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Technical Note

Dynamic behavior of infinite beams resting on elastic foundation under the action of moving loads

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1. Introduction

Moving loads have a significant influence on the dynamic behavior of elastic or inelastic solid elements, of an entire structure, or parts of a structure, while they may produce strong vibrations of such systems, especially in the high speeds spectra. The problem of finite beams under the action of a moving load has already been solved employing a number of methods, among which the modal superposition technique dominates. The aforementioned method, that is as a matter of fact a Fourier analysis, is highly effective for moving loads with low speeds.

A special category, among the problems involving moving loads, is the case of a load moving along an infinite beam resting on an elastic foundation. This type of problems is of great theoretical and practical significance. It is clear that the main factor, which drastically affects the problem, is the validity of the foundation model. The soil can be modeled by foundation models, such as the Winkler's model (with one parameter) or another one with two parameters (Pasternak 1954) that includes the shear parameter influence or by higher order models that include the influence of some secondary parameters. It has been proved (Mallik *et al.* 2006) that the effect of various models on the foundation behavior is insignificant, except the case of a semi-infinite elastic medium, where the wave field including surface waves does affect the response of the beam.

The present work deals with the aforementioned problem and attempts a solution using the modal superposition technique. The expressions obtained in this study are given in a general form that can be employed for any value of speed or damping parameters. The results gathered by the expressions obtained herein are compared to the ones presented by Frýba in the relative chapter of his classic textbook (Frýba 1972). Finally, the possible range of speeds for moving loads that may appear in the case of such a beam is estimated, and the application field of the gathered relations is determined.

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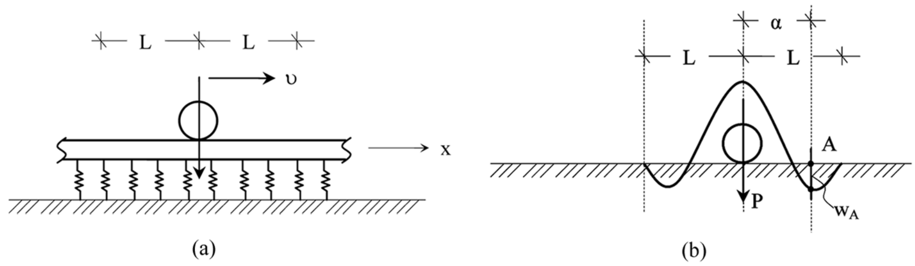

Fig. 1 (a) Moving load on an infinite beam, (b) dynamical response of a beam point A

2. Mathematical formulation

Let us consider an infinite beam subjected to a concentrated load P moving with a constant speed υ. The beam is made from a homogeneous and isotropic material with modulus of elasticity E and is resting on an elastic foundation. The elastic foundation is assumed to be of the Winkler type, i.e., the foundation reactions are directly proportional to the deflections of the beam, see Fig. 1(a).

According to the above assumptions, the transverse vibration of the beam is governed by the following differential equation

$$EIw'''' + c\dot{w} + m\ddot{w} + kw = P \cdot \delta(x - \upsilon t) \tag{1}$$

where c is the damping coefficient, k is the Winkler foundation coefficient, υ is the speed of the load, and δ is the Dirac-delta function.

For the above problem, the so-called quasi-stationary state exists in which the beam is at rest relatively to a coordinate system moving with the same speed υ as the moving load (Frýba 1972). This state is valid after a sufficiently long time of load travel (otherwise it is no longer time dependent) and thus, it depends only on the distance from the origin of the coordinate axes. With these assumptions and assuming that the load P affects at each time only a length $2L$ of the beam depending on the beam's and foundation's characteristics, we focus on studying the dynamic behavior of a random point A of the beam (see Fig. 1(b)) for different positions of the moving load P, considering that the moving load P passes over a finite beam with length $2L$ and constant speed υ.

Hence, we have the following cases:

(a) For $\alpha > L$: the point A is not yet affected by the load P

(b) For $\alpha < L$: the point A oscillates and the deflection is w_A

(c) For $\alpha > L$: the point A vibrates freely $(w_A \neq 0)$.

The influence length ($2L$) is determined by solving the static problem, and is

$$L = \frac{3\pi}{2} \cdot \sqrt[4]{\frac{4EJ}{k}} \tag{2}$$

3. Numerical results and discussion

In order for us to study a broad range of types of soils and beams, we shall consider the property data presented in the first two lines of Table 1, and in particular soils of grid (sand-gravel), sand soils, and argil-sand soils.

Table 1 Critical speeds of a simply supported beam and a beam on elastic foundation

BEAM EI (Nm^2) m (kg/m)	200 × 10^7 30			20 × 10^7 120			0.2 × 10^7 30		
SOIL E_s (N/m^2) K (N/m^2)	Grit 1.5 × 10^8 2 × 10^7	Sand 0.3 × 10^8 0.4 × 10^7	Argil-Sand 0.03 × 10^8 0.04 × 10^7	Grit 1.5 × 10^8 2 × 10^7	Sand 0.3 × 10^8 0.4 × 10^7	Argil-Sand 0.03 × 10^8 0.04 × 10^7	Grit 1.5 × 10^8 2 × 10^7	Sand 0.3 × 10^8 0.4 × 10^7	Argil-Sand 0.03 × 10^8 0.04 × 10^7
Equiv. length	42.15	63.03	112.08	23.70	35.44	63.02	7.49	11.21	19.93
υ_{cr} (km/h)	1 385	926	521	389	260	146	246	164	92
υ_{cr} (km/h)	4 157	2 780	1 563	1 169	782	439	739	494	278

Keeping in mind that the effect of difference in the foundation modeling is not significant (Mallik *et al.* 2006), we shall consider a Winkler-type soil. The constant k is obtained (Pasternak 1954) using the relation

$$k = \frac{E_s}{H(1+\nu_s)(1-2\nu_s)} \tag{3}$$

for a constant thickness of the soil layer $H \cong 12$ m and a Poisson's ratio $\nu_s = 0.25$.

