Structural Engineering and Mechanics, Vol. 34, No. 6 (2010) 739-753
DOI: http://dx.doi.org/10.12989/sem.2010.34.6.739

Pretension process control based on cable force observation values for prestressed space grid structures

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(Received May 19, 2009, Accepted December 28, 2009)

Abstract. Pointing to the design requirement of prestressed space grid structure being the target cable force, the pretension scheme decision analysis method is studied when there's great difference between structural actual state and the analytical model. Based on recursive formulation of cable forces, the simulative recursive system for pretension process is established from the systematic viewpoint, including four kinds of parameters, i.e., system initial value (structural initial state), system input value (tensioning control force scheme), system state parameters (influence matrix of cable forces), system output value (pretension accomplishment). The system controllability depends on the system state parameters. Based on cable force observation values, the influence matrix for system state parameters can be calculated, making the system controllable. Next, the pretension scheme decision method based on cable force observation values can be formed on the basis of iterative calculation for recursive system. In this way, the tensioning control force scheme that can meet the design requirement when next cyclic supplemental tension finished is obtained. Engineering example analysis results show that the proposed method in this paper can reduce a lot of cyclic tensioning work and meanwhile the design requirement can be met.

Keywords: prestress; space grid structure; pretension process; cable force; recursive system.

1. Introduction

In recent decades, the space grid structure designed for structures with large span and space is world-widely applied and quickly developed, because it has creative as well as elegant structural form and strong spanning ability. Since the space grid structure is usually a statically indeterminate structure, the introduced prestressing can result in the improvement of the stress distribution, reduction of structural deflection under service load and structural self-weight, so as to get a more economical structure design (Levy *et al.* 1994). The above advantages of the prestressed space grid structure make it possible to improve the structural span ability and space form variety. Therefore, such structures are widely used in public architectures like stadiums, exhibition halls, conference halls as well as waiting halls, and other large-scale industrial architectures.

The wide application of prestressed space grid structures has attracted a lot attention of many

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scholars and specialists, who make further study on its mechanical property and optimum design. In You's research (1997), the displacement control method for prestressed member structures was studied, and on the basis of force analysis method, the calculation method of prestressed member length regulation was presented so that the effective control of target displacement can be realized. Greschik (2008) focused on the mechanical property and design method when prestressed tendons are set along the diagonal of the prestressed truss structures. Based on effective form-finding methods of cable network structures, Tibert (2003) studied the shape optimal design of prestressed truss antenna structures. Han and Yuan (2005) used linear finite element method to analyze the static performance of prestressed composite latticed shell structures and the structural natural vibration characteristics were also investigated with subspace iterative method.

Various studies show that prestressed cable is a rather important bearing member of prestressed space grid structure, and it has important influence on the structural mechanic performance (Masao *et al.* 1999, Osamu *et al.* 1999). The way to ensure the internal force of prestressed cable meeting the design requirement through effective analysis and control method is crucial to the realization of excellent structural mechanic performance. As there are many prestressed cables in a large-scale prestressed space grid structure, it is impossible to stretch all the cables simultaneously under the limit of construction condition. So it is reasonable and necessary to carry out the batched tensioning. Meanwhile, the tensioning control forces of each batch cables should be analyzed so that internal forces of all the prestressed cables can meet the design requirements after tensioned. For this purpose, some scholars' researches focused on the pretension processs analysis of prestressed space grid structures. Dong and Yuan (2007) proposed initial internal force (IIF) method to analyze the pretension process of prestressed space grid structures. Zhuo and Ishikawa (2004) brought forth the tensile force compensation analysis method to analyze the construction process of hybridized structures. Li and Zhang (2004) used the cyclic progressing analysis method based on the cable initial strain to analyze the tensioning process of cable-supported lattice shell structures. Zhang and Ge (2007) proposed that one-time loading method can be used in the pretension process analysis of suspended dome structures, and such a method was applied into the construction of the badminton stadium for the Beijing Olympics Games.

The above pretension process analysis methods can solve the appropriate tensioning control force scheme according to design requirements, therefore all the cables can be ensured to meet the target requirement after tensioned. However, it should be pointed out that only when accurate analytical model is given in these methods, can the decision analysis be made. If there is great difference between the actual structural state in construction and the analytical model, the design requirement can not be realized accordingly after the pretension scheme analyzed beforehand is implemented. If the theoretical analysis model should be revised so as to make new pretension process analysis and to establish new tensioning control force scheme, the causes for the difference between actual structural state and theoretical analysis model should be identified. However, the influential factors in engineering construction are rather complicated and the structural finite element model updating technique is not mature enough to be used in practical engineering. Therefore, it's necessary to carry out subsequent cyclic supplemental tension to meet the design requirements. Meanwhile, the construction control should directly focus on the pretension process of the actual structure rather than on the theoretical analysis model of structure, and the scientific decision analysis method should be adopted so that negative effects of blindfold subsequent cyclic supplemental tension on the structure can be avoided.

From the perspective of system analysis, this paper focuses on the pretension process control of prestressed space grid structure which aims to meet the requirements of designed cable forces. On the basis of recursive formulation of cable forces, the recursive system of simulated pretension process is established, which is a mathematical model reflecting the mutual influence of the cable forces in pretension process. When the state parameters (influence matrix) are determined, the simulation analysis of actual pretension process of the structure can be directly carried out by using recursive system. Then, the pretension scheme decision method based on cable force observation values is proposed, therefore the tensioning control force scheme that meet the designed requirement when next cyclic supplemental tension finished can be solved.

