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Vibration analysis of asymmetric shear wall and thin walled open section structures using transfer matrix method

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Abstract. A method for vibration analysis of asymmetric shear wall and Thin walled open section structures is presented in this paper. The whole structure is idealized as an equivalent bending-warping torsion beam in this method. The governing differential equations of equivalent bending-warping torsion beam are formulated using continuum approach and posed in the form of simple storey transfer matrix. By using the storey transfer matrices and point transfer matrices which consider the inertial forces, system transfer matrix is obtained. Natural frequencies can be calculated by applying the boundary conditions. The structural properties of building may change in the proposed method. A numerical example has been solved at the end of study by a program written in MATLAB to verify the presented method. The results of this example display the agreement between the proposed method and the other valid method given in literature.

Keywords: vibration; asymmetric; wall; thin walled; transfer matrix.

1. Introduction

Number of methods, such as finite element method, has been developed for analyses of buildings. The continuum model is very simple and efficient method used in static and dynamic analysis of shear wall-frame buildings.

There are numerous studies (Rosman 1964, Heidebrecht and Stafford Smith 1973, Basu *et al.* 1979, Bilyap 1979, Balendra *et al.* 1984, Stafford Smith and Crowe 1986, Nollet and Stafford Smith 1993, Zalka 1994, Li and Choo 1996, Toutanji 1997, Miranda 1999, Mancini and Savassi 1999, Hoenderkamp 2000, 2001, 2002, Kuang and Ng 2000, 2008, Wang *et al.* 2000, Swaddiwudhipong *et al.* 2001, Zalka 2001, 2003, Miranda and Reyes 2002, Zalka 2002, Potzta and Kollar 2003, Tarjan and Kollar 2004, Savassi and Mancini 2004, Boutin *et al.* 2005, Miranda and Taghavi 2005, Reinoso and Miranda 2005, Georurgoussis 2006, Michel *et al.* 2006, Rafezy *et al.* 2007, Kaviani *et al.* 2008, Laier 2008, Meftah and Tounsi 2008, Savassi and Mancini 2008, Zalka 2008, Rafezy and Howson 2008, Bozdogan and Ozturk 2008, Bozdogan 2008) in the literature regarding continuum method.

Rosman (1964) proposed a continuum medium method for a pair of high rise coupled shear walls. Heidebrecht and Stafford Smith (1973) derived the differential equations of system for buildings with uniform stiffness along the height and then obtained closed-form solutions to uniform and

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triangular static lateral load distributions.

Zalka (2001) derived simplified expressions for the circular frequency of wall-frame buildings. Kuang and Ng (2000) considered the problem of doubly asymmetric structures; in which the motion is dominated by shear walls. For the analysis, the structure was replaced by an equivalent uniform cantilever whose deformation was coupled in flexure and warping torsion. An approximation method for estimating floor acceleration demands in multistory buildings subjected to earthquake ground motions has been developed in a recent study by Miranda and Taghavi (2005). The dynamic properties of multistory buildings were approximated by using equivalent continuum model consisting of a flexural cantilever beam and a shear cantilever beam that were assumed to be connected by an infinite number of axially rigid members in the proposed method. The dimensionless parameter, which controls the degree of overall flexural and overall shear deformations, was presented in the simplified model of buildings. In a companion paper (Taghavi and Miranda 2005), the accuracy of the methodology was evaluated by comparing the results of the approximation method with the computed response by using detailed finite element analyses for the case of the two generic buildings; and then the results were compared to recorded accelerations for the case of the four instrumented buildings.

Rafezy and Howson (2008) proposed a global approach to the calculation of natural frequencies of doubly asymmetric, three dimensional, multi bay, and multi storey frame structures. It was assumed that the primary frames of the original structure ran in two original directions and that their properties may have varied in a step-wise fashion at one or more storey levels. The structure therefore divided naturally into uniform segments according to changes in section properties.

