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Nonlinear analysis of the influence of increments amounts and history load on soil response

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Abstract. The soil response calculation is described, by which, threw the fictive path of stress, the stress-deformation diagrams are determined, considering the nonlinear soil behavior. The calculation are lead incrementally, by which is shown that in the presented soil model (modified Cam Clay), considering the influence of overconsolidated soil pressure OCR, the number of calculation steps may, but not necessarily, have a sufficient influence on the value of failure load and definite soil deformation. The simplicity and the practicalness of the procedure, the enables modeling the complex relations in soil.

Keywords: nonlinearity; average normal stress; deviation stress; plasticity; increment; calculation; volume and deviation deformation; modified Cam clay model.

1. Introduction

By the plasticity theory (Chen and Han 1988), the ideal elastic-plastic material is defined, also as the material with increase (hardening) or decrease (softening) of stress by reaching the liquid limit, with no consider if the issue is a linear or nonlinear law (Kanvinde *et al.* 2001).

The total material deformation is consisted of elastic and plastic components

$$\varepsilon = \varepsilon_e + \varepsilon_p \tag{1}$$

The elastic deformation component may be computed by the temporary amounts of stress and materials deformation module

$$\varepsilon_e = \frac{\sigma}{E} \tag{2}$$

The value and direction of the elastic deformation component εp depends on the criteria of relief, the defined relief surface f, also as the acquired law of material behavior. If the observed condition of stress is in the relief surface, only elastic deformations take place. The material in that area is idealistically elastic and the Hooks law is valid. By progressively increasing the load, the stress condition reaches the relief surface and generation of elastic and plastic deformations take place at the same time. If it is assumed that the direction of the flow law is defined with the vector h, then the following term for the incremental value of plastic deformation may be wrote

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$$d\varepsilon^{p} = d\lambda h \tag{3}$$

λ is a scalar size and defines the scalar value of material plastic deformation, and is determined from the stress condition at the relief surface. The h vector is a stress condition function and is defined as a gradient of plastic potential scalar function g, so one may be wrote

$$h = \frac{\partial g}{\partial \sigma} \tag{4}$$

The direction of plastic flow is always perpendicular to the surface of plastic potential, defined by plastic potential function g. The classic approach of the scalar field g determination, for ideal elastic-plastic materials, is the assumption that the plastic flow takes place in a direction that causes maximum work dissipation due to the activation of plastic deformations (plastic work). The mentioned work component is the irreversible part of total work developed by the cycles of load and relief.

The result of this assumption is that the vector of incremental plastic deformation must be perpendicular to the incremental stress vector. The stress condition is defined at the relief surface, which means that the incremental stress vector is in fact the tangent to the relief surface.

By other words, the vector of incremental plastic deformation is perpendicular to the relief surface, and in that case, the plastic potential function is also the function of material relief, that is, $g = f$. This kind of liquid law, where $g = f$, is called the associative liquid law. In the case when $g \neq f$, that is an no associative liquid law. For the ideal-plastic materials the associative liquid law maximizes the work developed on plastic deformations, while for the materials with increasing (hardening) or decreasing (softening) that can, but doesn't necessarily need to be in case.

2. The critical state model (CAM CLAY MODEL)

The Mohr-Coulomb law defines failure only threw the state of stress in the soil, not considering the value of deformation at failure. Threw the model of critical state (Atkinson and Bransby 1978, Wood 1990), it is possible to define the deformations in soil at soil failure, also as defining the real diagrams of stress-deformation.

For the stress state in the device for three-axis shear ($\sigma_2' = \sigma_3'$) the value of the middle normal effective stress p', and stress deviator q the following may be wrote

$$p' = \frac{1}{3}(\sigma_1' + 2\sigma_3') \tag{5}$$

$$q = \sigma_1' - \sigma_3' \tag{6}$$

The suitable deformations (volume ε_p and deviation ε_p) are determined from the condition of equality of total work, in no dependence of the choice of stress invariants

