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Technical Note

Force and stress for electromechanical integrated toroidal drive

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1. Introduction

Toroidal drive can transmit large torque in a small size and is suitable for technical fields such as aviation and space flight (Kuehnle 1966). As electrical and control techniques are utilized in mechanical engineering field, generalized composite drives become advancing edge of the mechanical science. In electromagnetic harmonic drive (Delin 1993) and piezoelectric harmonic one (Barth 2000), the mesh forces between flexible gear and rigid one are controlled by electromagnetic force or piezoelectric one, and drive and power are integrated. Based on investigating toroidal drive (Xu 2004), the Authors presented electromechanical integrated toroidal drive. Here, the toroidal drive, power and control are integrated.

The drive consists of four basic elements, Fig. 1: (a) the central worm, (b) planets, (c) a toroidal shaped stator, and (d) a rotor, which forms the central output shaft upon which the planets are mounted. The coils are mounted in helical grooves of the worm surface. The planets and the stator

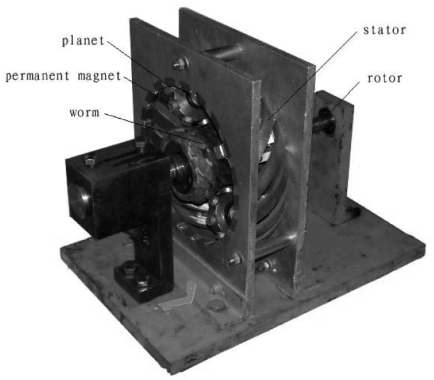

Fig. 1 Model machine of the drive

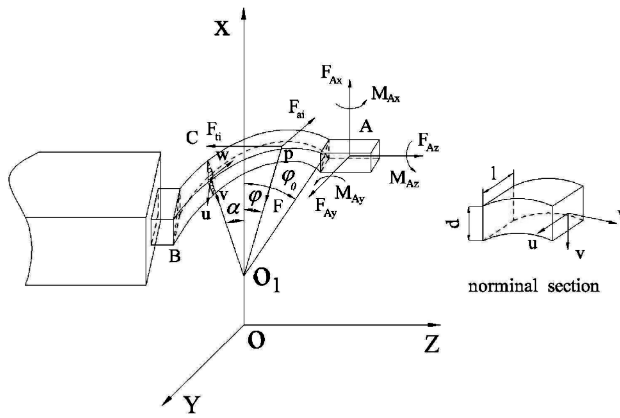

Fig. 2 Stator tooth under forces

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have permanent magnets instead of teeth.

If a specific parameter relation is given, N pole of one element will correspond to S pole of the other one all along. When the alternate current is made in the coils of the worm, a toroidal circular field is formed. It drives several planets to rotate about their own axes. With magnetic forces between teeth of the planet and stator, the rotor is driven to rotate about its own axis. Thus, a power of low speed and large torque is output.

In this paper, using virtual displacement principle and coordinate transformation principle, the internal force and torque distributions on the normal section of the stator tooth are calculated, the stress distributions on the normal section are given. The points at which the maximum normal stress or shear stress occur are obtained. These results are useful in design of the drive.

2. Basic equations and methods

A planet tooth meshes with the stator at an angular position α of the planet (see Fig. 2). F_{ti} and F_{ai} denote the tangential and axial components of the magnetic force between the planet and stator, respectively (Xu 2005). The radial force F between stator and planet is

$$F = \frac{10^7}{8\pi} B_0^2 S_0 \tag{1}$$

where B_0 is magnetic induction intensity, S_0 is section area of air gap

At support point A or B of the stator tooth, six support forces occur: F_{Ax}, F_{Ay} and F_{Az} for point A and F_{Bx}, F_{By} and F_{Bz} for point B, M_{Ax}, M_{Ay} and M_{Az} for point A and M_{Bx}, M_{By} and M_{Bz} for point B. There are twelve unknown parameters and only six balanced equations can be given. Hence, six compatibility equations are needed. All displacements of the stator tooth at point A are zero. Let $F_{Ax} = X_1$, $F_{Ay} = X_2$, $F_{Az} = X_3$, $M_{Ax} = X_4$, $M_{Ay} = X_5$, and $M_{Az} = X_6$, the six compatibility equations are

$$\delta_{i1}X_1 + \delta_{i2}X_2 + \delta_{i3}X_3 + \delta_{i4}X_4 + \delta_{i5}X_5 + \delta_{i6}X_6 + \Delta_{1p} = 0 \qquad (i = 1\text{-}6) \tag{2}$$

where δ_{ij} is displacement in ith direction at point A under unit force in jth direction. Δ_{ip} is displacement in ith direction at point A under all external forces. From Eq. (2), X_1, X_2, X_3, X_4, X_5, and X_6 can be given.

