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Shear locking-free earthquake analysis of thick and thin plates using Mindlin's theory

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Abstract. The purpose of this paper is to study shear locking-free parametric earthquake analysis of thick and thin plates using Mindlin's theory, to determine the effects of the thickness/span ratio, the aspect ratio and the boundary conditions on the linear responses of thick and thin plates subjected to earthquake excitations. In the analysis, finite element method is used for spatial integration and the Newmark-β method is used for the time integration. Finite element formulation of the equations of the thick plate theory is derived by using higher order displacement shape functions. A computer program using finite element method is coded in C++ to analyze the plates clamped or simply supported along all four edges. In the analysis, 17-noded finite element is used. Graphs are presented that should help engineers in the design of thick plates subjected to earthquake excitations. It is concluded that 17-noded finite element can be effectively used in the earthquake analysis of thick and thin plates. It is also concluded that, in general, the changes in the thickness/span ratio are more effective on the maximum responses considered in this study than the changes in the aspect ratio.

Keywords: shear locking-free parametric earthquake analysis; thick plate, Mindlin's theory; 17-noded finite element; thickness/span ratio; aspect ratio.

1. Introduction

Plates are structural elements which are commonly used in the building industry. A plate is considered to be a thin plate ate if this ratio is larger than 1/20 (Ugural 1981, Liew and Teo 1999).

The dynamic behavior of thin plates has been investigated by many researchers (Ugural 1981, Timoshenko and Krieger 1959, Leissa 1969, Leissa 1973, Providakis and Beskos 1989, Warburton 1954, Caldersmith 1984, Qiu and Feng 2000, Zhu and Gu 1991, Grice and Pinnington 2002, Sakata and Hosokawa 1988, Lok and Cheng 2001, Si *et al.* 2005, Ayvaz and Durmuş, 1995]. There are also many references on the behavior of the thick plates subjected to different loads. The studies made on the behavior of the thick plates are based on the Reissner-Mindlin plate theory (Reissner 1945, 1947, 1950, Mindlin 1951). This theory requires only C^0 continuity for the finite elements in the analysis of thin and thick plates. Therefore, it appears as an alternative to the thin plate theory which also requires C^1 continuity. This requirement in the thin plate theory is solved easily if Mindlin theory is used in the analysis of thin plates. Despite the simple formulation of this theory,

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discretization of the plate by means of the finite element comes out to be an important parameter. In many cases, numerical solution can have lack of convergence, which is known as "shear-locking". Shear locking can be avoided by increasing the mesh size, i.e., using finer mesh, but if the thickness/span ratio is "too small", convergence may not be achieved even if the finer mesh is used for the low order displacement shape functions.

In order to avoid shear locking problem, the different methods and techniques, such as reduced and selective reduced integration, the substitute shear strain method, etc., are used by several researchers (Hinton and Huang 1986, Zienkiewich *et al.* 1971, Bergan and Wang 1984, Ozkul and Ture 2004, Hughes *et al.* 1977). The same problem can also be prevented by using higher order displacement shape function (Özdemir *et al.* 2007). Wanji and Cheung (Wanji and Cheung 2000) proposed a new quadrilateral thin/thick plate element based on the Mindlin-Reissner theory. Soh *et al*. (2001) improved a new element ARS-Q12 which is a simple quadrilateral 12 DOF plate bending element based on Reissner-Mindlin theory for analysis of thick and thin plates. Brezzi and Marini (2003) developped a locking free nonconforming element for the Reissner-Mindlin plate using discontinuous Galarkin techniques. Belounar and Guenfound (2005) improved a nev rectangular finite element based on the strain approach and the Reissner-Mindlin theory is presented for the analysis of plates in bending either thick or thin. Cen *et al.* (2006) developped a new high performance quadrilateral element for analysis of thick and thin plates. This distinguishing character of the new element is that all formulations are expressed in the quadrilateral area co-ordinate system. Ayvaz (1992) derived the equations of motions for thick orthotropic elastic plates using Hamilton's principle, but did not present any results. Liew and Teo (1999) studied three-dimensional vibration analysis of rectangular plates based on differential quadrature method. Shen *et al.* (2001) studied free and forced vibration of Reissner-Mindlin plates with free edges resting on elastic foundations. Cai *et al.* (2002) presented the generalized mixed variational principle for Reissner plate analysis. Using finite element method, Raju and Hinton (1980) made significant contributions to the vibration analysis of plates including rotatory inertia effects for rhombic plates based on Mindlin's theory. Woo *et al.* (2003) found accurate natural frequencies and mode shapes of skew plates with and without cutouts by p-version finite element method using integrals of Legendre polynomial for $p = 1$-14. Qian *et al.* (2003) studied free and forced vibrations of thick rectangular plates using higher-order shear and normal deformable plate theory and meshless Petrov-Galarkin method. Morais *et al.* (2005) studied vibrations of thick plates using Lagrangean quadrilateral finite element with 16 nodes. However, no references have been found in the technical literature for the shear locking-free earthquake analysis of thick and thin plates using Mindlin theory by using 17-noded finite element.

