Structural Engineering and Mechanics, Vol. 33, No. 3 (2009) 357-371

DOI: http://dx.doi.org/10.12989/sem.2009.33.3.357

Behavior of reinforced concrete corbels

Wen-Yao Lu†

Department of Interior Design, China University of Technology, No. 56, Section 3, Xinglong Rd, Taipei, 11695 Taiwan

Ing-Jaung Lin‡

Department of Construction Engineering, National Taiwan University of Science and Technology, No. 43, Section 4, Keelung Road, Taipei 10672, Taiwan

(Received February 23, 2009, Accepted September 3, 2009)

Abstract. Test results of thirteen reinforced concrete corbels with shear span-to-depth ratio greater than unity are reported. The main variables studied were compressive strength of concrete, shear span-to-depth ratio and parameter of vertical stirrups. The test results indicate that the shear strengths of corbels increase with an increase in compressive strength of concrete and parameter of vertical stirrups. The shear strengths of corbels also increase with a decrease in shear span-to-depth ratio. The smaller the shear span-to-depth ratio of corbel, the larger the stiffness and the shear strength of corbel are. The higher the concrete strength of corbel, the higher the stiffness and the shear strength of corbel are. The larger the parameter of vertical stirrups, the larger the stiffness and the shear strength of corbel are. The softened strut-and-tie model for determining the shear strengths of reinforced concrete corbels is modified appropriately in this paper. The shear strengths predicted by the proposed model and the approach of ACI Code are compared with available test results. The comparison shows that the proposed model can predict more accurately the shear strengths of reinforced concrete corbels than the approach of ACI Code.

Keywords: reinforced concrete corbels; shear strength; strut-and-tie model.

1. Introduction

Owing to the increase use of precast concrete, reinforced concrete corbel is becoming a common feature in building construction. Previous experimental investigations (Kriz and Raths 1965, Mattock *et al*. 1976, Fattuhi and Hughes 1989, Her 1990, Yong and Balaguru 1994, Fattuhi 1994, Foster *et al*. 1996) focused on corbels with shear span-to-depth ratio (a/d) not greater than unity. Many parameters influence the behavior of reinforced concrete corbels, including shear span-to-depth ratio (a/d), compressive strength of concrete (f_c'), parameter of vertical stirrups ($\rho_v f_{yv}$) and parameter of horizontal stirrups ($\rho_h f_{yh}$). According to Hwang *et al*. (2000), the vertical stirrups are

† Professor, Corresponding author, E-mail: luwenyao@cute.edu.tw
‡ Professor, E-mail: ijlin@mail.ntust.edu.tw

not useful for shear strength of a corbel with $a/b \leq 1$. It is believed that the vertical stirrups are significant to the shear-carrying capacity for corbels with $a/d > 1$.

A nonlinear finite element analysis for reinforced concrete corbels was presented by Alfred *et al.* (2006). The failure mode of corbels with $a/d < 1$ is frequently dominated by shear. However, ductile flexural failure could occur if a corbel with $a/d > 1$ is carefully detailed. The shear strengths of reinforced concrete corbels with $a/d \leq 1$ have been accurately predicted by Hwang *et al.* (2000) and Russo *et al.* (2005), but very few, if any, theoretical models for predicting the shear strengths of reinforced concrete corbels with $a/d > 1$ were introduced. According to the current ACI Code (2008), corbels with $a/d < 1$ could be designed using the empirical formulas, and corbels with $1 < a/d < 2$ should be designed using the strut-and-tie model contained in Appendix A of the ACI Code (2008).

Beginning with the pioneering work of Ritter (1899) and Mörsch (1909) about a century ago, numerous researchers have examined the application of strut-and-tie concepts to structural design problems (Breen 1991, Schlaich and Schäfer 2001). In the conventional strut-and-tie model (Ritter 1899, Mörsch 1909, Breen 1991, Schlaich 2001), the stresses are usually determined by equilibrium condition alone, while the strain compatibility conditions are neglected. However, the softened strut-and-tie (SST) model (Hwang *et al.* 2000) which satisfies equilibrium, compatibility and constitutive laws of cracked reinforced concrete had been proposed for determining the shear strengths of reinforced concrete corbels with $a/d \leq 1$.

In this paper, thirteen tested corbels will first be presented, and then the SST model (Hwang and Lee 2002, Hwang *et al.* 2000) will be modified appropriately for predicting the shear strengths of corbels with $a/d > 1$. The precision of the modified SST model and the approach of ACI Code (2008) are gauged by the available experimental results.

2. Research significance

There is a wealth of test data on corbels with $a/d \leq 1$ (Kriz and Raths 1965, Mattock *et al.* 1976, Fattuhi and Hughes 1989, Her 1990, Yong and Balaguru 1994, Fattuhi 1994, Foster *et al.* 1996), but currently the available test data on corbels with $a/d > 1$ is still very limited. Further experimental works on corbels with $a/d > 1$ should be performed to enhance the credence of the current design model (ACI 2008) and the SST model (Hwang and Lee 2002, Hwang *et al.* 2000).

3. Experimental study

In this study, a total of 13 reinforced concrete corbels with $a/d > 1$ were tested under vertical load only. Variables considered in the tests were shear span-to-depth ratio (a/d), compressive strength of concrete (f_c') and parameter of vertical stirrups ($\rho_v f_{yv}$).

