Structural Engineering and Mechanics, Vol. 33, No. 2 (2009) 179-191 179
DOI: http://dx.doi.org/10.12989/sem.2009.33.2.179

Simulation of earthquake records using combination of wavelet analysis and non-stationary Kanai-Tajimi model

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(Received February 11, 2008, Accepted August 8, 2009)

Abstract. This paper is aimed at combining wavelet multiresolution analysis and nonstationary Kanai-Tajimi model for the simulation of earthquake accelerograms. The proposed approach decomposes earthquake accelerograms using wavelet multiresolution analysis for the simulation of earthquake accelerograms. This study is on the basis of some Iranian earthquake records, namely Naghan 1977, Tabas 1978, Manjil 1990 and Bam 2003. The obtained results indicate that the simulated records preserve the significant properties of the actual accelerograms. In order to investigate the efficiency of the model, the spectral response curves obtained from the simulated accelerograms have been compared with those from the actual records. The results revealed that there is a good agreement between the response spectra of simulated and actual records.

Keywords: earthquake ground motion; simulation; non-stationary model; wavelet analysis; Kanai-Tajimi model.

1. Introduction

Seismic design of structures requires a dynamic analysis procedure either response spectrum or time- history. The major drawback of response spectrum analysis lies in its inability to obtain time information of the structural responses. Such information is sometimes necessary in achieving a satisfactory design.

In many cases, structures' house equipment is sensitive to floor vibrations during an earthquake. It is sometimes necessary to develop the floor response. In addition, in designing critical or major structures such as power plants, dams, and high-rise buildings, the final design is usually based on the complete time-history analysis. To provide input excitations for structural models in sites with no strong ground motion data, it is necessary to generate artificial accelerograms.

The modeling and simulation of earthquake ground motion signals are important in structural earthquake engineering and may significantly facilitate the study of structural behavior under seismic excitation. The main difficulty in modeling such signals stems from their strongly non-stationary nature.

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Due to the complex nature of the formation of seismic waves and their travel path before reaching recording station, a stochastic approach may be most suitable for generating artificial accelerograms. To this purpose, different stochastic models, both stationary and non-stationary, have broadly been used in literature to simulate earthquake ground motions. The stationary filtered white noise model of earthquake ground motion of Kanai and Tajimi has attracted considerable attention, and was extensively used in random vibration analysis of structures (Kanai 1957, Tajimi 1960). More recent models were suggested to include the non-stationary variations in amplitude and frequency content by Fan and Ahmadi (1990) and Refooei *et al.* (2001).

The recently developed wavelet analysis has emerged as a powerful tool to analyze temporal variations in frequency content. Newland (1994) applied wavelets to analyze vibration signals, and developed special wavelets and techniques for engineering purposes. Ghodrati Amiri *et al.* (2008, 2007, 2006), Suarez and Montejo (2007, 2005), Rajasekaran *et al.* (2006), Hancock *et al.* (2006), Mukherjee and Gupta (2002a, 2002b) and Iyama and Kuwamura (1999) developed the wavelet analysis for generating artificial earthquake accelerograms. Using wavelet theory, Ghodrati Amiri *et al.* (2007, 2006) aimed at generating many artificial records compatible with the same spectrum. Also, Ghodrati Amiri *et al.* (2008) proposed a method of generating accelerograms from response spectra based on neural network and wavelet transform.

In this paper, first, the fundamentals of wavelet analysis are introduced. Then, the nonstationary Kanai-Tajimi model is explained. Next, an effective method for the generation of artificial accelerograms for an arbitrary site with at least one existing earthquake record is presented. Specific examples of Naghan 1977, Tabas 1978, Manjil 1990 and Bam 2003 earthquakes are illustrated in detail. It is shown that the generated accelerograms preserve the time dependent frequency content of the original records. The response spectra curves for the generated earthquakes are also compared with those of the original records and their results are discussed.

2. Wavelet analysis

The continuous wavelet transform is defined as

$$C_{a,b} = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} f(t)\,\psi^*\left(\frac{t-b}{a}\right)dt \tag{1}$$

where a and b are scale and translation parameters, respectively and ψ^* is the complex conjugate of ψ. The basis function ψ is represented as

$$\psi_{j,k}(t) = 2^{j/2}\,\psi(2^j t - k) \tag{2}$$

Eq. (1) can be represented as

$$C_{a,b} = \langle f(t), \psi^*_{a,b}(t)\rangle \tag{3}$$

Therefore, continuous wavelet transform is a collection of inner products of a signal $f(t)$ and the translated and dilated wavelets $\psi^*_{a,b}(t)$.