The purpose of this paper is to study shear locking-free parametric earthquake analysis of thick and thin plates using Mindlin's theory, to determine the effects of the thickness/span ratio, the aspect ratio and the boundary conditions on the linear responses of the thick and thin plates subjected to earthquake excitations. A computer program using finite element method is coded in C++ to analyze the plates clamped or simply supported along all four edges. In the program, the finite element method is used for spatial integration and the Newmark-β method is used for the time integration. Finite element formulation of the equations of the thick plate theory is derived by using higher order displacement shape functions. In the analysis, 17-noded finite element is used to construct the stiffness and mass matrices since shear locking problem does not occur if this element is used in the finite element modelling of the thick and thin plates (Özdemir *et al.* 2007). No matter what the mesh size is unless it is less than 4×4. This is a new element, details of its formulation are presented in (Özdemir *et al.* 2007) and this is the first time this element is used in the earthquake analysis of thick and thin plates. If this element is used in an analysis, it is not necessary to use finer mesh.

2. Finite element modeling

The governing equation for a flexural plate (Fig. 1) subjected to an earthquake excitation without damping can be given as (Ayvaz *et al.* 1995, Tedesco *et al.* 1999)

$$[M]\{\ddot{w}\}+[K]\{w\} = [F] = -[M]\{\ddot{u}_g\} \tag{1}$$

where $[K]$ and $[M]$ are the stiffness matrix and the mass matrix of the plate, respectively, w and $\ddot{w}$ are the lateral displacement and the second derivative of the lateral displacement of the plate with respect to time, respectively, $\ddot{u}_g$ is the earthquake acceleration.

In order to do forced vibration analysis of a plate, the stiffness, $[K]$, mass matrices, $[M]$, and equivalent nodal loads vector, $[F]$, of the plate should be constructed. The evaluation of these matrices is given in the following sections.

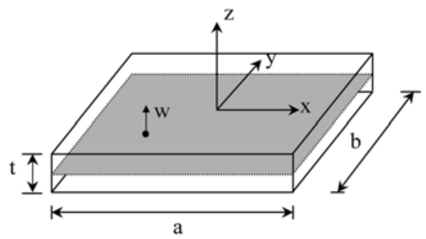


Fig. 1 The sample plate used in this study

2.1 Evaluation of the stiffness matrix

In this study, 17-noded quadrilateral serendipity element (MT17) (Fig. 2) is used. The stiffness matrix for this element can be obtained by the following equation (Cook *et al.* 1989).

$$[K] = \int_A [B]^T[D][B]dA = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]|J|drds \tag{2}$$

which must be evaluated numerically (Hughes *et al.* 1977).

As seen from Eq. (2), in order to obtain the stiffness matrix, the strain-displacement matrix, $[B]$, and the flexural rigidity matrix, $[D]$, of the element need to be constructed.