$$\varepsilon_p = \varepsilon_1 + 2\varepsilon_3 \tag{7}$$

$$\varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \tag{8}$$

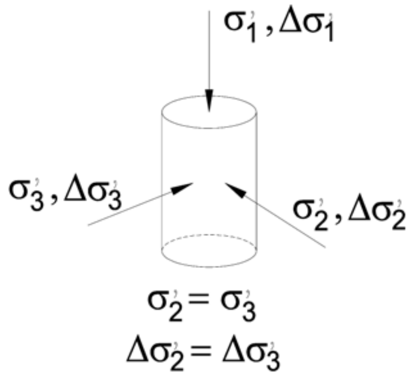

Fig. 1 State of stress on a sample in the device for three-axis shear

The relation between q i p' threw the parameters of Mohr-Coulombs model c and φ

$$q = \frac{6\sin\varphi'}{3-\sin\varphi'}p' + \frac{6\cos\varphi'}{3-\sin\varphi'}c \tag{9}$$

The term (9) presents the Mohr-Coulombs law in $q - p'$ plane. If the influence of cohesion is neglected, then the following term may be wrote

$$q = \frac{6\sin\varphi'}{3-\sin\varphi'}p' = Mp' \tag{10}$$

The term (10) defines the line of critical state (LKS) in $q - p'$ plane, that defines the state of soil failure. In the plane e-lnp' the line of critical state has a gradient λ and is parallel to the line of isotropic compression (LIK) (Fig. 2), where is e-pore coefficient. The line of isotropic relief, also as the repeated load in the plane e-lnp' has a gradient κ.

For the total incremental volume and deviation deformation (sum of elastic and plastic component) due to increment activeness of average main stress $\Delta p'$, and stress deviator Δq, the following may be wrote

$$\Delta\varepsilon_p = \Delta\varepsilon_p^e + \Delta\varepsilon_p^p \quad \text{total volume deformation} \tag{11}$$

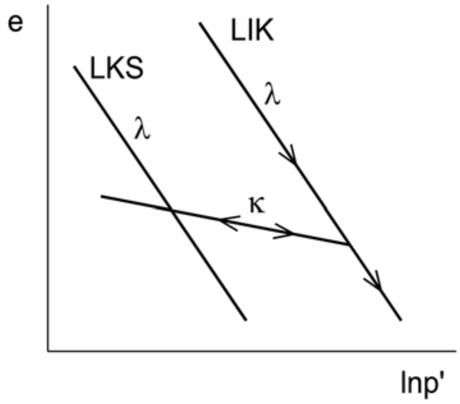

Fig. 2 The lines of critical states (LKS) and isotropic load (LIK), load removal and repeated load

$$\Delta\varepsilon_q = \Delta\varepsilon_q^e + \Delta\varepsilon_q^p \quad \text{total deviation deformation} \tag{12}$$

In undrained conditions the total volume change is zero, which means that for a volume deformation the following term is valid

$$\Delta\varepsilon_p^e = -\Delta\varepsilon_p^p \tag{13}$$

The deviator component of total incremental deformation will be determined by acquire the relief plane, and the associative liquid law. In that case the total increment of plastic deformation is perpendicular to the relief plane. In the modified Cam Clay model (Wood 1990) the relief plane is set by the following term

$$f = (p')^2 - p'p_c' + \frac{q^2}{M^2} = 0 \tag{14}$$

where p_c' over-consolidated stress, and the other marks according to the previous terms.

After deriving the term (14), for the load increments $\Delta p'$ i Δq term (15) may be wrote in a differential mode for a normal on the relief plane

$$\frac{\Delta p'}{\Delta q} = \frac{q}{M^2(p' - p_c'/2)} \tag{15}$$

If the term (15) is wrote in a function of plastic deformation increments (Fig. 3) then the term for plastic deviator deformation increment is

$$\Delta\varepsilon_q^p = \Delta\varepsilon_p^p \frac{q}{M^2(p' - p_c'/2)} \tag{16}$$

That way, by the known plastic volume deformation increment, and temporarily stress condition, the plastic incremental component of deviator deformation is gained.