The main idea of discrete wavelet transform is the same as that of continuous wavelet transform. In discrete wavelet transform the signals can be represented by approximations and details. The detail at level j is defined as

$$D_j(t) = \sum_{k \in Z} cD_{j,k}\psi_{j,k}(t) \tag{4}$$

where Z is the set of positive integers and $cD_{j,k}$ is *wavelet Coefficient*s at level j which is defined as

$$cD_j(k) = \int_{-\infty}^{\infty} f(t)\psi_{j,k}(t)dt \tag{5}$$

The approximation at level j is defined as

$$A_j(t) = \sum_{k=-\infty}^{\infty} cA_j(k)\phi_{j,k}(t) \tag{6}$$

where $cA_{j,k}$ is *scaling Coefficient*s at level j which is defined as

$$cA_j(k) = \int_{-\infty}^{\infty} f(t)\phi_{j,k}(t)dt \tag{7}$$

Finally, the signal $f(t)$ can be expressed by

$$f(t) = A_J + \sum_{j \le J} D_j \tag{8}$$

Since each $D_j(t)$ has a range of particular out of which the intensity is zero, an assumption is introduced here that the original function $f(t)$ is decomposed into a series of $D_j(t)$'s exclusively in the frequency domain; in other word, each $D_j(t)$ has non-zero components only in an exclusive range of frequency. This assumption is not theoretically exact but is justified later from an engineering practice viewpoint. The exclusive range of frequency of $D_j(t)$ is denoted as follows

$$\text{Frequency range of level } j = [f_{1j}, f_{2j}] \tag{9}$$

or

$$\text{Period range of level } j = [T_{1j}, T_{2j}] \tag{10}$$

From the nature of discrete wavelet transform that $D_j(t)$ has components of half frequency of $D_{j+1}(t)$ (Benedetto and Frazier 1994), f_{1j}, f_{2j}, T_{1j} and T_{2j} are expressed as follows

$$f_{1j} = 2^{-j-1}/\Delta t, \quad f_{2j} = 2^{-j}/\Delta t \tag{11}$$

$$T_{1j} = 2^{j}\Delta t, \quad T_{2j} = 2^{j+1}\Delta t \tag{12}$$

where Δt is the time step of digital data of $f(t)$.

3. Overview on Generalized Kanai-Tajimi model

The Kanai-Tajimi model for ground acceleration (Kanai 1957, Tajimi 1960) has been widely used in the analysis of engineering structures under earthquake excitation. In its original form, the ground acceleration is idealized as a stationary random process, having a spectral density function

$$G(\omega) = \frac{\omega_g^4 + (2\xi_g \omega \omega_g)^2}{(\omega_g^2 - \omega^2)^2 + (2\xi_g \omega \omega_g)^2} G_0 \tag{13}$$

This model corresponds to the acceleration of a mass, supported by liner spring and dashpot in parallel, whose base is undergoing a broad-band acceleration. The three parameters in Eq. (13) are ξ_g, ω_g site dominant damping coefficient and frequency, and G_0, the constant power spectral intensity of bedrock excitation. In practice, these parameters need to be estimated from the local earthquake records and geological features. Kanai-Tajimi power spectral density function may be commented as corresponding to an 'ideal white noise' excitation at bedrock level filtered through the overlaying soil deposits at a site. The most serious disadvantages of the original Kanai-Tajimi model is to consider the behavior of earthquake as a stationary random process. An improved version of the model was introduced by Fan and Ahmadi (1990) to capture the non-stationary feature of real earthquake records. This generalized non-stationary Kanai-Tajimi model is described in Eqs. (14) and (15)

$$\ddot{X}_f + 2\xi_g(t)\omega_g(t)\dot{X}_f + \omega_g^2(t)X_f = n(t) \tag{14}$$

$$\ddot{X}_g = -(2\xi_g(t)\omega_g(t)\dot{X}_f + \omega_g^2(t)X_f)e(t) \tag{15}$$

where X_f is filtered response, $\omega_g(t)$ is time dependent ground frequency, $\xi_g(t)$ is effective ground damping coefficient, $\ddot{X}_g$ is output ground acceleration, and $e(t)$ is amplitude envelope function. In Eq. (14), $n(t)$ is a stationary Gaussian white noise process with the parameters in Eq. (16).

$$E[n(t)] = 0, \quad E[n(t_1)n(t_2)] = 2\pi G_0 \delta(t_1 - t_2) \tag{16}$$

where $E[]$ stands for expected value, and $\delta(\ )$ is Dirac delta function. Eqs. (14) to (16) provide filtered white noise stochastic time series with appropriate frequency content and amplitude modulation for ground acceleration during earthquake.

4. Modeling method

This paper present the use of the discrate wavelet transform and nonstationary Kanai-Tajimi model for the simulation of earthquake accelerograms. This procedure decomposes earthquake accelerograms using wavelet multiresolution analysis for the simulation of earthquake accelerograms. In this method, the earthquake accelerogram is decomposed into a series of $D_j(t)$, since each $D_j(t)$ has non-zero components only in an exclusive range of frequency. Next, each $D_j(t)$ is simulated with the nonstationary Kanai-Tajimi model. Finally, the simulated record using summation of $D_j(t)$ is determined.

