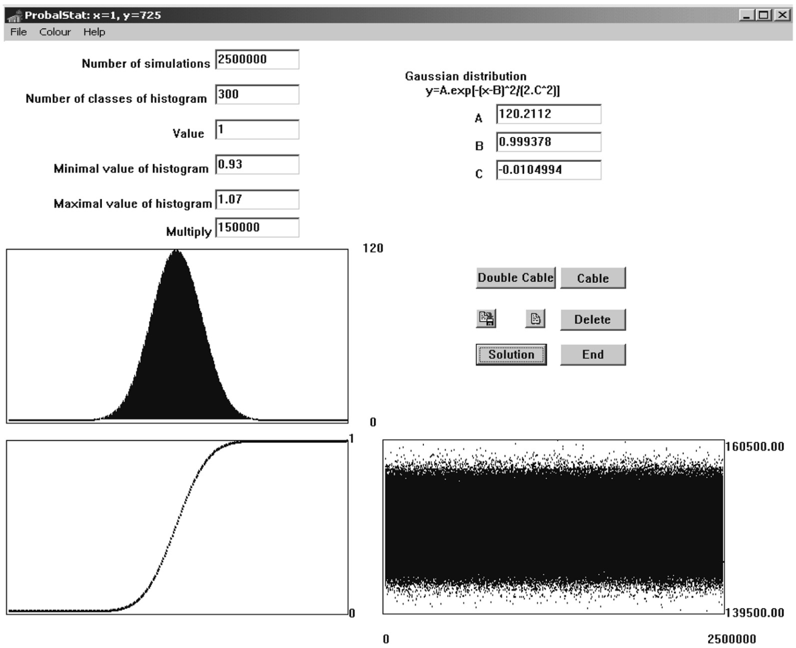


Fig. 16 Density function of the Young's modulus of elasticity $E_b(t)$ and $E_t(t)$ of the cables and the corresponding distribution function

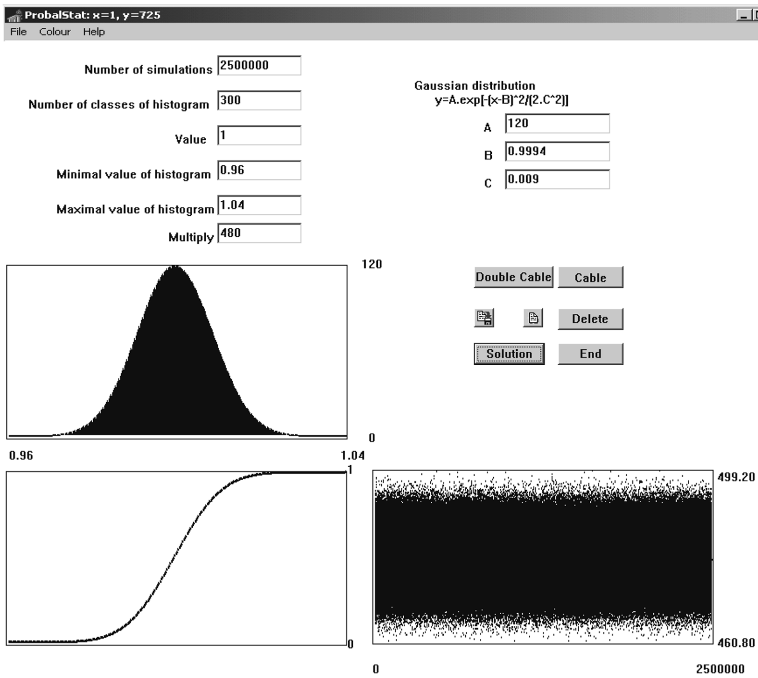


Fig. 17 Density function of the horizontal components of the initial pre-stressing forces $H_b(t_0)$ and $H_t(t_0)$ of cables and the corresponding distribution function

