

Numerical simulation of an external prestressing technique for prestressed concrete end block

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Abstract. This paper presents the details of finite element (FE) modeling and analysis of an external prestressing technique to strengthen a prestressed concrete (PSC) end block. Various methods of external prestressing techniques have been discussed. In the proposed technique, transfer of external force is in shear mode on the end block creating a complex stress distribution. The proposed technique is useful when the ends of the PSC girders are not accessible. Finite element modeling issues have been outlined. Brief description about material nonlinearity including key aspects in modeling inelastic behaviour has been provided. Finite element (FE) modeling including material, loading has been explained in depth. FE analysis for linear and nonlinear static analysis has been conducted for varying external loadings. Various responses such as out-of-plane deformation and slip have been computed and compared with the corresponding experimental observations. From the study, it has been observed that the computed slope and slip of the steel bracket under external loading is in good agreement with the corresponding experimental observations.

Keywords: concrete; external prestressing; finite element analysis; material nonlinearity.

1. Introduction

Concrete structural components require understanding into the responses of those components to a variety of loadings. There are number of methods for modeling the concrete structures through both analytical and numerical approaches. Finite element analysis (FEA) is a numerical technique widely employed to engineering design applications. FEA is a tool that can simulate and predict the responses of reinforced and prestressed concrete members. The use of FEA has increased because

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of progressing knowledge and capability of computer software and hardware. Developments in computer capabilities are making FEA of complex structural and mechanical components more achievable. This increases the expectations of capturing the physical and mechanical effects of component details and interactions. Typical example is the effects of bolts and the bolt pretensions on the distribution of stress in a bolted connection.

In post-tensioned prestressed concrete beams, the prestressing force is applied by anchoring the stressed cables at the end, thus leading to the application of discontinuous forces at the end. The discontinuous forces which are applied at the end while being transformed progressively to produce a continuous linear distribution develop transverse and shear stresses. The zone between the end of the beam and the section where only longitudinal stress exists is referred to as the anchorage zone or end block. It is well known that the transverse stresses developed in the anchorage zone are tensile in nature for a major portion of the length. There are two zones in the end block where the transverse stresses are tensile, are referred to as spalling and bursting zones respectively. The spalling zone is normally confined to the area near the corners of the end face of the block, while the bursting zone is a zone along the axis of the prestressing force normally confined to the interior of the end block. It is generally known that over a period of time, due to environmental conditions, bad detailing, loss of prestress due to creep and relaxation, there will be degradation in the strength of structural component. Hence, strengthening of the component is to be carried out in order to regain the strength and function, for the intended purpose. External prestressing has been extensively used in the construction of various engineering structures and is also considered as one of the most efficient approaches for strengthening existing bridges. Many experimental studies on the behaviour of externally prestressed members have been conducted (Haroyil 1993, Angel *et al.* 2000, Ayaho Miyamoto *et al.* 2000, Angel *et al.* 2002), whereas limited numerical analyses of these structures have been carried out.

Stallings and Hwarg (1992) described easy-to-apply method for modeling bolt pretensions in finite element analysis of bolted connections. Ziomek *et al.* (1992) carried out the finite element modeling of end-plate connections and compared with the corresponding experimental results. Bursi and Jaspart (1997) presented a methodology to calibrate finite element models which are able to reproduce the elastic-plastic behaviour of bolted connections to post failure. Elementary non-preloaded and preloaded tee stub connections were tested by Bursi and Jaspart and were proposed as benchmarks in the validation process of finite element software packages (Bursi and Jaspart 1998). Fanning *et al.* (2000) presented an ANSYS finite element model for flush end-plate joints. Material nonlinearity, large deflection analysis and contact surfaces were included in a non-linear solution. Bahaari and Shebourne (2000) developed a finite element methodology using ANSYS finite element code for equivalent three-dimensional analysis to investigate the behaviour of bolted end-plate connections. Double angle connections under tension and shear were considered using three-dimensional finite elements in ABAQUS by Yang *et al.* (2000). Contact was included between the bolt shank and hole was ignored. Padmarajaiah and Ramaswamy (2002) assessed the flexural behaviour of partially prestressed high strength concrete beams containing steel fibers. Non linear finite element analysis was carried out using ANSYS. Jung *et al.* (2006) presented experimental and finite element investigations of the load-deformation behaviour of tapered steel and fiber-reinforced plastic bridge camera poles subjected to cantilever bending type loading. Kim *et al.* (2007) described finite element analysis and modeling of structure with bolted joint using ANSYS. Zona *et al.* (2008) carried out finite element formulation for geometric and material nonlinear analysis of beams prestressed with external slipping tendons. Analytical model was developed based on a

nonlinear kinematic theory accounting for tendon eccentricity variation. Chen *et al.* (2009) conducted a comparative experimental study of moment redistribution and load carrying capacity of externally prestressed continuous composite beams. It was found that at the ultimate state, the moment redistribution in the prestressed continuous composite beams is greater than that in non-prestressed composite beams. Mukherjee and Rai (2009) presented the results of an experimental study to investigate the flexural behavior of reinforced concrete (RC) beams that have reached their ultimate bearing capacities and then retrofitted with externally prestressed carbon fiber reinforced composite (CFRC) laminates.