2. Cable force recursive system of repeatedly and batching pretension process

2.1 Establishment of cable force recursive system

In repeated and batched pretension process of prestressed space grid structure, the cable forces have mutual influence on each other, which can be simulated by the established cable force recursive system. Let P_0 be the initial cable force vector before pretension, $F_j^{(i)}$ be the cable force vector after the j^{th} cable is tensioned for the i^{th} round and the initial value F_0^0 equal to P_0, $F_{jk}^{(i)}$ be the cable force of cable k after the j^{th} cable is tensioned for the i^{th} round, $T^{(i)}$ be the tensioning control force vector in the i^{th} round tension, $T_j^{(i)}$ be the tensioning control force of cable j in the i^{th} round tension, $A^{(i)}$ be the cable force influence matrix in the i^{th} round tension, $a_{kj}^{(i)}$ be the force variable quantity of cable k once the force of cable j increases by 1 in the i^{th} round tension, $A_k^{(i)}$ be the column k of $A^{(i)}$, n be the total number of cables, P^t is the target cable force vector of design requirement. Hereby, according to the pretension order and mutual influence principle of cable forces in each-round tension, the cable force recursive system can be established as following

$$
\begin{aligned}
& i = 1 \qquad F_0^{(1)} = P_0 \\
& F_1^{(i)} = (T_1^{(i)} - F_{01}^{i})A_1^{(i)} + F_0^{i} \\
& F_2^{i} = (T_2^{(i)} - F_{12}^{(i)})A_2^{(i)} + F_1^{(i)} \\
& F_3^{(i)} = (T_3^{(i)} - F_{23}^{(i)})A_3^{(i)} + F_2^{(i)} \\
& \cdots \\
& F_n^{(i)} = (T_n^{(i)} - F_{n-1,n}^{(i)})A_n^{(i)} + F_{n-1}^{(i)} \\
& i = i + 1 \\
& F_0^{(i)} = F_n^{(i-1)}
\end{aligned} \tag{1}
$$

From the perspective of system control theory, through detailed analysis for mathematic model described in Eq. (1), it can be found that there are four kinds of parameters in this system, including system initial values, system input values, system state parameters, and system output values. System initial values are the initial cable force vector P_0 before pretension. System input values are the tensioning control force vector $T^{(i)}$. System state parameters are the influence matrix of cable forces $A^{(i)}$. System output values are the cable force vector at a certain tensioning step $F_j^{(i)}$. If system initial values P_0 and state parameters $A^{(i)}$ are determined, then system output values $F_j^{(i)}$ at any tensioning step can be obtained. Meanwhile, the system is completely known and can be called the white system, under which the system is totally controllable. When the system initial values P_0 or the state parameters $A^{(i)}$ can't be determined, the system will be totally unknown, which can be called black system. Under this condition, the system is uncontrollable. Obviously, the situation investigated in this paper belongs to the scope of black system for designers and constructors. Then, for the black system or uncontrollable system, can the target state be obtained? That is, can system output values meet the requirement of target cable forces? Here, the simplest condition of two cables is taken as an example for illustration.

2.2 Analysis of cable force recursive system

Suppose that tensioning control forces in each-round tension are equal to target cable forces, that is, let $T^{(i)} = P^t$, then for two cables, the following recursive formulation of cable forces can be established

$$
\begin{gathered}
F_1^{(1)} = (P_1^t - P_{01})A_1^{(1)} + P_0, \quad F_{11}^{(1)} = P_1^t, \quad F_{12}^{(1)} = (P_1^t - P_{01})a_{21}^{(1)} + P_{02} \\
F_2^{(1)} = (P_2^t - F_{12}^{(1)})A_2^{(1)} + F_1^{(1)}, \quad F_{22}^{(1)} = P_2^t \\
F_{21}^{(1)} = (P_2^t - F_{12}^{(1)})a_{12}^{(1)} + F_{11}^{(1)} = ((P_2^t - P_{02}) - (P_1^t - P_{01})a_{21}^{(1)})a_{12}^{(1)} + P_1^t \\
\cdots\cdots
\end{gathered}
\tag{2}
$$

According to the recursive formulation of cable forces, the expression of cable force values in the i^{th} $(i > 1)$ round tension can be obtained as

$$
F_{12}^{(i)} = P_2^t + (P_1^t - P_{01})a_{21}^{(i)} \prod_{m=1}^{i-1} (a_{12}^{(m)} a_{21}^{(m)}) - (P_2^t - P_{02}) \prod_{m=2}^{i} (a_{12}^{(m-1)} a_{21}^{(m)}) \tag{3}
$$

$$
F_{21}^{(i)} = P_1^t + (P_2^t - P_{02})a_{12}^{(1)} \prod_{m=2}^{i} (a_{21}^{(m)} a_{12}^{(m)}) - (P_1^t - P_{01}) \prod_{m=1}^{i} (a_{21}^{(m)} a_{12}^{(m)}) \tag{4}
$$

