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Estimation of structure system input force using the inverse fuzzy estimator

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Abstract. This study proposes an inverse estimation method for the input forces of a fixed beam structural system. The estimator includes the fuzzy Kalman Filter (FKF) technology and the fuzzy weighted recursive least square method (FWRLSM). In the estimation method, the effective estimator are accelerated and weighted by the fuzzy accelerating and weighting factors proposed based on the fuzzy logic inference system. By directly synthesizing the robust filter technology with the estimator, this study presents an efficient robust forgetting zone, which is capable of providing a reasonable trade-off between the tracking capability and the flexibility against noises. The period input of the fixed beam structure system can be effectively estimated by using this method to promote the reliability of the dynamic performance analysis. The simulation results are compared by alternating between the constant and adaptive and fuzzy weighting factors. The results demonstrate that the application of the presented method to the fixed beam structure system is successful.

Keywords: inverse estimation method; fuzzy kalman filter; least square method; fuzzy logic.

1. Introduction

In the structure design and reliability assessment of bridge system, the most important procedure is to obtain the values of active input forces to the system. However, in the practical engineering problem, there are always difficulties in installing the force transducers used to measure the active forces to the structure system (Yang *et al*. 1997). Furthermore, the force is sometimes large and transient so that the direct measurements will not be easily obtained. To resolve this situation, the active forces to the structure system can be estimated in real time by using the inversely technique. This will be verified as a great benefit on the design and reliability evaluation of the structure system by this means.

The input estimation is an analysis method for the dynamic structure system and is widely adopted to cope with the system with inputs that cannot be easily measured directly. This method does not need the load transducer to be equipped to directly measure the active loads. Instead, the structure system has actually been regarded as a load sensor (Inoue *et al*. 2001), and the results have verified this idea. There are various researches with regard to the input estimation of the structure system in recent years. For example, Michaels and Pao (1985) presented an iterative

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method of deconvolution which determines the orientation and time-dependent amplitude of the force from the transient response of the plate surface at a minimum number of two locations. Fabunmi (1986) presented the pseudoinverse technique to determine the effects produced on several structural modes due to the vibratory forces. Inoue *et al.* (1995) used the least square method, which is based on the wiener filtering theory, the mean square error, and the singular value decomposition (SVD), to improve the estimation precision and to obtain the optimal estimates. Martin *et al.* (1996) analyzed the motion of wave transmission to estimate the impact loads. Doyle (1997) developed the wavelet deconvolution to estimate the impact loads on the beam and plate structure. Recently, Haung (2001) adopted the conjugate gradient method (CGM) to estimate the force of the one-dimensional mass-spring-damper structure with the time-varying system parameters. Taking a comprehensive review of the above references, the estimation algorithms are all implemented in the batch forms. This kind of method is time-consuming and is not an on-line procedure for the unknown input estimation.

In order to solve the problem mentioned above, Tuan *et al.* (1996, 1997) presented an input estimation approach which can recursively solve the IHCPs in real time. Ma *et al.* (1998, 2003) presented an inverse method to estimate the excitation forces by analyzing the dynamic responses of structure system. The input estimation method is using the recursive form to process the measurement data. As opposed to the batch process, the recursive form is an on-line process and has higher effectiveness. However, although the processes in the studies mentioned above can be implemented in real time, they merely adopt the constant weighted estimator to estimate the unknown time-varying inputs. The access to the optimal constant weighting factor should go through the trial-and-error analyses, that is to say, the overall tracking performance of the estimator will be degraded or the measurement error will be magnified when an inappropriate weighting function is chosen in the estimation process.

The unknown inputs usually have slow or quick time-varying state which cannot be predicted in the estimation process. Therefore, it is difficult to choose an adaptive and efficient weighting function for any input variation. Tuan *et al.* (1998) presented an adaptive robust weighted input estimation method for the one-dimensional inverse heat conduction problem. Lee *et al.* (2008) utilized the adaptive weighted input estimation method to inversely solve the burst load of the truss structure system. Chen *et al*. (2008) investigated the adaptive input estimation method applied to the inverse estimation of load input in the multi-layer shearing stress structure and the identification of moving load in the bridge structure system. The overall input estimation performance is acceptable by analyzing the dynamic response of structure system. However, the estimates converge slowly in the initial state when the adaptive weighting function is used in the RLSE. The increase of the process noise variance will influence the estimation precision. With a larger process noise variance assumed, the better capability of tracking the time-varying force inputs can be obtained, but the overall measurement noise reduction effectiveness will be degraded. Therefore, Chen *et al.* (2007) developed an intelligent fuzzy weighted estimator with higher target tracking performance and better noise reduction effectiveness. Lee *et al.* (2010) applied an intelligent fuzzy weighted input estimation method to estimate the unknown input force in a plate structure system. The estimator proposed in this paper provides an efficient and robust estimation procedure to any unknown input situation. However, the overall measurement noise reduction effectiveness may be degraded when a larger initial process noise variance is assumed in the intelligent fuzzy weighted estimator. Besides, the estimates may be divergent in the high order of severity when an inappropriate initial process noise variance is assumed. In this study, the effective estimator are accelerated and weighted by adopting the fuzzy accelerating and weighting factor proposed based on the fuzzy logic inference system. The efficiency and robustness of the proposed method will be demonstrated through the simulation of case studies. The results will be compared with the results produced using other inverse estimation methods. The feasibility, adaptability and robustness of proposed method will be verified by implementing the case studies.