This paper presents details of FE simulation of an innovative external prestressing technique. FE modeling issues, various external prestressing techniques, material nonlinearity have been discussed. FE modeling and analysis of two end blocks have been carried out and the responses such as slope and slip of steel bracket have been compared with the corresponding experimental observations.

2. Methods to anchor external post-tensioning system

Method 1

In this method, additional support, either in concrete or steel for anchoring external prestressing is fixed at the girder ends as shown in Fig. 1. It is an easy method for applying external prestressing. It transfers the external prestressing force to the member in compression mode. The disadvantage of this method is that all the tendons have to run from one end to the other end of the girder thereby increasing the tendon length. External tendons are subjected to vibrations, leads to premature failure of anchorages and more space at the ends of girders is required for tensioning with prestressing jack. This method is not suitable for strengthening continuous span bridges and suspended span bridges.

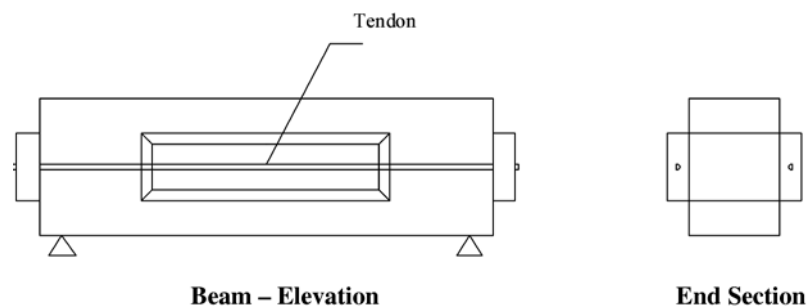


Fig. 1 Anchoring of External Prestressing (EP) at the ends

Method 2

In this method, external tendons are anchored to the web of bridge girders through additional supports, either in concrete or steel. The additional frictional supports are prestressed transversely and it is shown in Fig. 2. This method transfers the external prestressing force to the web in friction

that develops between the web and saddles. It depends on the contact surface and transverse prestress. This method provides a good distribution of the force in the external tendons, but creates high stresses locally where the prestressing force is introduced. Wherever strengthening is required, that portion alone can be strengthened by this method. Hence, length of tendon is small and subjected to fewer vibration problems. Transverse prestressing to the anchorage zones of external prestressing is very difficult and design of the anchorage zones is also difficult. If, it is located at the anchorage zones, it may break or disturb internal tendons, having through transverse prestressing bars. This method is more suitable to anchor at the webs of simply supported, continuous and suspended spans.

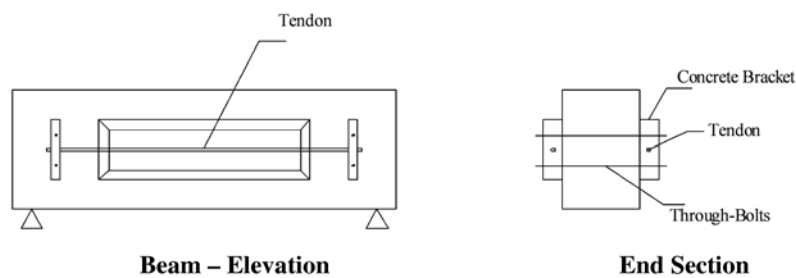


Fig. 2 Anchoring of EP at the sides with through-bolts

Proposed method

In this method, external prestressing tendons are anchored to the steel brackets as shown in Fig. 3. Steel brackets are attached to the sides of the end block of the bridge girders through torque-rated expansion type anchor bolts. Expansion anchors transfer the loads to the concrete principally by friction. The frictional resistance of the anchor depends directly on the normal forces generated by the anchor expansion mechanism during installation. This method has advantage of not requiring lateral/transverse prestressing as is necessary incase of conventional method and more suitable where ends of girders are not accessible for anchoring external prestressing. The deviators required for changing the profile of external tendons can be easily constructed with steel brackets and bolted to the existing girder by expansive type anchor bolts which act as effective shear keys in transferring prestress.

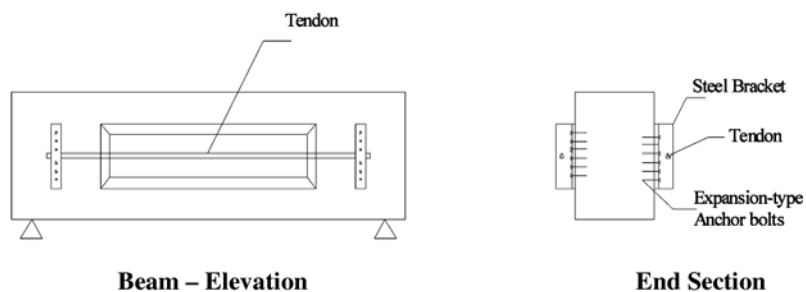


Fig. 3 Anchoring of EP at the sides with expansion-type anchor bolts

3. Finite element modelling issues

A large number of different FE formulations have been used for the analysis of concrete structural components. These may be categorized into facet plate/shell elements, thin-shell elements (Kirchoff assumptions), thin/thick shell elements (Reissner-Mindlin theory), three dimensional elements. The choice of an element for analysis of a structure/component depends on the geometry and the purpose for which the results of the analysis are to be used. The following are some of the key issues w.r.t modelling of prestressed concrete end block.

