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Technical Note

A new analytical method for deformation of composite steel-concrete straight box girders with interfacial slip

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1. Introduction

In past decades, composite steel-concrete I-girders have been widely used, because of their economic benefits and good structural behavior (Wang 1998, Jasim 1999, Nie *et al.* 1995). However, the composite steel-concrete box girders are a new structural form, and it has recently been becoming more popular in China, but relatively little to theoretical solutions for deformation of the composite steel-concrete box girders at home and abroad (Zhou *et al.* 2005). The deformation of composite steel-concrete straight box girders (CSCSBG) was studied in order to satisfy serviceability requirements. Based on the R. E. Goodman's elasticity intermediate layer hypothesis, a theoretical solution for the deformation of CSCSBG was derived by introducing rotation angle formulation of the axial deformation. Numerical simulations of CSCSBG subjected to different load cases were conducted and an example about engineering application was introduced to account for the deformation of CSCSBG. Analytical solutions show an excellent agreement with numerical results. It was found that deformation values of the CSCSBG increase with the increase of spacing D of shear connectors.

2. Proposed formulation for theoretical solutions of deformation

2.1 Establishment of differential equation

Using Goodman's elasticity intermediate layer hypothesis and introducing the rotation angle formula of the axial deformation, the calculation model for a simply supported composite steel-concrete box girder considering interface-slip is shown in Fig. 1. Six equilibriums both steel box girder and concrete flange plate can be derived from those of the force analysis respectively as

$$dQ_{cx}/dz + q_{cx} = 0 \qquad (1\text{-}1) \qquad dQ_{cy}/dz + q_y = t \qquad (1\text{-}2)$$

$$dN_c/dz = -q_u \qquad (1\text{-}3) \qquad dM_{cx}/dz = Q_{cy} - q_u h_c \qquad (1\text{-}4)$$

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$$dM_{cy}/dz = Q_{cx} - m_{cy} \quad (1\text{-}5) \qquad dT_c/dz = -m_{cz} \quad (1\text{-}6)$$

$$dQ_{sx}/dz - q_{cx} = 0 \quad (1\text{-}1^*) \qquad dQ_{sy}/dz = -t \quad (1\text{-}2^*)$$

$$dN_s/dz = q_u \quad (1\text{-}3^*) \qquad dM_{sx}/dz = Q_{sy} - q_u h_s \quad (1\text{-}4^*)$$

$$dM_{sy}/dz = -Q_{sx} - m_{sy} \quad (1\text{-}5^*) \qquad dT_s/dz = -m_{sz} \quad (1\text{-}6^*)$$

In addition, the relationship between internal force and displacement gives

$$M_{cx} = -E_c I_{cx} y'' \quad (2\text{-}1) \qquad M_{sx} = -E_s I_{sx} y'' \quad (2\text{-}1^*)$$

$$N_c = E_c A_c w_{c'} \quad (2\text{-}2) \qquad N_s = E_s A_s w_{s'} \quad (2\text{-}2^*)$$

$$T_c = -E_c I_{cw}\frac{d^3\Phi}{dz^3} + G_c I_{cd}\frac{d\Phi}{dz} \quad (2\text{-}3) \qquad T_s = -E_s I_{sw}\frac{d^3\Phi}{dz^3} + G_s I_{sd}\frac{d\Phi}{dz} \quad (2\text{-}3^*)$$

Combining Eqs. (1) and (2) yields

$$-E_c I_{cx} y''' = Q_{cy} - h_c q_z \quad (3\text{-}1) \qquad -E_s I_{sx} y''' = Q_{sy} - h_s q_z \quad (3\text{-}1^*)$$

$$E_c A_c w_{c''} = -q_u \quad (3\text{-}2) \qquad E_s A_s w_{s''} = q_u \quad (3\text{-}2^*)$$

The assumption gives

$$q_u = k_u S = k_u(w_s - w_c + (h_c + h_s)y') \quad (4) \qquad k_u = K/u \quad (5)$$

where k_u = equivalent stiffness; u = space of connectors; S = relative slip; $(w_s - w_c)$ = slip between flange plates and steel beams; $(h_c + h_s)y'$ = relative slip between flange plates and steel beams; Φ = torsion angle.