The elastic component of deviator deformation is gathered from the following term using Hooks law for valid invariants of stress and deformation

$$\Delta\varepsilon_q^e = \frac{\Delta q}{3G} \tag{17}$$

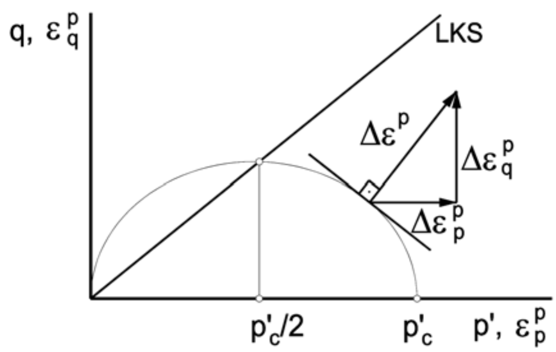

Fig. 3 The direction of total plastic deformation increment

2.1 Consolidated Isotropic Drain test (CID)

The initial elastic deformation, that is also the total deformation for the initial load increment, develops till reaching the relief ellipse

$$\Delta\varepsilon_{pi}^{e} = \frac{1}{1+e_0}\kappa\ln\frac{p_1'}{p_0'} \tag{18}$$

The line at gradient 3:1 determines the variation q in function of p' in $q - p'$ plane, and represents a mutual line of total and effective stresses. The suitable values of volume deformation, in differ from the term in (Budhu 2000) for the first increment are (according to the marks on Fig. 4)

$$\Delta\varepsilon_{p1} = \frac{1}{1+e_1}\left(\kappa\ln\frac{p_{c1}'}{p_1'} + \lambda\ln\frac{p_{c2}'}{p_{c1}'} - \kappa\ln\frac{p_{c2}'}{p_2'}\right) \tag{19}$$

$$\Delta\varepsilon_{p1}^{e} = \frac{1}{1+e_1}\kappa\ln\frac{p_2'}{p_1'} \tag{20}$$

$$\Delta\varepsilon_{p1}^{p} = \Delta\varepsilon_{p1} - \Delta\varepsilon_{p1}^{e} \tag{21}$$

where $e_1 = e_0 - \Delta\varepsilon_{pi}^{e}(1+e_0)$

The total volume deformation for the first load increment is now

$$\varepsilon_{p1} = \Delta\varepsilon_{pi}^{e} + \Delta\varepsilon_{p1} \tag{22}$$

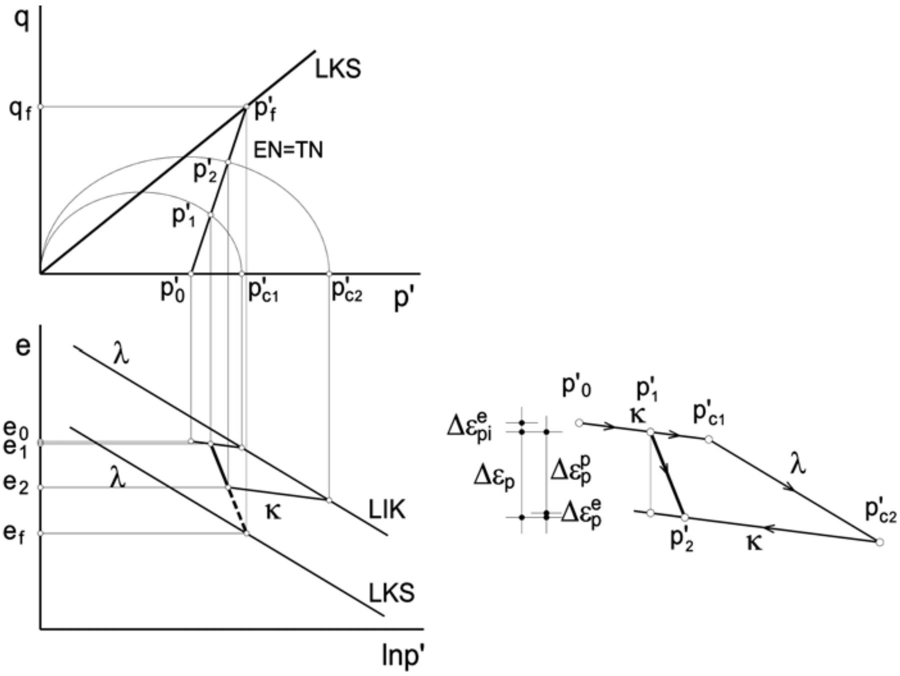

Fig. 4 Volume deformation calculation in CID test

When all components of incremental volume deformations are known, one is able to, threw the terms (16) i (17) determine the components of deviator incremental deformations. The total deviator deformation for the first step is

$$\varepsilon_{q1} = \Delta\varepsilon_{qi}^{e} + \Delta\varepsilon_{q1} \tag{23}$$

The components, also as the total values of volume and deviator deformations, for all load increments to failure, are calculated according to the proportional analogy with the shown terms. The real load increment is calculated threw suitable stresses. The direction of incremental volume load is gained by separating to the direction of isotropic load and relief with known values of average stresses.