2. Problem formulation

In this paper, a uniform dynamic loading on the fixed beam structural system is considered as shown in Fig. 1. The inverse estimation of the inputs by analyzing the active reaction of the structure system is investigated in this research. According to the Euler-Bernoulli theory, the equation of motion of the structure system can be expressed as (Mario 1986)

$$EI\frac{\partial^4 y}{\partial x^2}+\overline{m}\frac{\partial^2 y}{\partial t^2} = U(x,y) \tag{1}$$

where EI represents the flexural rigidity. $\overline{m}$ and $U(x,t)$ are the mass and the dynamic loading per unit length, respectively. The boundary condition of the fixed end beam can be described as

$$\text{for } x = 0,\ y(0,t) = 0, \quad \text{and} \quad y'(0,t) = 0 \tag{2}$$

$$\text{for } x = L, \quad y(L,t) = 0, \quad \text{and} \quad y'(L,t) = 0 \tag{3}$$

Assuming the general solution of motion equation is the sum of normal model shape function $\Phi_n(x)$ multiplied by the unknown model amplitude $z_n(t)$, the general solution can be described mathematically as

$$y(x,t) = \sum_{n=1}^{\infty}\Phi_n(x)z_n(t) \tag{4}$$

The normal model shape function must satisfy the solution of free vibration of Eq. (1). Therefore, the differential equation of position is concluded and given as

$$\Phi_n^{IV}(x)-a^4\Phi_n(x) = 0 \tag{5}$$

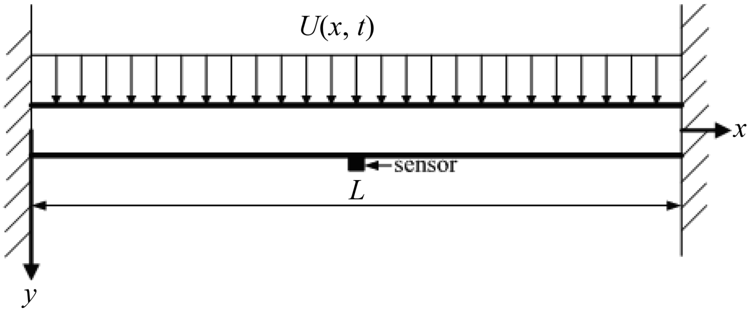

Fig. 1 Uniform dynamic loading on the fixed beam structural system

where $a^4 = \overline{m}\omega^2/EI$. The natural frequency, $\omega = C\sqrt{EI/\overline{m}L^4}$, where $C = (aL)^2$. aL can be solved by the equation of system frequency which satisfies the boundary condition (Tuan and Hou 1998).

Substituting Eq. (4) in Eq. (1) gives

$$EI\sum_{n=1}^{\infty}\Phi_n^{IV}(x)z_n(t) = U(x,t)-\overline{m}\sum_{n=1}^{\infty}\Phi_n(x)\ddot{z}_n(t) \tag{6}$$

Substituting Eq. (5) in Eq. (6), multiplying it by $\Phi_m(x)dx$, integrating it over the length of the beam, and then applying the orthogonal conditions, the equation of motion in terms of the model amplitude can be rewritten as

$$\omega_m^2 z_m(t)\int_0^L \overline{m}\Phi_m^2(x)dx = \int_0^L \Phi_m(x)U(x,t)dx-\ddot{z}_m(t)\int_0^L \overline{m}\Phi_m^2(x)dx \tag{7}$$

Eq. (7) can be rewritten as

$$M_n\ddot{z}_n(t) + \omega_n^2 M_n z_n(t) = F_n(t), \quad n = 1,2,3,\ldots,m,\ldots \tag{8}$$

where $M_n = \int_0^L \overline{m}\Phi_n^2(x)dx$ is the model mass, and $F_n(t) = \int_0^L \Phi_n(x)U(x,t)dx$ is the model force of the nth node. Eq. (8) represents the motion of the nth node.

The input estimation algorithm is a calculation method using the state space. Therefore, the state equation and the measurement equation have to be constructed before implementing this method. In order to satisfy this situation, the equality, $X = [z_n(t)\ \ \dot{z}_n(t)]^T$ is used to transfer the movement equation to the state space form. The continuous-time state equation and measurement equation of the structure system can be presented as follows (Tuan *et al.* 1996)

$$\dot{X}(t) = AX(t)+BU(t) \tag{9}$$

$$Z(t) = HX(t) \tag{10}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ I_n \end{bmatrix}, \quad I_n = \frac{\int_0^L \Phi_n(x)dx}{\int_0^L \overline{m}\Phi_n^2(x)dx}$$

$$H = [1\ \ 0]$$

$$X(t) = [X_1\ \ X_2]^T$$

A and B are both constant matrices composed of the nth natural frequency and the inertia moment of the structure system. $X(t)$ is the model state vector. $U(t)$ is the input dynamic loading. $Z(t)$ is the observation vector, and H is the measurement matrix. Generally speaking, there always exists the noise turbulence in the practical engineering environment. Nevertheless, Eqs. (9) and (10) do not take the noise turbulence into account. In order to construct the statistic model of the system state characteristics, a noise disturbance term, which can reflect these statistical characteristics of the state, will need to be added into these two equations. For this reason, the continuous-time state Eq. (9) can be sampled with the sampling interval, Δt, to obtain the discrete-time statistic model of the state equation shown as the following (Tuan *et al.* 1996)

$$X(k+1) = \Phi X(k) + \Gamma[U(k) + w(k)] \tag{11}$$

where

$$X(k) = [X_1(k) \;\; X_2(k)]^T$$

$$\Phi = \exp(A\Delta t)$$

$$\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp\{A[(k+1)\Delta t - \tau]\}Bd\tau$$