If viewed from the angle of mathematical equation, for the purpose that the cable force values can come to the target cable forces when the i^{th} round tensioning finished, then

$$
F_{12}^{(i)} = P_2^t, \text{ or } F_{21}^{(i)} = P_1^t
$$

That is

$$
(P_1^t - P_{01})a_{21}^{(i)} \prod_{m=1}^{i-1} (a_{21}^{(m)} a_{12}^{(m)}) - (P_2^t - P_{02}) \prod_{m=2}^{i} (a_{12}^{(m-1)} a_{21}^{(m)}) = 0 \tag{5}
$$

$$
\text{or} \quad (P_2^t - P_{02})a_{12}^{(1)} \prod_{m=2}^{i} (a_{21}^{(m)} a_{12}^{(m)}) - (P_1^t - P_{01}) \prod_{m=1}^{i} (a_{21}^{(m)} a_{12}^{(m)}) = 0 \tag{6}
$$

From the characteristic of influence matrix A, it can be known that $a_{ij} = a_{ji}$, therefore the following can be obtained by combining the above two equations

$$\frac{P_2^t - P_{02}}{P_1^t - P_{01}} = a_{21}^{(1)} = a_{12}^{(1)} \tag{7}$$

In this way, if the cable force vector before pretension meets the requirement of Eq. (7), the requirement of target cable forces can be fulfilled after one-round supplemental tension with target cable forces as the tensioning control forces. However, the restriction of Eq. (7) is too strict, since it's impossible that the system initial value can always meet the requirements of Eq. (7). So it can't effectively prove whether the system output can meet the target requirement when the cable force recursive system is unknown.

If viewed from the angle of mathematic limit instead of the angle of mathematical Equation, then take limit of Eq. (3) and Eq. (4), the results can be obtained as

$$\lim_{i\to\infty} F_{12}^{(i)} = P_2^t + (P_1^t - P_{01})\lim_{i\to\infty}\left(a_{21}^{(i)}\prod_{m=1}^{i-1}(a_{12}^{(m)}a_{21}^{(m)})\right) - (P_2^t - P_{02})\lim_{i\to\infty}\left(\prod_{m=2}^{i}(a_{12}^{(m-1)}a_{21}^{(m)})\right) \tag{8}$$

$$\lim_{i\to\infty} F_{21}^{(i)} = P_1^t + (P_2^t - P_{02})a_{12}^{(1)}\lim_{i\to\infty}\left(\prod_{m=2}^{i}(a_{21}^{(m)}a_{12}^{(m)})\right) - (P_1^t - P_{01})\lim_{i\to\infty}\left(\prod_{m=1}^{i}(a_{21}^{(m)}a_{12}^{(m)})\right) \tag{9}$$

It's not difficult to see from the above equation that as long as $|a_{12}^i| < 1$ or $|a_{21}^i| < 1$, then

$$\lim_{i\to\infty}\left(a_{21}^{(i)}\prod_{m=1}^{i-1}(a_{12}^{(m)}a_{21}^{(m)})\right) = 0 \qquad \lim_{i\to\infty}\left(\prod_{m=2}^{i}(a_{12}^{(m-1)}a_{21}^{(m)})\right) = 0$$

$$\lim_{i\to\infty}\left(\prod_{m=2}^{i}(a_{21}^{(m)}a_{12}^{(m)})\right) = 0 \qquad \lim_{i\to\infty}\left(\prod_{m=1}^{i}(a_{21}^{(m)}a_{12}^{(m)})\right) = 0$$

Thus, the following equation is easy to get

$$\lim_{i\to\infty} F_{12}^{(i)} = P_2^t \qquad \lim_{i\to\infty} F_{21}^{(i)} = P_1^t \tag{10}$$

Eq. (10) indicates that, under the condition of two cables, if the absolute value of the mutual influence factor $|a_{ij}|$ is less than 1, the actual cable force will always approximate the target cable force value when the cyclic supplemental tension is carried out with target cable forces as tensioning control forces.

In the condition of more than two cables, the formula derivation can be made in similar way, and the following conclusion can be made: if the cyclic supplemental tension is carried out with constant target cable forces as tensioning control forces, the precondition that the actual cable force can approximate the target cable force is

$$|a_{ij}| < 1, (i \neq j) \quad \text{and} \quad \sum_j a_{ij} > 0 \tag{11}$$

Obviously, the two cables only needs to meet one of the two requirements in Eq. (11) simultaneously, as $|a_{ij}| < 1, (i \neq j)$, it's certain that $\sum_j a_{ij} > 0$. However, if there are more than two cables, both of the requirements in Eq. (11) should be met simultaneously. Only under this precondition, can the actual cable force get close to the target cable force. For the well-designed prestressed space grid structure in practical engineering, the mutual influence of cable forces is generally relatively small. Even though sometimes such mutual influence is great at the beginning of the tension, it will grow less as tensioning steps increase. After several cyclic tensions, the cable force influential factor is in a stable and relatively low numerical level. Therefore, $|a_{ij}| < 1, (i \neq j)$ will generally be fulfilled, but whether $\sum_j a_{ij} > 0$ can be fulfilled still depends on the actual structure.

2.3 Analysis of actual cable forces approaching target cable forces

From the mathematical model in page 4, 12th line (Eq. (1)), the solving process of cable force recursive system is only related with the initial cable forces before pretension, mutual influence matrix of cable forces, tensioning control force scheme, and also the number of cables. In this part, the case investigated is that the cyclic supplemental tension is carried out with target cable forces as tensioning control forces. So the tensioning control forces are equal to the target cable forces P^t. As a result, when P^t is given before solving, only three central parameters should be considered, i.e., mutual influence of cable forces, initial cable forces before pretension, and the number of cables. The influence of the parameters on the approaching process of actual cable forces to target cable forces is then investigated.