3.1 Specimen details

Three classes of compressive strength of concrete were used in specimens. Specimen details and test results are given in Table 1. Each specimen consisted of a 1150-mm-long column with two corbels projecting from the column in a symmetrical fashion; the main reinforcement of corbels

Table 1 Specimen details

Specimen	f'_c		b		d		a		a/d	Main	Horizontal	Vertical	$V_{cv,test}$		Failure mode
	psi	MPa	in.	mm	in.	mm	in.	mm		bars	stirrups	stirrups	kips	kN	
H12230	9007	62.1	8.27	210	12.56	318.9	15.35	390	1.22	3-#7	3-#3	0	107.46	478	Diagonal compression failure
H16930	9007	62.1	8.27	210	12.56	318.9	21.26	540	1.69	3-#7	3-#3	0	62.72	279	Diagonal compression failure
H12233	9007	62.1	8.27	210	12.56	318.9	15.35	390	1.22	3-#7	3-#3	3-#3	145.46	647	Diagonal compression failure
H16933	9007	62.1	8.27	210	12.56	318.9	21.26	540	1.69	3-#7	3-#3	3-#3	87.68	390	Flexural failure
H12236	9007	62.1	8.27	210	12.56	318.9	15.35	390	1.22	3-#7	3-#3	6-#3	144.33	642	Flexural failure
H16936	9007	62.1	8.27	210	12.56	318.9	21.26	540	1.69	3-#7	3-#3	6-#3	95.77	426	Flexural failure
M12233	6265	43.2	8.27	210	12.56	318.9	15.35	390	1.22	3-#7	3-#3	3-#3	98.02	436	Diagonal compression failure
M12236	6265	43.2	8.27	210	12.56	318.9	15.35	390	1.22	3-#7	3-#3	6-#3	120.73	537	Flexural failure
L12220	3669	25.3	8.27	210	12.56	318.9	15.35	390	1.22	2-#7	3-#3	0	56.21	250	Diagonal compression failure
L16920	3669	25.3	8.27	210	12.56	318.9	21.26	540	1.69	2-#7	3-#3	0	29.23	130	Diagonal compression failure
L12222	3669	25.3	8.27	210	12.56	318.9	15.35	390	1.22	2-#7	3-#3	2-#3	62.95	280	Diagonal compression failure
L16922	3669	25.3	8.27	210	12.56	318.9	21.26	540	1.69	2-#7	3-#3	2-#3	43.62	194	Diagonal compression failure
L16924	3669	25.3	8.27	210	12.56	318.9	21.26	540	1.69	2-#7	3-#3	4-#3	57.33	255	Flexural failure

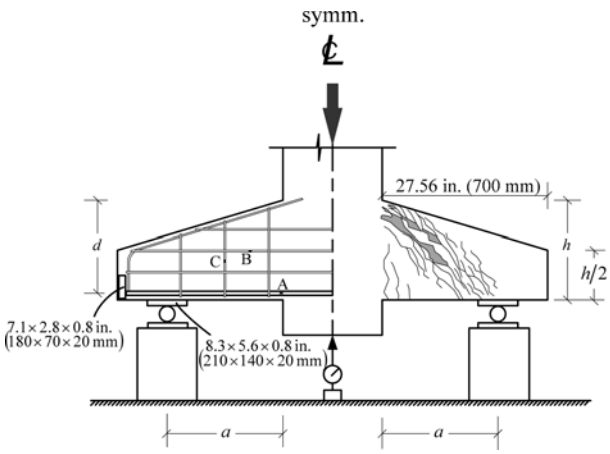


Fig. 1 Typical failures in corbels

consisted of parallel straight bars welded to the steel plates (180 × 70 × 20 mm) at the ends of corbels to prevent a local bond failure, as shown in Fig. 1. As shown in Fig. 1, each corbel in this study had a width of 210 mm and the overall depth of each corbel was 350 mm at the interface with the column and 175 mm at the cantilever end. The effective depth of each corbel was 318.9 mm at the interface with the column. For all the corbels in this study, the closed horizontal stirrups were 3-#3 deformed bars and were uniformly distributed in the two-thirds of the effective depth nearest the main reinforcement. The number of the closed vertical stirrups used in each corbel is shown in Table 1. The yield strengths of #3 and #7 deformed bars are 504 MPa and 517 MPa, respectively.

The corbel notation given in Table 1 includes four parts. The first part refers to the compressive strength of concrete: H is for high compressive strength ($f_c' = 62.1$ MPa), M is for median compressive strength ($f_c' = 43.2$ MPa) and L is for low compressive strength ($f_c' = 25.3$ MPa), respectively. The second part is used to identify the shear span-to-depth ratio: 122 for $a/d = 1.22$ and 169 for $a/d = 1.69$. The third part gives the amount of main reinforcement, e.g., 2 for 2-#7 main reinforcement and 3 for 3-#7 main reinforcement. The fourth part gives the number of vertical stirrups, e.g., 0 for no vertical stirrups and 2 for 2-#3 vertical stirrups.

