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Optimization of trusses under uncertainties with harmony search

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Abstract. In structural engineering there are randomness inherently exist on determination of the loads, strength, geometry, and so on, and the manufacturing of the structural members, workmanship etc. Thus, objective and constraint functions of the optimization problem are functions that depend on those randomly natured components. The constraints being the function of the random variables are evaluated by using reliability index or performance measure approaches in the optimization process. In this study, the minimum weight of a space truss is obtained under the uncertainties on the load, material and cross-section areas with harmony search using reliability index and performance measure approaches. Consequently, optimization algorithm produces the same result when both the approaches converge. Performance measure approach, however, is more efficient compare to reliability index approach in terms of the convergence rate and iterations needed.

Keywords: reliability based design optimization; reliability index approach; performance measure approach; harmony search.

1. Introduction

There are unavoidable uncertainties associated with the parameters, such as loading, material properties etc., used in the optimization process. Thus, due to the uncertainties it is not quite reliable to define the parameters as being deterministic. The results obtained at the end of the traditional (deterministic) optimization process ignoring uncertainties, generally satisfy the defined conditions at limit level. Therefore, the variation or fluctuation on parameters might cause the violation of the conditions satisfied at limit level in the deterministic optimization. When fluctuations of the loads, variability of the material properties, environmental data and the analytical models are taken into account in the optimization process in terms of probability theory the analysis discipline termed as Reliability-Based Design optimization (RBDO) (Enevoldsen and Sorensen 1994, Gasser and Schueller 1997, Choi *et al.* 2007) arises. The constraints of the optimization problem will be the functions of the parameters that are taken into account as being random. Hence it is possible to

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define RBDO as the minimization of an objective function under the conditions depending on the random variables.

The evaluation of constraints depended on random variables is required to use one of reliability analysis methods (Madsen *et al*. 1986, Melchers 2001, Karadeniz and Vrouwenvelder 2006) in the RBDO process. Failure probability (P_f) of the limit sate function defined for the probabilistic constraint is approximately calculated with those. However, in the RBDO, the evaluation of constraints is generally made with the reliability index (β) related to P_f. β is compared with the defined minimum level (β_t) and it is expected to be $\beta \geq \beta_t$. During the RBDO process the evaluation of probabilistic constraints with the stated approach is known as Reliability Index Approach (RIA). Another approach preferred to handle the constraint(s) is to determine the satisfaction of the corresponding constraint(s) by the sign of the target performance for β_t (Lee *et al*. 2002). This approach proposed first by Tu (1999), Tu *et al*. (1999) uses the inverse reliability analysis (Der Kiureghian *et al*. 19994, Li and Foschi 1998) in contrast RIA and it is called Performance Measure Approach (PMA).

For the solution of an optimization problem, one of the optimization method based on the mathematical theory or the meta-heuristic algorithms is generally carried out. From the definition, the methods based on the mathematical theory and the meta-heuristic algorithms follow the mathematical principles and mimic natural phenomena, respectively. In the last decade, the optimization methods developed by imitating the natural process or the natural phenomena have been encountered. One of those is the Harmony Search (HS) (Geem 2000, Geem *et al*. 2001) which is inspired by music phenomenon, namely the process of searching for better harmony. It has being successfully employed to solve the deterministic optimization of engineering systems (Geem 2000, Geem *et al*. 2001, Saka 2007, Degertekin 2008).

In this paper, the minimum weight of a space truss is obtained by using both reliability index approach (RIA) and performance measure approach (PMA) with harmony search (HS) under the uncertainties in the loading, material properties and cross-section of members. For this purpose, an algorithm including the integrants previously stated is developed for the RBDO of plane and space trusses. After showing the correctness of the algorithm in terms of a numerical example taken from technical literature it is employed to solve the RBDO problem of space truss.

2. Formulation of a reliability based design optimization problem

A reliability based design optimization (RBDO) problem can be defined as (Enevoldsen and Sorensen 1994, Gasser and Schueller 1997, Choi *et al*. 2007)

$$\begin{aligned} &\text{find} \quad d \\ &\text{min.} \quad W(d) \end{aligned} \tag{1a}$$

$$\text{subject to} \quad P_{f,k} = P(g_k(d,X) \leq 0) \leq \bar{P}_{f,k} \qquad k = 1, \ldots, p \tag{1b}$$

$$d_{lower} \leq d \leq d_{upper} \tag{1c}$$

in which, $d = [d_i]^T$ ($i = 1, \ldots, n$) is the design variables vector, $X = [X_j]^T$ ($j = 1, \ldots, m$) is the vector of random variables, W is the objective function, $g_k(d, X)$ is the k^{th} limit state function depending on both of d and X, p is the total probabilistic constraints number, $P_{f,k}$ is the failure probability of

$g_k(d, X)$ given in Eq. (1b), $\overline{P}_{f,k}$ is the specified upper level for $P_{f,k}$, P is the probability of an event, d_{lower} and d_{upper} are the vectors showing the lower and upper limit for the continuous design variables. However, if the design variables of the optimization problem are discrete d_{lower} is equal to one whereas d_{upper} is the maximum section number considered for design variables. d_{lower} is equal to one since adopted section list for the discrete design variables is numbered starting from one. d_{upper}, therefore, represents the maximum section number for the adopted list. In the RBDO applications, design variables of the optimization can be assumed as being random nature. In this case, the mean values of the distributions specified for their probabilistic models are considered as being design variables. The constraint(s) given in Eq. (1b) might be called as probabilistic or reliability constraints in the RBDO applications.

The existence of reliability constraint(s) requires a reliability analysis method to be inserted in the optimization process. Thus, an algorithm developed for RBDO includes three distinct components. First one is a structural analysis program to calculate structural response. An optimization program is the second integrant and it is used to find the design variables that minimize the objective function subjected to pre-specified conditions. The last integrant, a reliability analysis program helps the evaluation of the reliability constraints as functions of d and X. The RBDO process including three different programs mentioned above requires all three parts to be linked together interactively and effectively. Even though it suffers from the computation time when comparing with other strategies (Kuschel and Rackwitz 1997, Cheng *et al*. 2006, Kharmanda *et al*. 2002) the double loop strategy is preferred due to its simplicity. The optimization program initiates the design variables in the double loop strategy first. Then the reliability calculation for the reliability constraints is performed by the reliability analysis program for the current design. The structural analysis program is called whenever the structural response needs to be known during the process. The process continuous until an acceptable result is reached or a termination criterion is satisfied. In this paper, the double loop strategy is employed to implement in any general-purpose optimization and structural analysis software.