If it is assume that $a_g(t)$ is an accelerogram for simulation, accelerogram can be expressed by details using discrete wavelet transform. The wavelet coefficients at level j are

$$cD_j(k) = \int_{-\infty}^{\infty} a_g(t)\, \psi_{j,k}(t)\, dt \tag{17}$$

then, the detail at level j determines

$$D_j(t) = \sum_j cD_j(k)\,\psi_{j,k}(t) \tag{18}$$

After the decomposition of earhquake accelerogram using wavelet analysis, the model parametres should be determined. In this method, time-varying parameters for dynamic version of Kanai-Tajimi model are considered. In order to estimate the time-dependency of the filter parameters, at least one recorded accelerogram is needed. For this purpose, the 'Moving-Time-Window' technique (Fan and Ahmadi 1990) has been employed. This method is based on the assumption that a non-stationary process can approximately be assumed to be stationary within a time-window with appropriate size. The time-window should be sufficiently short to capture the rapid changes in frequency content; however, it should be long enough to be able to account for the stable estimation of parameters and the ability to capture significantly low frequency components. In this study, the optimal window size is selected on the basis of frequency content of earthquake using a trial and error method. In the current study, $\xi_{gj}(t)$ is assumed to be a constant, and the time-evolution of $\omega_{gj}(t)$ and $e_j(t)$ are determined using the following procedure:

Using a time-window that moves from the beginning to the end of $D_j(t)$, the standard deviation of the envelope function within each time window is calculated as follows

$$\sigma_j = (E[D_j^2] - E[D_j]^2)^{1/2} \tag{19}$$

This value is assigned to the center point of each time window. Then, a smooth algebraic time function $\sigma_{aj}(t)$ is fitted to the time variation of the standard deviation. Next, the amplitude envelope function is defined as

$$e_j(t) = C_{0j}\sigma_{aj}(t) \tag{20}$$

where, C_{0j} is a constant that is used to normalize the mean intensity of the synthetic accelerograms to the intensity of original record.

The time dependent ground frequency function is determined using the properties $D_j(t)$. Each $D_j(t)$ has non-zero components only in an exclusive range of frequency. For this reason, the time dependent ground frequency function at level j then is suggested as

$$\omega_{gj}(t) = 2\pi\left[\frac{f_{1j}+f_{2j}}{2} + \frac{f_{2j}-f_{1j}}{2}\sin(t)\right] \tag{21}$$

where f_{1j} and f_{2j} are frequency range of level j.

After determine the parameters of model, the nonstationary Kanai-Tajimi model is used to simulate the $D_j(t)$

$$\ddot{X}_{fj} + 2\xi_{gj}\omega_{gj}(t)\dot{X}_{fj} + \omega_{gj}^2(t)X_{fj} = n(t) \tag{22}$$

$$\ddot{X}_{gj} = -(2\xi_{gj}(t)\omega_{gj}(t)\dot{X}_{fj} + \omega_{gj}^2(t)X_{fj})e_j(t) \tag{23}$$

where, X_{fj} and $\ddot{X}_{gj}$ are filtered response and output ground acceleration at level j, respectively. Afterwards, $\ddot{X}_{gj}$ can be determined using the solutions of the Eqs. (22) and (23) for all of levels.

Finally, the simulated record may be obtained by summation of $\ddot{X}_{gj}$

$$a_{gs}(t) = \sum_j \ddot{X}_{gj}(t) \tag{24}$$

where a_{gs} is the simulated record.

5. Synthetic records

The proposed model is used to produce artificial records for several Iranian earthquake accelerograms with different characteristics. The considered earthquake events were Naghan 1977, Tabas 1978, Manjil 1990 and Bam 2003.

In this reserach, earthquake ground acceleration has been decomposed by discrete wavelet transform to level 8. In order to identify the model parameters, a time-window that moves from the beginning to the end of $D_j(t)$ has been applied. As stated previously, the window size should be selected in such a way that it captures the time evolutions of the significant frequency content and the amplitude of the record. The used values of ground damping coefficients $\xi_{gj}(t)$ were 0.35, 0.35, 0.3 and 0.35 for Naghan, Tabas, Manjil and Bam earthquakes, respectively (Refooei *et al.* 2001). For the sake of white noise process generation, a constant power spectral intensity of $G_0 = 1$ cm^2/s^3 and time interval of $\Delta t = 0.02$ s were used. For each record, ensembles of 700 samples were generated and the value of C_0 was determined in a way that the expected total energy of synthetic accelerograms became equal to that of original record within the predetermined duration.

An ensemble of 50 synthetic accelerograms was generated and statically studied for each earthquake. It should be noted that records have been decomposed with db-10 wavelet (the other wavelets could be applied). Figs. 1 to 4 show the comparison between the accelerograms of the actual Naghan, Tabas, Manjil and Bam earthquakes and the synthetically generated records.