The nodal displacements for 17-noded finite element can be written as follows

$$u = \{u, v, w\} = \{-z\varphi_x, z\varphi_y, w\}\left\{-z\frac{\partial\varphi_i}{\partial x}, z\frac{\partial\varphi_i}{\partial y}, w\right\} \tag{3}$$

$$u = z\varphi_x = -z\sum_1^{17} h_i\varphi_{xi},\ z = z\varphi_y = z\sum_1^{17} h_i\varphi_{yi},\ \ w = \sum_1^{17} h_i w_i \tag{4}$$

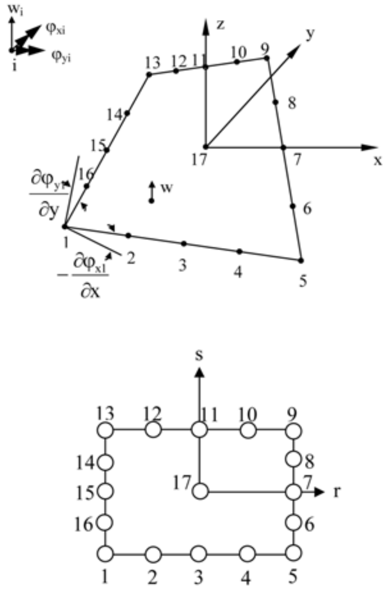


Fig. 2 17-noded quadrilateral finite element used in this study (Özdemir *et al*. 2007)

In these equations, w is the displacement of the plate in the vertical direction, x, y, and z are the co-ordinate axes (see Fig. 1).

The displacement function chosen for this element is

$$w = c_1 + c_2 r + c_3 s + c_4 r^2 + c_5 rs + c_6 s^2 + c_7 r^2 s + c_8 rs^2 + c_9 r^3 + c_{10} r^3 s + c_{11} rs^3 + c_{12} s^3 + c_{13} r^2 s^2 + c_{14} r^4 + c_{15} r^4 s + c_{16} rs^4 + c_{17} s^4 \tag{5}$$

From this assumption, it is possible to derive the displacement shape function to be

$$h = [h_1, \ldots, h_{17}] \tag{6}$$

Then, the strain-displacement matrix $[B]$ for this element can be written as follows (Cook *et al.* 1989)

$$[B] = \begin{bmatrix} 0 & 0 & -\dfrac{\partial h_i}{\partial x} & \ldots \\ 0 & \dfrac{\partial h_i}{\partial y} & 0 & \ldots \\ 0 & \dfrac{\partial h_i}{\partial x} & -\dfrac{\partial h_i}{\partial y} & \ldots \\ \dfrac{\partial h_i}{\partial x} & 0 & -h_i & \ldots \\ \dfrac{\partial h_i}{\partial y} & h_i & 0 & \ldots \end{bmatrix}_{5\times 51} \qquad i = 1, \ldots, 17 \tag{7}$$

The flexural rigidity matrix, [D], can be obtained by the following equation.

$$[D] = \begin{bmatrix} E_k & 0 \\ 0 & E_\gamma \end{bmatrix} \tag{8}$$

In this equation, [E_k] is of size 3×3 and [E_γ] is of size 2×2. [E_k], and [E_γ] can be written as follows (Bathe 1996, Weaver and Johston 1984)

$$[E_k] = \frac{t^3}{12}\begin{bmatrix} \frac{E}{(1-\nu^2)} & \frac{\nu E}{(1-\nu^2)} & 0 \\ \frac{\nu E}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0 \\ 0 & 0 & \frac{E}{2(1-\nu^2)} \end{bmatrix}; \quad [E_\gamma] = kt\begin{bmatrix} \frac{E}{2.4(1+\nu)} & 0 \\ 0 & \frac{E}{2.4(1+\nu)} \end{bmatrix} \tag{9}$$

where E, υ, and t are modulus of the elasticity, Poisson's ratio, and the thickness of the plate, respectively, k is a constant to account for the actual non-uniformity of the shearing stresses. By assembling the element stiffness matrices obtained, the system stiffness matrix is obtained.

2.2 Evaluation of the mass matrix

The formula for the consistent mass matrix of the plate may be written as

$$M = \int_\Omega H_i^T \mu H_i d\Omega \tag{10}$$

In this equation, m is the mass density matrix of the form (Tedesco *et al.* 1999)

$$\mu = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \tag{11}$$

where $m_1 = \rho_p t$, $m_2 = m_3 = 1/12(\rho_p t^3)$, and ρ_p is the mass density of the plate. and H_i can be written as follows

$$H_i = [dh_i/dx \;\; dh_i/dy \;\; h_i] \quad i = 1, \ldots, 17 \tag{12}$$

It should be noted that the rotation inertia terms are not taken into account. By assembling the element mass matrices obtained, the system mass matrix is obtained.