3. Reliability analysis

The rules and the conditions are expected to be satisfied when designing an engineering structure. Simply, those are stresses in elements and displacements of joints, which must be equal or lower the allowable stress and displacement, respectively. And reliability is the probability of a structure to perform its intended function. In other words, reliability is known as the probability of that the resistance of the structure (capacity) is greater than the action (load effect) on the structure. For a beam example, say R is the moment-carrying capacity and S is the bending moment due to the applied load. A performance function, or limit state function can be stated as

$$g(R, S) = R - S \tag{2}$$

It can be seen from Eq. (2) that when $g > 0$ beam performs its intended function. If $g < 0$ it does not perform desired performance. In addition, $g = 0$ represents the limit state corresponding to the boundary between desired and undesired performance. Mathematically, the probability of failure (P_f) of Eq. (2) can be defined as (Melchers 2001, Madsen *et al*. 1986, Karadeniz and Vrouwenvelder 2006)

$$P_f = P(g < 0) = P(R - S < 0) \tag{3}$$

in which $P(.)$ is the probability of an event. Due to uncertainties, if both R and S are considered as random variables Eq. (3) can be reformulated as

$$P_f = P(R < S|S = s)P(S = s) \tag{4a}$$

where $P(.|.)$ is the conditional probability of an event and if the events are independent Eq. (4a) can be written as

$$P_f = \int_{-\infty}^{+\infty} \int_{-\infty}^{s} f_R(r) f_S(s) dr ds \tag{4b}$$

Considering the definition of cumulative distribution function of a random variable Eq. (4b) is finally obtained to be

$$P_f = \int_{-\infty}^{+\infty} F_R(s) f_S(s) ds \tag{4c}$$

where $F_X(x)$ and $f_X(x)$ are the cumulative distribution and probability density functions of a continuous random variables respectively. Taking this integral is difficult in general because of indeterminacy or complexity of their probability density functions. Therefore, in practice, the probability of failure (P_f) is calculated using different approximate methods (Melchers 2001, Madsen *et al*. 1986, Karadeniz and Vrouwenvelder 2006), and the process is known as "reliability analysis".

The methods used to perform the reliability analysis can be separated into two main groups called as simulations and moments, respectively. The moment methods are widely used in RBDO applications due to their efficiency when comparing with the simulation methods that are computationally expensive, especially Monte Carlo method. The first order and second order reliability methods are the moment methods, which are commonly employed and known. The SORM (Second Order Reliability Method) is computationally expensive since it requires second order derivative of related limit state function with respect to variables of the problem. Therefore, FORM (First Order Reliability Method) is generally preferred in RBDO procedures.

4. Evaluation of reliability constraints

The evaluation of reliability constraints defined in Eq. (1b) can be carried out by two ways for RBDO problem.

4.1 Reliability index approach

A reliability based design optimization (RBDO) problem, which the constraints are evaluated by using reliability index, is called RBDO based on reliability index approach (RIA) and can be defined as

$$\begin{aligned} &\text{find} && d \\ &\min && W(d) \\ &\text{subject} && \beta_k(d, X) \geq \beta_{t,k} \qquad k = 1, \ldots, p \\ & && d_{lower} \leq d \leq d_{uppeer} \end{aligned} \tag{5}$$

in which β_k and $\beta_{t,k}$ are the structural and target reliability indexes for the kth limit state, respectively. The transformations between Eq. (1b) and β_k and $\beta_{t,k}$, as stated below, are valid (Melchers 2001, Madsen *et al.* 1986, Karadeniz and Vrouwenvelder 2006).

$$\begin{aligned} \beta_k &= -\Phi^{-1}(P_{f,k}) \Leftrightarrow P_{f,k} = \Phi(-\beta_k) \\ \beta_{t,k} &= -\Phi^{-1}(\bar{P}_{f,k}) \Leftrightarrow \bar{P}_{f,k} = \Phi(-\beta_{t,k}) \end{aligned} \tag{6}$$

where $\Phi(.)$ and $\Phi^{-1}(.)$ are the cumulative distribution function for the standard normal distribution and its inverse, respectively.

Reliability index (β) is defined as the minimum distance between the origin and the limit sate function where $g(.) = 0$ in the standard normal space (U-space). Thus, it is possible to formulate this definition as an optimization problem with an equality constraint in U-space as

$$\begin{aligned} &\text{find} && u \\ &\min && \beta = (u^T u)^{1/2} \\ &\text{subject} && g(u) = 0 \end{aligned} \tag{7}$$

where u are the uncorrelated normalized variables obtained by transforming the random variables, X, as

$$F_X(x) = \Phi(u) \Rightarrow u = \Phi^{-1}(F_X(x)) \tag{8}$$

Reliability index (β) is obtained by solving Eq. (7) with any optimization methods. In addition, it is also possible to obtain β using any of the reliability analysis method mentioned above. In the current work, β is calculated using FORM. The limit state function is expanded into Taylor series at the points in FORM. The determinacy of those is fulfilled with the subsequent steps running until satisfying an acceptable convergence. The updated formula proposed by Hasofer-Lind and Rackwitz-Fiessler (HL-RF) for seeking of point is as follows (Melchers 2001, Lee *et al.* 2002, Karadeniz and Vrouwenvelder 2006)

$$u^{(i+1)} = \frac{G^T_{u^{(i)}} u^{(i)} - g(u^{(i)})}{G^T_{u^{(i)}} G_{u^{(i)}}} G_{u^{(i)}} \tag{9}$$

where i is the iteration number, $u^{(i)}$ is the normalized variables at i th iteration, $g(u^{(i)})$ is the value of limit state function calculated for $u^{(i)}$, $G_{u^{(i)}}$ and $G^T_{u^{(i)}}$ are respectively the gradient vector of corresponding limit state function for $u^{(i)}$ and its transpose. $G_{u^{(i)}}$ is defined as

$$G_{u^{(i)}} = \left\{ \frac{\partial g}{\partial u_1}, \frac{\partial g}{\partial u_2}, \ldots, \frac{\partial g}{\partial u_n} \right\} \tag{10}$$