A typical segment was then replaced by an equivalent shear-flexure-torsion coupled beam; whose governing differential equations were formulated by using continuum approach and posed in the form of a dynamic member stiffness matrix.

Kuang and Ng (2008) derived the governing equation and the corresponding eigenvalue problem of asymmetric frame structures using continuum assumption. A theoretical method of solution was proposed and a general solution to the eigenvalue equation of the problem was presented for determining the coupled natural frequencies and associated mode shapes based on the theory of differential equations.

Bozdogan (2008) proposed the Transfer Matrix method for lateral static and dynamic analyses of wall-frame buildings. Step changes of properties along the height of the structure were allowed in none of the studies with the exception of Rafezy and Howson's and Bozdogan's papers.

A method for vibration analysis of non uniform asymmetric shear wall and thin walled open structures is suggested in this study. The following assumptions are made in this study; the behavior of the material is linear elastic, small displacement theory is valid, P-delta effects are negligible, the flexural rigidity center at each floor thus lies on vertical line through the height of structures, the shear deformations of walls are negligible, the storey mass acts on the storey (floor) level and the floor system is rigid in its plane.

2. Analysis

2.1 Transfer matrix method

The computations become more tedious and the possibility of making errors increases as the number of constants to be determined by the use of boundary conditions increases in various engineering problems. Therefore, ways of reducing the number of constants to a minimum are sought and the method of transfer matrix method makes this possible. The main principle of this theory, which is applied to problems with one variable, is to convert all the boundary value problems into problems of initial values. Thus new constants that may result from the use of intermediate condition are eliminated. Therefore, it can be stated as a method of expressing the equations in terms of the initial constants and this method makes no distinction between the so called determinate and indeterminate problems of elastomechanics (Inan 1968). Transfer matrix method is an efficient and easily computerized method and it also provides a fast and practical solution since the dimensions of the matrix for elements and system never changes (Pestel and Leckie 1963).

