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Optimal laminate sequence of thin-walled composite beams of generic section using evolution strategies

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Abstract. A problem formulation and solution methodology for design optimization of laminated thin-walled composite beams of generic section is presented. Objective functions and constraint equations are given in the form of beam stiffness. For two different problems one for open section and the other for closed section, the objective function considered is bending stiffness about x-axis. Depending upon the case, one can consider bending, torsional and axial stiffnesses. The different search and optimization algorithm, known as Evolution Strategies (ES) has been applied to find the optimal fibre orientation of composite laminates. A multi-level optimization approach is also implemented by narrowing down the size of search space for individual design variables in each successive level of optimization process. The numerical results presented demonstrate the computational advantage of the proposed method "Evolution strategies" which become pronounced to solve optimization of thin-walled composite beams of generic section.

Kerwords: evolution strategies; laminate sequence; optimization; composite; penalty approach.

1. Introduction

During the past 40 years, fibre composite materials have increased in use in many engineering applications. These materials can be engineered to have high stiffness and strength in a chosen direction. Fibre composites have many desirable characteristics such as high strength to weight ratio, corrosive resistance, magnetic transparency and excellent fatigue characteristic in the direction of fibres.

Research advances made in the analysis of fibre composites have been focussed primarily on plate and shell models. It is interesting to note that the beam theory is more difficult to develop than plate theories for an-isotropic materials. For isotropic materials, the Euler Bernoulli beam theory is developed by simply ignoring the dependence in the width direction and Poisson's ratio. Because of an-isotropic coupling, this cannot be done for fibre composite beams. The author has developed a method for the computation of mechanical properties of thin-walled composite beam of any generic section (Rajasekaran 2005). Due to an-isotropic nature of the beam walls, composite beams exhibit unconventional coupling behaviour between extension, bending, torsion and transverse shear. This

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elastic coupling can be tailored to enhance the structural response. For example, bending-twisting coupling can be used to eliminate zero elastic divergence instability of forward swept wings on advanced aircrafts. Hence designing a structure made up of composite materials is an optimization problem. It is possible to optimize a hybrid composite beam. Since design variables are many, in this paper attempt is made to optimize the design of composite open or closed generic section treating fibre orientations as design variables.

2. Mechanics of laminate thin-walled beam

Vlasov (1961), Timoshenko (1945) and Gjelsuk (1981) laid the foundation for all later research in this area. Vlasov (1961) developed a general theory for isotropic thin-walled beams of open and closed sections in 1930. The concept of sectorial area was introduced by him. Timoshenko (1945) developed a similar theory for isotropic beams with open sections also included primary warping effects. More recently, Gjelsuk (1981) extended Vlasov's theory to account for secondary warping for beam with both open and closed sections. Bhaskar and Librescu (1995) presented a geometrically nonlinear theory that poses no restriction on the lay up of the wall laminates and include the effects of secondary warping. More recently Maddur and Chathurvedi (1999) developed a first order deformation theory applicable to open section. Given a cross section, a stacking sequence for the walls, it is shown that effective stiffness can be calculated. Morton and Webber (1994) have described a procedure for obtaining an optimum (minimum area) design of a uniform composite I - beam with regard to structural failure, local buckling and central deflection constraint. The beam is subjected to lateral point load at its mid point and is assumed to be simply supported at each end and they have extended the method to fixed ends as well.