- FE modelling of post tensioned prestressed concrete end block
- FE modelling of steel bracket and expansive type anchor bolts
- Material modelling of concrete, steel and bolt
- Various loads for modelling such as compression load, reaction load on end block, pretension force in the bolt, external load on steel bracket
- Simulation of initial pretension and equivalent compression surrounding the bolt
- Modelling end connections such as bolt and steel bracket and bolt and concrete block
- Employing appropriate elements such as contact, gap, solid and link for effective load transfer and to simulate the realistic behaviour
- Simulation of material nonlinearity for larger load steps and post yield/crack regime.

4. Material nonlinearity

When a large force is applied on the structure/component, the resulting stresses may be greater than the yield strength of the material. In such a case, multilinear stress-strain relationship of the material can be used to consider the plastic deformation of the material. Key concepts in modeling inelastic behavior are:

The decomposition of strain into elastic and plastic parts

Under uniaxial loading, the strain at a given stress has two parts: a small recoverable elastic strain and a large, irreversible plastic strain (refer Fig. 4).

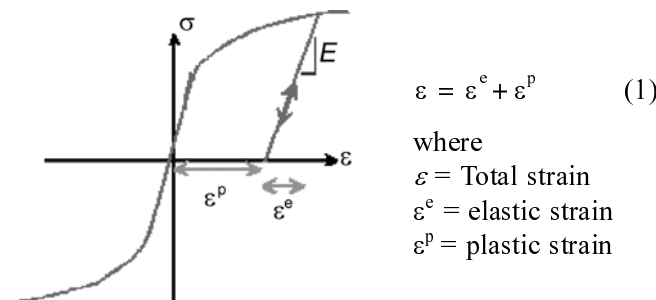


Fig. 4 Stress-strain plot under uniaxial tension

Yield criterion to predict whether the solid responds elastically or plastically

There are two assumptions under yield criteria, namely (a) Yield is independent of hydrostatic pressure; and (b) The solid is isotropic

Assumption (a) implies that the yield criterion can only depend on the deviatoric stress components

$$S_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3} \delta_{ij} \quad (2)$$

Assumption (b) implies that the onset of yield can only depend on the magnitudes of the principal stresses $\sigma_1, \sigma_2, \sigma_3$, and can't depend on the principal stress directions. Another way to state the same thing is to note that yield can only depend on the *invariants* of the deviatoric stress tensor

$$\begin{aligned} J_1 &= 0 \\ J_2 &= \frac{1}{2} S_{ij} S_{ij} \\ J_3 &= \frac{1}{3} S_{ik} S_{kj} S_{ji} \end{aligned} \quad (3)$$

Fig. 5 shows the visualization of yield criteria.

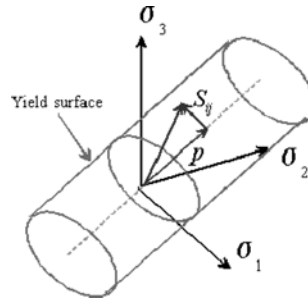


Fig. 5 Visualisation of yield criteria

Strain hardening rules, which control the shape of the stress-strain curve in the plastic regime

The easiest way to model strain hardening is to make the yield surface increase in size, but remain the same shape, as a result of plastic straining. A few of the more common forms of hardening stress-strain functions are illustrated below in Fig. 6.

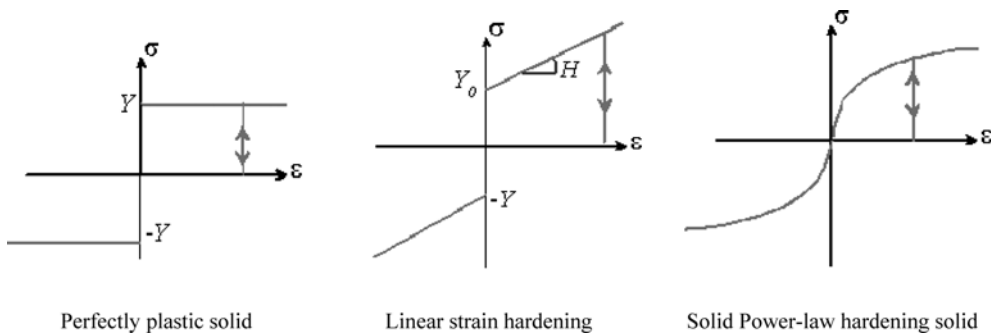


Fig. 6 Common forms of hardening functions

The plastic flow rule, which determines the relationship between stress and plastic strain

To complete the constitutive model, it is required to calculate the plastic strains induced by loading beyond yield. This can be achieved by applying a stress σ_{ij} that is just sufficient to reach yield. Now, increase the stress to $\sigma_{ij} + d\sigma_{ij}$. Then compute the resulting plastic strain increment $d\varepsilon_{ij}$. The magnitude of the plastic strain is completely determined by the hardening behavior of the solid. This is because during continued plastic flow, the stress must be on the yield surface at all times. Since the radius (or position, for kinematic hardening) of the yield surface is related to the magnitude of the plastic strain increment through an appropriate hardening law, this means the plastic strain magnitude must be related to the stress increment.