The beam with $EI = 200 \times 10^7$ Nm^2 and $m = 30$ kg/m corresponds to a steel profile (HE-M 1000), the beam with $EI = 20 \times 10^7$ Nm^2 and $m = 120$ kg/m to a concrete beam with cross-section area $A = 0.5$ m^2, and the beam with $EI = 0.2 \times 10^7$ Nm^2 and $m = 30$ kg/m to a concrete beam with cross-section area $A = 1.0 \times 0.125$ m^2.

3.1 Reaching the critical speed

In the 4th line of Table 1 on can see the corresponding critical speeds of a simply-supported single-span beam not resting on soil, and with span length equal to the equivalent length (3rd line) of the beam under study resting on elastic foundation, in order for us to compare the critical speeds of the other cases with this one.

In the 5th line of Table 1, the corresponding critical speeds for a beam on elastic foundation are presented. We see that the gathered critical speeds of beams on elastic foundations are much greater than the ones of a simply-supported beam, while it is impossible for one to achieve such values of speed for this kind of beams and for normal soil types.

Let us consider, for example, the case of a 2 m thick swampy layer of soil with $E_s = 0.005 \times 10^8$ N/m^2. The corresponding Winkler constant is: $k = 0.004 \times 10^8$. We assume that a very thin and feeble concrete beam with cross-section $A = 0.125$ m^2 and $EI = 0.20 \times 10^7$ Nm^2 is based on this kind of soil. The corresponding critical speed is 285 km/h. The above speed is close to the one of a landing jet, which, of course, is meaningless from a practical point of view.

3.2 The beam on elastic foundation

Keeping in mind that it is not possible to achieve the critical speed on a beam on elastic foundation, we proceed for the sub-critical speeds.

The plots in Fig. 2 show the deflections under point A of a low stiffness beam resting on a rather feeble soil, with and without damping. The plots in Fig. 3 show the deflections of a normal beam

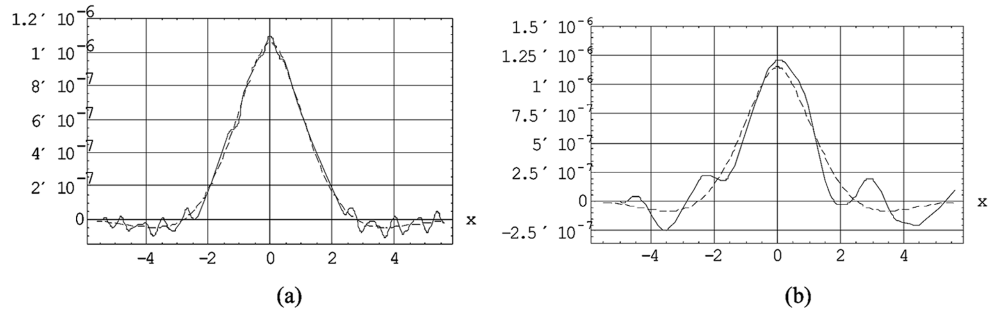

Fig. 2 Point A deflections according to the proposed equations (——) and the Frýba relations (- - - -) for speeds (in km/h): (a) 50 and (b) 200 km/h, for $EI = 0.2 \times 10^7$, $m = 30$, and $k = 0.04 \times 10^8$, and $\beta = 0$

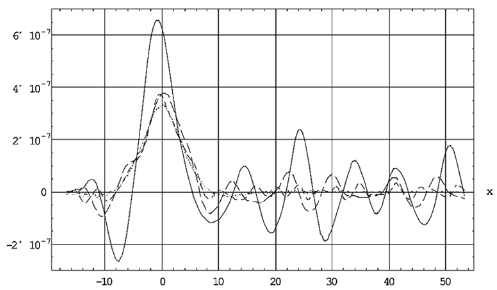

Fig. 3 Point A deflections according to the proposed equations for $EI = 20 \times 10^7$, $m = 120$, $k = 0.04 \times 10^8$ and $\beta = 1$ and speeds: (——) 500 km/h, (——) 350 km/h, (- - - -) 200 km/h, and (········) 50 km/h

with damping resting on a regular soil, for a range of sub-critical speeds.

4. Conclusions

From the above analysis it is concluded that for usual civil engineering structures, it is impossible for one to achieve the critical speeds, since they are significantly higher than the ones of the corresponding free single-span simply-supported beam even on cohesionless soils. The proposed formulae are accurate for subcritical speeds either with or without damping. This accuracy is significant for normal and well-designed beams even when they are founded on cohesionless soils, but it decreases for feeble beams on cohesionless soils as one approaches the critical speed.

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