$X(k)$ is the discrete state vector. Φ is the state transition matrix. Γ is the input matrix. Δt is the sampling interval. $U(k)$ is the sequence of deterministic dynamic input, and $w(k)$ is the processing error vector, which is assumed as the Gaussian white noise. In Eq. (11), when describing the active characteristics of the structure system, the additional term, $w(k)$, can be used to represent the uncertainty in a numerical manner. The uncertainty could be the random disturbance, the uncertain parameters, or the error due to the over-simplified assumption of numerical models. Note that $E\{w(k)w^T(k)\} = Q\delta_{kj}$, $Q = Q_w \times I_{2n\times 2n}$, Q is the discrete-time processing noise covariance matrix. δ_{kj} is the Kronecker delta function.

Generally speaking, the system state can be determined by measuring the output of the system. The measurement usually has a certain relationship with the system output. However, there is also the noise issue with the measurement. As a result, the discrete-time statistic model of the measurement vector can be presented as the following

$$Z(k) = HX(k) + v(k) \tag{12}$$

$X(k)$ is the discrete observation vector. $v(k)$ represents the measurement noise vector and is assumed as the Gaussian white noise with zero mean and the variance, $E\{v(k)v^T(k)\} = R\delta_{kj}$, $R = R_v \times I_{2n\times 2n}$, R is the discrete-time measurement noise covariance matrix.

3. Design of the fuzzy estimator

The fuzzy estimator are accelerated by the fuzzy accelerating factor in the processing noise covariance matrix and weighted by the weighting factor of the input estimation method proposed based on the fuzzy logic inference system. The presented method can inversely estimate the unknown inputs by analyzing the active reaction of the structure system. This method is composed of the fuzzy Kalman filter without the input term and the fuzzy weighted recursive least square estimator. The fuzzy Kalman filter can produce the residual innovation sequence, which contains the bias or systematic error caused by the unknown time-varying inputs and the variance or random error caused by the measurement error. Therefore, the estimator utilizes the innovation sequence to estimate the inputs over time by adopting the fuzzy weighted recursive least square method. The

Kalman filter without the input term is shown as follows (Tuan *et al.* 1996)

$$\overline{X}(k/k-1) = \Phi\overline{X}(k-1/k-1) \tag{13}$$

$$P(k/k-1) = \Phi P(k-1/k-1)\Phi^T + \Gamma Q\Gamma^T \tag{14}$$

$$\overline{Z}(k) = Z(k) - H\overline{X}(k/k-1) \tag{15}$$

$$S(k) = HP(k/k-1)H^T + R \tag{16}$$

$$K_a(k) = P(k/k-1)H^T S^{-1}(k) \tag{17}$$

$$\overline{X}(k/k) = \overline{X}(k/k-1) + K_a(k)\overline{Z}(k) \tag{18}$$

$$P(k/k) = [I - K_a(k)H]P(k/k-1) \tag{19}$$

In Eqs. (13) to (19), the superscript '–' indicates the value of filter estimation. $\overline{X}(k/k-1)$ is the state estimation. $P(k/k-1)$ is the state estimation error covariance. $\underline{Z}(k)$ is the residual of predictor. $S(k)$ is the innovation covariance. $K_a(k)$ is the Kalman gain. $\overline{X}(k/k)$ is the state filter. $P(k/k)$ is the state filter error covariance.

The related equations of the recursive least square estimator are shown as follows (Tuan *et al.* 1996)

$$B_s(k) = H[\Phi M_s(k-1) + I]\Gamma \tag{20}$$

$$M_s(k) = [I - K_a(k)H][\Phi M_s(k-1) + I] \tag{21}$$

$$K_b(k) = \gamma^{-1}P_b(k-1)B_s^T(k)[B_s(k)\gamma^{-1}P_b(k-1)B_s^T(k) + S(k)]^{-1} \tag{22}$$

$$P_b(k) = [I - K_b(k)B_s(k)]\gamma^{-1}P_b(k-1) \tag{23}$$

$$\hat{U}(k) = \hat{U}(k-1) + K_b(k)[\overline{Z}(k) - B_s(k)\hat{U}(k-1)] \tag{24}$$

$\overline{Z}(k)$ is the bias innovation produced by the measurement noise and the input disturbance. $K_b(k)$ is the correction gain. $B(k)$ and $M(k)$ are the sensitivity matrices. γ is the weighting factor. $P_b(k)$ is the error covariance of the input estimation process, and $\hat{U}(k)$ is the estimated dynamic inputs.

Some parameters of filter must be obtained before going through the filtering process. Those parameters are the state transition matrix of the structure system Φ, the measurement matrix H, the discrete-time processing noise covariance matrix Q, and the discrete-time measurement noise covariance matrix R. The on-line state estimate $\overline{X}(k/k-1)$ and state estimation error covariance $P(k/k-1)$ of the filter will be required when the observation vector is unceasingly inputted immediately after the initial conditions X_0 and P_0 are drawn into the estimator. $K_a(k)$ gets smaller as the processing noise covariance matrix Q and the state filter error covariance get smaller according to Eqs. (14) and (17). This indicates that the new measurement is utilized to contribute to the correction of predicted state. $K_a(k)$ gets smaller as the measurement noise covariance matrix R