The elastic unloading criterion, which models the irreversible behavior

In general it is known that the plastic flow is irreversible, and always dissipates energy. If the increment in stress $d\sigma_{ij}$ is tangent to the yield surface, or brings the stress below yield, then it induces no plastic strain. Fig. 7 shows the elastic unloading condition.

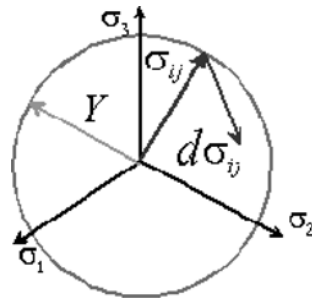


Fig. 7 Elastic unloading condition

5. Statement of the problem

The geometry of the PSC end block is shown in Fig. 8. Two steel Brackets are attached on either side of the block by means of expansion type anchor bolts with eccentricity in x -direction as shown in Fig. 8. There are four bolts on either side of the block. Initial pretension force is to be applied in the bolt which is the resultant of the torque applied to the bolt head. Due to the application of torque, an equivalent compression will be developed on the steel bracket, known as lateral compression. During application of torque at the bolt head, expansion sleeves of bolt at tail end, expands against surrounding concrete and develops perfect bond between bolt and concrete and resist tensile force due to external load. Further, there are two steel plates attached to the block. One is on top of block which is for pre compression and other one is on side of the block which is for transfer of shear load. External prestressing load is applied gradually on the projected part of the steel bracket. Due to this load, compression below the head of the bolt i.e., concrete surrounding the bolt element is initially nullified while tension will be developed on top row bolts adding to initial pretension. Further increase of external load, the bolt should deform and cause separation of bracket more at top and minimum at bottom. Slip of the bracket is also expected for larger external load. In the present study, FE modeling and analysis has been carried out for two PSC end blocks.

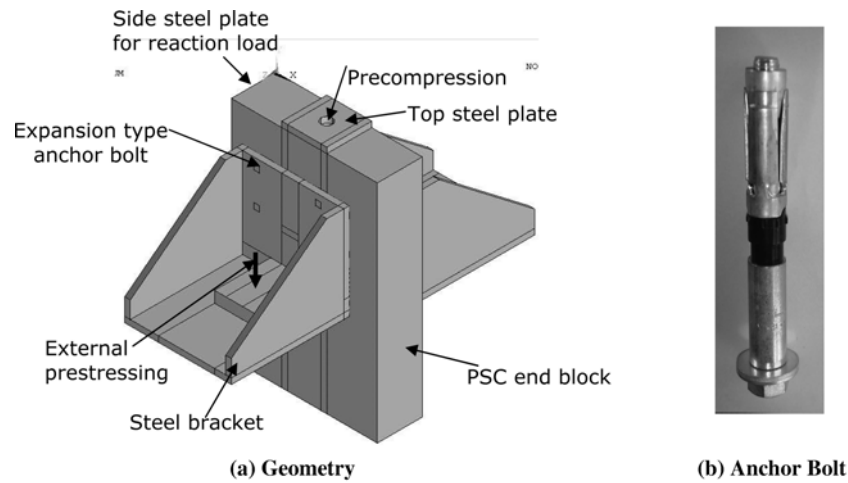


Fig. 8 Geometry of PSC end block

5.1 Force transfer mechanism in expansion anchors

Torque-rated expansion anchors are expanded by the application of a specified torque to the bolt head or nut. The applied torque serves to draw a conical wedge between the spreading elements, forcing them outwards against the sides of the hole in the concrete. The resultant of this pressure along the circumference and the length of the sleeve is called expansion force and is shown in Fig. 9. Application of torque generates a prestressing tensile force in the bolt which is counter balanced by compression in the base material. Expansion anchors transfer the tension loads to the concrete principally by friction. The frictional resistance of the anchor depends directly on the normal forces generated by the anchor expansion mechanism during installation and through out the life of the anchorage. In addition of the frictional force-transfer mechanism described above, torque-rated expansion anchors may transfer load to the concrete by means of mechanical interlock caused by inelastic concrete deformations at the expansion end of the anchor. Load capacity in tension and shear per anchor is given in Table 1.