Considering Eq. (8) it is also possible to express $G_{u^{(i)}}$ as

$$G_{u^{(i)}} = \left\{ \frac{\partial g}{\partial x_1}\frac{\partial x_1}{\partial u_1}, \frac{\partial g}{\partial x_2}\frac{\partial x_2}{\partial u_2}, \ldots, \frac{\partial g}{\partial x_n}\frac{\partial x_n}{\partial u_n} \right\} \tag{11}$$

The procedure of seeking of point known as the most probable point (MPP) is usually started assigning $u = 0$ and it is continued until the difference in successive u values is less than or equal to a prescribed error taken as $|(u^i - u^{i-1})/u^i| \leq \varepsilon = 0.001$. When the convergence is satisfied the reliability index $\beta = (u^T u)^{1/2}$ is computed. However, the possibility of a negative value of β is ignored. The sign of the reliability index, therefore, should be corrected according to the sign of g when $u = 0$.

4.2 Performance measure approach

If reliability constraints of a RBDO problem are evaluated by using performance measure approach (PMA), the RBDO can be expressed by

$$\begin{array}{ll} \text{find} & d \\ \min & W(d) \\ \text{subject} & g_k^* \geq 0 \qquad k = 1, \ldots, p \\ & d_{lower} \leq d \leq d_{uppeer} \end{array} \tag{12}$$

where g_k^* is the performance measure of kth reliability constraint corresponding to the target reliability $\beta_{t,k}$ (Lee *et al.* 2002, Tu 1999, Tu *et al.* 1999). In other words, g_k^* is the value of the kth limit state function calculated for $u_{\beta_k = \beta_{t,k}}$ in the normalized space. Whether the reliability constraint is satisfied or not can be controlled according to the sign of g_k^* as it can be seen from Eq. (12). Therefore, since the negative value of a limit state function indicates the failure the sign of g_k^* can be used as a measure to determine whether a reliability constraint is satisfied (Lee *et al.* 2002). Thus, it is necessary to seek for the point (u) where the distance from the origin in normalized space is equal to the target reliability index β_t, and which also makes the limit state function minimum. From the definition, it can be possible an optimization problem to be formulated with an equality constraint in U-space as

$$\begin{array}{ll} \text{find} & u \\ \min & g^* = g(u) \\ \text{subject} & \|u\| = (u^T u)^{1/2} = \beta_t \end{array} \tag{13}$$

u can be obtained by solving of Eq. (13) with any optimization methods. Besides, the inverse reliability analysis based on FORM (Lee *et al.* 2002, Tu 1999, Tu *et al.* 1999) is also used as a tool to calculate u. The updated formula of the algorithm developed to solve the problem in Eq. (13) is given by

$$u^{(i+1)} = -\beta_t \frac{G^T_{u^{(i)}}}{\sqrt{G^T_{u^{(i)}} G_{u^{(i)}}}} \tag{14}$$

The procedure continues until the difference in successive u values is less than a prescribed error taken as $|(u^i - u^{i-1})/u^i| \le \varepsilon = 0.001$. The value of limit state function calculated for the converged values is taken as g^* and the reliability constraint is evaluated (Tu 1999, Tu *et al*. 1999).

5. Sensitivity analysis

Sensitivity analysis quantifies the influence of each parameter on model, function, response, etc. It is crucial integrant both for the reliability analysis and the optimization methods based on the mathematical theory. For the reliability analysis based on FORM, the updated formula given in Eqs. (9) and (14) needs the gradient information ($G_{u^{(i)}}$) of the limit state function with respect to random variables.

Two distinct ways can be employed to calculate $G_{u^{(i)}}$. The related gradient information in Eq. (10) can be directly calculated in normalized space in the first way. In the second, applying the chain rule of differentiation, the gradient of the limit state function is calculated in the original space and then those are multiplied with the derivatives of corresponding random variables calculated in the normalized space (Eq. (11)). Since the value of limit state function is generally obtained after performing the structural analysis for a structural engineering problem the second way for obtaining the related gradient information is easily linked to the structural analysis program.

The calculation of the first term $\partial g/\partial x$ in Eq. (11) is performed by means of the structural analysis program for the engineering problems in general. Those used for this purpose are generally based on finite element method (FEM). The gradient information $\partial g/\partial x$ is consequently calculated using (Haug *et al*. 1986, Mohamed and Lemaire 1999) one of; 1) Finite difference method, 2) Direct differentiation and 3) Adjoin method.

The second term of Eq. (11) $(\partial x/\partial u)$ can be easily calculated considering Eq. (8) as

$$\frac{\partial x}{\partial u} = \frac{\partial F_X^{-1}(\Phi(u))}{\partial u} = \frac{\phi(u)}{f_X(x)} \tag{15}$$

in which $\phi()$ and $f_X(x)$ are respectively the probability density function of the standard normal distribution and the corresponding random variable.

The linear elastic static analysis of the structures under the external load can be stated as based on FEM terminology

$$Kq = F \tag{16}$$

where K is the structural stiffness matrix, q is the vector of nodal displacements and F is the vector of applied forces. The responses of the structure obtained after performing the linear elastic static analysis are used in the evaluation of the constraints that are generally given by

$$g_i = 1.0 - |\sigma_i|/\sigma_i^a \le 0 \qquad i = 1, 2, \ldots, m \tag{17}$$

$$g_{j+m} = 1.0 - |q_j|/q_j^a \le 0 \qquad j = 1, 2, \ldots, r \tag{18}$$

where σ_i is the stress in the ith member and σ_i^a is the allowable stress for the same member, q_j is the displacement of the jth node and q_j^a is its upper bound. Thus, the functions defined for the constraints are implicit functions of the variables, $s(=d \cup X)$. The derivatives of the constraint function with respect to s according to methods mentioned above are calculated (Haug *et al*. 1986, Mohamed and Lemaire 1999). However, since the adjoint method is employed in this study a short explanation is given below.