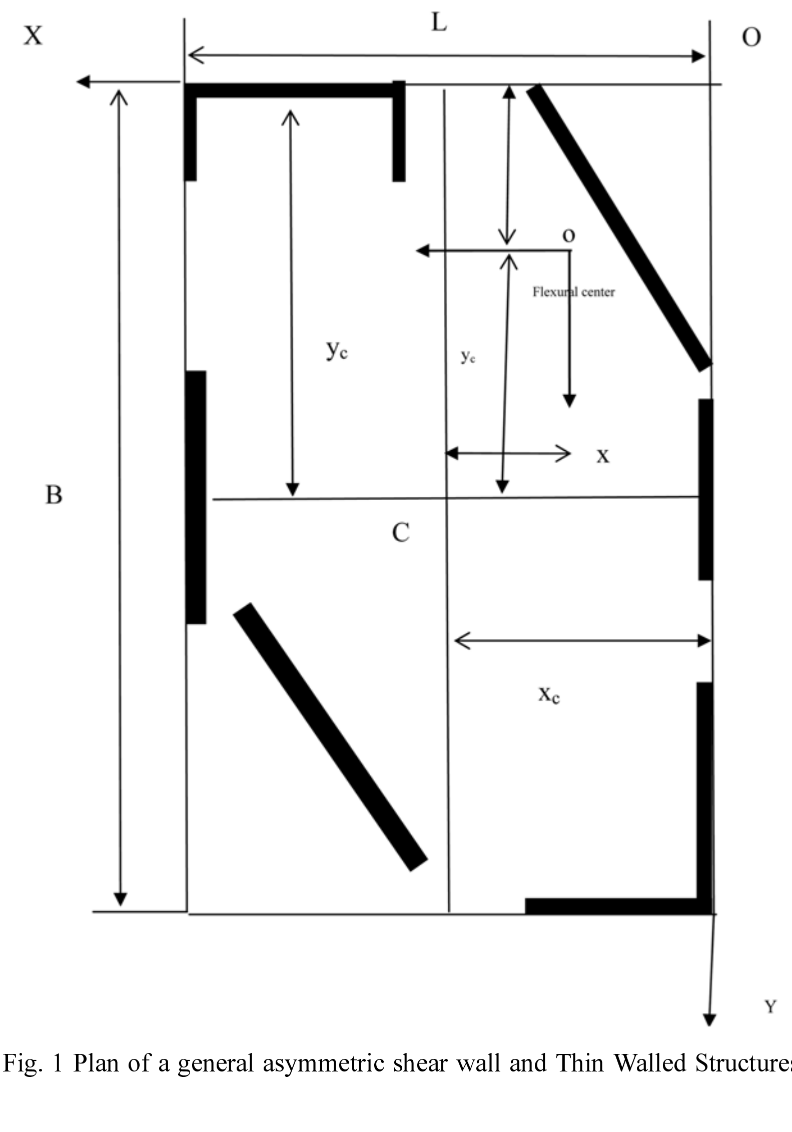

Fig. 1 Plan of a general asymmetric shear wall and Thin Walled Structures

2.2 Physical model

Fig. 1 shows a typical floor plan of asymmetric, three dimensional shear wall structures (Kuang and Ng 2000). Shear wall structures ignoring shear deformations, demonstrate Bending-warping torsional beam behavior. The differential equation of this equivalent bending-warping torsional beam can be initially written.

2.3 Storey transfer matrices

Under the horizontal loads governing equations of i.th storey can be written as

$$(EI)_{xi}\frac{d^4u_i}{dz_i^4} + (EI)_{xyi}\frac{d^4v_i}{dz_i^4} = 0 \tag{1}$$

$$(EI)_{yi}\frac{d^4v_i}{dz_i^4} + (EI)_{xyi}\frac{d^4u_i}{dz_i^4} = 0 \tag{2}$$

$$(EI)_{wi}\frac{d^4\theta_i}{dz_i^4} = 0 \tag{3}$$

where u_i and v_i are the lateral deflections of the flexural center, respectively, θ_i is the torsional rotation of the floor plan about flexural rigidity at the given height, and z_i is the vertical axis of each storey.

$(EI)_{xi}$, $(EI)_{yi}$ and $(EI)_{xyi}$ are the equivalent flexural rigidity of the storey for wall structures in x, y and xy directions and can be calculated as follows (Kuang and Ng 2000, Rafezy and Howson 2008)

$$EI_{yi} = \sum_j EI_{yi,j}, \quad EI_{xi} = \sum_j EI_{xi,j}, \quad EI_{xi} = \sum_j EI_{xyi,j} \tag{4}$$

$(EI)_{wi}$ are the warping stiffness of i.th storey and can be calculates as follows (Kuang and Ng 2000)

$$(EI)_{wi} = \sum_j [(\bar{y}_j - \bar{y}_c)^2(EI)_{xi,j} + (\bar{x}_j - \bar{x}_c)^2(EI)_{yi,j}] + \sum_c (EI)_{woc} \tag{5}$$

where $\bar{y}_j$ and $\bar{x}_j$ are the coordinates at the location of the center of flexural rigidity of the j-th bent at i-th storey in coordinate system $(\bar{y}_j, \bar{x}_j)$ and EI_{woc} is the warping torsional stiffness of a core about its own shear centre.

$\bar{y}_c$ and $\bar{x}_c$ are the coordinate of shear center and can be calculated as follows (Kuang and Ng 2000)

$$\bar{y}_c = \frac{\sum_j \bar{y}_j(EI)_{xj}}{\sum_j (EI)_{xj}} \tag{6}$$

$$\bar{x}_c = \frac{\sum_j \bar{x}_j(EI)_{yj}}{\sum_j (EI)_{yj}} \tag{7}$$

When Eqs. (1), (2) and (3) are solved with respect to the z, $u_i(z)$ and $v_i(z)$ and $\theta_i(z)$ can be obtained as follows

$$u_i(z_i) = c_4 z_i^3 + c_3 z_i^2 + c_2 z_i + c_1 \tag{8}$$

$$v_i(z_i) = c_8 z_i^3 + c_7 z_i^2 + c_6 z_i + c_5 \tag{9}$$

$$\theta_i(z_i) = c_{12} z_i^3 + c_{11} z_i^2 + c_{10} z_i + c_9 \tag{10}$$

where c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12} are integral constants.