For the convenience of comparison, let the target cable force of each cable $P_j^t = 1$, and the number of cables be n, the influential factor of cable forces $a_{ij} = -\alpha$, the difference between the actual force in each cable before tensioning and the target cable force $P_{0j} - P_j^t = d$. Then, suppose that the central parameters are $n = 7, \alpha = 0.16, d = 0.5$, and only one parameter changes for one time. In this way, the situation that the actual cable force approaches the target cable force can be found under the influence of every parameter. Fig. 1- Fig. 3 show the analysis results under various parameters. Hereby, the formula for the error between actual cable forces and target cable forces can be defined as following

$$e = \sqrt{\sum_{j=1}^{n} (P_j^t - F_{nj})^2 / n} \tag{12}$$

Based on the above analysis, the conclusion can be summarized as the following two aspects:

(1) When the cable force recursive system status is unknown, and the cyclic tension is carried out with the target cable force as the constant tensioning control force, the actual cable force will get rather closer to the target cable force gradually under the precondition that the mutual influence between cables meet the requirement of Eq. (11).

(2) The speed for the actual cable force approaching target cable force, or in other words, the cyclic tension times necessary for the fulfillment of target cable force, depends on three factors: cable numbers, the difference between the cable force before tensioning and the target cable force, and the mutual influential factor between cables. If there are more cables and there is bigger cable force mutual influence as well as difference between actual cable force and target cable force before tensioning, the speed for the actual cable force approaching the target cable force will be slower, and the cycling times that is necessary for the fulfillment of target cable force will increase too.

3. Tensioning scheme decision-making based on cable force observation values

Although under the condition that cable force mutual influence can meet the requirement of

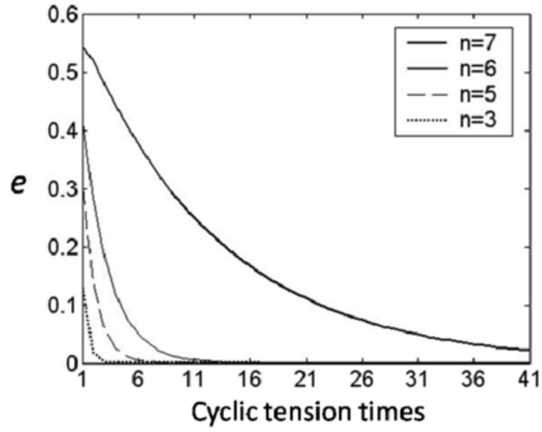

Fig. 1 $\alpha = 0.16$, $d = 0.5$

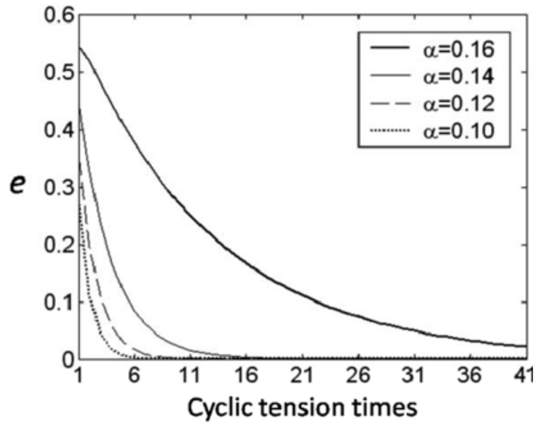

Fig. 2 $n = 7$, $d = 0.5$

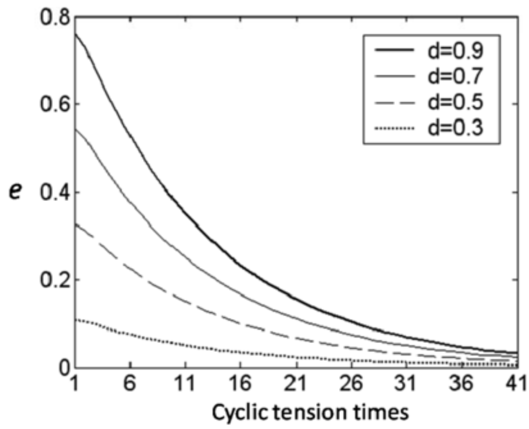

Fig. 3 $\alpha = 0.16$, $n = 7$

Eq. (11), the actual cable force will approach the target cable force approximately, if the target cable force is used constantly as the tensioning control force. However, the speed of approaching is largely connected with the cable force mutual influence, number of cables, and the difference between actual cable force and the target cable force before tension. It can be seen from Fig. 1-Fig. 3 that under the condition that there are more cable numbers, and there is larger cable force mutual influence as well as larger difference between the actual cable force and target cable force before tensioning, the cyclic tension should be carried out many times in order to obtain the target cable force. However, in practical engineering, because of the restriction of tensioning equipments and engineering progress, the cyclic tension with target cable force as the tensioning control force can be usually carried out for only 4-5 times, while after that the difference between actual cable forces and target cable forces may still be large. Under such condition, the monitoring unit will always intervene in, so as to ensure that the construction unit has worked to meet the designed requirement. But if the monitoring unit only plays the above role, its intervention can only inspect the result of tensioning, and can not offer guidance to the pretension process. In fact, the monitoring unit can make full use of its own equipments and technique in order to get the cable force data in every tensioning process as much as possible, the analysis of which can help to offer guidance for the following tensioning task. For the above reason, the concept 'pretension process scheme decision-making based on cable force observation values' is suggested in this paper.