5.1 Adjoint method

Using the chain rule of differentiation, the total derivative of g with respect to s may be calculated as

$$\frac{dg}{ds} = \frac{\partial g}{\partial s} + \frac{\partial g}{\partial q}\frac{dq}{ds} \tag{19}$$

Differentiating both sides of Eq. (16) with respect to s, dq/ds can be stated as

$$\frac{dq}{ds} = K^{-1}\left[\frac{\partial F}{\partial s} - \frac{\partial K}{\partial s}q\right] \tag{20}$$

This result is substituted into Eq. (19) to obtain

$$\frac{dg}{ds} = \frac{\partial g}{\partial s} + \frac{\partial g}{\partial q}K^{-1}\left[\frac{\partial F}{\partial s} - \frac{\partial K}{\partial s}q\right] \tag{21}$$

If an adjoint variables vector λ is introduced as

$$\lambda \cong \left[\frac{\partial g}{\partial q}K^{-1}\right]^T = K^{-1}\frac{\partial g^T}{\partial q} \tag{22}$$

and both sides of Eq. (22) is multiplied by the matrix K, Eq. (23) is obtained as follows.

$$K\lambda \cong \frac{\partial g^T}{\partial q} \tag{23}$$

After λ in Eq. (23) is solved and substituted into Eq. (21), it becomes

$$\frac{dg}{ds} = \frac{\partial g}{\partial s} + \lambda^T\left[\frac{\partial F}{\partial s} - \frac{\partial K}{\partial s}q\right] \tag{24}$$

6. Numerical examples

The reliability based design optimization (RBDO) of a 10 bar truss example from the technical literature is investigated to demonstrate the applicability and the accuracy of the RBDO algorithm explained above with the integrants. Since the design variables of the 10 bar truss example are considered as continuous in Lee *et al*. (2002) they are also assumed as continuous in the current study for the comparison. Later, the proposed RBDO algorithm is applied to minimize the weight of 200 bar space truss under the uncertainties associated with the loading, material properties and cross-section areas of members. The reliability constraints are evaluated by using the reliability index approach (RIA) and the performance measure approach (PMA). For this example, the design variables, d, of the optimization is considered as discrete since the discrete design offers an applicable design from the practice point of view.

Different methods can be preferred for the integrants of an RBDO algorithm. In this study, first order reliability method (FORM) and inverse reliability method based on FORM are employed for the reliability analysis. A structural analysis program based on the matrix displacement method is coded and used to calculate the responses of the structures. For the optimization, instead of the sequential quadratic programming (SQP), that is preferred much in RBDO application, a meta-heuristic algorithm called harmony search is used.

Harmony search (HS) developed by Geem (2000) and Geem *et al*. (2001) is a population-based optimization technique inspired by the idea of seeking the musician with a better state of harmony. HS algorithm sets up firstly a randomly generated harmony memory (HM) matrix. The total number of design variables n and a pre-selected parameter m represent the number of column and row of the HM. The number of the individuals in the population is adjusted by a variable called harmony memory size (HMS). The matrix is sorted in descending order according to the objective function value of the row showing a possible candidate solution. Three rules are applied for composing a new harmony. The values of design variable i ($i = 1, 2, ..., n$) can be randomly selected from the set of all candidate values with a probability of P_{Random} (random selection); it can be selected from the set of good values stored in computer memory with a probability of HMCR (harmony memory consideration rate); or it can be slightly adjusted by moving to neighboring values once the value is selected from the set of stored good values, with a probability of PAR (pitch adjusting rate). The HM is updated with better design vectors with iteration. If newly generated vector is better than the worst vector stored in the HM in terms of objective function value, the new vector is swapped with the worst one. This process ends up when predetermined termination criteria is satisfied (Geem 2000, Geem *et al*. 2001, Saka 2007, Degertekin 2008).

The objective function value of each candidate solution also includes a penalty function showing the fitness of it.

$$\Omega(d) = W(d)(1 + CPE(d)) \tag{25}$$

where $\Omega(d)$ is the modified objective function, $PE(d)$ is the constraint violation function and C is a coefficient taken as 100. The constraint violation function composes the sum of all constraint violation values. If there is no violation for considering constraint the value of violation becomes zero.

An algorithmic procedure to solve a RBDO problem outlined above can be summarized as follows:

Step 1 Specify the target reliability index and initial values of the continuous design variables. If the variables are discrete then define the section list.

Step 2 Form an initial harmony memory matrix in a random manner.

Step 3 Perform reliability analysis depended on the RIA or PMA.

Step 4 Check the constraints and calculate the objective function.

Step 5 Repeat Step 3 and 4 for all candidate solution in the harmony memory matrix.

Step 6 Generate new vector according to HS rules and run over Step 3 and 4.

Step 7 Refresh or save the harmony memory matrix depending on the objective value of new generated vector and worst vector stored in the harmony memory matrix.

Step 8 Return to Step 6 and 7 until a stopping criterion has been accomplished.

6.1 Ten bar truss

The 10-bar truss as shown in Fig. 1 is one of the standard test problems used by researchers to demonstrate the efficiency and accuracy of their algorithm. The accuracy of the proposed RBDO algorithm and the efficiency of the harmony search are examined by comparing the results obtained in the study with those obtained by Lee *et al*. (2002).

In the RBDO of ten bar truss structure performed by Lee *et al*. (2002) using the RIA and PMA, the cross-section area, A_i, of each member of ten bar truss structure was treated as a normal random variables and its mean value was adopted as the design variable of the optimization. In addition, Young's modulus, E, and the external force, P, were also considered as random variables with the normal distribution. It is required that the mean of each section area should not be less than 64.516 mm^2. The limit state function given in Eq. (26) is considered as a reliability constraint.

$$g(d,X) = 50.8 - q_2 \leq 0 \tag{26}$$

where q_2 is the vertical deflection at node 2, which should be less than 50.8 mm. The statistical properties of A_i, E, and P are presented in Table 1. The prescribed acceptable reliability index, β_t, is 3.0. The weight of structure is taken as the objective function. A value of 2.770E-06 kg/mm^3 is assumed for the material density.

In Table 2, the results obtained by the RIA and PMA as well as those from Lee *et al*. (2002) are summarized. The iteration histories of RBDO process are respectively shown in Figs. 2 and 3 for the PMA and RIA.