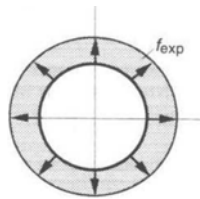


Fig. 9 Expansion force

Table 1 Load capacity of torque-controlled expansion Anchor (non cracked concrete)

Type of force	Characteristic load (kN)	Ultimate load (kN)
Tension	50.4	71.3
Shear	101.1	128.6

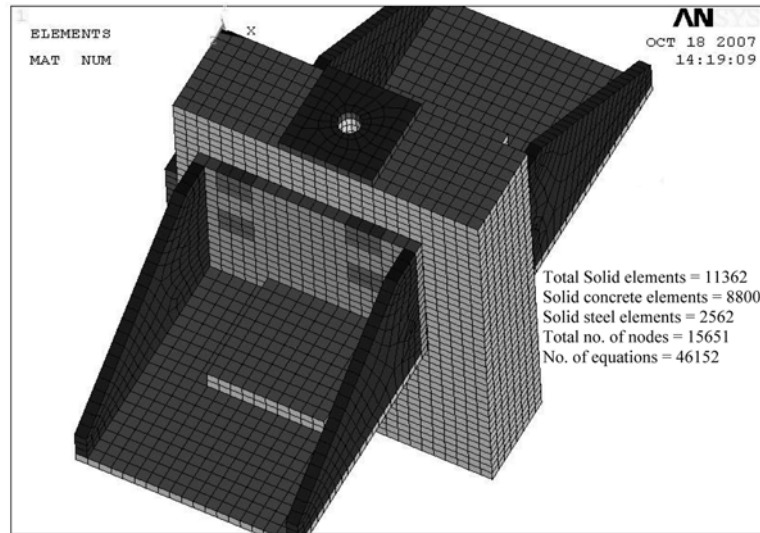


Fig. 10 FE mesh and model characteristics

6. FE modelling and analysis of Prestressed Concrete (PSC) end block

6.1 PSC End Block 1 (EB1)

Creation of geometry and finite element mesh, modeling of all loads, defining material properties, simulation of material nonlinearity and application of boundary conditions have been carried out using a general purpose finite element software, ANSYS 6.0 (Yang *et al.* 2000). The details of modeling are furnished below. Volume has been created as per the geometry shown in Fig. 8. FE mesh along with model characteristics is shown in Fig. 10.

Hexahedral eight noded solid elements (SOLID 45) have been employed to model concrete block, bolts and bracket connection. The use of these elements provides the same number of integration point density as the higher order elements but requires much less computational effort. This element can be defined by orthotropic material properties correspond to the element coordinate directions. Each node has three translational degrees of freedom, namely, U_x , U_y and U_z in x , y and z directions respectively. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. Bolt is idealized as square in shape instead of circular to avoid modeling issues. The bolt is embedded 125 mm in the concrete block. Bolts are not symmetrically placed on either side of the concrete block. At the end of the bolt, a link element has been created connecting the surfaces of bolt and concrete. The principal function of link element is to transfer the forces arising due to external loads. The 3-D spar element (link element) is a uniaxial tension-compression element with three degrees of freedom at each node: translations in the nodal x , y , and z directions (refer Fig. 11). This element has plasticity, creep, swelling, stress stiffening, and large deflection capabilities.

An imaginary surface has been created between the concrete block and the bracket in order to allow the bolt movement in the out of plane direction under the external load.

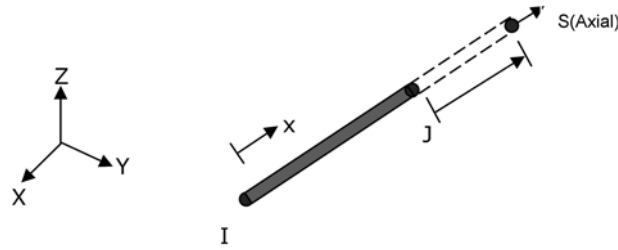


Fig. 11 Link 3-D Spar element

Table 2 Material properties

Material	Modulus of elasticity, MPa	Poisson's ratio, ν
Concrete	31623	0.12
Steel	2×10^5	0.3
Bolt	2×10^5	0.3