3. Manufacturing consideration

Several different techniques such as pultrusion, resin transfer molding (RTM) or lay up of pre impregnated (Pre -preg) material have been used for manufacture of a composite I section. According to Savic *et al.* (2001) three separate panels have been cured and then bonded together to form the I section as shown in Fig. 1(a). An advantage of the application is that the I section may have three distinct laminates in the walls because the fibre orientation in one wall does not depend on the fibre orientation in the other walls. Therefore it is possible to produce a beam with any desired lay up in each wall without restriction on fibre angles. A disadvantage is that the fibres are discontinuous across the web/flange interface and hence this may represent a potential failure site. A second technique, which is more commonly used, is to have two C- shaped laminates as shown in Fig. 1(b) and put back to back to form the web of the I section and additional plies are put on the top and bottom to complete the flange laminates. In this application fibres are continuous across flange web junction. In this paper it is assumed that the sections are manufactured using the second approach. Hence it is assumed that all plies present within the web laminate will be extended through web/flange interface becoming a part of both top and bottom flange laminates. There are two possibilities such as web symmetric cross ply and web un-symmetric angle ply as shown in Figs. 2(a) and (b).

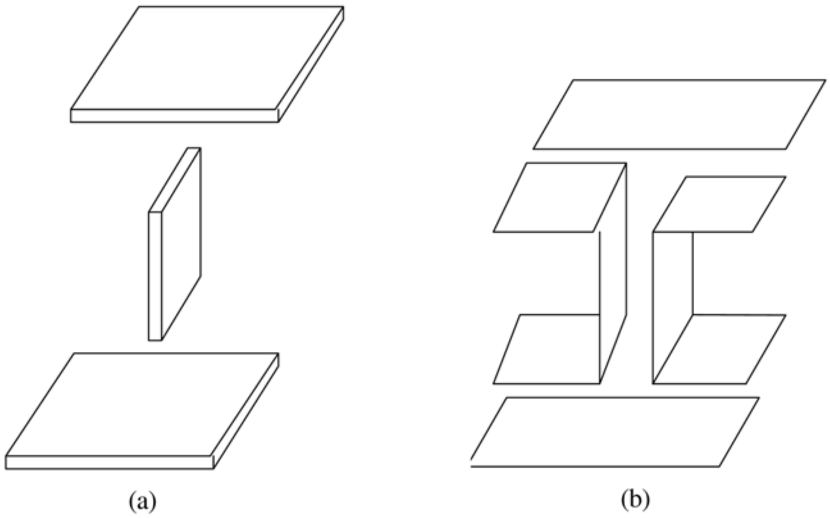


Fig. 1 (a) Three component plates to make I beam, (b) I beam made up of 2 C and two plates

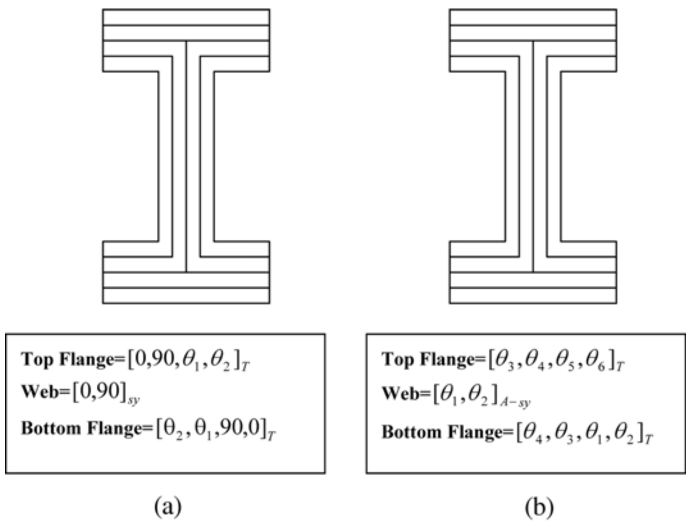


Fig. 2 (a) Symmetric I beam (Web symmetric cross ply), (b) Unsymmetric I beam (Web Anti-symmetric ply)

4. Design optimisation

Composite materials offer more possibility for a design than isotropic material. It can be seen that varying fibre orientations in each ply or a number of plies can produce a large number of acceptable designs that satisfy some specific loading condition. Hence it is necessary to find the structure with the best possible proportion for a specific application. An optimal structure for one specific application may not be optimal for another application. Hence optimum technique can help the design engineer to show various dependencies between design variables and structural properties and identify the best structure for a specific application. Even though numerous optimum techniques have been developed, but many of them may not be appropriate for composite design problem. In this paper we will apply the technique of Evolution Strategies.