2.3 Evaluation of equivalent nodal loads vector

Equivalent nodal loads, [F], can be obtained by the following equation.

$$[F] = \int H_i^T \bar{q} d\Omega \tag{13}$$

In this equation, H_i can be obtained by Eq. (13), and $\bar{q}$ denotes

$$-[M]\{\ddot{u}_g\} \tag{14}$$

It should be noted that, the Newmark-β method is used for the time integration of Eq. (1) by using the average acceleration method.

3. Numerical examples

3.1 Data for numerical examples

In the light of the results given in references (Özdemir *et al.* 2007, Özdemir 2007), the aspect ratios, b/a, of the plate are taken to be 1, 1.5, 2.0, and 3.0. The thickness/span ratios, t/a, are taken as 0.01, 0.05, 0.1, 0.2, and 0.3 for each aspect ratio. The shorter span length of the plate is kept constant to be 3 m. The mass density, Poisson's ratio, and the modulus of elasticity of the plate are taken to be 2.5 kN s^2/m^2, 0.2, and 2.8×10^7 kN/m^2.

In order to obtain the response of each plate by using the time history analysis, the East-West component of March 13 1992 Erzincan earthquake in Turkey is used. Duration of this earthquake is 21 s, but, in this study, the first 8 s of the earthquake is used since the peak value of the record occurred in this range (Fig. 3).

For the sake of accuracy in the results, rather than starting with a set of a finite element mesh size and time increment, the mesh size and time increment required to obtain the desired accuracy were determined before presenting any results This analysis was performed separately for the mesh size and time increment. It was concluded that the results have acceptable error when equally spaced 4×4 mesh sizes are used for a 3 m $\times$ 3 m plate even if it is a thin plate, if the 0.01 s time increment is used. Length of the elements in the x and y directions are kept constant for different aspect ratios as in the case of square plate.

In order to illustrate that the mesh density used in this paper is enough to obtain correct results, the first six frequency parameters of the thick simply supported plate with $b/a = 1$ and $t/a = 0.1$ is presented in Fig. 4 by comparing with the result obtained using 8-noded quadrilateral finite element (MT8). It should be noted that the results presented for MT8 element are obtained by using equally spaced 16×16 mesh size. As seen from Fig. 4, the results obtained by using 17-noded quadrilateral

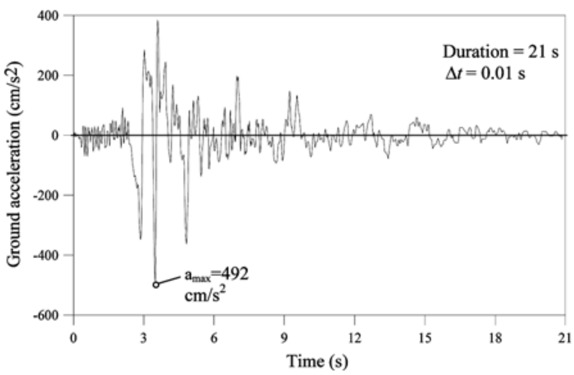


Fig. 3 East-West component of the March 13, 1992 Erzincan earthquake in Turkey

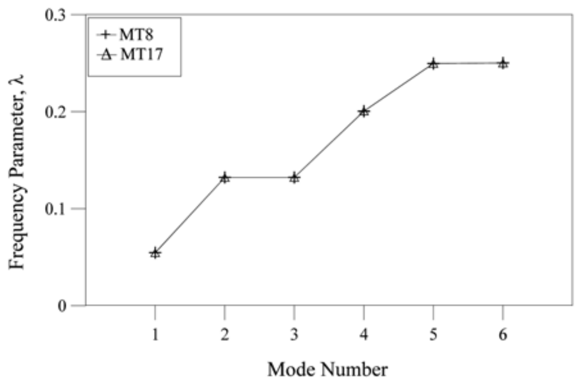


Fig. 4 The first six frequency parameters of the thick simply supported plates modeled with 8- and 17-noded finite elements ($b/a = 1$, $t/a = 0.1$)

finite element have excellent agreement with the results obtained by using MT8 element even if 4×4 mesh size is used for MT17 element.