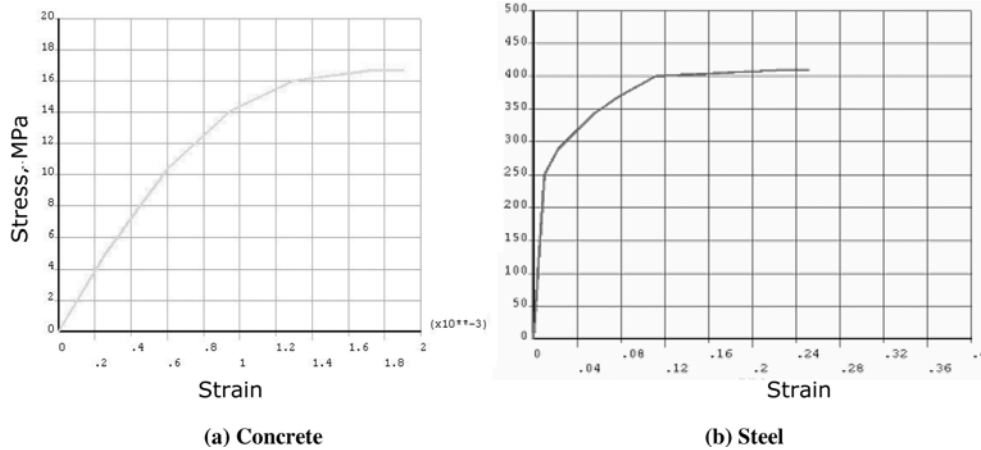


Fig. 12 Multilinear stress strain

Material properties

Material properties for the concrete, steel and bolt are given in Table 2. For linear static analysis, modulus of elasticity, poisson's ratio are the input values whereas for nonlinear static analysis, multilinear elastic model available in ANSYS have been used. The material behavior is described by a piece-wise linear stress-strain curve, starting at the origin, with positive stress and strain values. Successive slopes can be greater than the preceding slope; however, no slope can be greater than the elastic modulus of the material. The slope of the first curve segment usually corresponds to the elastic modulus of the material, although the elastic modulus can be input as greater than the first slope to ensure that all slopes are less than or equal to the elastic modulus. Fig. 12 shows multilinear stress strain plot adopted for steel and concrete material.

Newton-Raphson procedure has been employed for nonlinear static analysis where the load is divided into series of load increments applied in several load steps. Before each solution step, out-of-balance load vector which is difference between the restoring forces corresponding to element

stresses and the applied load is evaluated. Then a linear solution is carried out using out-of-balance loads and convergence is checked. When the convergence criterion is not satisfied, out-of-balance load is reevaluated, the stiffness matrix is updated and a new solution is obtained. This iterative procedure continued until the solution converges.

Loading conditions

As already stated, there are various loadings, namely internal prestress, Reaction force, pretension in the bolt, and external prestress. Internal prestress is applied as compressive load on the top of the PSC block. Reaction force is applied horizontal to the member. Initial pretension force obtained in bolt by applying torque on the bolt, is applied to all the bolts on the face of the element and an equivalent compressive force is applied to all the surrounding elements of the bolt to simulate the head of the bolt. It is assumed that there is perfect frictional bond between bolt and concrete. Prestress load on the block and initial pretension to the bolt are retained to be same throughout the analysis. External prestressing load is applied incrementally on the projected part of the bracket to capture nonlinearities induced by material yielding at larger load. The magnitudes of the loads are given below.

Prestress load (on PSC block)	: 438 kN
Initial pretension (to each bolt)	: 37.5 kN
External prestress load (on bracket)	: 0 to 400 kN
Reaction load (to PSC block)	: 231 kN

Boundary conditions

All the degree of freedom, were constrained on the bottom surface of the block.

Analysis

Linear static and nonlinear static analysis has been carried out depending on the level of external prestressing load. At each incremental load, out-of-plane deformation and slip have been noted