By using Eqs. (8), (9) and (10), the rotation angles in x and y direction (u_i', v_i'), the rate of twist (θ_i'), Bending Moments in x and y directions (M_{xi}, M_{yi}) and bi-moment (M_{wi}), Shear forces in x and y direction (V_{xi}, V_{yi}) and torque (T_i) for i.th storey can be obtained as follows

$$u_i'(z_i) = 3c_4 z_i^2 + 2c_3 z_i + c_2 \tag{11}$$

$$v_i'(z_i) = 3c_8 z_i^2 + 2c_7 z_i + c_3 \tag{12}$$

$$\theta_i'(z_i) = 3c_{12} z_i^2 + 2c_{11} z_i + c_{10} \tag{13}$$

$$M_{xi}(z_i) = (EI)_{xi}\frac{d^2 u_i}{dz_i^2} + (EI)_{xyi}\frac{d^2 v_i}{dz_i^2} = [(EI)_{xi}(2c_3 + 6c_4 z_i) + (EI)_{xyi}(2c_7 + 6c_8 z_i)] \tag{14}$$

$$M_{yi}(z_i) = (EI)_{yi}\frac{d^2 v_i}{dz_i^2} + (EI)_{xyi}\frac{d^2 u_i}{dz_i^2} = [(EI)_{yi}(2c_7 + 6c_8 z_i) + (EI)_{xyi}(2c_3 + 6c_4 z_i)] \tag{15}$$

$$M_{wi}(z_i) = (EI)_{wi}\frac{d^2 \theta_i}{dz_i^2} = (EI)_{wi}(2c_{11} + 6c_{12} z_i) \tag{16}$$

$$V_{xi}(z_i) = (EI)_{xi}\frac{d^3 u_i}{dz_i^3} + (EI)_{xyi}\frac{d^3 v_i}{dz_i^3} = (EI)_{xi}6c_4 + (EI)_{xyi}6c_8 \tag{17}$$

$$V_{yi}(z_i) = (EI)_{yi}\frac{d^3 v_i}{dz_i^3} + (EI)_{xyi}\frac{d^3 u_i}{dz_i^3} = (EI)_{yi}6c_8 + (EI)_{xyi}6c_4 \tag{18}$$

$$T_i(z_i) = (EI)_{wi}\frac{d^3 \theta_i}{dz_i^3} = (EI)_{wi}(6c_{12}) \tag{19}$$

Eq. (20) shows the matrix form of Eqs. (8)-(19)

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ u_i'(z_i) \\ v_i'(z_i) \\ \theta_i'(z_i) \\ M_{xi}(z_i) \\ M_{yi}(z_i) \\ M_{wi}(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ T_i(z_i) \end{bmatrix} = \begin{bmatrix} 1 & z_i & z_i^2 & z_i^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & z_i & z_i^2 & z_i^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & z_i & z_i^2 & z_i^3 \\ 0 & 1 & 2z_i & 3z_i^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2z_i & 3z_i^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2z_i & 3z_i^2 \\ 0 & 0 & 2EI_{xi} & 6zEI_x & 0 & 0 & 2EI_{xyi} & 6zEI_{xyi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 2EI_{xyi} & 6z_iEI_{xyi} & 0 & 0 & 2EI_{yi} & 6z_iEI_{yi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2EI_{wi} & 6z_iEI_{wi} \\ 0 & 0 & 0 & 6EI_{xi} & 0 & 0 & 0 & 6EI_{xyi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6EI_{xyi} & 0 & 0 & 0 & 6EI_{yi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6EI_{wi} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} = A_i(z_i) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \tag{20}$$

At the initial point of the storey for $z_i = 0$, Eq. (20) can be written as;

$$\begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} = A_i(0) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \end{bmatrix} \tag{21}$$

The vector in right-hand side of Eq. (21) can be shown as follows

$$c = [c_1 \;\; c_2 \;\; c_3 \;\; \dots \;\; c_{11} \;\; c_{12}]^t \tag{22}$$

When vector c is solved by implementing Eq. (21) and substituted in Eq. (20), then Eq. (23) would be obtained.