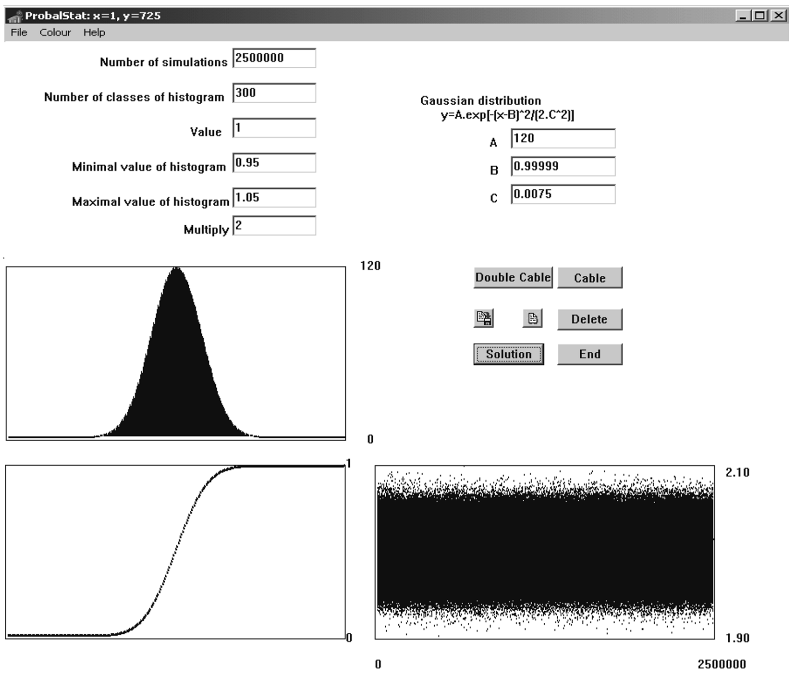


Fig. 18 Density function of loading q of the cable truss and the corresponding distribution function

• $E_b(t) = E_t(t)$ in the interval <1.395 . 10^8, 1.605 . 10^8> kNm^{-2} (Fig. 16)

$$f(x) = 90{,}2113 e^{\left[\frac{-(x-0.9994)^2}{2(-0.0105^2)}\right]} \quad (17)$$

• $H_b(t_0)$ and $H_t(t_0)$ in the intervals <460,500> kN and <580,620> kN, respectively, (the density functions for the bottom and top cables are similar, the differences are accounted through a multiplication factor) (Fig. 17)

$$f(x) = 120 e^{\left[\frac{-(x-0.9994)^2}{2(0.009^2)}\right]} \quad (18)$$

• q in the interval <1, 9, 2.1> kN·m^{-1} (Fig. 18)

$$f(x) = 120 e^{\left[\frac{-(x-0.9999)^2}{2(0.0075^2)}\right]} \quad (19)$$

The random variables are considered as independent quantities. The program developed simulates the uncertainty of geometry of the vertical positions of cables by varying the mid-span sag of carrying (top) cable and the corresponding camber of the stabilizing (bottom) cable of the truss. The dependence between sag and camber of the cables at mid-span of the truss is expressed as $d_b = 2.0 - d_t$, where d_t is a variable quantity.

It is necessary to note, that the assumptions based on the mutual independences of random variables and on their Gaussian distributions are in this case only “academic” and not realistic.

3.4 Simulation-based time-dependent reliability assessment of cable truss

The direct Monte Carlo technique is applied to the time-dependent reliability assessment of the cable truss shown in Fig. 1. In this study the serviceability limit states are assessed.

Serviceability assessment refers to the vertical deflection limit value $w_{\lim}(t) = 0.315$ m at the mid-span of the cable truss. This limit deflection was calculated for $P_{fd} = 7.10^{-2}$ ($\beta_d = 1.48$) and $P_{sd} = 1 - P_{fd} = 1 - 7.10^{-2} = 0.93$.

One million and two-and-half million simulations were considered. The simulation systems response, i.e., the resulting vertical deflections at the investigated times ($t = 0$ initial time); 0.003; 0.04; 1; 5; 10; 25; 50; 80 and 100 years, are compared with the specified limit deflection. The serviceability reliability function of the pre-stressed cable truss is shown, as an illustration, in Fig. 19.