By progressive load increase one reaches the failure of average stresses p'_f, with a suitable value of porosity factor, that is, the final failure deformation. In that way the real value of variation of the total volume deformation till failure is calculated.

2.2 Consolidated undrain test (CIU)

At the CIU test there is no variation of volume while applying load, which means that the change of total deformation is preformed only by the deviator components. The line of total stresses no longer matches with the line of effective stresses, as it was in CID test.

A failure occurs by progressive decreasing the average normal stress p' all to the failure values p'_f , with increasing the stress deviator q_f to failure, which is shown on Fig. 5. At the CID test the

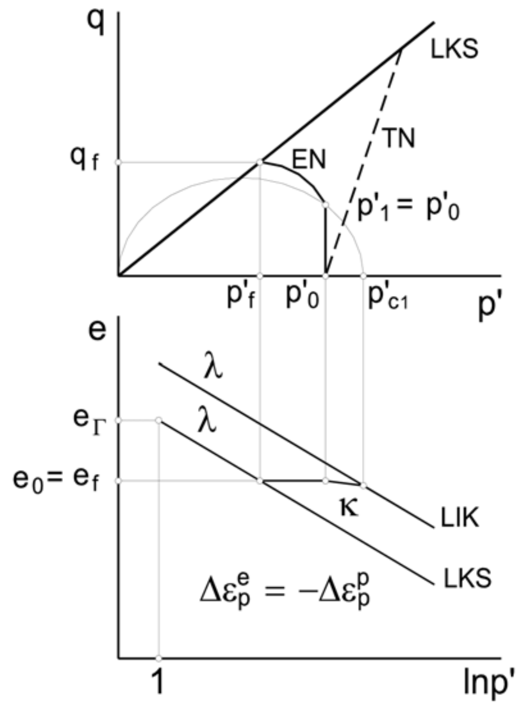

Fig. 5 The stress and deformation path in the CIU test

variation of volume is characteristic, while at the CIU test the variation of pore pressure value while load applying is competent.

The average failure stress p'_f is determined from the term

$$p'_f = e^{\frac{e_\Gamma - e_0}{\lambda}} \tag{24}$$

where e_Γ is the value of the porosity coefficient on the critical state line in the e-lnp' plane for $p'_f = 1$ and equals

$$e_\Gamma = e_0 + (\lambda - \kappa) ln\frac{p'_c}{2} + \kappa ln p'_0 \tag{25}$$

When the initial value of average stress p'_0 is known, its possible, with the chosen value of increment increase $\Delta p'$, to lead the calculation till the failure value p'_f . If the following term is generally

$$p'_{i+1} = p'_i - \Delta p' \tag{26}$$

And if p'_{ci} is an over-consolidated stress for p'_i , then it can be shown that for undrained conditions the following term for over-consolidated stress for p'_{ci+1} is valid

$$p'_{ci+1} = p'_{ci}\left(\frac{p'_i}{p'_{i+1}}\right)^{\frac{\kappa}{\lambda - \kappa}} \tag{27}$$

The value of stress deviator q for a known average normal stress is determined for the term (14) based on the condition stress state on the relief ellipse. From the term (20) the elastic volume deformation increment is determined, which is equal to the plastic volume deformation increment. When the components of incremental volume deformation are known, the incremental plastic deviator deformation is defined by using the term (16). The axial deformation equals the deviator deformation

$$\varepsilon_1 = \Delta\varepsilon^e_{qi} + \varepsilon_{q1} \tag{28}$$

The pore pressure is determined as a horizontal distance between the line of total stress at gradient 3:1 (TN) and the curve of effective stresses (EN).

$$p = p'_o + q/3 \tag{29}$$

$$\Delta u = p - p' \tag{30}$$

3. Numerical examples

Using the presented terms, an algorithm is made which enables an analysis of the soil response according to the critical condition model (modified Cam Clay) for a proizvoljna stress variation path.