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ u_i'(z_i) \\ v_i'(z_i) \\ \theta_i'(z_i) \\ M_{xi}(z_i) \\ M_{yi}(z_i) \\ M_{wi}(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ T_i(z_i) \end{bmatrix} = A_i(z_i)A_i(0)^{-1} \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} = T_i(z_i) \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} \tag{23}$$

T_i represents the storey transfer matrix for $z = h_i$ in Eq. (23).

The storey transfer matrices obtained from Eq. (23) can be used for the dynamic analysis of asymmetric shear wall and thin walled open structures. Therefore, when considering the inertial forces in the storey levels, the relationship between the ith and the (i+1)th stories can be shown by the following matrix equation

$$\begin{bmatrix} u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ u_i'(h_i) \\ v_i'(h_i) \\ \theta_i'(h_i) \\ M_{xi}(h_i) \\ M_{yi}(h_i) \\ M_{wi}(h_i) \\ V_{xi}(h_i) \\ V_{yi}(h_i) \\ T_i(h_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \omega^2 m_i & 0 & -\omega^2 m_i y_c & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \omega^2 m_i & \omega^2 m_i x_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\omega^2 m_i y_c & \omega^2 m_i x_c & \omega^2 m_i r_m^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} T_i(h_i) \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ u_i'(0) \\ v_i'(0) \\ \theta_i'(0) \\ M_{xi}(0) \\ M_{yi}(0) \\ M_{wi}(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ T_i(0) \end{bmatrix} \tag{24}$$

where, m_i is the mass of the ith storey and ω are the natural frequencies of the system and r_m^2 is the inertial radius of gyration; and can be calculated as (Kuang and Ng 2000, Rafezy and Howson 2008)

$$r_m^2 = \frac{L^2 + B^2}{12} + y_c^2 + x_c^2 \tag{25}$$

y_c and x_c are the dimensions of the location of the geometric center and can be calculated as follows

$$y_c = \bar{y}_c - \bar{y}_s \tag{26}$$

$$x_c = \bar{x}_c - \bar{x}_s \tag{27}$$

where the coordinate $(\bar{y}_c, \bar{x}_c)$ is the location of the geometric center C in the coordinate system $(\bar{y}, \bar{x})$.

Dynamic transfer matrix can be shown as T_{di}.

$$T_{di} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \omega^2 m_i & 0 & -\omega^2 m_i y_c & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \omega^2 m_i & \omega^2 m_i x_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -\omega^2 m_i y_c & \omega^2 m_i x_c & \omega^2 m_i r_m^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} T_i \tag{28}$$

The displacements - internal forces relationship between the base and the top of the structure can be found as follows

$$\begin{bmatrix} u_{top} \\ v_{top} \\ \theta_{top} \\ u'_{top} \\ v'_{top} \\ \theta'_{top} \\ M_{xtop} \\ M_{ytop} \\ M_{wtop} \\ V_{xtop} \\ V_{ytop} \\ T_{top} \end{bmatrix} = T_{dn} * T_{d(n-1)} * \ldots T_{di} * \ldots T_{d2} * T_{d1} \begin{bmatrix} u_{base} \\ v_{base} \\ \theta_{base} \\ u'_{base} \\ v'_{base} \\ \theta'_{base} \\ M_{xbase} \\ M_{ybase} \\ M_{wbase} \\ V_{xbase} \\ V_{ybase} \\ T_{base} \end{bmatrix} \tag{29}$$