The mid-span deflections $w_{\min}(t)$ and $w_{\max}(t)$ of the cable truss obtained from one set of the corresponding number of simulations at the investigated times are shown in Fig. 20 and Fig. 22. The corresponding time-dependent reliabilities $P_s(t)$ are shown in Fig. 21 and Fig. 23. The deflections of the cable truss increased with the time increment due to the creep effects and the reliability of the cable structure is decreased. Influence of the different number of simulations is clear in Fig. 24. Higher number of simulations resulted into a lower value of reliability at the time $t = 80$ years and more. At the time $t < 80$ years the differences are not significant.

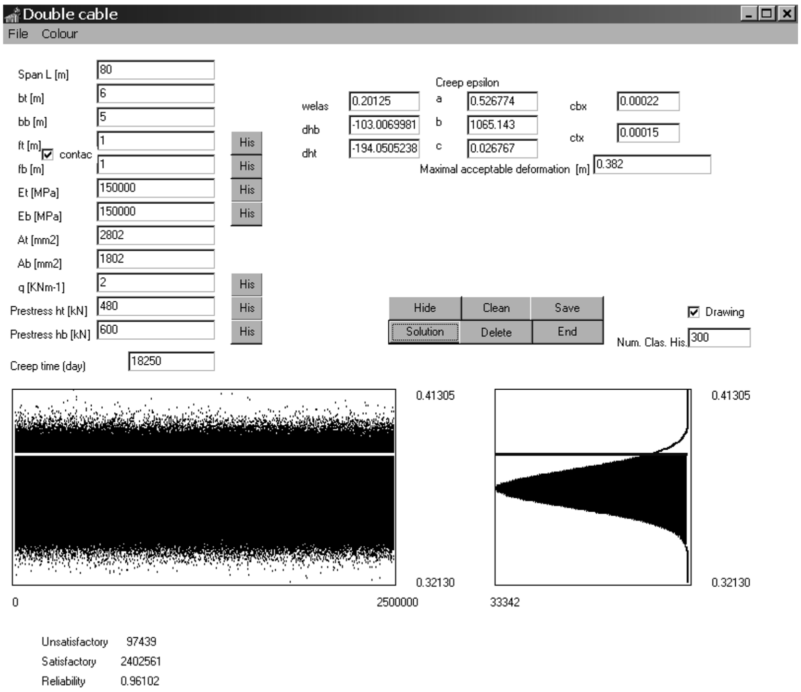


Fig. 19 *ProbabStat* program output, i.e., the serviceability function of pre-stressed biconcave cable truss

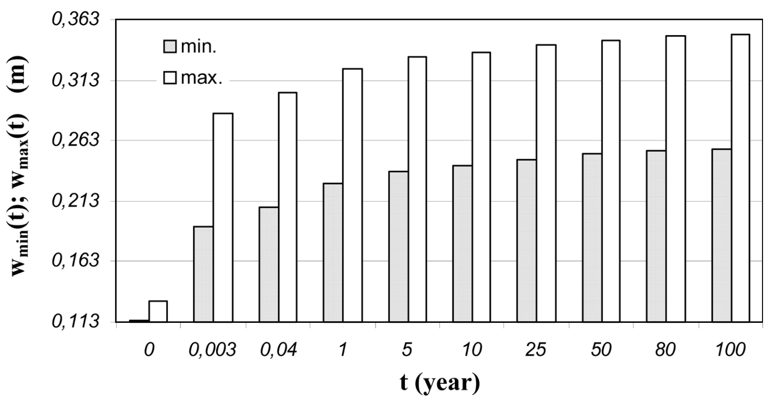


Fig. 20 Minimum $w_{\min}(t)$ and maximum $w_{\max}(t)$ deflections at the mid-span of the cable truss with fixed supports at the investigated times t, under 1 000 000 simulations

3.5 FORM-based time-dependent reliability assessment of cable truss

In order to compare some of the results obtained by simulation-based reliability assessment of the cable truss with fixed supports, reliability based on the first-order second-moment analysis has been performed for the cable truss previously examined. Let the deflection of the cable truss due to an external load w_{load} follows the Gaussian distribution of the mean $\mu_{load} = 0.25$ m and of the standard