gets larger according to Eqs. (16) and (17), that is to say, the measurement error is utilized to contribute to the state estimation. In other words, the value of Kalman gain $K_a(k)$ depends on R_v and Q_w. The above-mentioned is an important principle and a key problem that the appropriate R_v and Q_w can be chosen in accordance with the system property and the magnitude of noise interference in the estimation process. R_v can be chosen in accordance with the precision of the measurement instrument. Q_w can be chosen in accordance with the modular error of the system. The Kalman gain can be slightly corrected with the higher precision of the measurement instrument, this is to say, the modular error of the system can be reduced. For this reason, the processing noise covariance can be defined as following

$$Q_w(k+1) = Q_w(k) \times 10^{\alpha(k)} \tag{25}$$

where $\alpha(k)$ is the fuzzy accelerating factor, which is chosen within the interval, $[-1, 1]$. The estimation precision gets better as the $\alpha(k)$ gets smaller. On the contrary, the estimation precision gets worse as the $\alpha(k)$ gets larger.

The weighting factor $\gamma(k)$ is another important parameter which will affect the estimation precision in the estimation process. It also plays the role as an adjustable parameter to control the bandwidth or the gain magnitude of recursive least square estimator. It can operate at each step based on the innovation produced by the Kalman filter. Furthermore, the weighting factor $\gamma(k)$ is employed as the tradeoff between the tracking capability and the estimation precision. The fuzzy estimator in this paper is proposed based on the fuzzy logic inference system. The processing noise covariance and the weighting factor can be adjusted by using the innovation produced by the Kalman filter in each time step. The fuzzy logic system includes four basic components, which are the fuzzy rule base, the fuzzy inference engine, the fuzzifier, and the defuzzifier. The value of fuzzy logic system input, $\theta(k)$, may be chosen within the interval, $[0, 1]$. The Pythagorean theorem with the transverse axle (time, t) and the vertical axle (residual of predictor, $\bar{Z}$) can be used to solve the length of the hypotenuse. In other words, the length of the hypotenuse is the variation rate of the residual in the sampling interval. The dimensionless input variable is defined as the following

$$\theta(k) = \frac{\left|\dfrac{\Delta\bar{Z}(k)}{\bar{Z}(k)}\right|}{\sqrt{\left(\dfrac{\Delta\bar{Z}(k)}{\bar{Z}(k)}\right)^2 + \left(\dfrac{\Delta t}{t_f}\right)^2}} \tag{26}$$

where $\Delta\bar{Z}(k) = \bar{Z}(k) - \bar{Z}(k-1)$, and Δt is the sampling interval. $t_f = 1$ is assumed. The fuzzy sets for $\theta(k)$, $\alpha(k)$ and $\gamma(k)$ are labeled in the linguistic terms, such as EL (extremely large), VL (very large), LV (large value), MV (medium value), SV (small value), VS (very small), and ES (extremely

Table 1 The fuzzy rule base

		Input variable $\theta(k)$						
		ES	VS	SV	MV	LV	VL	EL
Output variables	$\alpha(k)$	EL	VL	LV	MV	SV	VS	ES
	$\gamma(k)$	EL	VL	LV	MV	SV	VS	ES

small). A fuzzy rule base is a collection of fuzzy IF-THEN rules which are shown in Table 1.

where $\theta(k)$ is input variable, and $\alpha(k)$ and $\gamma(k)$ are the output variables of the fuzzy logic system. The fuzzier maps a crisp point $\theta(k)$ into a fuzzy set A. Therefore, the nonsingleton fuzzier can be expressed as the following (Wang 1994).

$$\mu_A(\theta(k)) = \exp\left(-\frac{(\theta(k) - \bar{x}_i^l)^2}{2(\sigma_i^l)^2}\right) \tag{27}$$

$\mu_A(\theta(k))$ decreases from 1 as $\theta(k)$ moves away from $\bar{x}_i^l$. $(\sigma_i^l)^2$ is a parameter characterizing the shape of $\mu_A(\theta(k))$.

The Mamdani maximum-minimum inference engine is used in this paper. The max-min-operation rule of fuzzy implication of the output variable, $\alpha(k)$, is shown as the following (Wang 1994).

$$\mu_B(\alpha(k)) = \max_{j=1}^{c}\left\{\min_{i=1}^{d}[\mu_{A_i^j}(\theta(k)), \mu_{A_i^j \to B^j}(\theta(k), \alpha(k))]\right\} \tag{28}$$

The output variable $\gamma(k)$ can be similarly shown as following (Wang 1994)

$$\mu_B(\gamma(k)) = \max_{j=1}^{c}\left\{\min_{i=1}^{d}[\mu_{A_i^j}(\theta(k)), \mu_{A_i^j \to B^j}(\theta(k), \gamma(k))]\right\} \tag{29}$$

where c is the fuzzy rule, and d is the dimension of input variables.

The defuzzifier maps a fuzzy set B to a crisp point $\alpha \in V$. The fuzzy logic system with the center of gravity is defined as the following (Wang 1994).

$$\alpha(k) = \frac{\sum_{l=1}^{n} \bar{y}^l \mu_B(\alpha^l(k))}{\sum_{l=1}^{n} \mu_B(\alpha^l(k))} \tag{30}$$

The defuzzifier of the output variable $\gamma(k)$ can be similarly shown as following

$$\gamma^*(k) = \frac{\sum_{l=1}^{n} \bar{y}^l \mu_B(\gamma^l(k))}{\sum_{l=1}^{n} \mu_B(\gamma^l(k))} \tag{31}$$

n is the number of outputs. $\bar{y}^l$ is the value of the lth output. $\mu_B(\alpha^l(k))$ and $\mu_B(\gamma^l(k))$ represent the membership of $\alpha^l(k)$ and $\gamma^l(k)$ in the fuzzy set B, respectively. Substituting $\alpha(k)$ of Eq. (30) in Eq. (25) and $\gamma^*(k)$ of Eq. (31) in Eqs. (22) and (23) allows us to configure the fuzzy estimator. A design flow chart of the fuzzy estimator is given in Fig. 2.