It has been pointed out previously that when the system state parameters are known, the cable force recursive system will be totally known. In pretension process, the equipments and technique of the monitoring unit can be used to record cable forces in the cyclic pretension process. Based on these cable force observation values, the cable force influence matrix A for each tensioning will be calculated through simple algebraic operation. For example, for n cables, suppose that after finishing the $(i\text{-}1)^{\text{th}}$ tensioning, the cable force vector obtained by the monitoring unit is

$$F_{i-1} = \{F_{i-1,1}, \ldots, F_{i-1,j}, \ldots, F_{i-1,n}\}$$

Then, after the tensioning of cable j for the i^{th} time, the cable force vector at this time is

$$F_i = \{F_{i,1}, \ldots, F_{i,j}, \ldots, F_{i,n}\}$$

The column j of influence matrix A can be obtained as following

$$A_j^{(i)} = \{a_{1j}^{(i)}, \ldots, a_{mj}^{(i)}, \ldots, a_{nj}^{(i)}\}$$

Among them, $a_{mj}^{(i)}$ can be calculated as the following

$$a_{mj}^{(i)} = \frac{F_{i,m} - F_{i-1,m}}{F_{i,j} - F_{i-1,j}}, \quad m = 1, \ldots, n$$

Generally, with progress of cyclic pretension process, A will tend to be stable, therefore, the previously obtained value A can be used approximately as state parameters of the system, while the initial value of the system is the actual cable force vector after finishing the previous tensioning, which is already contained in cable force observation values. Therefore, in the following tensioning, the system changes from black to white, being the completely known and controllable state. Meanwhile, it is expected that in the following tensioning process decision, the proper tensioning control force vector could be chosen so that target cable force can be fulfilled after finishing the following tension.

Assuming that under current status, the actual cable force vector (namely system initial value) is F_0, cable force influence matrix (namely system state parameters) is A, cable tensioning control force vector (namely system input value) is T, based on the recursive formulation of cable forces, the expression of actual cable force vector after tensioning once is as following

$$F_n = F_0 + \sum_{j=1}^{n}\left((T_j - F_j)\left(A_j - \sum_{k=j+1}^{n} A_k \vec{a}_{k\to j}\right)\right) \tag{13}$$

Where the subscript of $\vec{a}_{k\to j}$ $k \to j$ means all the paths from k to j, $\vec{a}_{k\to j}$ refers to the algebraic sum of all the influential matrix factors consisting of these paths, the odd term is positive and the even term is negative.

For instance: $\vec{a}_{3\to 3} = a_{33}$, $\vec{a}_{3\to 1} = a_{31} - a_{32}a_{21}$, $\vec{a}_{4\to 1} = a_{41} - a_{42}a_{21} - a_{43}a_{31} + a_{43}a_{32}a_{21}$

In fact, the purpose of tensioning scheme decision-making is to choose appropriate T to make the actual cable force equal to the target cable force, that is $F_n = P^t$ when F_0 and A are already known, therefore the following equation can be established

$$P^t = F_0 + \sum_{j=1}^{n}\left((T_j - F_{0j})\left(A_j - \sum_{k=j+1}^{n} A_k \vec{a}_{k\to j}\right)\right) \tag{14}$$

that is $$\sum_{j=1}^{n}\left((T_j - F_{0j})\left(A_j - \sum_{k=j+1}^{n} A_k \vec{a}_{k\to j}\right)\right) + (F_0 - P^t) = 0 \tag{15}$$

Eq. (15) is the linear equation group about T, as long as the factors in these groups are solved based on F_0 and A, and the tensioning control force vector T of next time tensioning scheme can be obtained by solving the equation groups. Then the tensioning scheme decision-making is completed.

When there are more cables, the coefficient calculation of Eq. (15) is rather complicated with many calculations. Actually, because the recursive formulation of cable forces is easy for programming, based on the cable force recursive system, the iterative solving method can be established as following:

(1) Let $i = 1$, $T^i = P^t$, and input them to the cable force recursive system, then F_n^i can be obtained;

(2) There is deviation between F_n^i and P_t, the following equation can be used to revise T^i

$$T^{i+1} = T^i + (P^t - F_n^i) \tag{16}$$

(3) Judge whether $\varepsilon = \left|P^t - F_n^i\right|/P^t$ is less than the given tolerance $[\varepsilon]$. If $\varepsilon \le [\varepsilon]$, then the iteration will stop, T^i is the expected cable tension vector. Otherwise, let $i = i + 1$ and turn back to Step (2), continue the iteration until convergence.

4. Engineering example

4.1 Background of the example

As shown in Fig. 4, Yangtze River Flood-control Mode-testing Hall in Wuhan, Hubei province, is taken as the engineering background. This engineer is an arch-supported prestresssed latticed shell structure, which is composed by steel truss arches, latticed shells, horizontal cables and vertical hangers. With vertical hangers, steel truss archers hang the latticed shells. Meanwhile, by tensioning the hangers to make them rigid enough, effective elastic support is offered for the latticed shells, making it the four-edge supported and bidirectional forced system. Fig. 5 shows the analysis model of the whole structure established in finite element program ANSYS.