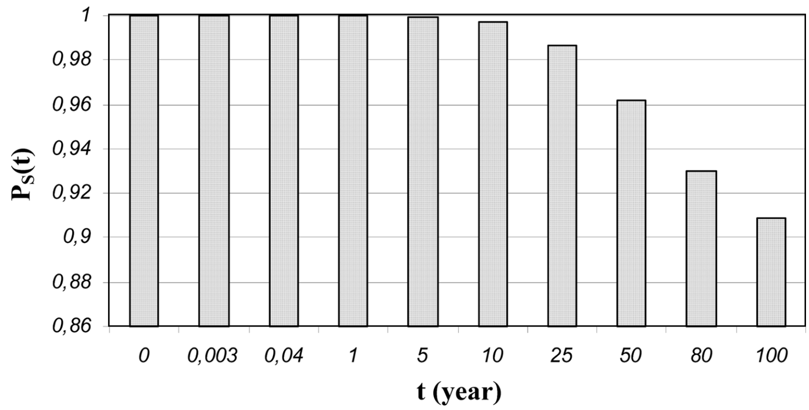


Fig. 21 Time-dependent reliability $P_s(t)$ of the cable truss with fixed supports at the investigated times t, under 1 000 000 simulations

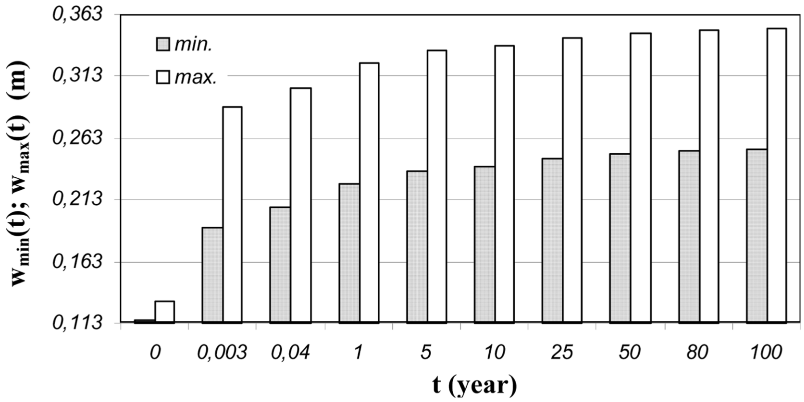


Fig. 22 Minimum $w_{min}(t)$ and maximum $w_{max}(t)$ deflections at the mid-span of the cable truss with fixed supports at the investigated times t, under 2 500 000 simulations

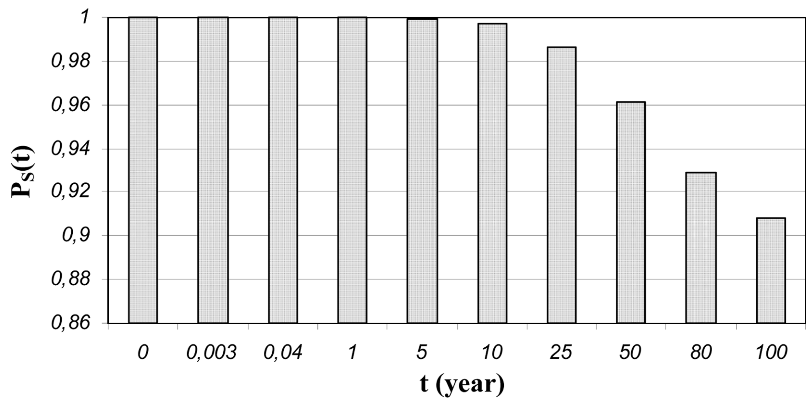


Fig. 23 Time-dependent reliability $P_s(t)$ of the cable truss with fixed supports at the investigated times t, under 2 500 000 simulations