4. Results and discussion

To verify the practicability and robustness of the presented approach in estimating the uniform dynamic loading, the fixed beam structural system is considered to evaluate the inverse algorithm. A simulated sensor was installed under the middle of the beam as shown in Fig. 1. The material data and physical properties of the beam are shown as follows: $EI = 8.6 \times 10^6\ \mathrm{N-m^2}$, $L = 6.096$ m and $\bar{m} = 17.51\mathrm{N-sec^2/m}$. Assuming each side of the beam is fixed, the normal model shape

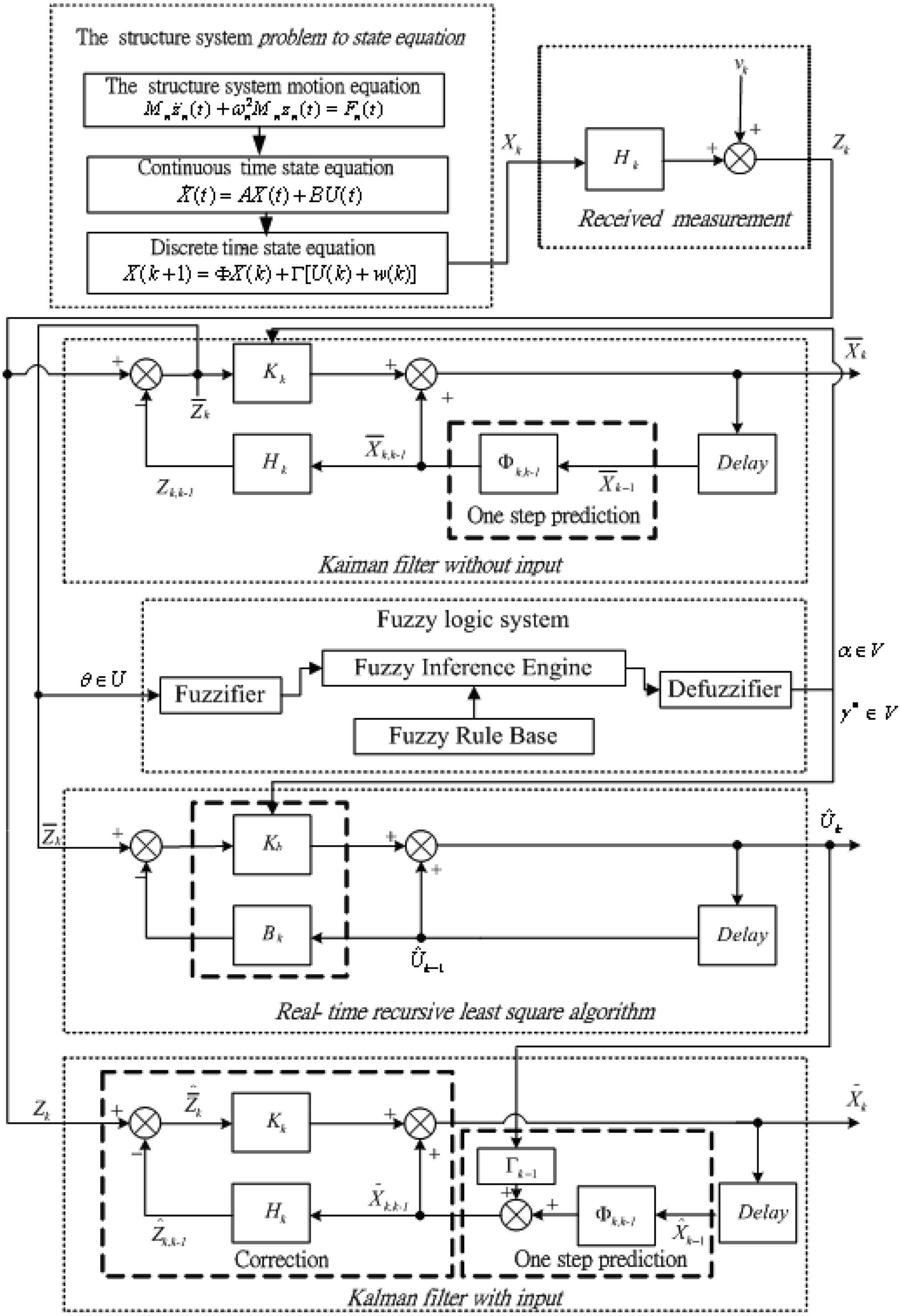

Fig. 2 The design flowchart of the fuzzy estimator

function satisfying the boundary condition can be shown as (Mario 1986)

$$\Phi_n(x) = \cosh a_n x - \cos a_n x - \sigma_n(\sinh a_n x - \sin a_n x) \tag{32}$$

where $\sigma_n = \dfrac{\cos a_n L - \cosh a_n L}{\sin a_n L - \sinh a_n L}$, and $a_n L = 4.73$.