5. Evolution strategies (ES)

Evolution strategies were proposed for parameter optimization problems in the seventies by Rechenberg (1973) and Schwefel (1981). ES imitate biological evolution in nature and have three characteristics that make them differ from other conventional optimization algorithms. (i) In place of the usual deterministic operators, they use randomized operators: mutation, selection as well as recombination; (ii) instead of single design point, they work simultaneously with a population of design points in the space of variables; (iii) they can handle continuous, discrete or mixed optimization problems. The second characteristic allows for a natural implementation of ES on parallel computing environments. The ES, however, achieve a high rate of convergence compared to other nontraditional techniques due to their self-adaptation search mechanism and are considered more efficient for solving real world problems. The ES were initially applied for continuous optimization problems, but recently they have also been implemented in discrete and mixed optimization problems.

5.1 ES for discrete optimization problems

In engineering practice the design variables are not continuous because usually the structural parts are constructed with certain variation of their dimensions. Thus design variables can only take values from a predefined discrete set. For the solution of discrete optimization problems a modified algorithm has been proposed by Cai and Thierauf (1993). The basic differences between discrete and continuous concern the mutation and recombination operators. The mutation operator ensures that each parent $s_p^{(g)}$ of the current generation g produces an offspring $s_o^{(g)}$ whose genotype is slightly different from that of the parents as

$$s_o^{(g)} = s_p^{(g)} + z^{(g)} \tag{1}$$

where $z^{(g)} = [z_1^{(g)}, z_2^{(g)}, \ldots, z_n^{(g)}]^T$ is a random vector. The mutation operator in the continuous version of ES produces a normally distributed random change vector $z^{(g)}$. Each component of this vector has small standard deviation value σ_i and zero mean value. As a result of this there is a possibility that all components of the parent vector may be changed but usually the changes are small. In the discrete version of ES the random vector $z^{(g)}$ is properly generated in order to force the offspring vector to move to another set of discrete values.

The fact that the difference between any two adjacent values can be relatively large and this is against the requirement that the variance σ_1^2 should be small. For this reason it is suggested that not all the components of the parent vector but only a few of them (eg l) should be randomly changed in every generation. This means that $(n-l)$ components of the randomly changed vector $z^{(g)}$ will have zero value. In other words the terms of the vector $z^{(g)}$ are derived from

$$z_1^{(g)} = \begin{cases} (k+1)\delta_{si} & \text{for } l \text{ randomly chosen components} \\ 0 & \text{for } (n-l) \text{ other components} \end{cases} \tag{2}$$

where δ_{si} is a small change in the design variable (say 1/10 of design variable). This of course violates the discreteness of the sections and once the final optimal design is arrived at we can make necessary corrections by choosing the available sections.

k is a random integer, which follows the Poisson distribution

$$p(\kappa) = \frac{(\gamma)^{\kappa}}{\kappa!}e^{-\gamma} \tag{3}$$

γ is the standard deviation as well as the mean value of the random number k. The choice of l depends on the size of the problem and here half of the total number of design variables is considered. The l components are selected randomly in every generation from the set of design variables.

In both versions of continuous and discrete optimisation of multi-membered ES there are two different types of selection:

$(\mu+\lambda)$**-ES**: The best μ individuals are selected from a temporary population of $(\mu+\lambda)$ individuals to form the parents of the next generation.

(μ, λ)**-ES**: The μ individuals produce λ offspring $(\mu \le \lambda)$ and the selection process defines a new population of μ individuals from the set of λ offspring only.