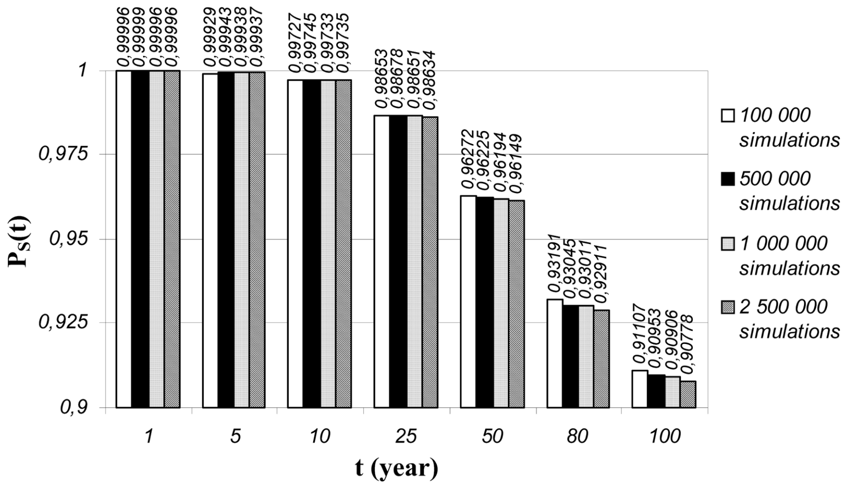


Fig. 24 Comparison of the time-dependent reliability $P_s(t)$ of the cable truss with fixed supports at the investigated times t, calculated under various number of simulations, i.e., 100 000, 500 000, 1 000 000 and 2 500 000 simulations

deviation $\sigma_{load} = 0.017$ m at the time $t = 5$ years. This can be transformed into the standard normal distribution of type $N(0, 1)$. A deflection (as serviceability limit states are considered) has the deterministic limitation $w_{\lim} = 0.315 \text{ m} = \mu_{\lim}$ while $\sigma_{\lim} = 0$. Deflections w_{load} and $w_{\lim}$ are independent.

Consequently, the reliability of the cable truss is

$$P_s = \Phi\left(\frac{\mu_{lim} - \mu_{load}}{(\sigma_{lim}^2 + \sigma_{load}^2)^{1/2}}\right) = \Phi\left(\frac{\mu_{lim} - \mu_{load}}{\sigma_{load}}\right) =$$

$$= \Phi\left(\frac{0.315 - 0.25}{0.017}\right) = \Phi(3.82) = 0.99993327 \qquad (20)$$

which is comparable with the values obtained by simulation-based reliability assessment of the cable truss at the time $t = 5$ years as is shown in Fig. 24.

4. Conclusions

The substance of a probabilistic structural simulation-based time-dependent reliability assessment method based on the Monte Carlo technique was explained and the strategy of its application was outlined using serviceability assessment of the pre-stressed biconcave cable truss with rheological properties as an example. The influence of an excessive deflection of the cable truss (caused by creep of cables and rheologic changes) on its time-dependent serviceability was investigated. The cable truss with fixed and elastic flexible supports was considered.

This approach serves to determine the reliability, i.e., serviceability of the cable truss, considering the basic random variables and their statistical functions. Basic, mutually dependent random variables such as the pre-stressing forces of the bottom and top cable, structural geometry, the

Young's modulus of elasticity of the cables, and the independent variables, like permanent load, wind, snow and thermal actions, were considered. Hence, random effects of geometrical, physical and structural imperfections can be modelled.

Results obtained by the First order reliability method were compared with those obtained by the Monte Carlo simulation technique.

It is believed that the presented time-dependent solution will lead to an improved closed-form analysis of the pre-stressed cable truss with rheological properties and to an improvement of its reliability assessment.

The approach holds promise in solving more complex time-dependent reliability problems of cable trusses. Future work will consider the application of the proposed simulation-based reliability assessment approach to dynamic behaviour of nonlinear cable structures, including post-elastic and rheological effects. A new principle for the analysis of nonlinear time response of structures based on the energy approach will be applied (Tesar and Tvrda 2006).

Acknowledgements

This work is a part of Research Project No. 1/0400/09, partially founded by the Scientific Grant Agency of the Ministry of Education of Slovak Republic and the Slovak Academy of Sciences. The present research has been carried out within the project *Centre of excellent integrated research of the progressive building structures, materials and technologies*, supported from the Structural funds of the European Union.