3.2 Testing procedure

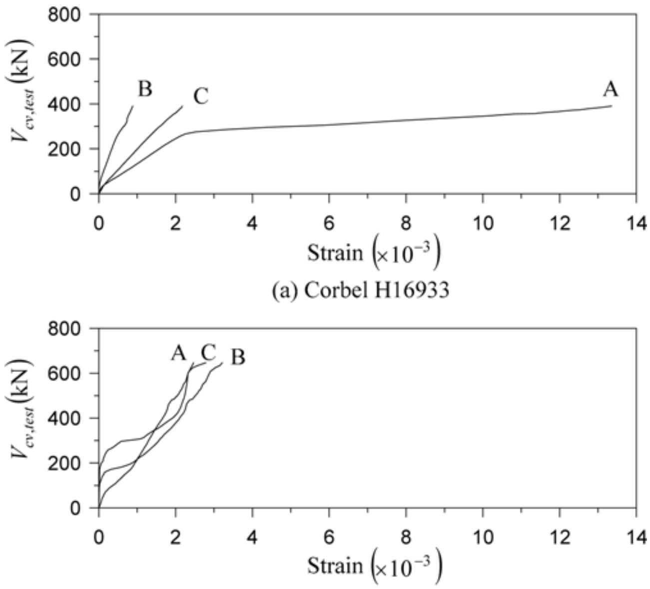
During the test, the strains in the main reinforcement, horizontal stirrups and vertical stirrups were measured using electrical resistance gauges at locations A, B and C, respectively (Fig. 1). Displacement was measured using a dial-gauge connected to the center of the column. Both surfaces of the corbels were whitewashed to aid on the observation of crack development during testing. The typical arrangement for the test is shown in Fig. 1. The shear span (a) was measured from the center of the support to the face of the column. Vertical loading was applied through the column to each corbel using a 6000 kN capacity universal testing machine. Each corbel was seated on a bearing plate (210 × 140 × 20 mm) to prevent bearing failure (Fig. 1). At each load increment, the test data were captured by a data logger and automatically stored.

3.3 Test results

As shown in Fig. 1, the shear action in the corbel leads to compression in a diagonal direction and tension in a perpendicular direction. The first crack suddenly developed in the diagonal direction at the mid-height of corbels, and then a vertical flexural crack was formed in the sagging region at the interface with the column. As the load increased, more flexural and diagonal cracks were formed. However, the corbels did not fail immediately due to the formation of the diagonal cracks. After diagonal cracking, the concrete between diagonal cracks can be represented as a concrete compression strut, the external shear is assumed to be transferred by the concrete compression strut, and the possible failure mode will be the diagonal compression failure or the flexural failure.

As shown in Table 1, a total of thirteen reinforced concrete corbels with $a/d > 1$ were tested, and eight of them failed because of diagonal compression while five of them failed because of flexure. The failure mode of corbels with $a/d = 1.22$ is dominated by diagonal compression, except corbels H12236 and M12236 which failed because of flexure (Table 1). This may be due to more vertical stirrups (6-#3) used in corbels H12236 and M12236, which can significantly enhance the shear strengths of corbels and may avoid diagonal compression failure. The failure mode of corbels with $a/d = 1.69$ is dominated by flexure, except corbels H16930, L16920 and L16922 which failed because of diagonal compression (Table 1). This may be due to no vertical stirrups or less vertical stirrups used in corbels H16930, L16920 and L16922, which can not provide sufficient shear strengths to avoid diagonal compression failure. If a corbel with $a/d > 1$ is detailed with sufficient vertical stirrups, then the failure mode of corbel will be converted from diagonal compression failure into flexural failure (Table 1).

The measured shear strength, $V_{cv,test}$, for each specimen obtained in the tests is summarized in Table 1. Test results show that the shear strengths of corbels increase with an increase in compressive strength of concrete and parameter of vertical stirrups (Table 1). Test results in Table 1



(a) Corbel H16933

(b) Corbel H12233

Fig. 2 Shear force versus steel strain for corbel also show that the shear strengths of corbels increase with a decrease in shear span-to-depth ratio. The contribution of parameter of vertical stirrups on the shear strengths of corbels with $a/d = 1.69$ is more effective than those of corbels with $a/d = 1.22$ (Table 1).

The typical shear force versus steel strain for corbels failed by flexure is presented in Fig. 2(a). It can be seen that the strain of main reinforcement of corbel H16933 increases rapidly beyond the yielding strain of the reinforcing bar before the ultimate state was reached (Fig. 2(a)). Also, it can be seen that the strain of the vertical stirrup is larger than that of the horizontal stirrup (Fig. 2(a)). The vertical stirrup is thus more effective than the horizontal one for corbels with $a/d = 1.69$ in this study. The typical shear force versus steel strain for corbels failing by diagonal compression is presented in Fig. 2(b). It can be seen that the strain of main reinforcement of corbel H12233 is below the yielding strain of reinforcing bars at the ultimate state (Fig. 2(b)). The strain of horizontal stirrup of corbel H12233 increases beyond the yielding strain of reinforcing bar before the ultimate state was reached (Fig. 2(b)). However, the vertical stirrup of corbel H12233 did not yield at the ultimate state (Fig. 2(b)). It is likely that the horizontal stirrup is more effective than the vertical stirrup for corbels with $a/d = 1.22$ in this study.

Fig. 3 shows the effects of shear span-to-depth ratio, compressive strength of concrete and parameter of vertical stirrups on the load-displacement relationship of corbels. As shown in Fig. 3, both the stiffness and the ultimate load of corbel L12220 are larger than those of corbel L16920. The smaller the shear span-to-depth ratio of corbel, the larger the stiffness of corbel and the ultimate load of corbel are. As shown in Fig. 3, both the stiffness and the ultimate load of corbel H12233 are larger than those of corbel M12233. The higher the concrete strength of corbel, the larger the stiffness of corbel and the ultimate load of corbel are. As shown in Fig. 3, both the stiffness and the ultimate load of corbel L16924 are larger than those of corbel L16922. The higher the parameter of vertical stirrups of corbel, the larger the stiffness of corbel and the ultimate load of corbel are.