Statistical response spectra of the synthetic accelerograms have also been compared with those of the actual records. Figs. 5 to 8 compare the pseudo-acceleration response spectra and the related statistics for the ensemble of 50 simulated accelerograms with those for the actual records.

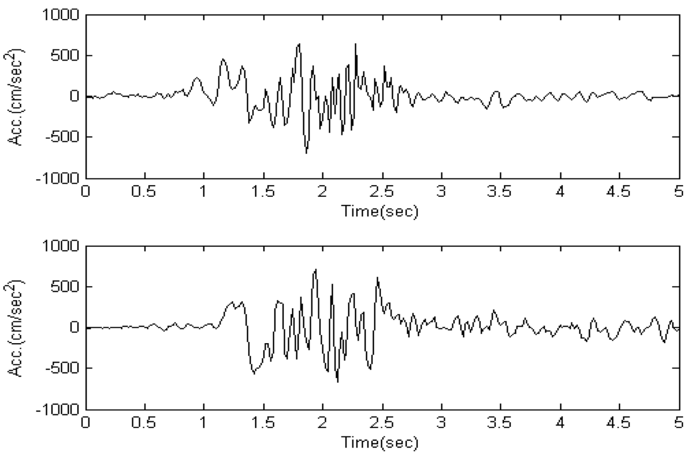

Fig. 1 Artificial (top) and actual (bottom) earthquake ground motion accelerograms of Naghan record

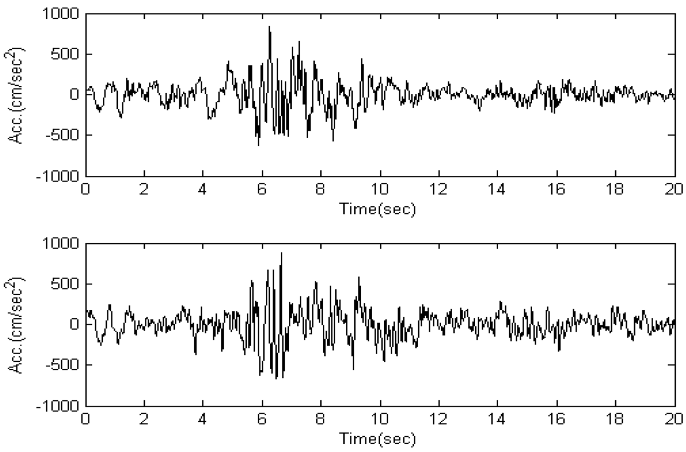

Fig. 2 Artificial (top) and actual (bottom) earthquake ground motion accelerograms of Tabas record

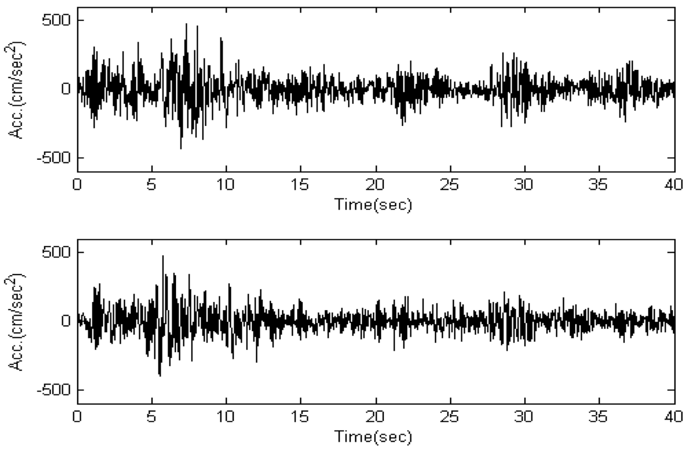

Fig. 3 Artificial (top) and actual (bottom) earthquake ground motion accelerograms of Manjil record

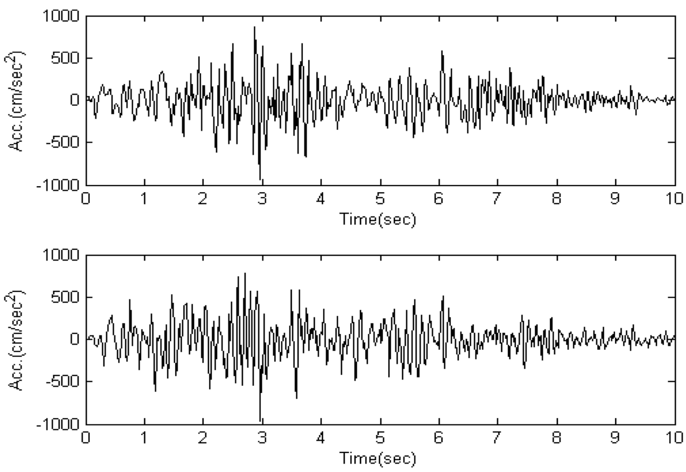

Fig. 4 Artificial (top) and actual (bottom) earthquake ground motion accelerograms of Bam record