References

Ahammed, M. and Melchers, R.E. (2009), "A convenient approach for estimating time-dependent structural reliability in the load space", *Probabilist. Eng. Mech.,* **24**(3), 467-472.

Benedetti, A. and Ceccoli, C. (1987), "A Monte Carlo method study on probabilistic analysis of spatial cable nets", *Costruzioni Metalliche*, 3-21.

Ditlevsen, O. and Madsen, H.O. (1996), *Structural Reliability Methods*, John Wiley & Sons, Chichester.

EN 1993-1-11 (2006), *Eurocode 3: Design of Steel Structures, Part 1.11: Design of Structures with Tension Components*, CEN, Brussels.

Faber, M.H., Engelund, S. and Rackwitz, R. (2003), "Aspects of parallel wire cable reliability", *Struct. Saft.*, **35**(2), 201-225.

Fishman, G.S. (1996), *Monte Carlo: Concept, Algorithm and Application*, Springer-Verlag, Berlin.

Guan, X.L. and Melchers, R.E. (2001), "Effect of response surface parameter variation on structural reliability estimates", *Struct. Saft.*, **23**, 429-444.

Imai, K. and Frangopol, D.M. (2000), "Response prediction of geometrically non-linear structures", *J. Struct. Eng.*, **126**(11), 1348-1355.

ISO 6946 (1996), *Building Components and Building Elements, Thermal Resistance and Thermal Transmittance, Calculation Methods*, ISO, Brussels.

JCSS (2001), *Probabilistic Model Code. Part 1 - Basis of Design*.

Kadlcak, J. (1995), *Statics of Suspension Cable Roofs*, (Ed. Balkema, A.A.), Rotterdam.

Kmet, S. (1994), "Rheology of pre-stressed cable structures", *Proceedings of the International Conference on Computational Structures Technology*, (Eds. Papadrakakis, M. and Topping, B.H.V.), Athens, September.

Kmet, S. and Tomko, M. (2011), "Time-dependent analysis of cable trusses Part I. Closed-form computational model", *Struct. Eng. Mech.*, **38**(2), ~~XX-XX.~~

157-169.

Lewis, W.J. (2003), *Tension Structures. Form and Behaviour*, Thomas Telford.
Mahadevan, S. and Raghothamachar, P. (2000), "Adaptive simulation for system reliability analysis of large structures", *Comput. Struct.*, **77**(6), 725-734.
Marek, P., Brozzetti, J. and Gustar, M. (2001), *Probabilistic Assessment of Structures Using Monte Carlo Simulation*, ITAM CAS, Prague.
Marek, P., Gustar, M. and Anagnos, T. (1995), *Simulation-based Reliability Assessment for Structural Engineers*, CRC Press, Inc., Boca Raton, Florida.
Melcher, J., Kala, Z., Holicky, M., Fajkus, M. and Rozlivka, L. (2004), "Design characteristics of structural steels based on statistical analysis of metallurgical products", *J. Construct. Steel Res.*, **60**(3-5), 795-808.
Melchers, R.E. (1999), *Structural Reliability: Analysis and Prediction*, 2nd edition, John Wiley & Sons, Chichester.
Melchers, R.E., Ahammed, M. and Middleton, C. (2003), "FORM for discontinuous and truncated probability density functions", *Struct. Safe.*, **25**, 305-313.
Nowak, A.S. and Collins, K.R. (2000), *Reliability of Structures*, McGraw-Hill, Boston.
Onoufriou, T., Dan, M. and Frangopol, D.M. (2002), "Reliability-based inspection optimization of complex structures: a brief retrospective", *Comput. Struct.*, **80**(12), 1133-1144.
Raizer, V.D. (1998) *Theory of Reliability in Structural Design*, ACEU Press, Moscow. (in Russian)
Shi, Y., Deodatis, G. and Betti, R. (2007), "Random field-based approach for strength evaluation of suspension bridge cables", *J. Struct. Eng.*, **133**(12), 1690-1699.
Tesar, A. and Tvrda, K. (2006), "Energy approach for analysis of nonlinear time response", *Build. Res. J.*, **54**(2), 101-122.