The active model amplitude reaction of the beam structure system under various frequency dynamic load inputs has to be determined first. Furthermore, by applying the active model amplitude reaction to the presented estimation algorithm, the inverse load estimation of the structure system can be simulated numerically. The estimation algorithm includes the fuzzy Kalman filter technique and the fuzzy weighted recursive least square method. The initial conditions and other parameters of simulation are shown as follows: $P(0/0) = \mathrm{diag}[10^4]$, $\hat{U}(0) = 0$, $P_b(0) = 10^4$, and $M(0)$ is assumed to be a zero matrix. The sampling interval, $\Delta t = 0.001$ secs. The weighting factor, γ, is assumed as a fuzzy weighting factor, an adaptive weighting function, and a constant weighting factor, respectively.

Example: Periodic sinusoidal dynamic loading

The uniform periodic sinusoidal dynamic loading is applied on the beam structure system. These sinusoidal dynamic loading is shown as follows

$$U(x, t) = U_0 \sin\overline{\omega} t \tag{33}$$

where $U_0 = 200$ N/m, and $\overline{\omega} = 10$ Hz. The uniform periodic sinusoidal dynamic loading of the structure system is determined by using the presented approach when considering the influence due to the initial processing noise and the measurement noise of the system. The initial processing noise variance, $Q_w(0) = 10^8$. The measurement noise variance, $R_v = \sigma^2 = 10^{-8}$. By applying the active dynamic reaction which contains noise to the presented algorithm, the estimation result of the uniform periodic sinusoidal dynamic loading can be obtained and plotted in Fig. 3. The coarse estimation result in the initial response on account of the larger initial processing noise variance is shown in Fig. 3. The presented estimator has the property of faster convergence with the regulated processing noise variance in the estimation process.