In the second type, the life of each individual is limited to one generation. This allows the (μ, λ)-ES selection to perform better on dynamic problems where the optimum is not fixed, or on problems where the objective function is noisy. The next important thing in the optimization procedure is the termination criteria. The optimization procedure terminates when one of the following termination criteria is satisfied: (i) when the best value of the objective function in the last $4 \times n \times \mu/\lambda$ generations remains unchanged, (ii) when the mean value of the objective values from all parent vectors in the last $2 \times n \times \mu/\lambda$ generations has not been improved by less than a given value ε_b (=0.0001) (iii) when the relative difference between the best objective function value and the mean value of the objective function values from all parent vectors in the current generation is less than a given value ε_c (=0.0001), (iv) when the ratio μ_b/μ has reached a given value ε_d (=0.5 to 0.8) where μ_b is the number of the parent vectors in the current generation with the best objective function value. The last criterion is used for all the problems considered in this paper.

6. ES in structural optimization problems like composite beams

There are four main components in the operations of ES and they are

1) Creation of Initial pool of designs.
2) Combination of the designs in a pool in order to produce better designs.
3) Mutation for giving a small change to the offspring from the parents.
4) Obtain a new generation of designs.

6.1 Creation of pool of designs

In ES, the pool of designs is called population. Here the individuals consist of binary strings (there may be other representations) representing design parameters such as fibre orientation of laminates in the case of optimal stacking sequence. Assuming we represent fibre orientation by 4 bit binary string $(X_i)_{\min}$ is represented by 0000 and $(X_i)_{\max}$ is represented by 1111. The increment $(X_i)_{\mathrm{inc}}$

in the design variable is calculated as

$$(X_i)_{\text{inc}} = \frac{\{(X_i)_{\max} - (X_i)_{\min}\}}{(2^{nd} - 1)} \tag{4}$$

where '*nb*' is the number of bits.

To get the corresponding fibre orientation for a bit string of '1011' the decoded value is

$$1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 11 \tag{5}$$

and the area corresponding to '1011' is given by

$$X_i = (X_i)_{\min} + 11(X_i)_{\text{inc}} \tag{6}$$

Eq. (4) again violates the discreteness of the sections. Once the final optimum design is arrived at, fibre orientations very near to the integer values can be chosen for a practical design.

In the studies by Papadrakakis *et al.* (1998, 2002) and Lagaros Papadrakakis and Kokossalakis (Lagaros *et al.* 2002), it was found that probabilistic search algorithms are computationally efficient even if greater number of analyses are needed to reach the optimum. These analyses are computationally less expensive than in the case of mathematical programming algorithms since they do not need gradient information. Furthermore, probabilistic methodologies were found to converge to global optimum in due course whereas mathematical programming algorithms may be trapped in local optima. Finally the natural parallelism inherent in probabilistic search algorithms makes them very attractive for application in parallel computer architectures.

To start with the algorithm, the initial population is created randomly. If the fibre orientations consists of orientation of fibres of 4 layers and the value of each group is represented by 4 bits then any one individual contains 16 bits and there will be *n* such individuals. Care is taken that all parents are selected in the vicinity of the best parent. Mating the best parent with all other parents through crossing over operation generates the offspring. Offspring thus generated are mutated and checked if they are in the feasible region. According to $(\mu + \lambda)$ selection scheme, in every generation the values of the objective function of the parent and the offspring vectors are compared and the worst vectors are rejected, while the remaining ones are considered to be the parent vectors of the new generation. On the other hand, according to (μ, λ) selection scheme, only the offspring vectors of each generation are used to produce the new generation. This procedure is repeated until the chosen termination criterion is satisfied. The ES algorithm for structural optimization of composite beam applications can be stated as follows:

6.2 Evolution algorithm

1. Selection step:
   Selection of $S_i(i = 1, 2, \ldots, \mu)$ parent vectors of the design variables
2. Analysis step :
   Find the objective function for each population
3. Constraints check :
   All parent vectors become feasible
4. Offspring generation:
   Generate $S_j(i = 1, 2, \ldots, \lambda)$ offspring vectors of the design variables

5. Mutation
   The offspring thus generated are mutated to give a small variation from the parents
6. Analysis step :
   Find the objective function for each population
7. Nominal convergence check :
   Nominal convergence occurs when the mean value of the objective function of the designs of the current population is relatively close to the best design achieved until the current generation. If satisfied according to the current level of violation, continue. Else change S_j and return to step 4.
8. Selection step :
   Selection of the next generation parents according to $(\mu + \lambda)$ or (μ, λ) selection schemes
9. Convergence check :
   If satisfied stop, else go to step 4.