Fig. 4 Yangtze river flood-control mode-testing hall structure effect

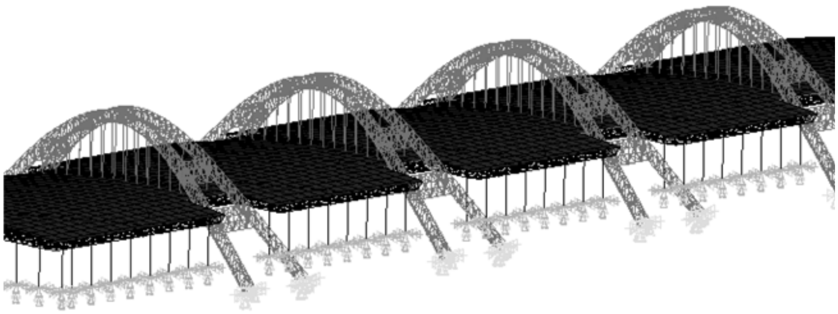

Fig. 5 Analysis model of the whole structure

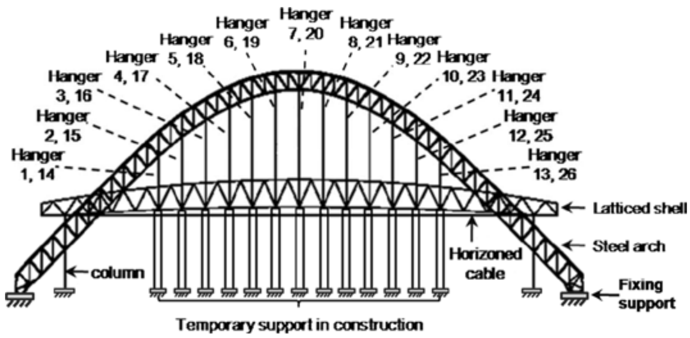

Fig. 6 Hanger numbering and temporary support

Concerning the requirement of pretension process, because the hangers haven't been tensioned during the installation construction of latticed shells, temporary supports (Shown in Fig. 6) are set for the hangers to maintain the bidirectional forced state of latticed shells. After finishing installing steel truss arches and tensioning hangers, the hangers will provide the latticed shells with vertical supports. Therefore, temporary supports can be removed to realize the designed state.

According to the pretension scheme determined by design requirement, tensioning hangers of every single steel truss arch follows the following order: tensioning 7, 20→tensioning 6, 19, 8, 21→tensioning 4, 17, 10, 23→tensioning 3, 6, 11, 24→tensioning 2, 15, 12, 25→tensioning 1, 14, 13, 26. In the batched tensioning process, the deformation of steel arches, latticed shells and vertical supports has effect on the forces of hangers. If the force analysis method is adopted, the effect of tensioning a hanger on the forces of previously tensioned hangers needs be computed, which makes the tensioning process analysis rather complicated. As a result, the pretension process analysis method based on hanger initial deformation is adopted in this paper. In this method, the initial deformation of hangers, which are determined according to the target hanger forces, is taken as state variables to compute the hanger force change in pretension process automatically (Chen and Zhou 2005). The analysis results are shown in Table 1, in which the data framed are the tensioning control force of hangers and then the tensioning control force vector T is as following

$$T = \{175.91, 126.40, 123.38, 96.79, 80.73, 60.74, 65.54\}\text{kN}$$

Considering the symmetry of the structure, taking hanger 1-7 as the analysis object, the tensioning sequence is from hanger 7 to hanger 1. Therefore, based on the analysis result in Table 1, if all the

Table 1 The force value of the hangers under batch tensioning

Hanger number	Tension 7,20	Tension 6, 19 8, 21	Tension 5, 18 9, 22	Tension 4, 17 10, 23	Tension 3, 16 11, 24	Tension 2, 15 12, 25	Tension 1, 14 13, 26
1	-	-	-	-	-	-	65.54
2	-	-	-	-	-	60.74	44.25
3	-	-	-	-	80.73	66.57	61.44
4	-	-	-	96.79	84.237	80.06	80.03
5	-	-	123.38	109.76	106.40	107.07	109.48
6	-	126.40	103.71	97.44	98.51	101.54	105.06
7	175.91	127.38	109.74	105.14	107.41	111.11	115.01

hangers are tensioned according to the tensioning control force vector T, the target hanger force vector P^t for the design requirement should be fulfilled as

$$P^t = \{115.01, 105.06, 109.48, 80.03, 61.44, 44.25, 65.54\}\text{kN}$$

However, in the actual construction process, the number and strength of the temporary supports set by the construction unit is in shortage. Moreover, there's bad influence of the great support settlement caused by the bad temporary support foundation condition. The above factors, together with other negative influences, cause the complete invalidation of most of the temporary supports before hanger pretension. At this time, there is a big difference between the actual forced state and the theoretical analysis model in Fig. 5. If the construction unit still adopt the original tensioning control force vector to stretch the hangers, many hangers will still be slack after tension, which varies greatly from target cable force vector P^t that the design requires. Therefore, it's necessary to make a new decision of the supplemental tensioning scheme so that the target hanger force of design requirements can be reached.