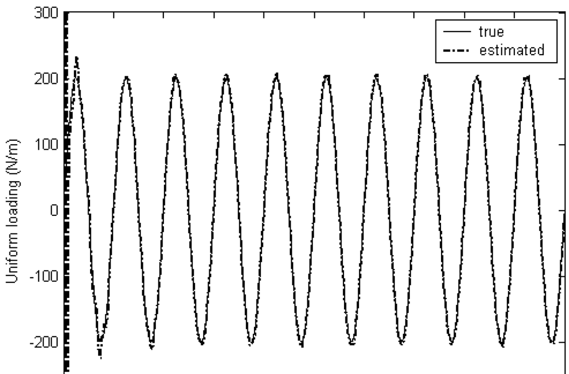

Fig. 3 The estimation result using the periodic sinusoidal load input

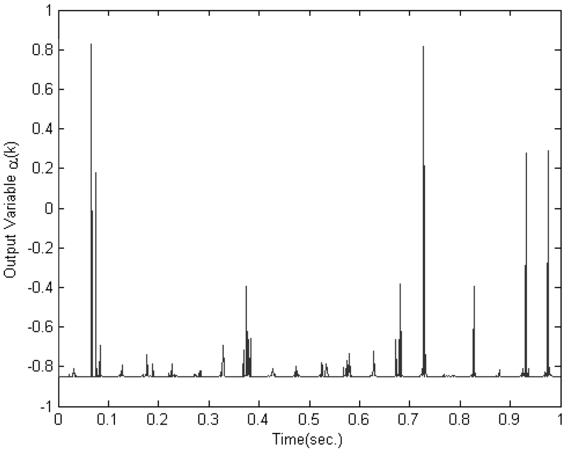

Fig. 4 The variance of the output variable $\alpha(k)$

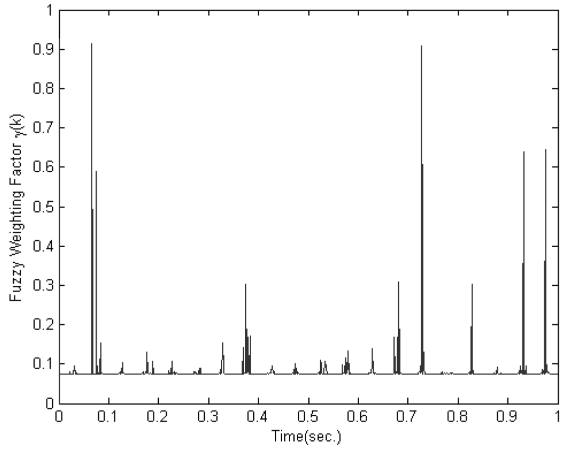

Fig. 5 The variance of the fuzzy weighting factor $\gamma(k)$

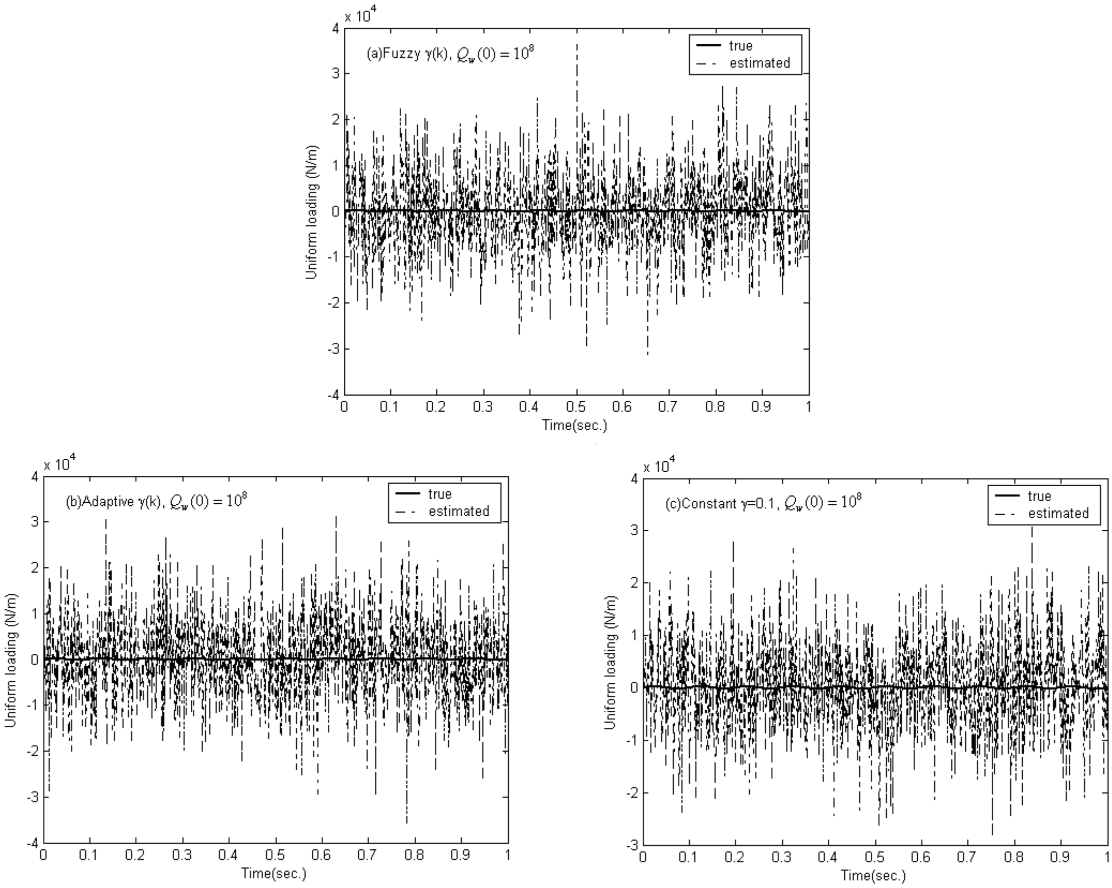

Fig. 6 Comparison of the estimation results using different weighting factors

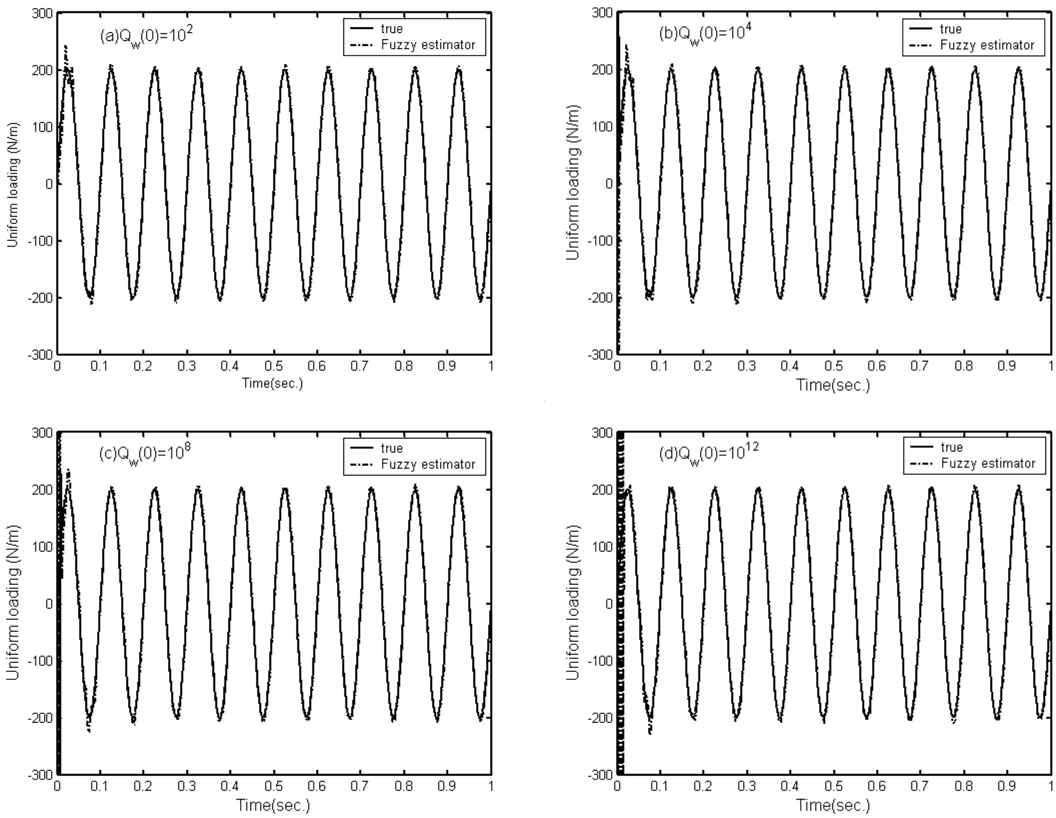

Fig. 7 Comparison of the estimation results using different initial modeling errors

Fig. 4 shows that the estimator has the greater tracking performance, so that the larger output variable, $\alpha(k)$, can be chosen to generate the larger processing noise variance, $Q_w(k)$, according to Eq. (25). The estimator has the capability of reducing the effect of noise, so that the smaller output variable, $\alpha(k)$, can be chosen to generate the smaller processing noise variance, $Q_w(k)$, when the unknown system input is steady. Fig. 5 shows that the smaller weighting factor can be chosen in the fuzzy recursive least square method when the unknown system input is larger. $K_b(k)$ gets larger as $\gamma(k)$ gets smaller according to Eq. (22). $P_b(k)$ gets larger as the forgetting effect becomes more conspicuous to adapt the fast input state change according to Eq. (23). It should be noted that the faster the forgetting effect is, the lower the smoothing effect will be, that is, it introduces oscillation. The fuzzy weighting factor $\gamma(k)$ is employed as the trade-off between the upgrade of tracking capability and the loss of estimation precision.