The boundary conditions of the bending- warping beam are

1) $u_{base} = 0$ 2) $v_{base} = 0$ 3) $\theta_{base} = 0$ 4) $u'_{base} = 0$ 5) $v'_{base} = 0$ 6) $\theta'_{base} = 0$
7) $M_{xtop} = 0$ 8) $M_{ytop} = 0$ 9) $M_{wtop} = 0$ 10) $V_{xtop} = 0$ 11) $V_{ytop} = 0$ 12) $T_{top} = 0$

When boundary conditions are considered for Eq. (29) for the nontrivial solution of $t_d = T_{dn}T_{dn-1}T_{dn-2}\dots T_{d1}$, Eq. (30) can be attained

$$f = \begin{bmatrix} t(7,7) & t(7,8) & t(7,9) & t(7,10) & t(7,11) & t(7,12) \\ t(8,7) & t(8,8) & t(8,9) & t(8,10) & t(8,11) & t(8,12) \\ t(9,7) & t(9,8) & t(9,9) & t(9,10) & t(9,11) & t(9,12) \\ t(10,7) & t(10,8) & t(10,9) & t(10,10) & t(10,11) & t(10,12) \\ t(11,7) & t(11,8) & t(11,9) & t(11,10) & t(11,11) & t(11,12) \\ t(12,7) & t(12,8) & t(12,9) & t(12,10) & t(12,11) & t(12,12) \end{bmatrix} \tag{30}$$

The values of ω, which set the determinant to zero, are natural frequencies of the asymmetric wall building.

3. Process of computation

The process of the computation for transfer matrix method is presented step by step as below:
1) The equivalent rigidities of each storey are calculated by using the geometric and material properties of the structure.
2) Storey transfer matrices are calculated for each storey by using equivalent rigidities.
3) System transfer matrix (Eq. (29)) is obtained with the help of storey transfer matrices and inertia forces effecting to the storey levels with the procedure told in section 3.
4) The nontrivial equation is obtained by using Eq. (30) as a result of the application of the boundary conditions.
5) The angular frequencies and relevant periods are found with the help of a method obtained from numerical analysis.
6) The modes are found with the help of angular frequency and Eq. (24).
7) The effective mass ratio and participation factor is found by using the modes.
8) With the help of the acceleration and displacement spectrums, obtained from an earthquake record or design spectrum from codes, the displacement and internal forces are found by using effective mass and participation factor.

4. Numerical example

A numerical example has been solved by a program written in MATLAB to verify the presented method in this part of the study. The results are then compared with those given in the literature.

Example 1.

A typical asymmetric building braced by shear-walls and angle type thin- walled open section structure (Fig. 2) (Meftah and Tounsi 2008) is analyzed as an example. The structure has 25 storeys with total height H = 75 m, and floor dimensions L = 24 m and B = 24 m. The thickness of of the shear wall and thin walled angle cross sections ia 0.3 m. An elastic modulus E = 25000 MN/m^2, the weight per unit volume of building is 25 kN/m^2 and the mass density per unit length is m = 106.056