4.2 Simulation of the cyclic pretension process for hangers

In order to simulate the influence of the hanger cyclic tension on cable forces after invalidation of temporary supports, the temporary support element is killed in ANSYS analysis model, and the analysis is made based on the original tensioning control force vector T. In finite element analysis, decreasing temperature method is used to simulate the tensioning of hangers, in another word, to cause the hanger's initial deformation by decreasing the environmental temperature of the hanger, therefore the prestress effect of the hanger realized (Chen and Zhou 2005). When simulating hanger cyclic pretension process, the tensioning control force of every hanger for each time tension is known, therefore, it's necessary to change it into the corresponding hanger decreasing temperature values, which can be done according to the following steps.

When tensioning the j^{th} batch hangers for the i^{th} time, the decreasing temperature values of all the hangers at the $(i\text{-}1)^{\text{th}}$ time and the ones of the former j-1 batchers hangers at the i^{th} time are already known, then in Eq. (17), only ${}^{i}w_j$ is the unknown value that needed to be obtained while all the others are known

$$W_j^{(i)} = \{w_1^{(i)}, w_2^{(i)}, \ldots, w_j^{(i)}, w_{j+1}^{(i-1)}, \ldots, w_n^{(i-1)}\} \tag{17}$$

Where n is the number of the hanger batches, $W_j^{(i)}$ is the hanger temperature vector when tensioning the j^{th} batch hangers for the i^{th} time. Hereby, the tensioning control force $T_j^{(i)}$ at the i^{th} time can be used as the target value in order to obtain $w_j^{(i)}$ with the following iterative method.

Step (1): Suppose that p is the iteration times, at the beginning, let $p = 1$, ${}^{p}w_j^{(i)} = w_j^{(i-1)}$, the cable force of the j^{th} batch hangers ${}^{p}F_j^{(i)}$ can be obtained through structural analysis.

Step (2): ${}^{p}F_j^{(i)}$ is not equal to $T_j^{(i)}$, therefore ${}^{p}w_j^{(i)}$ needs to be revised. Assume that

$${}^{p+1}w_j^{(i)} = {}^{p}w_j^{(i)} + \Delta w_j^{(i)} \tag{18}$$

$$\Delta w_j^{(i)} = \frac{({}^{p}F_j^{(i)} - T_j^{(i)})}{\alpha E_s A_s} \tag{19}$$

Where E_s is elastic modulus of the hanger, A_s is the section area of the hanger, α is the linear expansion coefficient of the hanger, being assumed as $1 \times 10^{-5}/^{\circ}\mathrm{C}$.

Step (3): Make the new structural analysis and the new ${}^{p+1}F_j^{(i)}$ can be obtained. Assume that

$$\varepsilon_F = \frac{\left|{}^{p+1}F_j^{(i)} - T_j^{(i)}\right|}{T_j^{(i)}} \tag{20}$$

If ε_F is less than the set tolerance, the iteration will stop, ${}^{p+1}w_j^{(i)}$ is the decreasing temperature value of the j^{th} batch hangers when tensioned at the i^{th} time. Otherwise, let $i = i + 1$ and go back to step (2) to continue the next iteration until convergence.

The APDL programming language (Swanson 2002) is adopted in ANSYS to program the simulation analysis of the cyclic pretension process. Through finite element analysis, the actual hanger force after tension according to the original tensioning control force vector is as following

$$F_0^{(0)} = \{0,\ 3.32,\ 45.85,\ 34.78,\ 22.46,\ 0,\ 65.54\}$$

It can be known that, because of the invalidation of temporary supports, after tensioning according to the original tensioning scheme, cable 2 is completely slack, and cable 6 is nearly slack too, which cause the big difference between the actual hanger force and the target hanger force. At this time, the cyclic supplemental tension is necessary. If the cyclic supplemental tension analysis is continued with target hanger force vector P^t as the tensioning control force vector, the actual hanger force after finishing the first time cyclic supplemental tension $F_7^{(1)}$ is as the following

$$F_7^{(1)} = \{0,\ 36.88,\ 59.46,\ 35.39,\ 15.91,\ 2.82,\ 65.54\}$$

When the first round cyclic supplemental tension is finished, cable 7 is still completely slack and cable 2 is nearly slack too. The actual hanger force still varies greatly from the target hanger force. Therefore the cyclic supplemental tension still should be continued. In practical engineering, the supplemental tension generally can only be carried out for four times by the construction unit. Let's see the actual cable force $F_7^{(4)}$ after finishing the fourth time cyclic supplemental tension

$$F_7^{(4)} = \{43.10,\ 60.32,\ 73.92,\ 47.19,\ 28.58,\ 14.30,\ 65.54\}$$

It can be found that the actual hanger force still varies greatly from the target cable force. Therefore, it may be necessary to carry out the cyclic supplemental tension many times so that the actual hanger force can be equal to the target hanger force.

4.3 The hanger pretension scheme analysis based on cable force observation values

As being pointed out in the previous analysis, under the condition the monitoring unit intervenes, the hanger force data in every-time tensioning process can be used to calculate the hanger force influence matrix in each-time tension. In fact, the finite element analysis can play the role of monitoring unit here. The hanger force influence matrix in each-time tension can also be calculated based on the analysis result of previous tensioning process. It can be found that when finishing the cyclic tension for the fourth time, the influence matrix of hanger forces is stable as following

$$A = \begin{bmatrix} 1 & -0.489 & -0.267 & -0.146 & -0.071 & -0.024 & 0.013 \\ -0.214 & 1 & -0.300 & -0.167 & -0.084 & -0.032 & 0.005 \\ -0.134 & -0.345 & 1 & -0.239 & -0.131 & -0.061 & -0.02 \\ -0.082 & -0.214 & -0.267 & 1 & -0.226 & -0.125 & -0.079 \\ -0.045 & -0.12 & -0.163 & -0.252 & 1 & -0.256 & -0.207 \\ -0.017 & -0.05 & -0.084 & -0.154 & -0.283 & 1 & -0.502 \\ 0.007 & 0.008 & -0.024 & -0.086 & -0.203 & -0.446 & 1 \end{bmatrix}$$