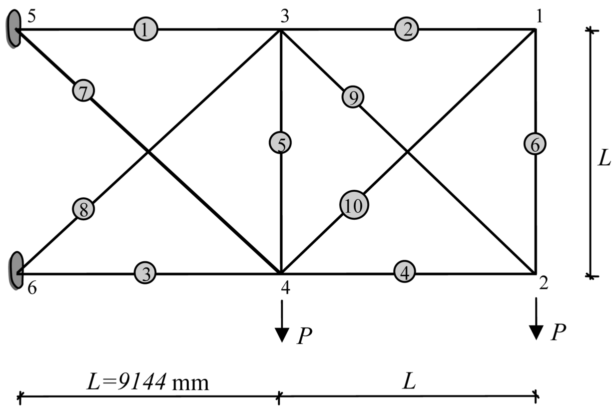
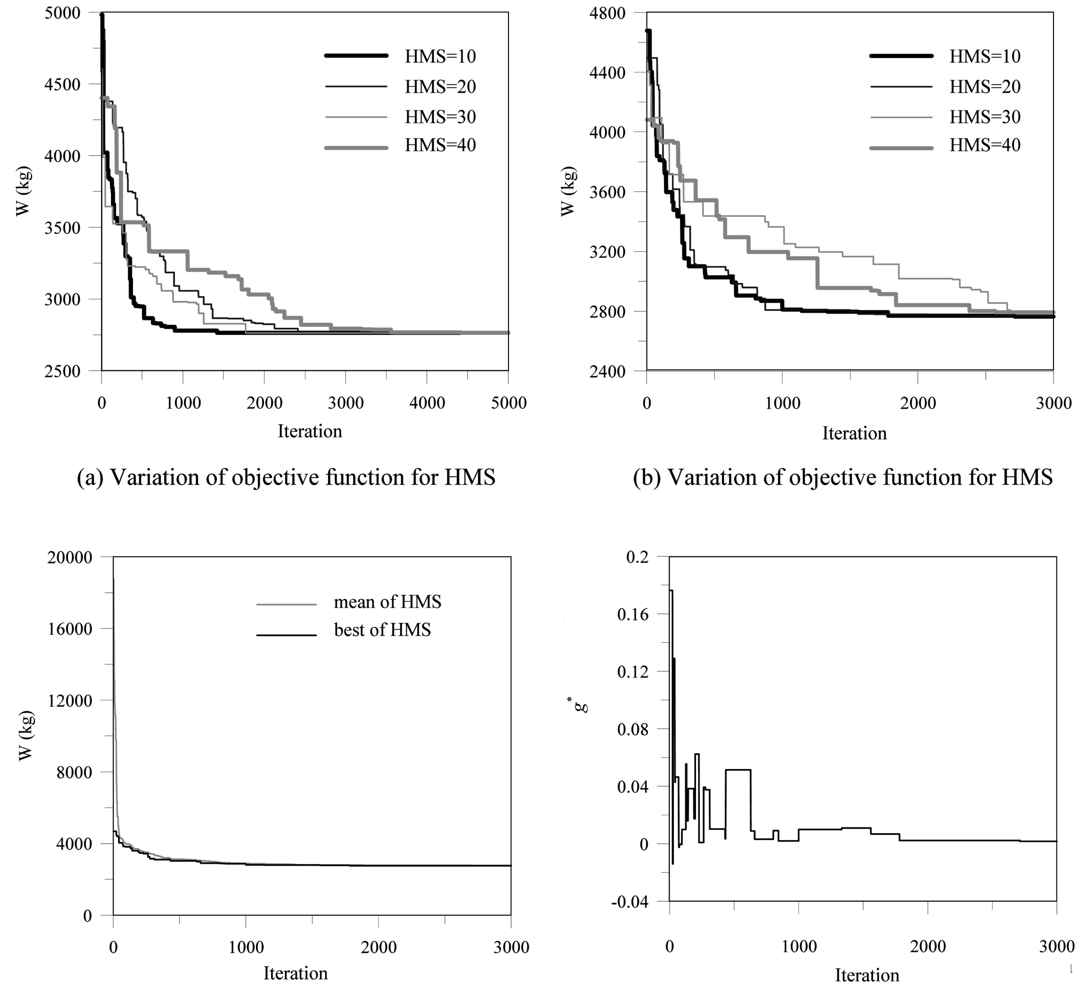


Fig. 1 10-bar plane truss

Table 1 Statistical property for the 10-bar truss example

	Description	Distribution	Mean	CoV
E	Young's modulus (kN/mm^2)	Normal	68.950	0.05
P	External load (kN)	Normal	444.80	0.05
A_i	Area i = 1, …, 10 (mm^2)	Normal	μ_{Ai}	0.05

μ_{Ai} the design variables of the optimization



(a) Variation of objective function for HMS

(b) Variation of objective function for HMS

(c) Variation of objective function for the best solution

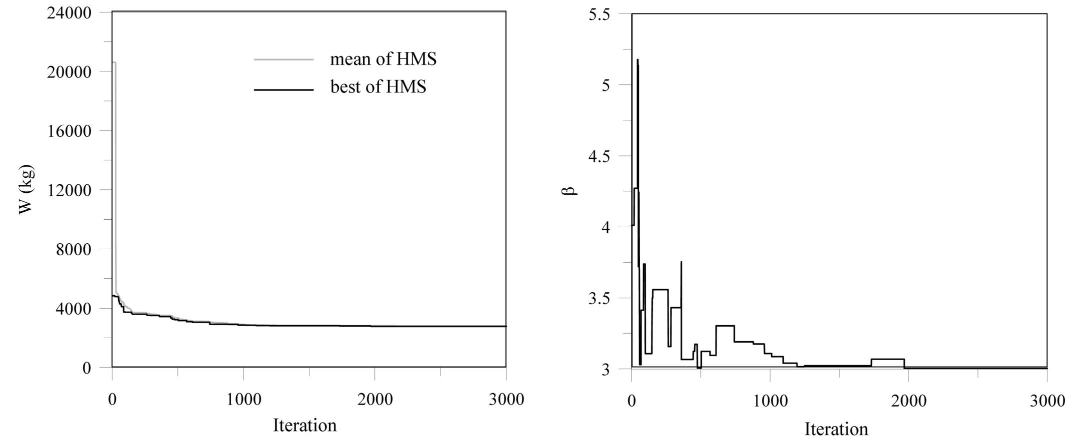
(d) Variation of performance function for the best solution