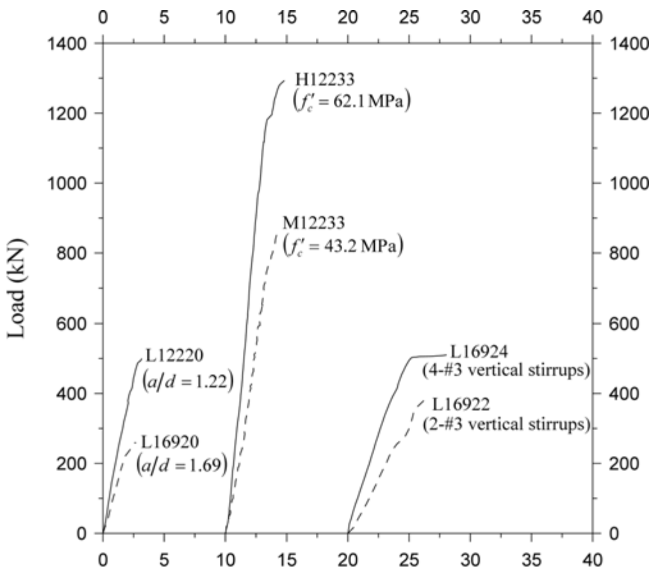


Fig. 3 Load versus displacement

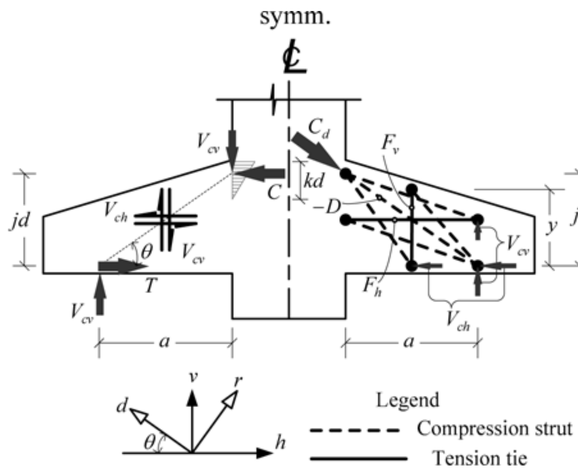


Fig. 4 SST model for internal forces

4. Proposed model

Fig. 4 shows the loads acting on the corbel and the force transferring mechanisms of the proposed SST model. By considering the distances between force couples (Fig. 4), it will be sufficiently accurate to express the following relationship between vertical and horizontal shears.

$$\frac{V_{cv}}{V_{ch}} \approx \frac{jd}{a} \tag{1}$$

where V_{cv} is the vertical shear force, V_{ch} is the horizontal shear force and jd is the length of the lever arm from the resultant compressive force to the centeroid of the flexural reinforcement. According to the linear bending theory, the lever arm jd can be estimated as

$$jd = d - kd/3 \tag{2}$$

where d is the effective depth of the corbel, kd is the depth of compression zone at the section and coefficient k can be defined as

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{3}$$

where n is the modular ratio of elasticity and can be defined as

$$n = \frac{E_s}{E_c} \tag{4}$$

where E_s is the elastic modulus of the steel, E_c is the elastic modulus of the concrete and ρ is the ratio of the main reinforcement.

Fig. 4 shows the proposed SST model, which is composed of the diagonal, horizontal and vertical mechanisms (Hwang *et al.* 2000). The diagonal mechanism is a diagonal compression strut whose angle of inclination θ is defined as (Hwang *et al.* 2000)

$$\theta = \tan^{-1}\left(\frac{jd}{a}\right) \tag{5}$$

The effective area of the diagonal strut, A_{str}, can be estimated as

$$A_{str} = t_s \times b_s \tag{6}$$

where t_s is the thickness of the diagonal strut and b_s is the width of the diagonal strut which can be taken as the width of the corbel.

The thickness of the diagonal strut is dependent on its end condition provided by the compression zone at the column face. It is intuitively assumed that (Hwang *et al.* 2000)

$$t_s = kd \tag{7}$$

The horizontal mechanism consists of one horizontal tie and two flat struts (Hwang *et al.* 2000). The horizontal tie is made up of horizontal stirrups. When computing the area of the horizontal tie, A_{th}, it is roughly assumed that the horizontal stirrups within the center half are fully effective, and the rest are at 50% effectiveness (Hwang *et al.* 2000). If the horizontal stirrups are uniformly distributed in the two-thirds of the effective depth nearest the main reinforcement, then $A_{th} = 0.8A_h$, where A_h is the area of the horizontal stirrups. The vertical mechanism consists of one vertical tie and two steep struts (Hwang *et al.* 2000). The vertical tie is made up of vertical stirrups. The area of the vertical tie, A_{tv}, is computed in the same way as that of the horizontal tie. If the vertical stirrups are uniformly distributed within the shear span, then $A_{tv} = 0.75A_v$ in which A_v is the area of the vertical stirrups within the shear span.

4.1 Evaluation of shear strength

According to Hwang and Lee (2002), the diagonal compression strength of corbel can be estimated as follows

$$C_d = (K_h + K_v + 1)\zeta f_c' A_{str} \tag{8}$$

where C_d is the predicted diagonal compression strength, K_h is the horizontal tie index, K_v is the vertical tie index, f_c' is the compressive strength of concrete and ζ is the softening coefficient of concrete in compression.