3.2 Results

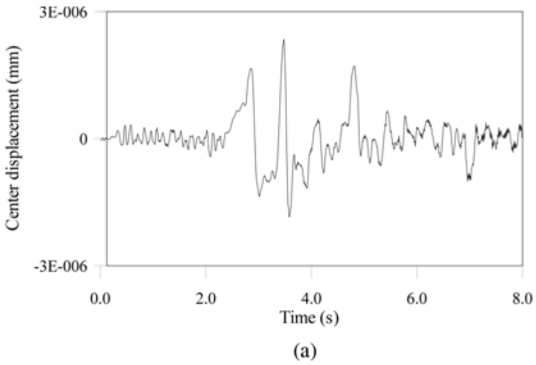
One of the purposes of this paper was to determine the time histories of the displacements and the bending moments at different points of the thick and thin plates subjected to earthquake excitations, but presentation of all of the time histories would take up excessive space. Hence, only the absolute maximum displacements and bending moments for different thickness/span ratio and aspect ratio are presented after two time histories are given. This simplification of presenting only the maximum responses is supported by the fact that the maximum values of these quantities are the most important ones for design. These results are presented in graphical rather than in tabular form.

The time histories of the center displacements of the thick clamped plates for $b/a = 1.0$, and 2.0 when $t/a = 0.2$ are given in Figs. 5(a), and 5(b), respectively.

As seen from Figs. 5(a), and 5(b), the center displacements of the thick clamped plates for $b/a = 1$, and $t/a = 0.2$, and for $b/a = 2$, and $t/a = 0.2$, reached their absolute maximum values of 0.00236 mm at 3.48 s, and of 0.00427 mm at 3.48 s, respectively. These absolute maximum values are different even with the same occurring time as the dynamic characteristics of the thick plates affect the response. It is also understandable that the system becomes more flexible as the aspect ratio increases.

The absolute maximum displacements of the thick and thin plates for different aspect ratios, and thickness/span ratios are given in Fig. 6 for the thick and thin plates simply supported along all four edges and in Fig. 7 for the thick and thin plates clamped along all four edges.

As seen from Figs. 6, and 7, the absolute maximum displacements of the thick and thin plates increase with increasing aspect ratio for a constant t/a ratio. The same displacements decrease with increasing t/a ratio for a constant b/a ratio. As also seen from these figures, the decrease in the absolute maximum displacement for a constant b/a ratio increases with increasing b/a ratio. The curves for a constant value of the aspect ratio, b/a are fairly getting closer to each other as the value of t/a increases. This shows that the curves of the absolute maximum displacements will almost coincide with each other when the value of the thickness/span ratio, t/a, increases more. In other words, the increase in the thickness/span ratio will not affect the absolute maximum displacements after a determined value of t/a.



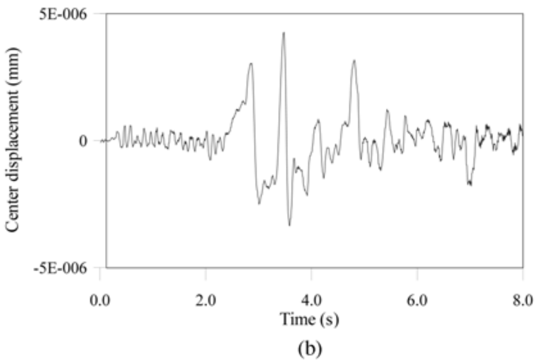


Fig. 5 The time history of the center displacement of the thick clamped plate for (a) $b/a = 1.0$ and $t/a = 0.2$, and (b) $b/a = 2.0$ and $t/a = 0.2$