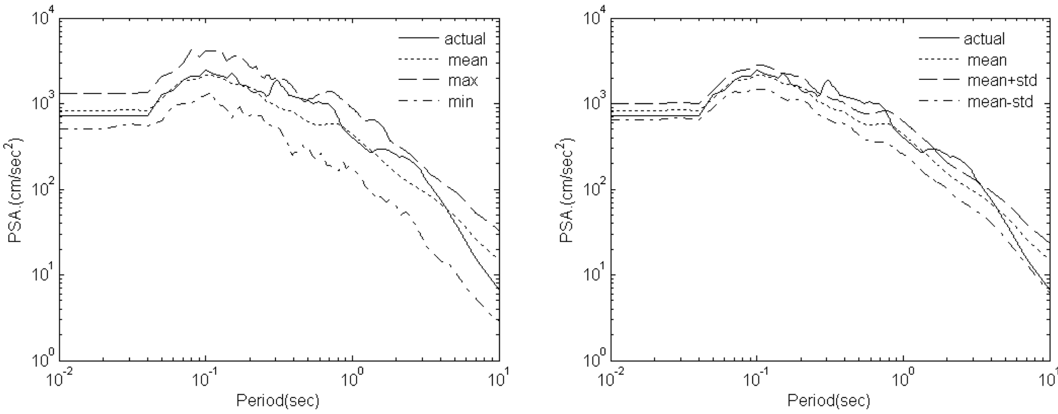

Fig. 5 Comparison between pseudo-acceleration response spectra of original and generated accelerograms for Naghan earthquake

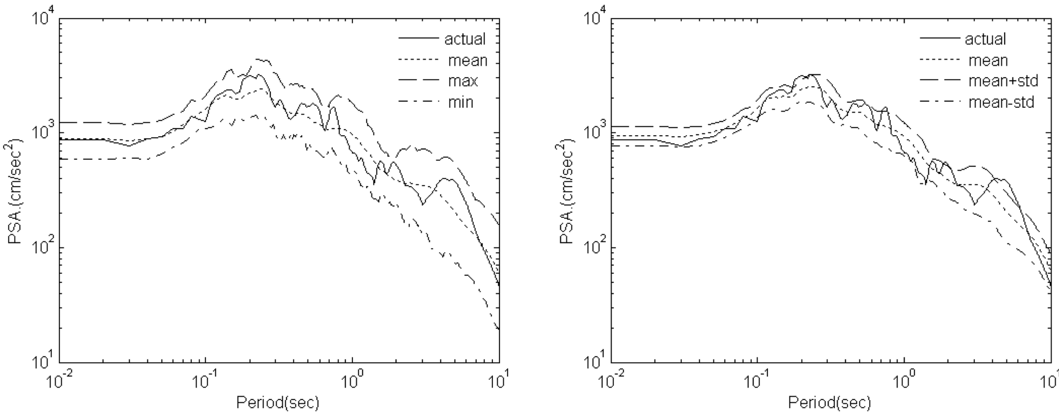

Fig. 6 Comparison between pseudo-acceleration response spectra of original and generated accelerograms for Tabas earthquake

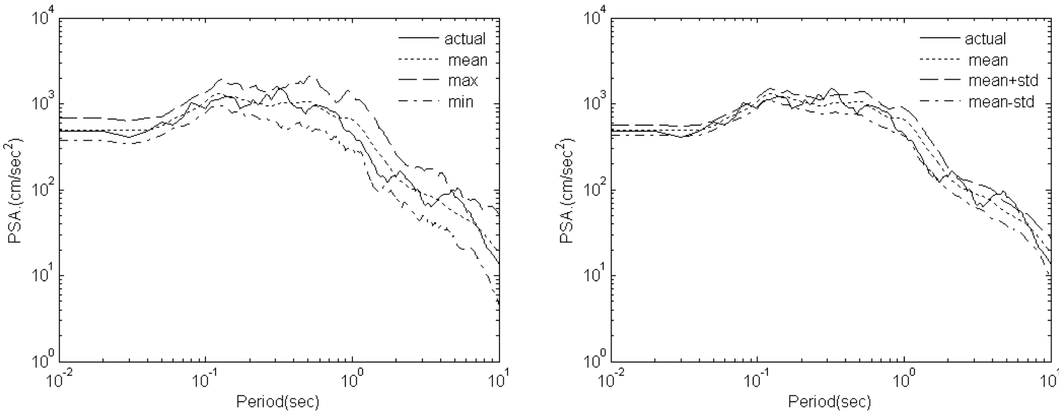

Fig. 7 Comparison between pseudo-acceleration response spectra of original and generated accelerograms for Manjil earthquake