7. Multilevel optimization

In this work, a multilevel optimization approach is implemented and proved to eliminate the effect of a drawback such as trapping in a local optimum. In this approach, an initial optimization named the first level optimization is carried out with $(X_i)_{\min} < X_i < (X_i)_{\max}$ by judiciously assuming the values of minimum and maximum values for the design parameters. After a few iterations, one can get X_i for near optimum solution. In the second level optimization, we can narrow down the search space for each variable X_i as

$$(\bar{X}_i)_{\min} < X_i < (\bar{X}_i)_{\max} \tag{7}$$

where the minimum value of the design variable of the present iteration is greater than that used in the first level and the maximum value of the design variable must be less than that used in the first level thus narrowing down the search space.

The process is continued in a similar fashion by narrowing down the search space. The proposed approach has two important characteristics: firstly it encourages the optimization process to investigate better solutions in more restricted favourable regions of search space and secondly each level may be interpreted as one step climbing down the hill towards the foot of the hill (minima). During the tests it is observed that five to six levels of optimization are adequate for the convergence to the true optimum. This multi-level optimization procedure encourages the optimization process to investigate better solutions in more restricted favourable regions of the search space. This novel approach is implemented in the determination of optimal fibre orientation of composite beams.

8. Transformation of constrained optimization problem to unconstrained optimization problem

Genetic Algorithm (GA) and Evolution Strategies (ES) initially developed employed penalty function approach both static and dynamic in majority of the cases for treating cost optimization problem. In Static Penalty approach

$$F_X = \phi_X \tag{8}$$

if constraints are satisfied where ϕ_X is the objective function to be minimised.

If constraints are not satisfied

$$F_X = \phi_X(1 + pC) \tag{9}$$

where C is the sum of the constraints given by

$$C = \sum_{j=1}^{k} C_j \tag{10}$$

In Eq. (9), 'p' is called the static penalty parameter. The main advantage is its simplicity. It is a traditional method and is still used in practice since it also exploits the information from infeasible points to guide search using. Usually 'p' value is chosen as 10 by Rajeev and Krishnamoorty (1992) for practical purposes. There is no guidance how to choose single penalty parameter 'p'. If 'p' is chosen too small, the search will converge to an infeasible solution and otherwise a feasible solution may be located but it could be far from the global optimum In dynamic penalty approach proposed by Joines and Houck (1994)

$$F_X = \phi_X + p(cg)^{\alpha} \tag{11}$$

where 'p', α and β are constants. A reasonable choice of these parameters proposed by Joines and Houck are $C = 0.5\text{-}2.0$ and α and β are taken as 1 and 'g' is the generation number. The term $(pg)^{\alpha}$ is the penalty term, which takes extreme large values to make even slight violation design not to be selected in subsequent generations. In this paper we use static and the dynamic penalty approach. There are many more penalty approaches recently developed as a) annealing penalty b) adoptive penalty c) co-evolutionary penalty d) death penalty e) use of non-dominance and the reader may refer to the paper by Coello (2002).