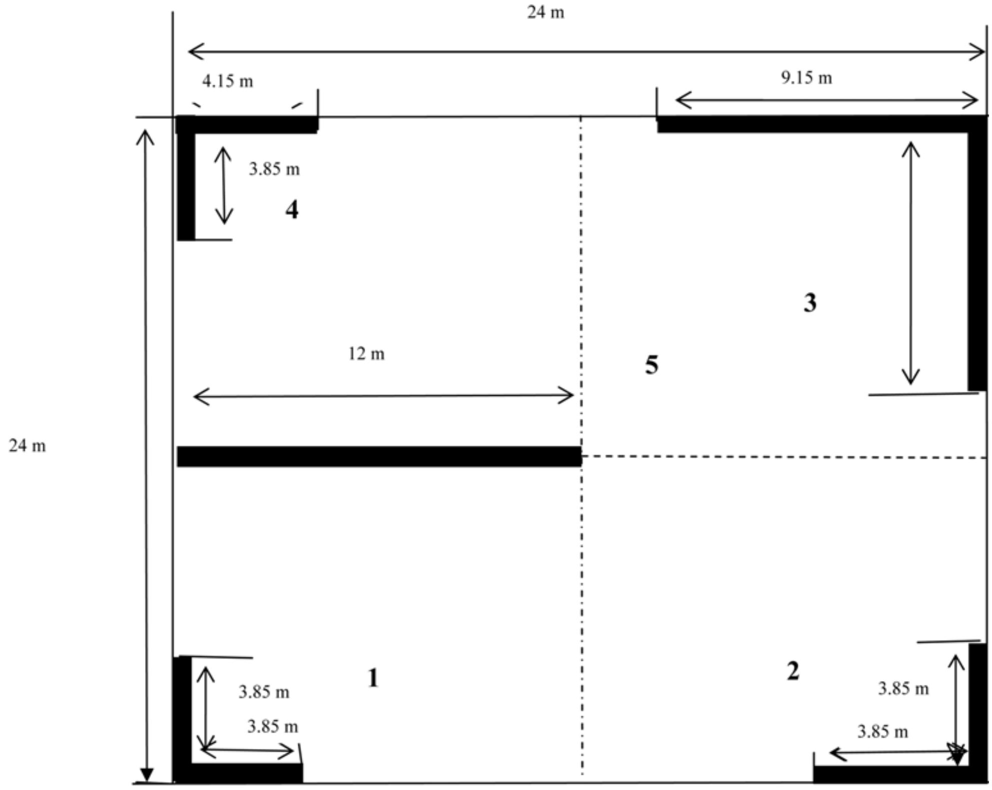

Fig. 2 A typical asymmetric building braced by shear-walls and angle type thin- walled open section structure (Meftah and Tounsi 2008)

Table 1 Structural Properties of Asymmetric Shear Wall and Angle Typed thin Walled Structures

Structural Properties of Asymmetric Shear Wall and Angle Typed Thin Walled Building	
$(EI)_x$	$2521.125*10^3$ MNm^2
$(EI)_y$	$1441.8*10^3$ MNm^2
$(EI)_{xy}$	623.2*MNm^2
$(EI)_w$	$221983*10^3$ MNm^4
x_c	6.94 m
y_c	3.971 m
m	324.33 kNm/sn^2
r_m	12.646 m^2

t/m. The structural properties are given in Table 1. The natural frequencies calculated by this method are compared with the results in the reference (Meftah and Tounsi 2008). The results are presented in Table 2.

Table 2 Comparison of natural frequencies in Example 1

Mode	Natural frequencies of the first three modes (s^{-1})								
	Proposed Method			Meftah and Tunsi (2008)			Finite element method (Meftah and Tounsi 2008)		
	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
1	1.788	2.406	4.365	1.822	2.388	4.451	1.746	2.622	5.646
2	11.187	15.053	27.306	11.419	14.939	27.894	10.430	15.517	27.423
3	31.269	42.077	61.161	31.977	41.919	78.112	30.714	40.275	72.485

5. Conclusions

This paper presents a method for vibration analysis of asymmetric shear wall and thin walled open structures. The whole structure is idealized as an equivalent bending-warping torsion beam in this method. The governing differential equations of equivalent bending-warping torsion beam are formulated using continuum approach and posed in the form of simple storey transfer matrix. By using the storey transfer matrices and point transfer matrices which consider the inertial forces, system transfer matrix is obtained. Natural frequencies can be calculated by applying the boundary conditions. Example solved in this study shows that results obtained from the proposed method are in close agreement with the solution which was developed in literature. The structural properties of building may change in the proposed method and different numerical examples can also be solved. The proposed method is simple and accurate enough to be used both at the concept design stage and for final analyses.

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