The horizontal tie index can be estimated as follows (Hwang and Lee 2002)

$$K_h = 1 + (\overline{K}_h - 1)\frac{A_{th} f_{yh}}{\overline{F}_h} \le \overline{K}_h \tag{9}$$

where

$$\overline{K}_h \approx \frac{1}{1 - 0.2(\gamma_h + \gamma_h^2)} \tag{10}$$

$$\gamma_h = \frac{2\tan\theta - 1}{3}, \text{ but } 0 \le \gamma_h \le 1 \tag{11}$$

$$\overline{F}_h = \gamma_h \times (\overline{K}_h \zeta f_c' A_{str}) \times \cos\theta \tag{12}$$

$$\zeta = \frac{3.35}{\sqrt{f_c'}} \le 0.52 \tag{13}$$

where $\overline{K}_h$ is the horizontal tie index with sufficient horizontal stirrups, f_{yh} is the yield stress of horizontal stirrups, γ_h is the fraction of horizontal shear transferred by the horizontal tie in the absence of the vertical tie and $\overline{F}_h$ is the balance amount of horizontal tie force.

The vertical tie index can be estimated as follows (Hwang and Lee 2002)

$$K_v = 1 + (\overline{K}_v' - 1)\frac{A_{tv} f_{yv}}{\overline{F}_v} \le \overline{K}_v' \tag{14}$$

where $\overline{K}_v'$ is the modified vertical tie index with sufficient vertical stirrups, f_{yv} is the yield stress of vertical stirrups and $\overline{F}_v$ is the balance amount of vertical tie force.

As shown in Fig. 4, the effective length of vertical tie, y, which is less than the original length of vertical tie, jd. It is assumed to be multiplied by y/jd to account for the reduction in vertical tie index.

$$\overline{K}_v' = \overline{K}_v \times \frac{y}{jd} \tag{15}$$

where

$$\overline{K}_v \approx \frac{1}{1 - 0.2(\gamma_v + \gamma_v^2)} \tag{16}$$

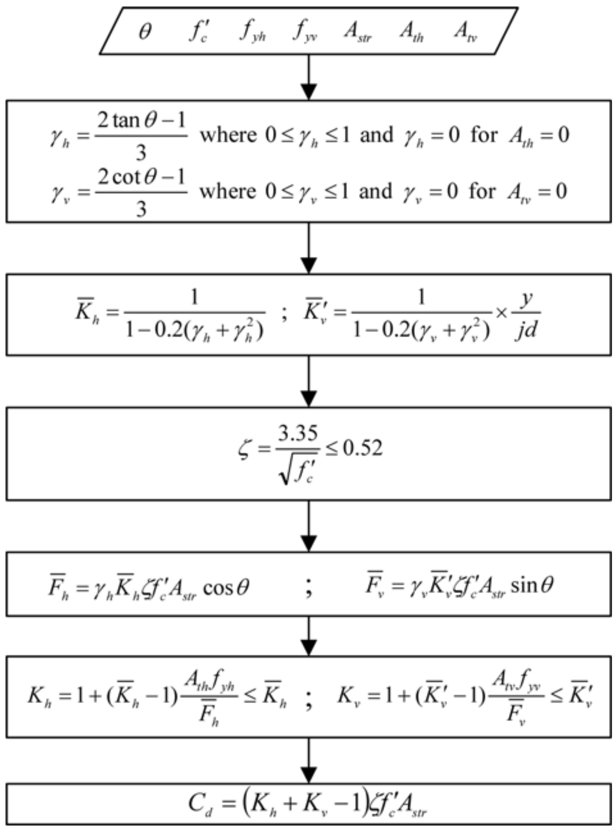


Fig. 5 Flow chart showing solution procedure

$$\gamma_v = \frac{2\cot\theta - 1}{3}, \text{ but } 0 \le \gamma_v \le 1 \tag{17}$$

where $\overline{K}_v$ is the vertical tie index with sufficient vertical stirrups and γ_v is the fraction of vertical shear transferred by the vertical tie in the absence of the horizontal tie.

The balance amount of vertical tie force can be estimated as

$$\overline{F}_v = \gamma_v \times (\overline{K}_v' \zeta f_c' A_{str}) \times \sin\theta \tag{18}$$

The solution algorithm for C_d is summarized in Fig. 5.

In the proposed model, the predicted shear strength of the corbel cannot be greater than the shear force based on the flexural strength of the corbel. Thus, the predicted shear strength of the corbel can be determined as follows

$$V_{cv,calc} = MIN\left(C_d \sin\theta, \frac{M_n}{a}\right) \tag{19}$$

where M_n is the nominal flexural strength of corbel, it can be defined as

$$M_n = A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c' b}\right) \tag{20}$$

where A_s is the area of the main reinforcement, f_y is the yield strength of the main reinforcement and b is the width of the corbel.

5. Experimental verification

A total of 191 test specimens and their results are used to verify the proposed model. Thirteen of them are the corbels with $a/d > 1$ tested in this study while 178 of them were corbels with $a/d \le 1$ previously tested by Kriz and Raths (1965), Mattock *et al.* (1976), Fattuhi and Hughes (1989), Her (1990), Yong and Balaguru (1994), Fattuhi (1994) and Foster *et al.* (1996).