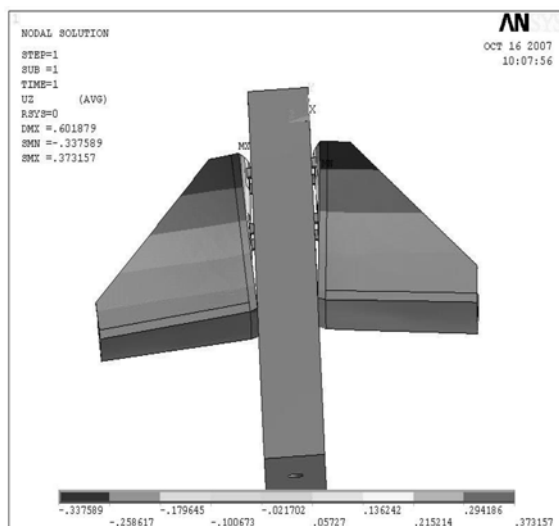


Fig. 13 Out-of-plane deformation for design load

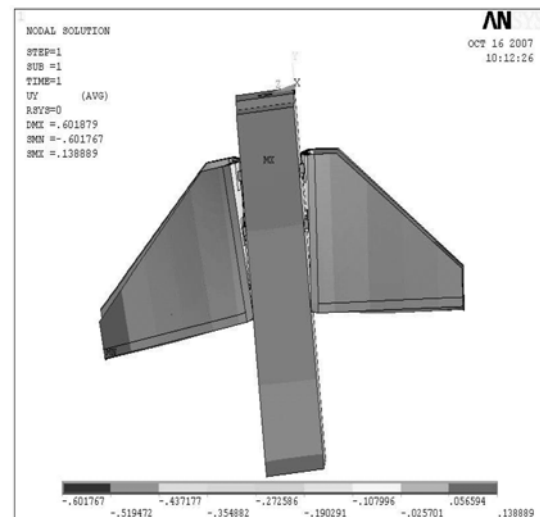


Fig. 14 Vertical deformation (Slip) for design load

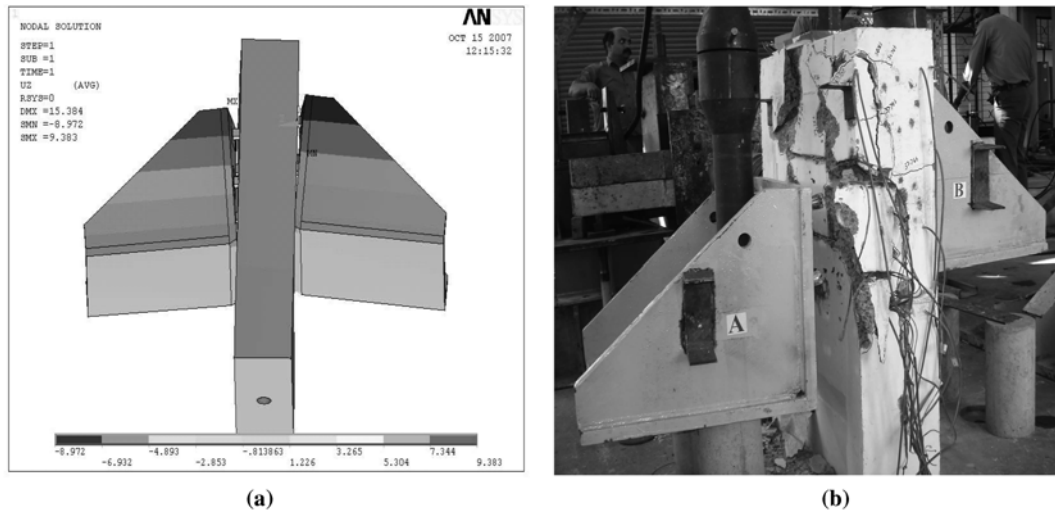


Fig. 15 (a) Out-of-plane deformation for ultimate load, (b) Failure pattern of EB1 at ultimate-experimental

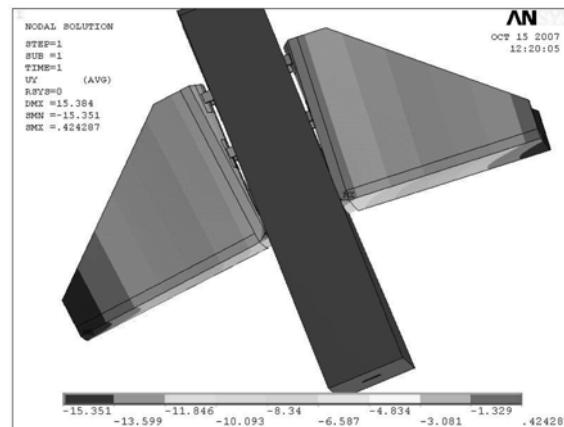


Fig. 16 Vertical deformation (Slip) for ultimate load

down. Rotation of the bracket has been calculated by using the computed out-of-plane deformation. Figs. 13 and 14 show the out-of-plane deformation and vertical deformation (slip) respectively for the design load of 95 kN on each side of EB1.

Figs. 15(a) and 15(b) show the computed out-of-plane deformation and experimentally measured deformation respectively ANSYS 6.0 (2002). From Figs. 15(a),(b), it can be observed that the trend of predicted behaviour is similar to experimental observation. Fig. 16 shows the vertical deformation (slip) for the ultimate load of 400 kN. The computed slope and slip values of steel bracket have been compared with the corresponding experimental observations. Figs. 17 and 18 show the slope and slip plots of computed and experimental values. From Fig. 17, it can be observed that the computed slopes and the corresponding experimental observations are in very good agreement with each other.

From Fig. 18, it can be observed that there is reasonable agreement between the computed and experimental slip values. The difference in the slip may be attributed to the noncoincident of nodal location and experimentally measured point.

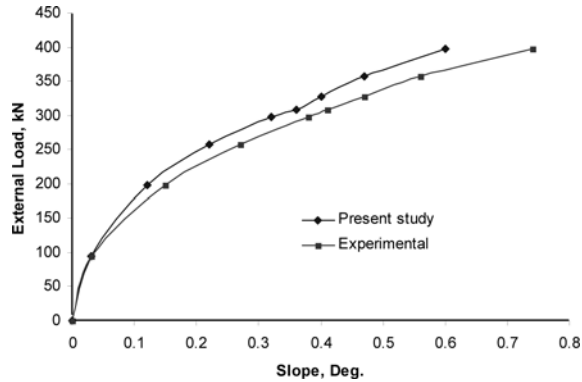


Fig. 17 Slope of Steel bracket under external load for EB1

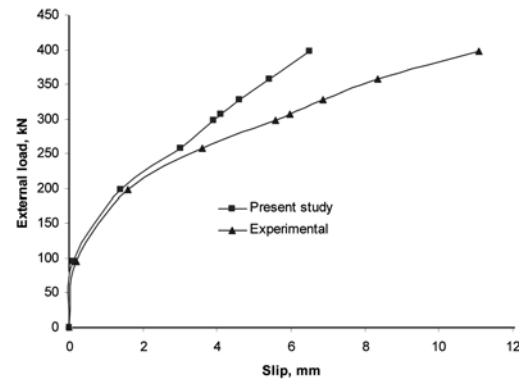


Fig. 18 Slip of Steel bracket under external load for EB1

6.2 PSC End Block 2 (EB2)

The details of EB2 are same as EB1 except the applied loadings. There is no internal prestress load on the top of the concrete block to represent as RCC block. The details of other loadings are given below.