Fig. 2 Histories of RBDO process of 10-bar truss example for PMA

In Fig. 2(a), the variation of the objective function is illustrated for the harmony memory size (HMS) from 10 to 40 for a total number of searches taken as 5000. It is noticed that the results remain the same. It is also observed that the result is almost the same when the number of maximum iteration taken as 3000, Fig. 2(b). Therefore, a value of 10 and 3000 are respectively considered for HMS and the total number of searches in order to shorten the computational cost of RBDO process. The example is designed several times using different harmony memory considering rate (HMCR) and pitch adjusting rate (PAR). The results reported here correspond to the best that are having the least weight and they are obtained when HMCR = 0.90 and PAR = 0.35. Fig. 2(c) shows the histories of the best solution and the mean value of the corresponding HMS. Besides, the variation of the reliability constraint for PMA is presented in Fig. 2(d). When the RIA is adopted in RBDO process of the 10-bar truss example the corresponding histories for the objective function and the reliability constraints are illustrated in Figs. 3(a) and 3(b), respectively.

The RBDO process based on PMA takes 14.936 sc and the total number of iterations by the reliability analysis is 11802. In the case of using RIA in RBDO to evaluate the reliability constraint, to reach the results takes 15.722 sc and the total number of iterations by the most probable point (MPP) search is 11980. When the forward finite difference is employed to calculate the derivatives of limit state function instead of adjoin method the RBDO process based on PMA takes 130.766 sc.

The results obtained in this study shows an agreement with those reported in Lee *et al.* (2002). Therefore, it can be stated that the RBDO algorithm proposed in this study is accurate enough and works well. The sequential linear programming (SLP) was used by Lee *et al.* (2002) for the optimization. Since harmony search is employed in this study the performance of proposed RBDO process according to computational cost is different from Lee *et al.* (2002).



(a) Variation of objective function for the best solution (b) Variation of reliability index for the best solution

Fig. 3 Histories of RBDO process of 10-bar truss example for RIA

Table 2 Summary of results for the 10-bar truss example

Design variables	Reliability index approach (RIA)		Performance measure approach (PMA)	
	Lee *et al.* (2002)	This study	Lee *et al.* (2002)	This study
A_1 (mm^2)	24845.112	25516.723	25264.466	25699.948
A_2	64.520	64.520	64.520	64.520
A_3	17341.901	16703.257	17851.577	17245.127
A_4	12303.201	11205.139	12361.266	12045.137
A_5	64.520	64.520	64.520	64.520
A_6	64.520	64.520	64.520	64.520
A_7	3987.089	2360.640	1703.223	2020.641
A_8	17645.126	18451.576	18083.835	17432.223
A_9	17503.191	18465.770	17735.448	18564.479
A_{10}	64.520	64.520	64.520	64.520
W (kN)	27.872	27.654	27.543	27.600
β	3.0	3.0	3.0	3.0

Actually, the heuristic algorithms such as harmony search, genetic algorithm, evolution strategy etc., do not need any gradient information related to objective function and constraints. Although they have a good feature, the heuristic algorithms generally require more iteration to reach the result due to randomness compare to the optimization methods based on mathematical programming. This causes of losing the performance, especially the increasing computational time.

It is seen from the results presented in Table 2 that the uses of PMA and RIA in RBDO process introduce slightly different results. Even though they produce same results when the two approaches converge, PMA is more robust and efficient compared to the RIA since the reliability constraint is determined along the line $\beta = \beta_t$ by the PMA (Tu 1999, Tu *et al*. 1999, Lee *et al*. 2002). As indicated in Lee *et al*. (2002), when the big value is taken as the population size for the population based heuristic algorithms, abnormal terminations are sometimes encountered in the RBDO process. The rationale of this termination is due to generation of population randomly within the entire design space, in which some points in the solution space may cause numerical instabilities depending on initial values. In that case, to handle the mentioned drawback, either the population size is decreased or the population is reinitialized.

6.2 200 bar space truss

The space truss, a 200-bar roof truss shown in Fig. 4, is investigated as another design problem to demonstrate the accuracy of the proposed RBDO algorithm and the efficiency of the harmony search. The top chord joints of the space truss are subjected to vertical loading of *P*, which is treated as a lognormal random variable. The cross-sections of members are collected into three groups. One of the groups contains the bottom chord members. Diagonals are grouped together as another one, and finally top chord members are collected in the third group. The cross-section areas, A_i, of each member of 200-bar space truss structure are treated as normal random variables and its mean values are the design variables of the optimization. Circular hollow sections given in Table 3 are adopted for the members of the space truss in the study. Young's modulus, *E*, the allowable stress, σ^a, are also considered as random variables with the lognormal distribution. A value of 7.85E-06 kg/mm^3 is assumed for the material density.

The first limit state is defined as nodal maximum vertical displacement, which should be less than

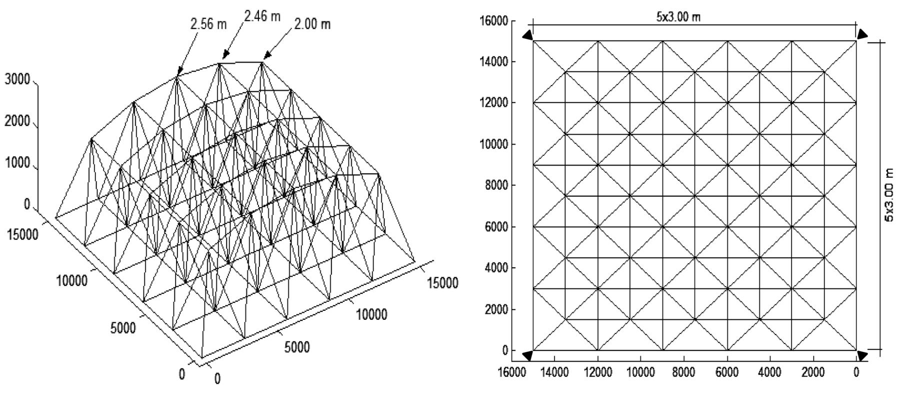


Fig. 4 200-bar space truss

Table 3 List of circular hollow sections used for the 200-bar space truss

Section no	A (mm^2)	r (mm)	Section no	A (mm^2)	r (mm)	Section no	A (mm^2)	r (mm)
1	430.0	16.0	9	1120.0	25.0	17	2040.0	38.0
2	560.0	16.0	10	810.0	30.0	18	2670.0	38.0
3	680.0	15.0	11	1070.0	30.0	19	1170.0	44.0
4	540.0	20.0	12	1320.0	30.0	20	1550.0	44.0
5	710.0	20.0	13	1560.0	29.0	21	1920.0	43.0
6	870.0	20.0	14	1050.0	39.0	22	2280.0	43.0
7	690.0	26.0	15	1390.0	39.0	23	2390.0	43.0
8	910.0	26.0	16	1720.0	39.0	24	2990.0	42.0