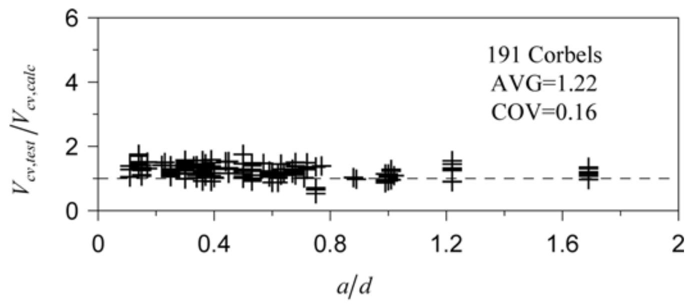
Accuracy for the proposed model is evaluated in terms of a strength ratio, which is defined as the ratio of the measured strength to the calculated strength. The test-to-theory comparisons of 13 corbels with $a/d > 1$ are presented in Table 2 to examine the validity and accuracy of the proposed model and the approach of ACI Code (2008) to the corbels with $a/d > 1$. It can be found in Table 2, the mean of the measured-to-calculated strength ratios is 1.26 with a coefficient of variation of 0.15 for predictions using the proposed model, and the mean of the measured-to-calculated strength ratios is 1.30 with a coefficient of variation of 0.21 for predictions using the approach of ACI Code (2008). Both approaches can accurately predict the shear strengths of corbels with $a/d > 1$.

According to experimental results of this study, the major factors influencing the shear strength of corbels have been found to be shear span-to-depth ratio (a/d), compressive strength of concrete (f_c'), parameter of main reinforcement (ρf_y) and parameter of vertical stirrups ($\rho_v f_{yv}$). As shown in Table 2, the proposed model can consistently predict the shear strengths of corbels with different shear span-to-depth ratio, compressive strength of concrete, parameter of main reinforcement and parameter of vertical stirrups.

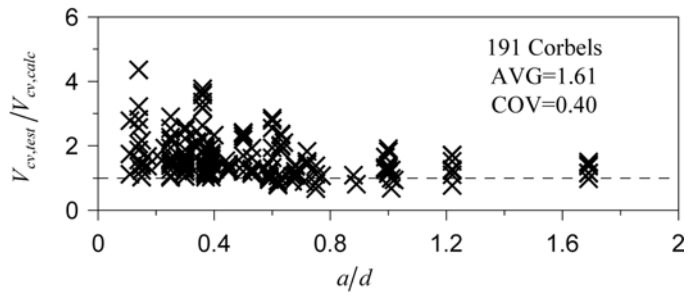
The test-to-theory comparisons use parametric study to further assess the suitability of the proposed model and the approach of ACI Code (2008) to corbels with shear span-to-depth ratios

Table 2 Experimental verification

Specimen	a/d	f'_c		$\rho_h f_{yh}$		$\rho_v f_{yv}$		ρf_y		$V_{cv,test}$		$V_{cv,test}/V_{cv,calc.}$	
		psi	MPa	psi	MPa	psi	MPa	psi	MPa	kips	kN	SST	ACI
H12230	1.22	9007	62.1	467	3.22	0	0	1301	8.97	107.46	478	1.28	1.27
H16930	1.69	9007	62.1	467	3.22	0	0	1301	8.97	62.72	279	0.97	0.98
H12233	1.22	9007	62.1	467	3.22	381	2.63	1301	8.97	145.46	647	1.58	1.72
H16933	1.69	9007	62.1	467	3.22	276	1.90	1301	8.97	87.68	390	1.20	1.37
H12236	1.22	9007	62.1	467	3.22	763	5.26	1301	8.97	144.33	642	1.51	1.71
H16936	1.69	9007	62.1	467	3.22	551	3.80	1301	8.97	95.77	426	1.31	1.50
M12233	1.22	6265	43.2	467	3.22	381	2.63	1301	8.97	98.02	436	1.32	1.10
M12236	1.22	6265	43.2	467	3.22	763	5.26	1301	8.97	120.73	537	0.90	0.77
L12220	1.22	3669	25.3	467	3.22	0	0	867	5.98	56.21	250	1.32	1.23
L16920	1.69	3669	25.3	467	3.22	0	0	867	5.98	29.23	130	1.10	1.15
L12222	1.22	3669	25.3	467	3.22	254	1.75	867	5.98	62.95	280	1.29	1.52
L16922	1.69	3669	25.3	467	3.22	184	1.27	867	5.98	43.62	194	1.18	1.16
L16924	1.69	3669	25.3	467	3.22	367	2.53	867	5.98	57.33	255	1.40	1.43
Total											AVG	1.26	1.30
13											COV	0.15	0.21



(a) SST



(b) ACI

Fig. 6 Effect of shear span-to-depth ratio on shear strength prediction between 0.11 and 1.69. Fig. 6 shows the effect of shear span-to-depth ratio a/d on the shear strength predictions for 191 corbels. The proposed model consistently predicts the shear strengths of corbels with a/d ratios between 0.11 and 1.69 (Fig. 6). However, a greater scattering is found for the ACI (2008) predictions, with more conservative estimates for low a/d ratios (Fig. 6).

6. Case study

A case study was performed to demonstrate the variation in shear-carrying capacity of a reinforced concrete corbel caused by various parameters (Fig. 7). The studied corbel was shown in Fig. 7(a), having compressive strength of concrete of 30 MPa, width of 200 mm, overall depth of 600 mm at the interface with the column and 300 mm at the cantilever end. The effective depth of