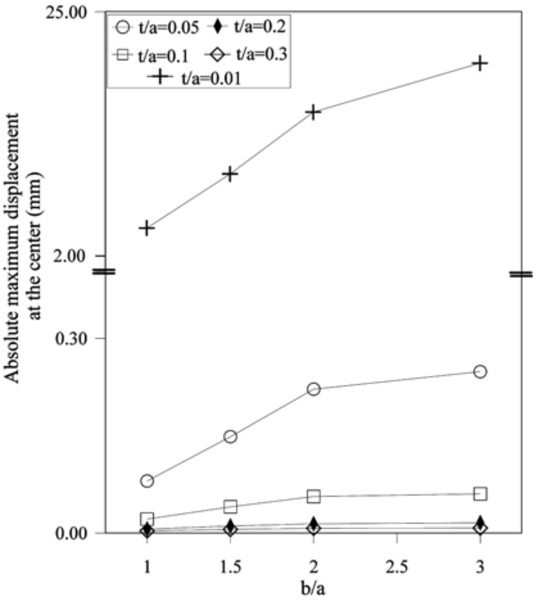


Fig. 6 Absolute maximum displacement of the thick and thin simply supported plates for different aspect ratios and thickness/span ratios

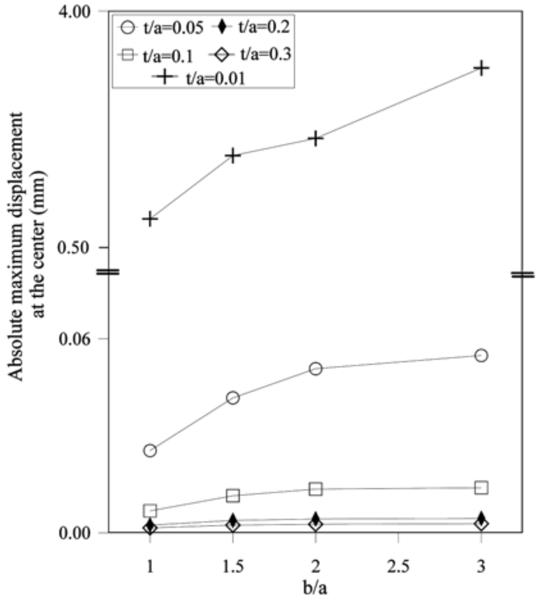


Fig. 7 Absolute maximum displacement of the thick and thin clamped plates for different aspect ratios and thickness/span ratios

As also seen from Figs. 6, and 7, the absolute maximum displacements of the thick and thin simply supported plates are larger than those of the thick and thin clamped plates for the same aspect and thickness/span ratios. In general, the effects of the changes in the thickness/span ratios on the absolute maximum displacement are larger than the changes in the aspect ratios.

The absolute maximum bending moments M_x at the center of the thick and thin plates for different aspect ratios and thickness/span ratios are given in Fig. 8 for the thick and thin simply supported plates and in Fig. 9 for the thick and thin clamped plates, respectively.

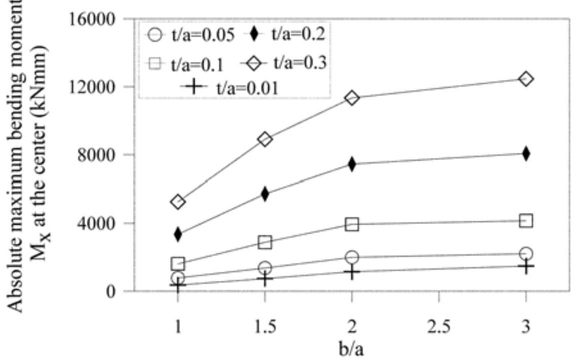


Fig. 8 Absolute maximum bending moment M_x at the center of the thick and thin simply supported plates for different aspect ratios and thickness/span ratios

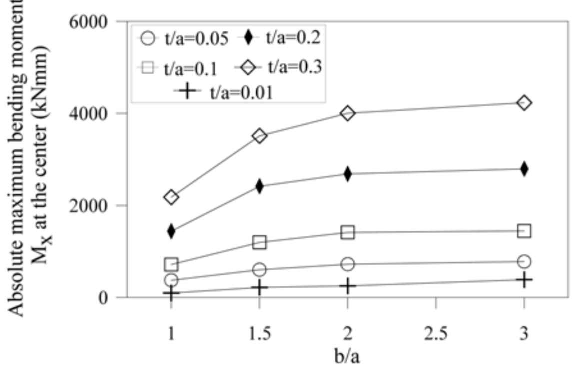


Fig. 9 Absolute maximum bending moment M_x at the center of the thick and thin clamped plates for different aspect ratios and thickness/span ratios