A cross-section area; r radii of gyration

Table 4 Statistical property for the 200-bar truss example

	Description	Distribution	Mean	CoV
A_i	Area i = 1-3 (mm^2)	Normal	μ_{Ai}	0.05
E	Young's modulus (kN/mm^2)	Lognormal	210.0	0.05
P	External load (kN)	Lognormal	13.50	0.09
σ^a	Allowable stress (N/mm^2)	Lognormal	150.0	0.06

μ_{Ai} the design variables of the optimization

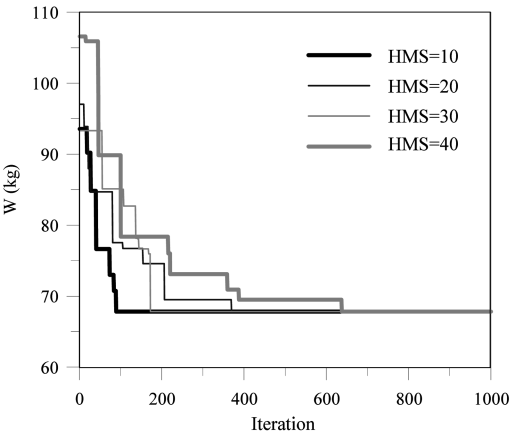
Table 5 Results for the 200-bar truss example

Design variables	Reliability index approach (RIA)	Performance measure approach (PMA)
A_1 (mm^2)	690.0	690.0
A_2	1920.0	1920.0
A_3	1170.0	1170.0
W (kN)	67.8013	67.8013

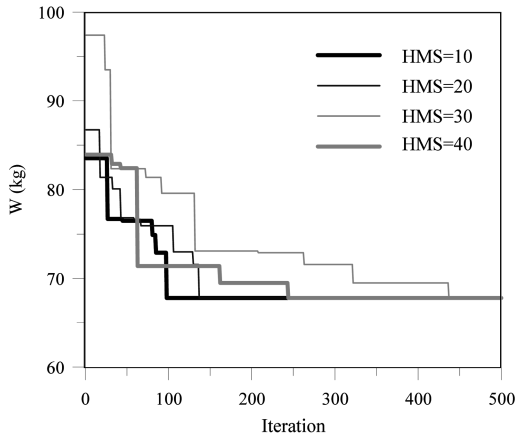
50 mm. the second, third, and fourth limit sates are such that the maximum tensile and compressive stress in each element group should not be grater than allowable stress, σ^a, and the allowable compressive stress, σ_c, computed depending on the Turkish design code for each member (see Appendix). The prescribed acceptable reliability index, β_t, is 3.70 (JCSS 2000). The weight of structure is taken as the objective function. The statistical properties of A_i, E, P and σ^a are presented in Table 4.

The results obtained by the proposed RBDO algorithm using RIA and PMA are presented in Table 5. The iteration histories of RBDO process are respectively shown in Figs. 5 and 6 for the PMA and RIA.

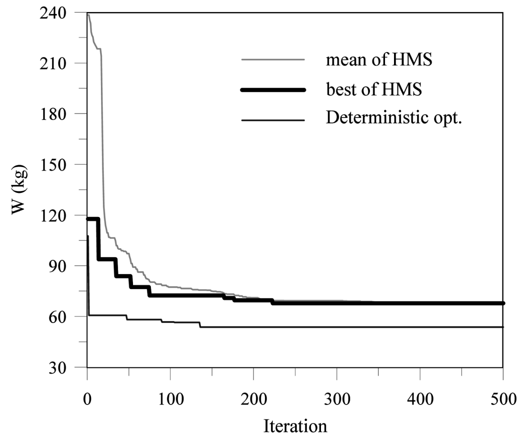
In Fig. 5(a), the variation of the objective function is illustrated for the harmony memory size (HMS) changing from 10 to 40 and the total number of searches taken as 1000. It is noticed that the results remain the same. When the RBDO process of the 200-bar space truss example is



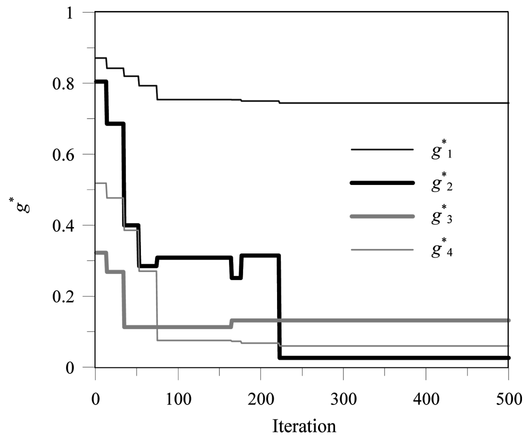
(a) Variation of objective function for HMS



