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Technical Note

Profiled sheets - the optimum vs the oft used

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1. Introduction

Shape, as a primary design variable, warrants consideration in structural optimization. The mode of the transmission of force is a function of configuration. The technical success of optimization depends on how efficiently the load flow is accomplished. Decision-making that leads to final geometry represents the highest level of structural engineering.

Shape optimization of structures is an ever-growing area of research that has attracted the attention of several researchers. Shape searching problem formulation and solution techniques have been detailed in numerous articles (Beveridge and Schechter 1970, Gallagher and Zienkiewicz 1977, Richard Courant and Frite John 1989). The optimization of curved structures, such as arches and shells under mechanism constraints have been extensively studies (Rozvany 1992). Parametric surfaces such as Bezier surfaces considering stress deviation or fundamental frequency, have been employed to optimize shells (Ramm 1993). Reanalysis techniques expediting design process without compromising accuracy for boundary element systems have been detailed by Leu (1999). The efficacy of fuzzy optimum design has been demonstrated by Kang *et al.* (1999).

Ohsaki and Hayashi (2000) presented the shape optimization of round ribbed shells, wherein the number of ribs should be defined in advance by employing fairness metrics. The flexural and shear behaviors of profiled double skin composite elements have been studied (Hossain and Wright, 2004). Finite element (FE) modelling of the shear behavior of walls with particular emphasis on the simulation of steel-concrete interface has been attempted by Hossain and Wright (2005). Analytical and experimental investigations on profiled steel sheets to develop self-supporting roofing element have been conducted (Islam *et al.* 2005). Mezzomo *et al.* (2009) presented studies on the mechanical behavior of trapezoidal roofing sheets employing GA (Genetic Algorithm).

Profiled sheets are used extensively in roofing, cladding, sheet piling and containments. Profile development is increasing rapidly owing to advancement in materials and construction technology. The resistance to longitudinal bending is proportional to the depth and thickness and depends greatly on the profile itself.

The problem of finding the best form for profiled sheets has been formulated, and the objective function to be maximized has been shown to be a functional. A solution to the problem has been

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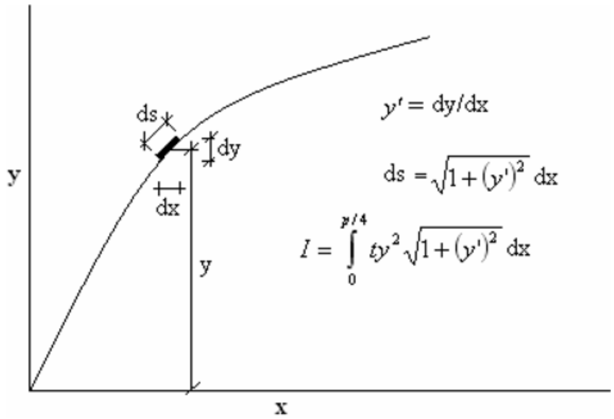

Fig. 1 Profiled sheet parameters

sought by the calculus of variations. Comparison of the best profile obtained with the traditional sine wave form most widely used, for structural efficacy, suggests that the popularity of sine wave form is not unwarranted.

2. Form finding -problem formulation

The strength of the profiled sheet in longitudinal bending is proportional to the depth of corrugation, thickness and on the profile itself. More often than not profiles are repetitive and most popular being of the sine wave form (Steel Designer's Manual 1985). Hence it may be assumed that if a quarter of the best form is determined, the most optimum profile is known.

If 'p' is the pitch of the corrugations and 'd' the depth and 't' the thickness of the sheet and if $y = f(x)$ a continuous function that defines the sheet profile (Fig. 1), for quarter pitch we can write I the inertia of the section about x-axis as,

$$I = \int_0^{p/4} ty^2\sqrt{1+(y')^2}\,dx$$

The problem of finding the best form is now reduced to extremising 'I'.

3. Solution by Calculus of Variations

The inertia of the profile section about the axis of bending is as follows;

$I = \int_0^{p/4} ty^2\sqrt{1+(y')^2}\,dx$ is a functional, and an extremum can be obtained by satisfaction of the famous Euler-Lagrange Equation.

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\frac{\partial F}{\partial y'} = 0$$

where
$$F = y^2\sqrt{1+(y')^2}$$

We approximate F as,

$$F = y^2\left[1+\frac{(y')^2}{2}\right]$$

And since F is independent of x, satisfaction of the Euler-Lagrange Equation is reduced to the following condition (Shames and Clive 1995).

$$y'\frac{\partial F}{\partial y'} - F = C_1$$

where, C_1 is a constant.

From this condition we obtain

$$y' = \sqrt{\frac{2(C_1+y^2)}{y^2}}$$

Separating variables and integrating yields

$$k + x = \sqrt{\frac{C_1+y^2}{2}}$$

This can be reduced to

$$\frac{\sqrt{C_1}}{2} + \frac{y^2}{2\sqrt{2C_1}} = x + k$$

and from boundary conditions, $x = 0$ when $y = 0$ and $x = p/4$ when $y = d/2$ we get

$$x = \left(\frac{p}{d^2}\right)y^2 \quad \text{or} \quad y = \sqrt{\frac{d^2x}{p}}$$

4. Comparison with the popular sine wave profile

The best profile as obtained from the calculus of variations is $y = \sqrt{d^2x/p}$, and the popular sine wave follows $y = \frac{d}{2}\frac{\sin 2\pi x}{p}$. Fig. 2 shows the profiles.

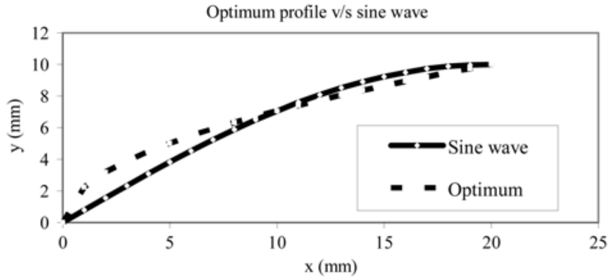

Fig. 2 Comparison of profiles

It can be derived that the length and inertia for a sine wave per quarter pitch are as follows:

$$l_1 = \frac{p}{4}\left(1 + \frac{\pi^2 d^2}{4p^2}\right) \quad \text{and} \quad I_1 = \frac{td^2 p}{32}\left[1 + \frac{\pi^2 d^2}{4p^2}\right]$$

And for the profile $y = \sqrt{\frac{d^2 x}{p}}$

$$l_2 = \frac{2p}{d^2}\left[\frac{d^2}{8}\left(\sqrt{1 + \frac{d^2}{p^2}}\right) + \frac{d^4}{8p^2}\lambda n\left(\frac{1 + \sqrt{1 + \frac{d^2}{p^2}}}{\frac{d}{p}}\right)\right] \quad \text{and} \quad I_2 = \frac{td^2 p}{32}\left[1 + \frac{2d^2}{p^2}\right]$$

If for instance d = 20 mm, t = 1 mm and p = 80 mm, the section modulus and length for sine wave per pitch are 461.685 mm^3 and 92.33 mm and the corresponding values for the best profile obtained are 450 mm^3 and 92.936 mm. The strength in terms of section modulus/unit length are 5 mm^3/mm and 4.842 mm^3/mm for sine wave and the best profile, respectively. The strength/unit length of the most widely used sine wave form is in agreement with the best profile suggested by the calculus of variations. The popularity of the sine wave form, hence, is justified.

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