As seen from Fig. 8, the absolute maximum bending moment, M_x, at the center of the thick and thin simply supported plates increases with increasing aspect ratio and thickness/span ratio. The increases in the absolute maximum bending moment, M_x, increase with increasing aspect and thickness/span ratios. This is understandable that increasing the aspect ratio makes the plate stiffer in the short span, the x axis, direction. As also seen from this figure, in general, the effects of the changes in the aspect ratios on the absolute maximum bending moment, M_x, are larger than the changes in the thickness/span ratios.

As seen from Fig. 9, the absolute maximum bending moment, M_x, at the center of the thick and thin clamped plates, as in the case of the absolute maximum bending moment, M_x, at the center of the thick and thin simply supported plates, increases with increasing aspect ratio and thickness/span ratio. The increases in the absolute maximum bending moment, M_x, increase with increasing aspect and thickness/span ratios. This is also understandable that increasing the aspect ratio makes the plate stiffer in the short span, the x axis, direction. As also seen from this figure, in general, the effects of the changes in the aspect ratios on the absolute maximum bending moment, M_x, are larger than the changes in the thickness/span ratios.

The absolute maximum bending moments M_y at the center of the thick and thin plates for different aspect ratios and thickness/span ratios are given in Fig. 10 for the thick and thin simply supported plates and in Fig. 11 for the thick and thin clamped plates, respectively.

As seen from Fig. 10, the absolute maximum bending moment, M_y, at the center of the thick and thin simply supported plates decreases with increasing aspect ratio and increases with increasing thickness/span ratio. The decrease in the absolute maximum bending moment, M_y, increase with increasing aspect ratio. The increase in the absolute maximum bending moment, M_y, increases with increasing thickness/span ratios. This is understandable that increasing the aspect ratio makes the thick and thin plates more flexible in the long span, the y axis, direction. As also seen from this figure, in general, the effects of the changes in the thickness/span ratios on the absolute maximum bending moment, M_y, are larger than the changes in the aspect ratios.

As seen from Fig. 11, the absolute maximum bending moment, M_y, at the center of the thick and thin clamped plates, as in the case of the absolute maximum bending moment, M_y, at the center of the thick and thin simply supported plates, decreases with increasing aspect ratio and increases with

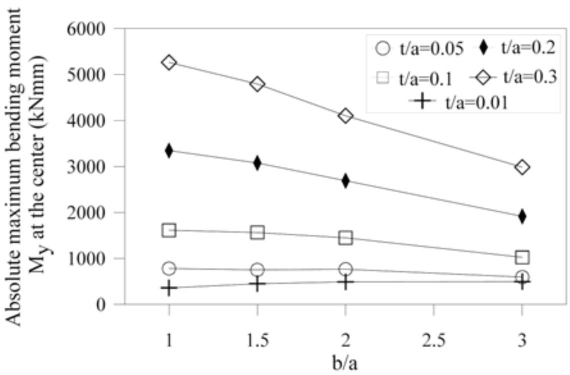


Fig. 10 Absolute maximum bending moment M_y at the center of the thick and thin simply supported plates for different aspect ratios and thickness/span ratios