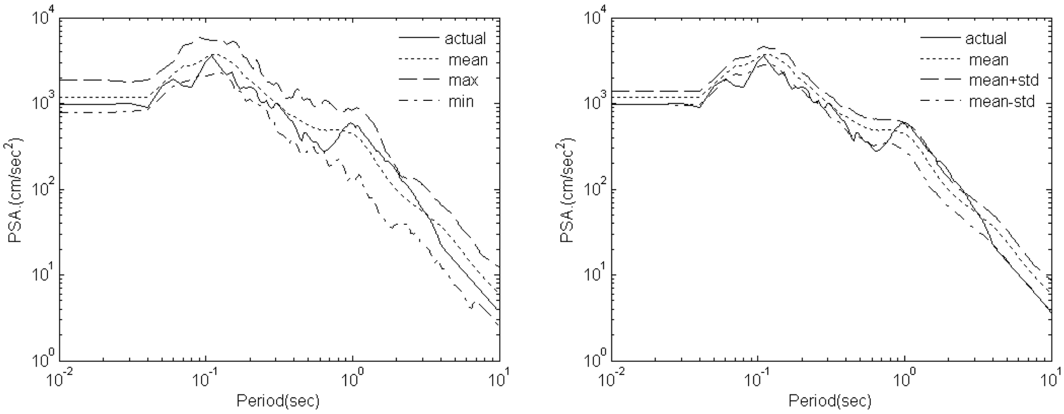

Fig. 8 Comparison between pseudo-acceleration response spectra of original and generated accelerograms for Bam earthquake

Traditional measures of earthquake energy input are as follows:

- Root mean square of the strong phase of the ground acceleration
- Maximum energy response of a single degree-of-freedom system subjected to ground motions with no damping
- Fourier amplitude spectrum

Zhou and Adeli (2003) proposed Eq. (25) for wavelet energy spectrum to represent the time-frequency evolution of earthquake energy input

$$E_{a,b} = \frac{|C(a,b)|^2}{\pi} \tag{25}$$

where the coefficients $C_{a,b}$ are obtained by applying the continuous Mexican hat wavelet transform to the ground acceleration $a_g(t)$ as follows

$$C(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} a_g(t)\,\psi\left(\frac{t-b}{a}\right)dt \tag{26}$$

In Eq. (26), a is used for the scaling parameters to distinguish it from the ground acceleration a_g. The Mexican hat wavelet is used as the basis wavelet function ψ in Eq. (26). The constant π is included in Eq. (25) for the Mexican hat wavelet so that the Parseval's theorem can be extended to include the wavelet transform.

The wavelet energy spectra of the actual and artificial accelerograms are shown in Figs. 9 to 12 (MATLAB 1999). According to these figures, the evolutions of the frequency content with time of the actual and generated accelerograms are comparable.

Finally, pseudo-acceleration response spectra and the related statistics for the ensemble of 50 simulated accelerograms have been obtained using propsed model in this paper the model presented by Rofooei *et al.* (2001) for the following earthquakes records: Naghan's, 1977; Tabas', 1978; and Manjil's, 1990. The results are illustrated in Figs. 13 to 15. As it is shown, the pseudo-acceleration response spectra obtained by the proposed model are closer to actual earthquake records compared to Rofooei model. Therefore, the performance of the proposed method using wavelet transform and generalized Kanai-Tajimi model was evaluated by comparison with the other model. It was

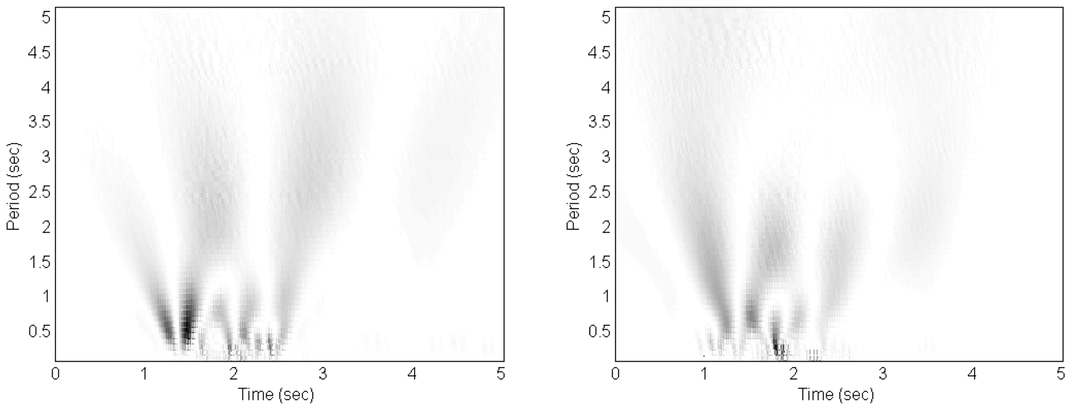

Fig. 9 Wavelet energy spectrum of actual (left) and artificial (right) earthquake accelerograms of Naghan record