9. Problem formulation

It is required to maximize beam mechanical bending stiffness E_{33} about X-axis subjected to 1) axial beam stiffness - $E_{11} \geq$ equivalent Aluminum beam axial stiffness $(E_{11})_{Al}$ 2) torsional beam stiffness – $E_{44} \geq$ equivalent Aluminum beam torsional stiffness $(E_{44})_{Al}$ 3) Beam mechanical bending stiffness E_{22} about Y-axis $\geq$ equivalent Aluminum beam bending stiffness $(E_{22})_{Al}$. Hence the function to be maximized is

$$F = E_{33} \tag{12}$$

In order to convert to minimization problem consider function ϕ to be minimized as

$$\phi = 10^{14}/E_{33} \tag{13}$$

The constraints can be written as

$$C_1 = \left(1 - \frac{E_{11}}{(E_{11})_{Al}}\right)$$
$$If - ve\ C_1 = 0 \tag{14a}$$

$$C_2 = \left(1 - \frac{E_{22}}{(E_{22})_{Al}}\right)$$
$$If - ve\ C_2 = 0 \tag{14b}$$

$$C_3 = \left(1 - \frac{E_{44}}{(E_{44})_{Al}}\right)$$
$$If - ve\ C_3 = 0 \tag{14c}$$

$$C = \sum_{i=1}^{3} C_i \tag{15}$$

Then constrained optimization problem is converted to unconstrained optimization problem as shown in Eq. (9) as

$$F_X = \phi_X(1 + pC) \tag{16}$$

where p is taken as 10.

10. Examples of optimization analysis

A Fortran program "COMPOPT" has been developed to determine the fibre orientations for optimal design. The input parameters for the program are the material properties, dimensions of the cross section, closed or open and number of plies in each wall. The user has to select a section from a design family by choosing symmetry, anti-symmetry or un-symmetric fibre orientation.

10.1 Example.1

Composite I section optimized for transverse (bending) loads (see Fig. 2(a)). The objective is to identify the flange / web stacking sequence that maximizes the mechanical bending stiffness E_{33} of the composite section about X-axis. However in all practical applications, other structural properties such as axial and torsional stiffness must be kept above minimum level so that structure does not deflect or twist excessively if exposed to random loads in different directions. I section of size 50.8×50.8 mm with 4 layers of laminate (each) thickness of 0.127 mm have been used. The material selected is High Performance Graphite with properties as shown in Table 1. The optimization problem can be described in words as follows.

Table 1 gives the assumed material properties and Table 2 shows the structural properties of equivalent Aluminum section used as a comparison.

Table 1 Assumed material properties

Material	E_{LL}	E_{TT}	G_{LT}	γ_{LT}	Ply thickness
High performance Graphite epoxy	470 GPa	6.2GPa	5.58GPa	0.31	0.127
Aluminum	70.0	70.0	26.0	0.33	t = 0.508

Table 2 Properties of equivalent aluminum section (Example 1)

Axial Stiffness E_{11}	6.081MN
Mechanical Bending Stiffness about y axis E_{22}	0.8718e09 Nmm^2
Torsional Mechanical Stiffness E_{44}	1.752E05 Nmm^2

Table 3 E_{33} values for Example 1

θ_1/θ_2	0-45	45-90
0-45	$\theta_1 = 5$; $\theta_2 = 27.6$ 1.1607E10	$\theta_1 = 2.35$; $\theta_2 = 45$ 1.024E10
45-90	$\theta_1 = 45.64$; $\theta_2 = 22.84$ 0.901E10	$\theta_1 = 45.71$; $\theta_2 = 57.8$ 0.667E10

Table 4 Multi-level optimization (Example 1)

	θ_1	θ_2	E_{33}
Range	0-10	20-30	1.172E10
Result	4.0	26.5	
Range	3-5	24-28	1.174E10
Result	4.01	26.28	

The influence of the optimal fibre orientation on the beam cross section is to improve the structural performance. Before optimization process begins, let us assume $\theta_1 = \theta_2 = 0$ and calculate the following properties as
$E_{33} = 1.8284e10$; $E_{22} = 5.2247e9$; $E_{44} = 3.7161e4$; $E_{11} = 36.43\text{MN}$. It is seen that E_{44} (3.7161e4) of the composite section < E_{44} of equivalent aluminum section (1.752e05) and hence the constraint is not satisfied. Hence optimal fibre orientations have to be found out to maximize E_{33} so as to satisfy the constraints. After carrying out ES the value of E_{33} is arrived at satisfying the constraints.