The estimates of $U(x,t)$ using the fuzzy weighting function, the adaptive weighting function, and the constant weighting factor, $\gamma = 0.1$, are plotted in Fig. 6. The estimation results show that the tracking performance of estimators is not good enough, and they are not suitable in reducing the effect of the noise. This case has been compared by using presented estimator as shown in Fig. 3. It shows that if the initial process noise variance increases, it will have greater performance in tracking and in reducing the effect of measurement noise. This case has been compared by using different values of initial process noise variance, such as $Q_w(0)$ = $10^2, 10^4, 10^8$, and 10^{12} as shown

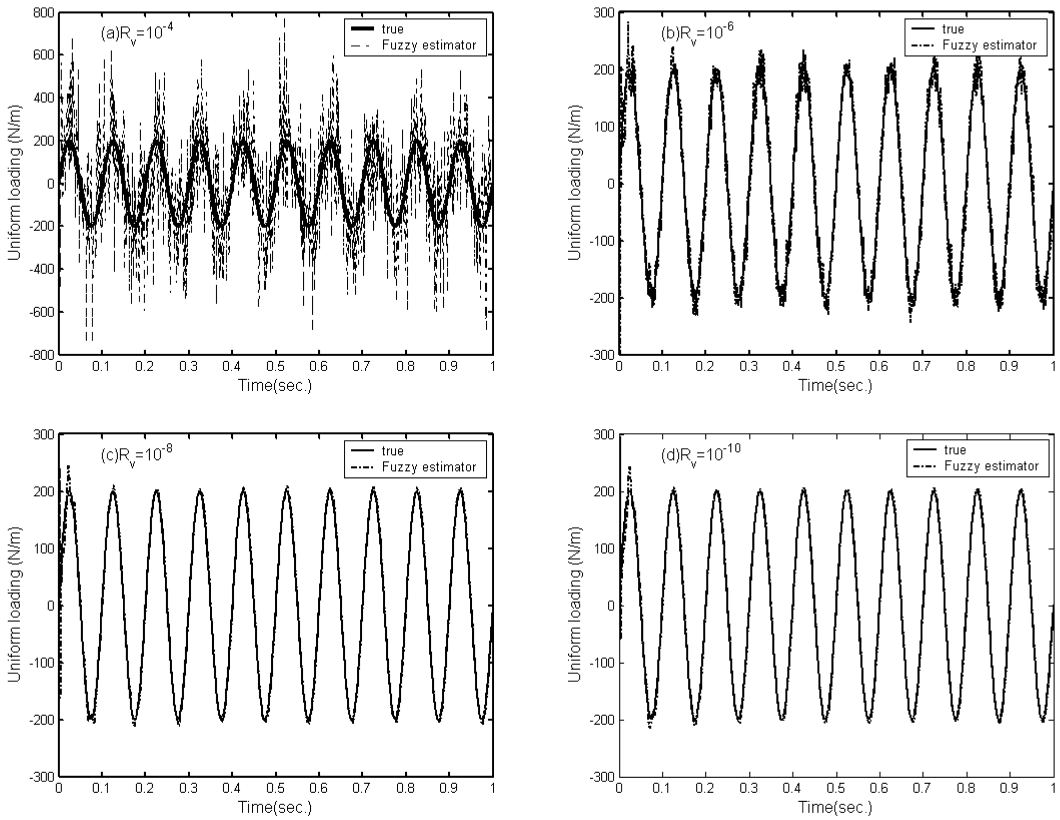

Fig. 8 Comparison of the estimation results using different measurement errors

in Fig. 7. It shows that if the initial process noise variance $Q_w(0)$ increases, it will influence the estimation resolution. A larger initial process noise variance will affect the capability of tracking the time-varying force inputs. Fig. 8 shows the estimation results of the fuzzy estimator with the initial process noise variance fixed ($Q_w(0) = 10^8$), and with different measurement error variances ($R_v = 10^{-4}, 10^{-6}, 10^{-8}$ and 10^{-10}). The result shows that when R_v is small, the transient performance of the estimator will be better against the noise effect. On the other hand, the fluctuation will become severer when R_v increases. The transient performance of the estimator will be poorer with more influence caused by the noise. The smaller R_v indicates that the measurement is more precise. The effort made to obtain a more precise measurement will be higher.

5. Conclusions

This paper proposes the fuzzy Kalman filter technology combined with the fuzzy weighted recursive least square method to develop the fuzzy estimator, which can estimate the active loads of the structure system over time by analyzing the active reaction of the system. The assumption of the inaccurate initial process noise variance and the measurement noise are considered, that is to say, the larger random variable will be added into the statistic model of the system state characteristics in the estimation process. The fuzzy estimator has the properties of fast tracking and effective noise reduction, since it is accelerated and weighted by the fuzzy accelerating factor $\alpha(k)$ of the processing noise covariance matrix and the weighting factor $\gamma(k)$ of the method proposed based on the fuzzy logic inference system. The simulation results are compared by alternating between the fuzzy weighting, adaptive and constant factors. The results demonstrate that this method has the properties of faster convergence in the initial response, better target tracking capability, and more effective noise and measurement bias reduction.

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