For convenience of analysis, the rotation angle formula w' of the axial deformation was introduced as Liu (2007)

$$w' = (w_c - w_s)/h \quad (6)$$

On the basis of the euqations mentioned above, one gets

$$\frac{d^6 y}{d\xi^6} - \beta^2\frac{d^4 y}{d\xi^4} = -\frac{\beta(\alpha-1)l^4}{EI_x\alpha}q_y \quad (7)$$

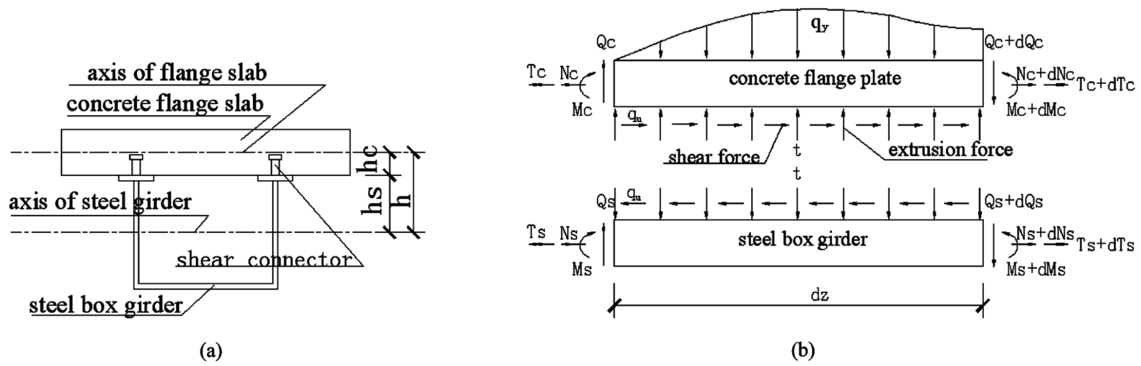

Fig. 1 Calculation model for simply supported beam (a) Cross-section, and (b) deformation of finites length

$$\beta = l\sqrt{(C^* \alpha/(EI_x))} \quad (8) \qquad \alpha = 1 + EI_x/EI_0 \quad (9)$$

Where β = connection degree coefficient; and α = composite degree coefficient; $\xi = z/l$.

2.2 Deformation-formula subjected to uniform load

In the current calculation model for simply supported beam, uniform load q_y was applied. Solving Eq. (7), one gets

$$y = \frac{l^4 q_y}{24EI_x\alpha}\left[(\alpha-1)\xi(1-2\xi^2+\xi^3) + \frac{12}{\beta}\xi(1-\alpha\xi) + \frac{24}{\beta^4 sh\beta}(sh\beta\xi + sh\beta(1-\xi) - sh\beta)\right] \quad (10)$$

Where q_y = vertical uniform load; and y = deformation of the simply supported composite box beams.

2.3 Deformation-formula subjected to concentrated load

For the case of simply supported composite box girders subjected to concentrated load, calculation model was divided into two parts. Namely, when $0 \le z \le L1$, one gets

$$y_1(\xi) = \left\{\frac{pL^3\xi_0}{6EI_0\alpha}(1-\xi_0)(7\xi_0-4\xi_0^2-2) - \frac{pL3}{\alpha(1-\alpha)\beta^3 EI_0}\left[\frac{\xi_0(sh\beta(1-\xi_0)-\beta(1-\xi_0)ch\beta(1-\xi_0))}{ch\beta(1-\xi_0)}\right.\right.$$
$$\left.\left. + \frac{(1-\xi_0)(ch\beta\xi_0-\beta(1-\xi_0)ch\beta\xi_0)}{ch\beta\xi_0}\right]\right\}\xi - \frac{p(1-\xi_0)L^3\xi^3}{6EI_0\alpha} + \frac{p(1-\xi_0)L^3 sh\beta\xi}{\alpha(1-\alpha)\beta^3 EI_0 ch\beta\xi_0} \quad (11)$$