Initial pretension (to each bolt)	: 37.5 kN
External prestress load (on bracket)	: varies from 0 to 270 kN
Reaction load (to PSC block)	: varies from 0 to 100.17 kN

Initial pretension to each bolt and compression on all the surrounding concrete elements are same throughout analysis whereas the reaction load and external prestress load is applied in steps. Finite element analysis has been carried out for the above load combinations and results for ultimate load are presented below.

Figs. 19 and 20 show the out-of-plane deformation and vertical deformation (slip) respectively for the ultimate load of 270 kN (one side load). The computed slope and slip values of steel bracket

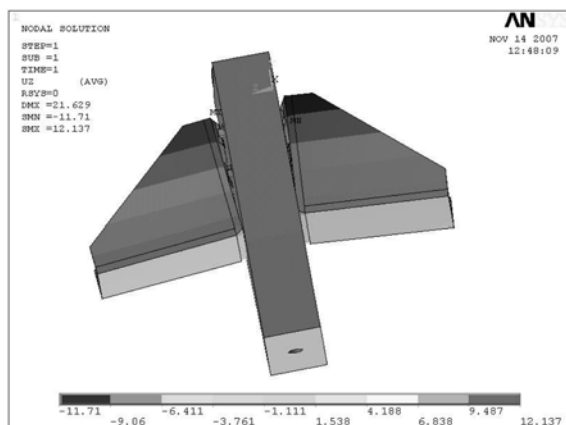


Fig. 19 Out-of-plane deformation for ultimate load

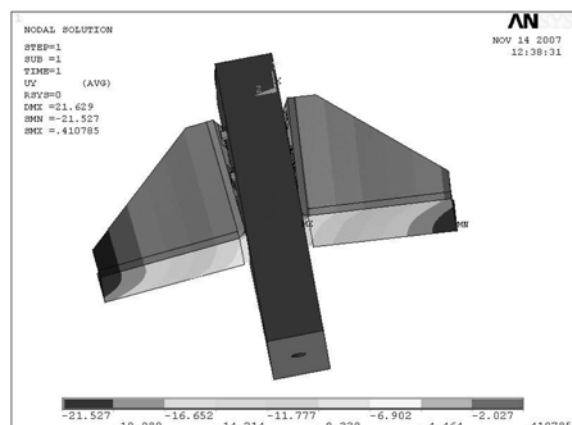


Fig. 20 Vertical deformation (Slip) for ultimate load

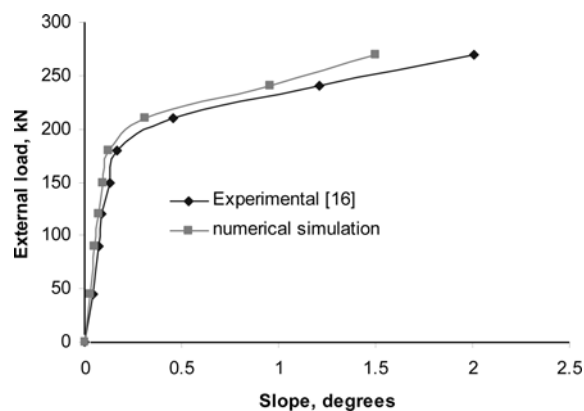


Fig. 21 Slope of Steel Bracket (EB2)

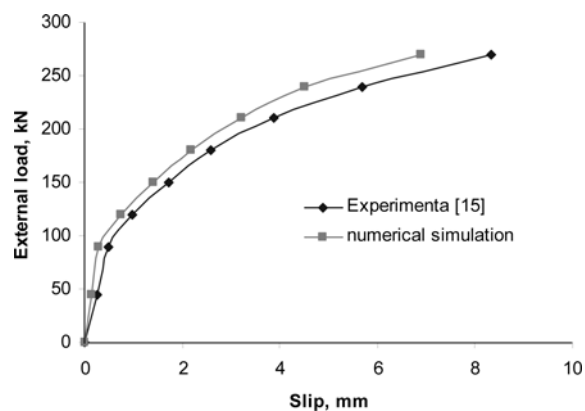


Fig. 22 Slip of the steel bracket (EB2)

have been compared with the corresponding experimental observations. Figs. 21 and 22 show the slope and slip plots of computed and experimental values. From Figs. 21 and 22, it can be observed that the computed slopes and measured slips are in good agreement with the corresponding experimental observations.

7. Conclusions

Finite element modeling and analysis of prestressed concrete (PSC) end block (EB) have been presented. Finite element modeling issues have been outlined. Brief description about material nonlinearity including key aspects in modeling inelastic behaviour has been discussed. Finite element (FE) modeling including material, loading, boundary condition of PSC end concrete block has been explained in depth. Multilinear elastic material model has been used for bolt material in order to predict nonlinear behaviour. PSC EB1 and EB2 have been considered for FE analysis. Linear and nonlinear static analysis has been conducted for two PSC end blocks for various external prestressing loadings. Various responses such as out-of-plane deformation and slip have been computed and compared with the corresponding experimental observations. From the study, it has been observed that the computed out-of-plane deformation is in very good agreement with the corresponding experimental observation whereas the computed slip is in reasonably agreement with the corresponding experimental observation. The analytical model is quite realistic for further simulation and parametric studies on external prestressing load.

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