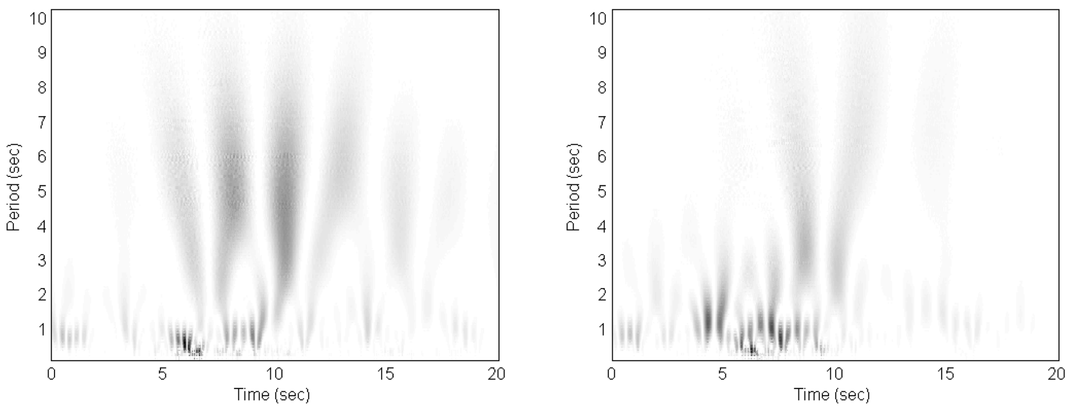

Fig. 10 Wavelet energy spectrum of actual (left) and artificial (right) earthquake accelerograms of Tabas record

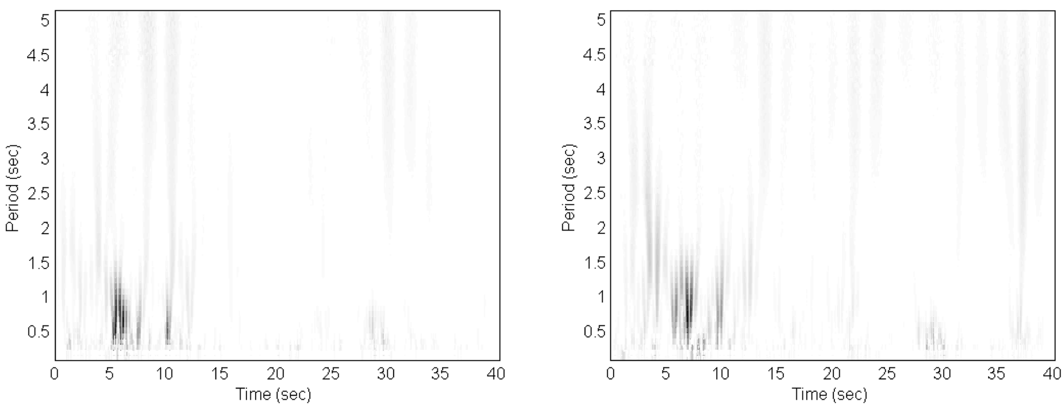

Fig. 11 Wavelet energy spectrum of actual (left) and artificial (right) earthquake accelerograms of Manjil record

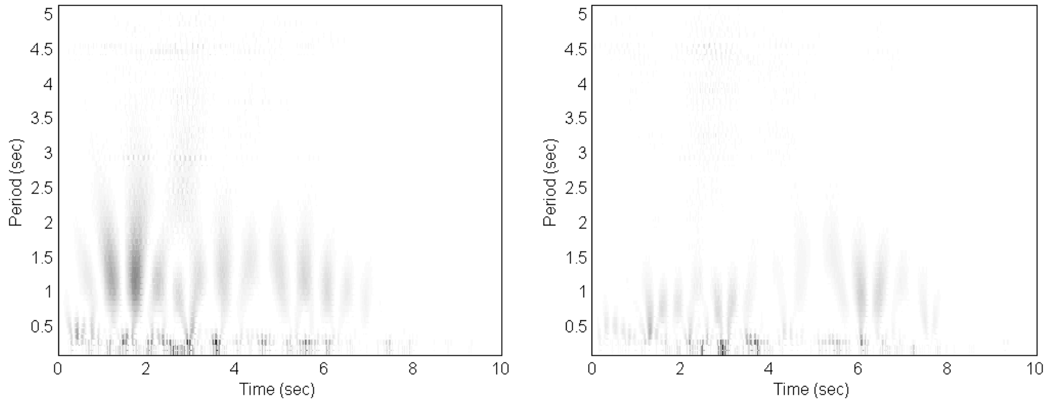

Fig. 12 Wavelet energy spectrum of actual (left) and artificial (right) earthquake accelerograms of Bam record