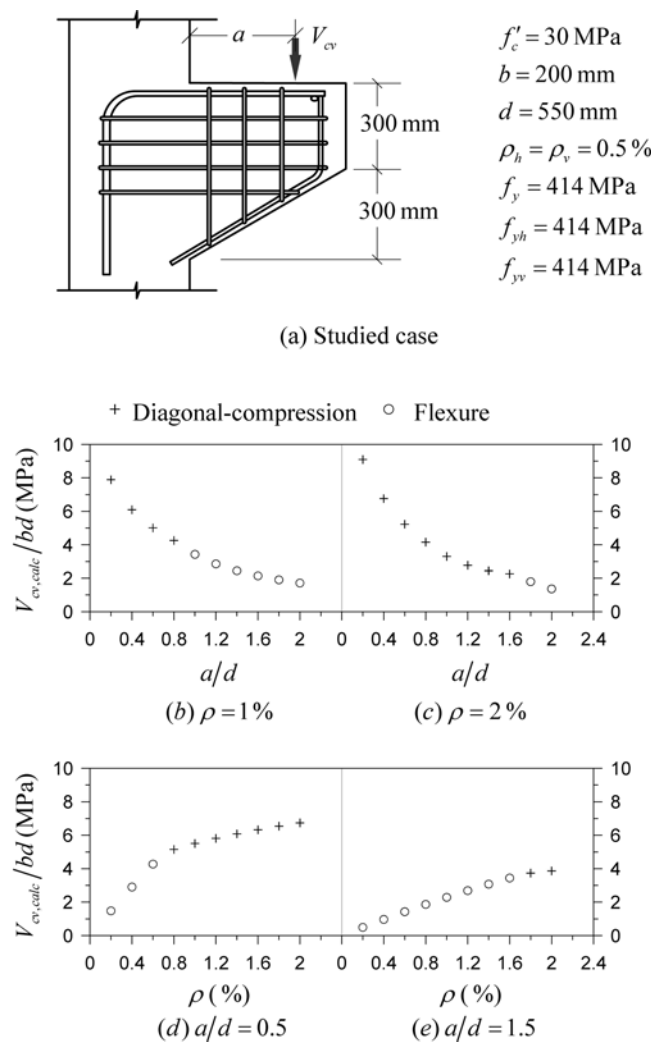


Fig. 7 Variation in shear-carrying capacity

the corbel was 550 mm at the interface with the column. The yield strengths of main reinforcement, horizontal and vertical stirrups are equal to 414 MPa. As shown in Fig. 7(a), the ratio of horizontal stirrups $\rho_h = A_h/bd$ is equal to 0.5 %. The ratio of vertical stirrups $\rho_v = A_v/ab$ is equal to 0.5%.

The effect of a/d on the shear-carrying capacities of corbels with ρ = 1% and ρ = 2% are shown in Figs. 7(b) and 7(c), respectively. The shear-carrying capacity of the corbel decreases with an increase in a/d. When a/d is low, the shear-carrying capacity of the corbel is governed by diagonal compression. As shown in Fig. 7(b), the critical value of a/d is 0.8 for the corbel with ρ = 1%, if a/d is higher than 0.8, the failure mode of corbel will be converted from diagonal compression into flexure. The shear-carrying capacity of a corbel with ρ = 2% is dominated by the diagonal compression (Fig. 7(c)).

As shown in Figs. 7(d) and 7(e), the shear-carrying capacity of a corbel increases with an increase in ρ. When ρ is low, the shear-carrying capacity of the corbel is governed by flexure. As shown in Fig. 7 (d), the critical value of ρ is 0.6% for a corbel with a/d = 0.5, if ρ is higher than 0.6%, the failure mode of the corbel will be converted from flexure into diagonal compression. For a corbel with ρ > 0.6%, the shear-carrying capacity is increased with increasing ρ at a much lower rate (Fig. 7(d)). As shown in Fig. 7(e), the shear-carrying capacity of corbel with a/d = 1.5 is dominated by flexure.

7. Conclusions

In this study, a total of 13 reinforced concrete corbels with a/d > 1 were tested under vertical load only. The softened strut-and-tie model for predicting the shear strengths of reinforced concrete corbels is modified appropriately in this paper. According to the available test results in this study (Table 1) and the literature (Kriz and Raths 1965, Mattock *et al.* 1976, Fattuhi and Hughes 1989, Her 1990, Yong and Balaguru 1994, Fattuhi 1994, Foster *et al.* 1996), and the comparison of predictions by the proposed model and the approach of ACI Code (2008) (Tables 1 and 2, Figs. 3, 6 and 7), the following conclusions can be made:

1. The shear strengths of corbels increase with an increase in compressive strength of concrete and parameter of vertical stirrups. The shear strengths of corbels also increase with a decrease in shear span-to-depth ratio.
2. The smaller the shear span-to-depth ratio of corbel, the larger the stiffness and the ultimate load of a corbel are. The higher the concrete strength of corbel, the larger the stiffness and the ultimate load of corbel are. The higher the parameter of vertical stirrups of a corbel, the larger the stiffness and the ultimate load of a corbel are.
3. Both the proposed model and the approach of ACI Code (2008) can accurately predict the shear strengths of corbels with a/d > 1.
4. The proposed model can consistently predict the shear strengths of corbels with a/d ratios between 0.11 and 1.69; wherever experimental tests have been done for $1.22 \le a/d \le 1.69$.
5. A ductile flexural failure is likely to occur in a corbel with a high a/d and a low ρ. While, a diagonal compression failure is likely to occur in a corbel with a low a/d and a high ρ. The shear-carrying capacity of a corbel increases with an increase in ρ. When ρ is higher than a critical value, the failure mode of the corbel will be converted from flexure into diagonal compression and the shear-carrying capacity is increased with increasing ρ at a much lower rate.

References

Alfred Strauss, Andrea Mordini and Konrad Bergmeister (2006), "Nonlinear finite element analysis of reinforced concrete corbels at both deterministic and probabilistic levels", *Comput. Concr.*, **3**(2), 123-144.