In fact, the influence matrix of hanger forces, A, reflects the mutual influence of forces in hangers when tensioning the hangers in batches. Concretely speaking, the element a_{ij} in A is the force variation of hanger i once the force in hanger j increase by one unit. If A is determined, the simulative recursive system in this paper can be used to solve the hanger forces after tensioned when the structural initial state and tensioning scheme known.

First of all, the hanger force influence matrix A can be used as the state parameter of cable force recursive system and $F_7^{(4)}$ is used as the initial value. By this mean, after carrying out cyclic tension four times with target cable force as tensioning control force, the additional times of cyclic tension that is needed to make actual cable force equal to target cable force can be estimated. Analysis results of the recursive system are shown in Fig. 7. If the relative tolerance of target hanger force is 5%, the tolerance [e] can be calculated in the following

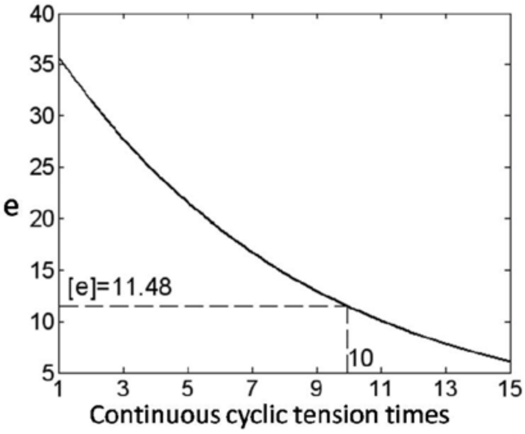

Fig. 7 Continuous cyclic pretension process

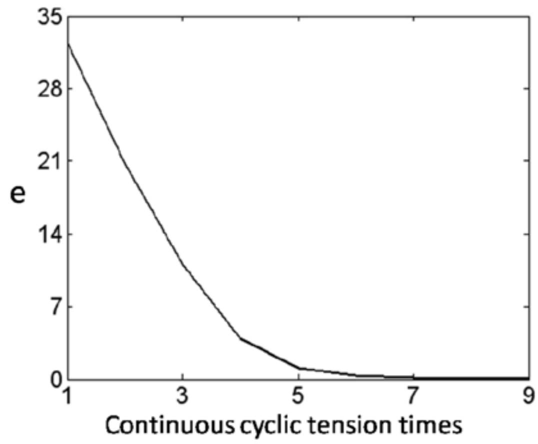

Fig. 8 Iterative solving method process

$$[e] = 0.05\sqrt{\sum_{j=1}^{7}(T_j)^2/7} = 11.48$$

As known from Fig. 7, if the cyclic supplemental tension is carried out with target cable forces as the constant tensioning control forces, at least another ten times cyclic tension will be necessary in order to control the difference between actual cable force and target cable force within 5%. By adopting the pretension scheme decision-making method based on hanger force observation values, the tensioning control force can be solved to make the actual hanger forces after the next click tension equal to the target hanger force. Herby, the iterative method is used, which is shown in Fig. 8. It can be seen that through the 9 times iterative calculation, the error is very small, and T obtained at this time can be used as the tensioning control force vector for the next cyclic tension. The detailed value of T is as the following

$$T = \{263.32, 181.16, 155.53, 116.17, 94.26, 76.40, 65.54\}\text{kN}$$

Then the final actual hanger force obtained by simulating the next cyclic tension in the finite element analysis is as following

$$F = \{114.25, 103.93, 109.08, 78.86, 60.12, 43.48, 65.54\}\text{kN}$$

It can be seen that the actual hanger force varies little from the target hanger force, and the pretension result has meet the design requirement. Therefore, the pretension scheme decision-making method based on hanger force observation values can save a lot of cyclic supplemental tension work and ensure that the pretension result can meet the design requirement.

5. Conclusions

This paper investigates the pretension process control with the target cable force as the design requirement. The cable force recursive system is established based on recursive formulation. Theoretically, if the cyclic tension is carried out with the target cable force as the constant tensioning control force and the requirement in Eq. (11) is met, no matter what initial value the actual cable force is, it will approach the target cable force endlessly. The cable force recursive system shows that the rate for the actual cable force to approach the target cable force is affected by three factors, including cable mutual influence, quantity of cable, the difference between actual cable force and target cable force before cyclic supplemental tension.

If there is larger cable force mutual influence as well as more cables, and there is big difference between actual cable force and target cable force before tensioning, the speed of the actual cable force approaching the target cable force will be slower, and the cyclic number that is necessary for the fulfillment of target cable force will increase too. Under the condition that the monitoring unit intervenes, the detective equipment can be used to get the cable force data of each supplemental tension. The proposed pretension scheme decision-making method based on cable force observation values can be used to calculate the tensioning control force that make the actual cable force equal to the target cable force after the next time cyclic tension. In this way, a great deal of cyclic tensioning work will be saved and the pretension result can meet the design requirement.

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