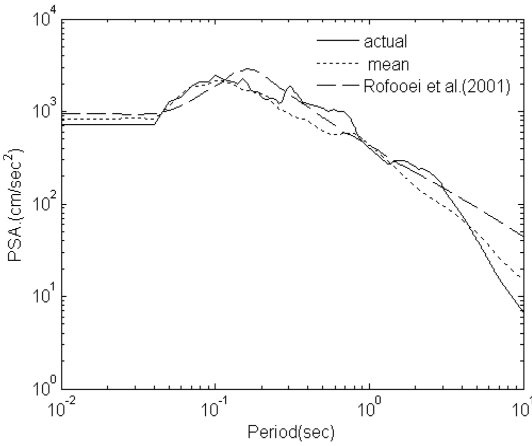

Fig. 13 Comparison between pseudo-acceleration response spectra of original and generated accelerograms with other model for Naghan earthquake

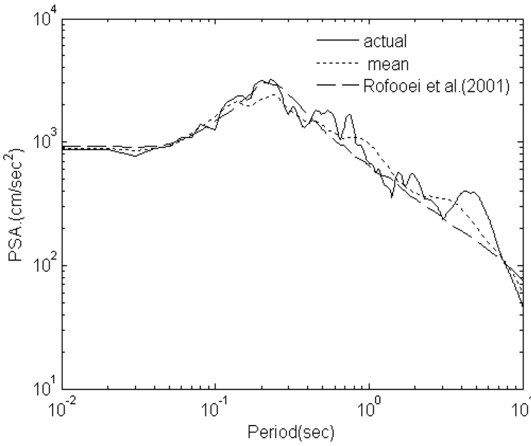

Fig. 14 Comparison between pseudo-acceleration response spectra of original and generated accelerograms with other model for Tabas earthquake

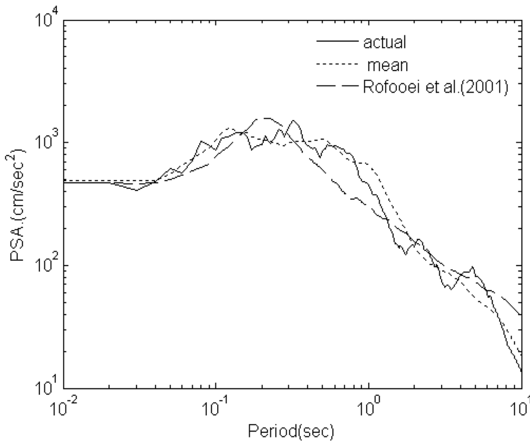

Fig. 15 Comparison between pseudo-acceleration response spectra of original and generated accelerograms with other model for Manjil earthquake

concluded that the time domain characteristics and the response spectra of the generated accelerograms are similar to the original recorded accelerograms. Hence, the proposed model can be used in the simulation of earthquake accelerograms for time history analysis of linear and nonlinear structures.

6. Conclusions

In this study, a model has been developed on the basis of Kanai-Tajimi model and wavelet multiresolution analysis to generate artificial accelerograms in an arbitrary site with at least one existing earthquake record. The presented approach is on the basis of using the generalized Kanai-Tajimi model to include the non-stationary evaluation of amplitude and dominant frequency of ground motion and properties of wavelet transform. The study considers four Iranian earthquakes with different characteristics, namely Naghan 1977, Tabas 1978, Manjil 1990 and Bam 2003. It is shown that the time characteristics and the response spectra obtained from of generated accelerograms are similar to that obtained from original recorded accelerograms. The efficiency and applicability of the model have been studied using comparison of the obtained results from actual and simulated records. Moreover, the proposed model has been compared with the presented model by Rofooei *et al.* (2001). The results revealed that the obtained results by proposed model in this paper are closer to actual records compared to Rofooei model.

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Notation

A_j	: Approximation at level j
a	: Scale parameter
$a_g(t)$	: Ground acceleration
b	: Translation parameter
C_0	: Constant
$C(a, b)$	: Continuous wavelet transform coefficients
cA	: Scaling coefficients
cD	: Wavelet coefficients
D_j	: Detail at level j
$\delta()$	: Dirac delta function
E	: Earthquake energy input
$E[]$	: value
$e(t)$	: Amplitude envelope function
f	: Frequency
$n(t)$	: A stationary Gaussian white noise process
ξ_g	: Site dominant damping coefficient
$\xi_g(t)$	: Effective ground damping coefficient
PSA	: Pseudo-acceleration response spectrum
G	: Spectral density function
G_0	: Constant power spectral intensity of bedrock excitation
σ	: Standard deviation
$\sigma_a(t)$	: A smooth algebraic time function is fitted to the time variation of the Standard deviation
T	: Period
t	: Time
t_w	: Time-window size
$\phi(t)$	: Scaling function
X_f	: Filtered response
$\ddot{X}_g$	: Output ground acceleration
y	: Basis function
ψ^*	: Complex conjugate of ψ
ω	: Frequency
ω_g	: Site dominant frequency
$\omega_g(t)$	: Time dependent ground frequency