When $L1 \le z \le L1 + L2$, one gets

$$y_2(\xi) = \left\{\frac{pL^3\xi_0}{6EI_0\alpha}(1-\xi_0)(4\xi_0^2-\xi_0-1) + \frac{pL3}{\alpha(1-\alpha)\beta^3 EI_0}\left[\frac{\xi_0(sh\beta(1-\xi_0)+\xi_0\beta ch\beta(1-\xi_0))}{ch\beta(1-\xi_0)}\right.\right.$$
$$\left.\left. + \frac{(1-\xi_0)(sh\beta\xi_0-\beta\xi_0 ch\beta\xi_0)}{ch\beta\xi_0}\right]\right\}(1-\xi) + \frac{pt_0L^3(1-\xi)^3}{6EI_0\alpha} - \frac{pt_0L^3 sh\beta(1-\xi)}{\alpha(1-\alpha)\beta^3 EI_0 ch\beta(1-\xi_0)} \quad (12)$$

Where p = vertical concentrated loading; ξ_0 = acting position of vertical concentrated loading; y_1 = deformation for left side; and y_2 = deformation for right side.

3. Comparison of proposed theory with numerical simulations

In the current model, the longitudinal spacing D of studs includes $D1$ = 150 mm, $D2$ = 300 mm, and $D3$ = 450 mm, and planned loads include uniform load (q_y = 100 kN/m) and concentrated load (P = 300 kN). The material of flange plate used here is the reinforced concrete with mechanical properties: Young's modulus $E_c = 3.49 \times 10^4$ MPa, Poisson's ratio μ_c = 0.2 and grade of the concrete adopts $C30$. For the steel material, Young's modulus $E_s = 2.48 \times 10^5$ MPa and Poisson's ratio μ_s = 0.3. For the section size of calculation model of simply supported beam, web thickness (8 mm), bottom flange thickness (10 mm), and thickness (80 mm) for flange plate of the reinforced concrete.

Theoretical solutions and numerical results for deformation of simply supported beam are shown

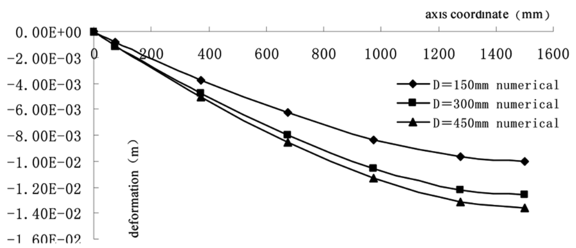

Fig. 2 Deformation distributions of simply supported beam subjected to concentrated load

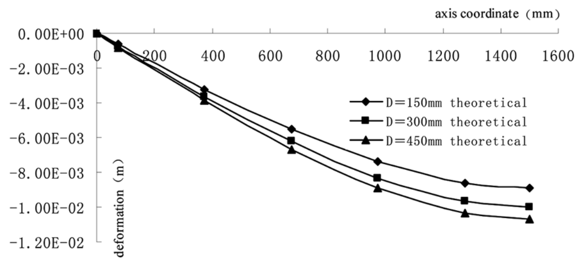

Fig. 3 Deformation distributions of simply supported beam subjected to concentrated load

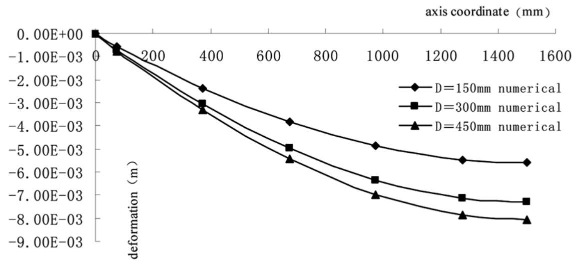

Fig. 4 Deformation distributions of simply supported beam subjected to concentrated load

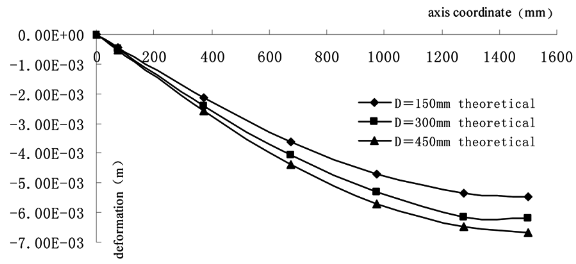

Fig. 5 Deformation distributions of simply supported beam subjected to concentrated load

in Figs. 2-5. Analytical solutions show an excellent agreement with numerical results. By analyzing a group of size with same section, one can see that theoretical solution of deformation for simply supported beam considering interface-slip subjected to different loads can be greatly improved by using Goodman's elasticity intermediate layer hypothesis and introducing the rotation angle formula of the axial deformation.

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