As the stator teeth are helical surface, the coordinate transformation principle should be used to obtain the internal forces on normal section of the stator tooth. Let $\mathbf{X}$ denote coordinate vector of one point in $s(x,y,z)$, using coordinate transformation, the coordinate vector $\mathbf{X}_p$ of this point in $s_p(u,v,w)$ can be given

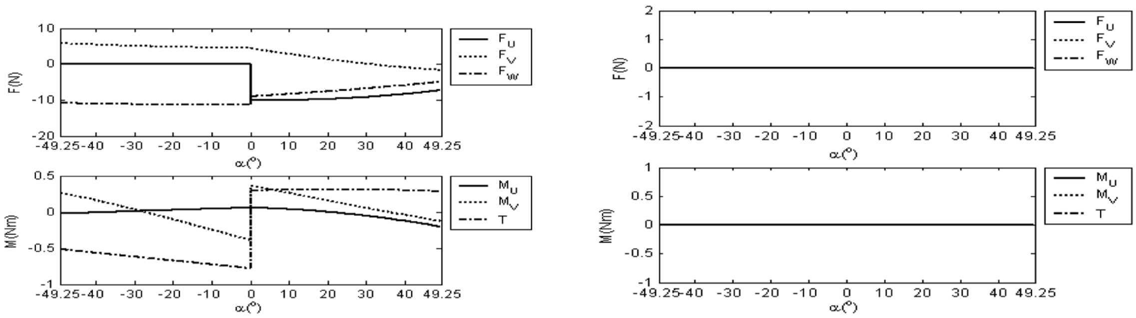
$$\mathbf{X}_p = M_{0p}^{-1}\mathbf{X} \tag{3}$$

where coordinate system $s(x,y,z)$ is attached to the stator, $s_1(x_1,y_1,z_1)$ to the planet, $s_2(x_2,y_2,z_2)$ to the worm, $s_r(x_r,y_r,z_r)$ to the rotor, $s_{10}(x_{10},y_{10},z_{10})$ to the rotor, $s_p(u,v,w)$ to the normal section of the stator tooth; M_{0p} is coordinate transformation matrix from system s_p to s, $M_{0p} = M_{0r} M_{r10} M_{101} M_{1p}$, M_{1p} is transformation matrix from s_p to s_1, M_{101} from s_1 to s_{10}, M_{r10} and M_{0r} from s_{10} to s_r, and from s_r to s, respectively (Xu 2004).

In a same manner, the force vector and the torque vector can also be transformed to the coordinate system $s_p(u,v,w)$.

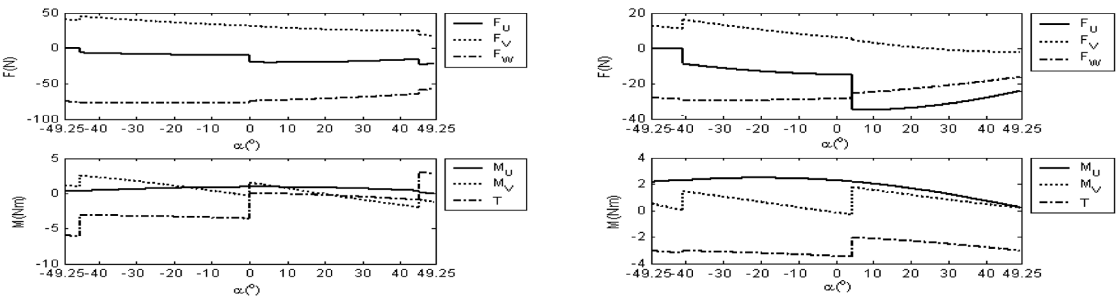
Table 1 Parameters of the example system

a/R	R (mm)	z_3	z_1	p	φ_0 (°)	l (mm)	d (mm)
2	60	20	8	2	49.25	12	6



(a) $\varphi = 0^0$ (b) $\varphi = 49.25^0$

Fig. 3 The changes of the internal forces along with φ and α for single tooth mesh