(b) Variation of objective function for HMS



(c) Variation of objective function for the best solution

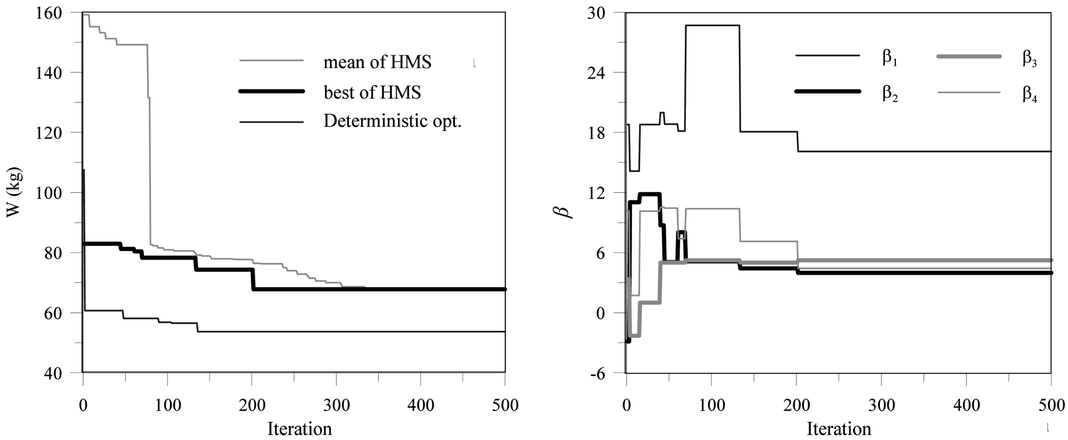


(d) Variation of performance functions for the best solution

Fig. 5 Histories of RBDO process of 200-bar truss example for PMA

repeated for the number of maximum iteration taken as 500 it is also observed that almost the same results are obtained, Fig. 5(b). Therefore, a value of 10 and 500 are considered for HMS and the total number of searches respectively in order to shorten the computational cost of RBDO process. The example is designed several times using different harmony memory considering rate (HMCR) and pitch adjusting rate (PAR). The results reported here correspond to the best having the least weight and they are obtained when HMCR = 0.90 and PAR = 0.45. Fig. 5(c) shows the history of the best solution and the mean value of the corresponding HMS. Besides, the variation of the reliability constraints for PMA is presented in Fig. 5(d). When the RIA is adopted in RBDO process of the 200-bar space truss example the corresponding histories for the objective function and the reliability constraints are illustrated in Figs. 6(a) and 6(b), respectively.

The RBDO process based on PMA takes 319.153 sc and the total number of iterations by the reliability analysis is 8012. In case of using RIA in RBDO to evaluate the reliability constraint, it



(a) Variation of objective function for the best solution

(b) Variation of reliability indices for the best solution

Fig. 6 Histories of RBDO process of 200-bar truss example for RIA

takes 499.571 sc to reach the results and the total number of iterations by the most probable point (MPP) search is 13734. When the forward finite difference is employed to calculate the derivatives of limit state function instead of adjoin method the RBDO process based on PMA takes 972.083 sc.

It is seen from the results presented in Table 5 that the uses of PMA and RIA in RBDO process introduce the same results. As mentioned above and indicated in Tu (1999), Tu *et al*. (1999), and Lee *et al*. (2002), PMA is more robust and efficient compare to the RIA. The minimum weight of the 200-bar space truss obtained as 53.6938 kN [690 mm^2 1390 mm^2 1050 mm^2] after performing its deterministic optimization by using harmony search. Deterministic optimization of the example takes 13.074 sc. Thus, it can be stated that the uncertainties in parameters affects the results obtained by the deterministic optimization and those do not satisfied the intended reliability. The RBDO process requires more computational time than the deterministic one.

It is apparent from Figs. 5(d) and 6(b) that the reliability constraints related to the second and fourth limit state are dominant in the RBDO problem of 200-bar space truss. While none of first and third reliability constraints is close to their lower bounds, the maximum value of second and fourth reliability constraints are close to $g^* = 0$ for PMA and $\beta_t = 3.70$ for RIA. The value of reliability indices (Fig. 5(d)) show large fluctuation compared to performance functions (Fig. 6(b)). This situation arises from the difference of reliability analysis performed for PMA and RIA. For the former, an inverse reliability analysis working on a fixed surface in *U*-space during the entire optimization is employed while that working on the position of surface ($g = 0$) varies with the design point (d) during the entire optimization is performed for the later.

The harmony memory size (HMS) is varied for harmony search in order to observe its influence on the design (Figs. 5(a), (b)). When HMS is increased the number of search required to reach the solution might be greater than the number of search needed for lower HMS. This is because the solutions generated within the solution space show more diversity for big values of HMS. However, an abnormal termination might be encountered during the RBDO process when a larger value is adopted for the population for the population based heuristic algorithms (Lee *et al*. 2002). The smallest possible population and the bigger search number might be used to overcome the abnormal termination. The variation of the harmony memory considering rate (HMCR) and the pitch adjusting rate (PAR) affect the results obtained by harmony search. As mentioned above, the RBDO processes of the examples are repeated with the different values defined for HMCR and PAR. It is apparent from this study that even though the selection of HMCR and PAR is problem dependent it is noticed in Geem *et al*. (2001) using 0.90 and 0.45 for HMCR and PAR respectively have produced the optimum results.

7. Conclusions

In this study, an algorithm is presented to obtain the minimum value of intended objective function under the uncertainties in the parameters taken in the optimization process. The proposed algorithm can evaluate the reliability constraints by two different methods. The performance measure approach (PMA) is more robust and efficient compare to reliability index approach (RIA) in terms of the convergence rate and total number of iteration for the convergence. According to forward finite difference method, the use of direct differentiation method or adjoin method for the calculation of gradient vector ($G_{u^{(i)}}$) required for the reliability analysis increases the computational cost performance of the RBDO process. The RBDO, in contrast deterministic optimization, can be taken into account the uncertainties associated with the parameters and can produce the results with intended reliability. The performance of harmony search used for the optimization method varies depending on the parameters defined for employing it. To shorten the computational cost of RBDO process, the harmony search should be used with the smallest possible population and the enough search number.

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Appendix

According to Turkish design code (TS 648), the permissible compressive stress is calculated as follows,

$$\text{If} \quad \lambda > \lambda_p, \quad \text{Elastic buckling,} \quad \sigma_c = \frac{2\pi^c E}{5\lambda^2}$$

$$\text{If} \quad \lambda < \lambda_p, \quad \text{Elastic buckling,} \quad \sigma_c = \frac{[1-(1/2)(\lambda/\lambda_p)^2]\sigma_y}{n}$$

$$n = 1.5 + 1.2(\lambda/\lambda_p) - 0.2(\lambda/\lambda_p)^3$$

where σ_c = allowable compressive stress; σ_y = yield stress; E = modulus of elasticity; λ = slenderness ratio; λ_p is taken as $\sqrt{2\pi^2 E/\sigma_y}$.