American Concrete Institute (2008), "Building code requirements for structural concrete (ACI 318-08) and Commentary (ACI 318R-08)", Farmington Hills, MI.

Breen, J.E. (1991), "Why structural concrete? p.15-26 in: IABSE Colloquium Structural Concrete", Stuttgart, IABSE Rep.

Fattuhi, N.I. (1994), "Reinforced corbels made with plain and fibrous concrete", *ACI Struct. J.*, **91**(5), 530-536.

Fattuhi, N.I. and Hughes, B.P. (1989), "Ductility of reinforced concrete corbels containing either steel fibers or stirrups", *ACI Struct. J.*, **86**(6), 644-651.

Foster, S.J., Powell, R.E. and Selim, H.S. (1996), "Performance of high-strength concrete corbels", *ACI Struct. J.*, **93**(5), 555-563.

Her, G.J. (1990), "A study of reinforced high-strength concrete corbels", Master's Thesis, Department of Construction Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan (in Chinese).

Hwang, S.J. and Lee, H.J. (2002), "Strength prediction for discontinuity regions failing in diagonal compressions by softened strut-and-tie model", *J. Struct. Eng.*, ASCE, **128**(12), 1519-1526.

Hwang, S.J., Lu, W.Y. and Lee, H.J. (2000), "Shear strength prediction for reinforced concrete corbels", *ACI Struct. J.*, **97**(4), 543-552.

Kriz, L.B. and Raths, C.H. (1965), "Connections in precast concrete structures - strength of corbels", *PCI J.*, **10**(1), 16-61.

Mattock, A.H., Chen, K.C. and Soongswang, K. (1976), "The behavior of reinforced concrete corbels", *PCI J.*, **21**(2), 52-77.

Mörsch, E. (1909), *Concrete-steel Construction*. New York: McGraw-Hill.

Ritter, W. (1899), Die bauweise hennebique, Schweizerische Bauzeitung, **33**(7), 59-61.

Russo, G., Venir, R., Pauletta M. and Somma, G. (2005), "Reinforced concrete corbels-shear strength model and design formula", *ACI Structu. J.*, **102**(3), 429-437.

Schlaich, J., Schäfer, K. Konstruieren im Stahlbetonbau (Detailing of reinforced concrete). Betonkalender 90, 2001. Teil II, 311-492. Ernst & Sohn Verlag, Berlin.

Yong, Y.K. and Balaguru, P. (1994), "Behavior of reinforced high-strength--concrete corbels", *J. Struct. Eng.*, ASCE, **120**(4), 1182-1201.

Notations

α	: shear span, measured from the center of support to the face of column
A_h	: area of the horizontal stirrups
A_s	: area of the main reinforcement
A_{th}	: area of the horizontal tie
A_{tv}	: area of the vertical tie
A_v	: area of the vertical stirrups within shear span
A_{str}	: effective area of the diagonal strut
b	: width of the corbel
b_s	: width of the diagonal strut
C	: resultant compressive force at section due to flexure
C_d	: predicted diagonal compression strength
d	: effective depth of the corbel at the interface of the column
	: assumed direction of principal compressive stress of concrete
	: direction of the diagonal concrete strut
D	: compression force in the diagonal strut (negative for compression)
E_c	: elastic modulus of the concrete

E_s : elastic modulus of the steel
f_c' : compressive strength of the standard concrete cylinder
F_h : tension force in the horizontal tie (positive for tension)
$\overline{F}_h$: balance amount of horizontal tie force
F_v : tension force in the vertical tie (positive for tension)
$\overline{F}_v$: balance amount of vertical tie force
f_{yh} : yield stress of the horizontal stirrups
f_{yv} : yield stress of the vertical stirrups
h : direction of the horizontal stirrups
: overall depth of the corbel
j, k : coefficients
jd : length of the lever arm from the resultant compressive force to the centroid of the flexural tension reinforcement
: original length of vertical tie
kd : depth of compression zone at the section
K_h : horizontal tie index
$\overline{K}_h$: horizontal tie index with sufficient horizontal hoops
K_v : vertical tie index
$\overline{K}_v$: vertical tie index with sufficient vertical hoops
$\overline{K}_v'$: modified vertical tie index with sufficient vertical hoops
M_n : nominal moment strength of the corbel
n : modular ratio of elasticity
: E_s/E_c
r : direction perpendicular to d
: assumed direction of principal tensile stress
T : resultant tensile force at section due to flexure
t_s : thickness of the diagonal strut
v : direction of the vertical stirrups
V_{ch}, V_{cv} : horizontal and vertical shear forces, respectively
$V_{cv,calc}$: predicted shear strength of corbels
$V_{cv,test}$: shear strength of corbels measured in the test
y : effective length of vertical tie
γ_h : fraction of horizontal shear transferred by the horizontal tie in the absence of the vertical tie
γ_v : fraction of vertical shear transferred by the vertical tie in the absence of the horizontal tie
ε_d, ε_r : average principal strains in the d- and r- directions, respectively (positive for tensile strain)
ε_h, ε_v : average normal strains in the h- and v- directions, respectively (positive for tensile strain)
θ : angle of inclination
ρf_y : parameter of the main reinforcement
ρ_h : ratio of horizontal stirrups
: A_h/bd
$\rho_h f_{yh}$: parameter of the horizontal stirrups
ρ_v : ratio of vertical stirrups
: A_v/ab
$\rho_v f_{yv}$: parameter of the vertical stirrups
ζ : softening coefficient of concrete in compression