(a) $\varphi = 0^0$ (b) $\varphi = 49.25^0$

Fig. 4 The changes of the internal forces along with φ and α for multi-teeth mesh

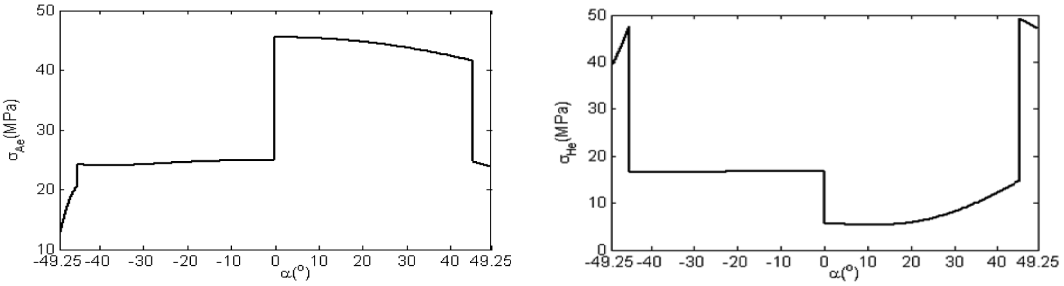
3. Results

The parameters of the numerical example are shown in Table 1. Here, load torque is 34.5 Nm, R is radius of the planet, a is center distance, z_1 is tooth number of planet, z_3 is tooth number of stator, p is pole pair number of the worm coils, l and d are length and thickness of the rectangle normal section for stator tooth, respectively, φ_0 is half of the contact angle between planet and the stator, φ is the position angle of the stator tooth.

Changes of internal forces on any normal section of the stator tooth with angles φ and α are given (Fig. 3 for single tooth mesh, Fig. 4 for multi-teeth mesh). Figs. 3 and 4 show:

(1) For single tooth mesh, at initial and final mesh points between planet and stator, the internal forces and torques in stator tooth are all zero. For multi-teeth mesh, at initial and final mesh points, they are not zero. It is because one tooth meshes with stator at initial or final mesh points, another tooth or other two teeth mesh with the stator at other mesh points.

(2) At mesh points, there are abrupt changes of the internal forces and torques. For multi-teeth



(a) σ_{Ae} (b) σ_{He}

Fig. 5 Stress distributions at dangerous points of the normal section ($\varphi = 0^o$)

mesh, the internal forces and torques in stator tooth are much larger than those for single tooth mesh. The three abrupt changes occur at three teeth mesh regions and the two abrupt changes occur at two teeth mesh regions.

(3) The internal forces and torques in stator tooth are much larger at three teeth mesh regions than those at two teeth mesh regions. When a tooth of planet is meshing with stator near $\varphi = 0^o$, other two teeth of the planet are meshing with stator near initial and final mesh points, respectively. Therefore, these mesh points are most dangerous for the drive.

Stresses are analyzed for the dangerous mesh points. On the normal section of the stator tooth, there are two dangerous points. The first is the mid-point of the long side of the rectangle section, here the equivalent stress is σ_{He}. The second is the crank point of the rectangle section, here the equivalent stress is σ_{Ae}. Fig. 5 shows changes of σ_{Ae} and σ_{He} with angle α. It shows:

(1) In equivalent stress σ_{Ae}, the bending stress is its main portion. The change of the bending stress decides on σ_{Ae} distribution. The maximum equivalent stress occurs at $\varphi = 0^o$.

(2) In equivalent stress σ_{He}, the shear stress is its main portion. The change of the shear stress decides on σ_{He} distribution. σ_{He} at $\varphi = -45^o$ and $\varphi = 45^o$ is much larger than that at point $\varphi = 0^o$.

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