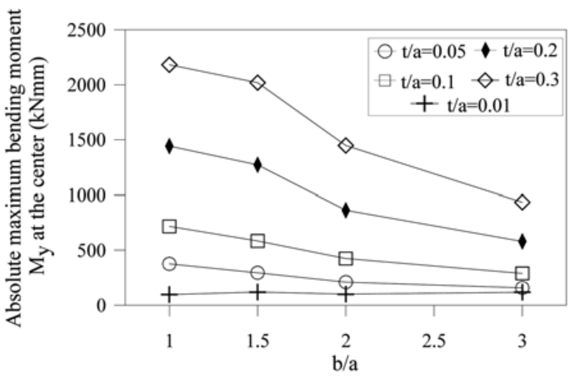


Fig. 11 Absolute maximum bending moment M_y at the center of the thick and thin clamped plates for different aspect ratios and thickness/ span ratios

increasing thickness/span ratio. The decrease in the absolute maximum bending moment, M_y, increase with increasing aspect ratio. The increase in the absolute maximum bending moment, M_y, increases with increasing thickness/span ratios. This is also understandable that increasing the aspect ratio makes the thick and thin plates more flexible in the long span, the y axis, direction. As also seen from this figure, in general, the effects of the changes in the thickness/span ratios on the absolute maximum bending moment, M_y, are larger than the changes in the aspect ratios.

In this study, the absolute maximum bending moments M_x at the center of the edge in the y direction and the maximum bending moment M_y at the center of the edge in the x direction are not presented for the thick and thin plates clamped along all four edges. It should be noted that the variations of these moments are similar to the absolute maximum bending moments M_x at the center of the thick and thin clamped plates.

4. Conclusions

The purpose of this paper was to study shear locking-free parametric analysis of thick and thin plates subjected to earthquake excitations and to determine the effects of the thickness/span ratio, the aspect ratio and the boundary conditions on the linear responses of thick and thin plates by using Mindlin's theory. It is concluded that 17-noded finite element can be effectively used in the earthquake analysis of thick and thin plates without shear locking-problem and that if this element is used in an analysis, it is not necessary to use finer mesh. No matter what the mesh size is unless it is less than 4×4. The coded program can be effectively used in the earthquake analyses of any thick and thin plates. It is also concluded that, in general, the changes in the thickness/span ratio are more effective on the maximum responses considered in this study than the changes in the aspect ratio.

For a thick or thin plates subjected to the earthquake excitations, it is somewhat difficult to interpret the effects of the thickness/span ratio, the aspect ratio, and the boundary conditions on the responses because both the frequency content of the earthquake excitation and the exact natural frequency of the particular thick and thin plates can make a difference to its response. In order to generalize the results obtained in this study, the responses of the different thick and thin plates subjected to different earthquake excitations should be evaluated all together. Therefore, the curves presented herein can help the designer to anticipate the effects of the thickness/span ratio, the aspect ratio, and the boundary conditions on the earthquake response of a thick plate.

The following conclusions can also be drawn from the results obtained in this study.

The absolute maximum displacements of the thick and thin plates increase as the aspect ratio increases for a constant t/a ratio. The same displacements decrease as the t/a ratio increases for a constant b/a ratio.

The changes in the aspect ratios are generally less effective on the absolute maximum displacement than the changes in the thickness/span ratios.

The absolute maximum bending moment, M_x, at the center of the thick and thin simply supported plates increases as the aspect ratio and thickness/span ratio increase.

The changes in the aspect ratios are generally more effective on the absolute maximum bending moment, M_x, of the thick and thin simply supported plates than the changes in the thickness/span ratios.

The absolute maximum bending moment, M_x, at the center of the thick and thin clamped plates increases with increasing aspect ratio and thickness/span ratio.

The changes in the aspect ratios are generally more effective on the absolute maximum bending moment, M_x, of the thick and thin clamped plates than the changes in the thickness/span ratios.

The absolute maximum bending moment, M_y, at the center of the thick and thin simply supported plates decreases as the aspect ratio increases and increases as the thickness/span ratio increases.

The changes in the thickness/span ratios are generally more effective on the absolute maximum bending moment, M_y, of the thick and thin simply supported plates larger than the changes in the aspect ratios.

The absolute maximum bending moment, M_y, at the center of the thick and thin clamped plates decreases with increasing aspect ratio and increases with increasing thickness/span ratio.

The changes in the thickness/span ratios are generally more effective on the absolute maximum bending moment, M_y, of the thick and thin clamped plates than the changes in the aspect ratios.

In general, degrees of decreases and increases depend on the changes in the aspect and thickness/span ratios, and the changes in the thickness/span ratio are more effective on the maximum responses considered in this study than the changes in the aspect ratio.

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