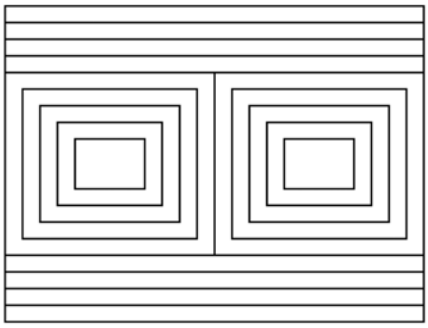
First optimization problem is performed in four regions as shown in Table 3. From Table 3 it is clear that E_{33} is maximum in the first range i.e., when

$$\theta_1 \leq 45; \quad \theta_2 \leq 45 \tag{17}$$

Then multilevel optimization is performed as shown in Table 4. For the problem the fibre orientations of $\theta_1 = 4.01^{\circ}$; $\theta_2 = 26.28^{\circ}$ will give maximum E_{33} of 1.174E10 satisfying all the constraints.

10.2 Example. 2

It is required to design optimal twin cell section of 101.6 × 50.8 with the laminate sequence as shown below using high performance Graphite (see Fig. 3).



Bottom Flange= $[90/0_3/\theta_1/\theta_2/\theta_3/\theta_4]_T$

Top Flange = $[\theta_4/\theta_3/\theta_2/\theta_1/0_3/90]_T$

Exterior Web= $[90/0_3]_T$

Interior Web= $[90/0_3]_{sy}$

Fig. 3 Twin - cell laminated beam

Table 5 Properties of equivalent aluminum section (Example 2)

Axial Stiffness E_{11}	0.2432E8
Mechanical Bending Stiffness about y axis E_{22}	0.2441E11
(Torsional Mechanical Stiffness E_{44})/3	0.2309E10

Bottom flange = $[90/0/_3\theta_1/\theta_2/\theta_3/\theta_4]_T$

Top flange = $[\theta_4/\theta_3/\theta_2/\theta_1/0/_3/90]_T$

Exterior web = $[90/0/_3]$

Interior web = $[90/0/_3]_s$

The properties of equivalent aluminum section are given in Table 5. Since the box section is considered, the torsional mechanical property is assumed to be greater than 1/3 (St.Venant constant of equivalent Aluminum section.). The optimal fibre orientations are obtained as

$$\theta_1 = 24.71^{\circ}; \quad \theta_2 = 13.71^{\circ}; \quad \theta_3 = 5.71^{\circ}; \quad \theta_4 = 1.42^{\circ} \tag{18}$$

with properties of

$$\begin{aligned} E_{33} &= 0.58395\text{E}11 \\ E_{44} &= 0.23126\text{E}10 \\ E_{11} &= 0.11484\text{E}9 \\ E_{22} &= 0.114599\text{E}12 \end{aligned} \tag{19}$$

11. Conclusions

The optimal design of composite beam of open and closed section manufactured using uni-directional pre-preg tape is considered in this study. Promising beam designs are identified by combining with this the theory of composite beam with Evolution Strategies. Constraints imposed by manufacturing technique commonly used to produce the sections using unidirectional tape are included in the problem formulation. Two different example analyses a) for open section b) for closed section are presented to illustrate the optimal process. In both the examples the objective is to maximize the beam mechanical stiffness E_{33}. Similarly results could be obtained for sections by maximizing axial or torsional stiffness depending on the situation. In both the examples high performance Graphite epoxy has been used. The optimal stacking sequence depends on whether the beam is designed to support axial load/or transverse bending load as would be expected. This study shows that optimal stacking sequence is very sensitive to material type as well. The analysis presented in this paper demonstrates that the combination of global optimization technique such as Evolution Strategies and appropriate mechanical model can produce practical insight into optimal design of composite structures in general and composite beams in particular of generic open or closed cross section.

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