A calculation is lead for normally and lightly consolidated soil samples, that is, for OCR ≤ 2 for undrained soil conditions, according to the accepted numerical model. Considering the values OCR ≤ 2, the curves without residual values of stress deviators are calculated, that is, failure occurs with the first reach of line critical conditions. For the drained conditions the limit value is OCR ≤ 6/(3-M), due to the line gradient (3:1) of total and effective stresses.

The calculation in both cases is lead close to value of relief limit (i.e., $0.99\times p_f$) due to numerical instability in the surround of the relief point, where generating movement with no load increase takes place. As a solution, one looks for a standard diagram of stress-deformation q-ε_1 where $\varepsilon_1 = \varepsilon_q + \varepsilon_p/3$, and the relation diagrams of axial deformation ε_1 and volume deformation ε_p or the axial deformation and pore pressure Δu relation, depending on that if the issue is drained or undrained state.

The calculation is lead for n = 2, 5, 10, 20, 50 i 100 steps, and for OCR = 1, 1.5, i 2.0, while for the undrained state also for OCR=2.0×3/(3-M) > 2, because it is neccesery M < 3. For example, it the following soil parameters are set, also as the values of initial pore coefficient and over-consolidated stress:

$$\lambda = 0.25, \quad \kappa = 0.05, \quad \varphi' = 28^{\circ}, \quad \nu = 0.3, \quad e_o = 0.85, \quad p_c' = 300 \text{ kN/m}^2$$

Fig. 6 shows the results of the deviation failure calculation for various calculation steps for OCR=1.5 in CID test. The relief for the minimum number of steps takes place at the failure value ≈60% of the value of failure deviator reached with the maximum number of steps. The failure deviator value is equal in al calculation steps, but the deformation values at smaller steps become unacceptably great by reaching the limit values of deviator stress, therefore they are compared with the failure deformation gathered with the maximum number of steps.

At the CIU test the influence of the initial elastic deformation (Fig. 7) for all analysed values for the number of calculation steps is notable. The initial value at the ellipse relief and failure stress deviator value don't differ much. The absolute value of the stress deviator is smaller comparing to the CID test, which is also for the deformation value, due in case for the deformation values, the absolute deformation is a result of the distortion component only (the volume component is zero).

In generally, with a greater value of initial elastic deformation comparing to the final deformation or analogy, with a greater ratio of the initial (at the first relief ellipse) and the final value of the

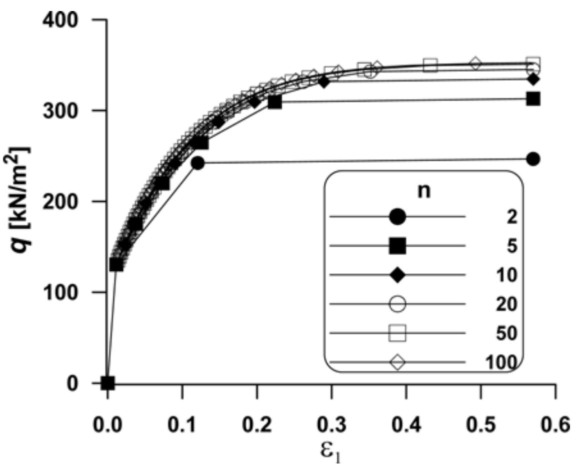

Fig. 6 Diagrams q-ε_1 for CID test and OCR=1.5, for various increments load amounts

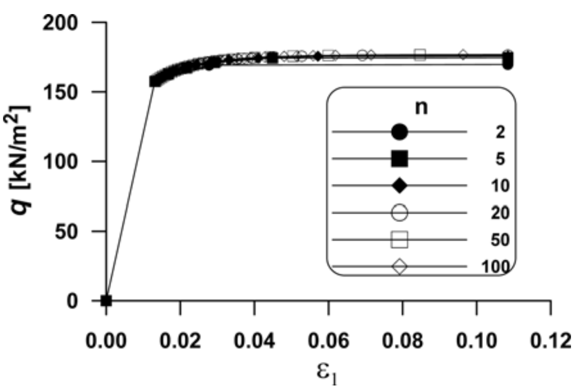

Fig. 7 The q-ε_1 diagrams for the CIU test and OCR=1.5, for different amounts of load increments

stress deviator, the influence of the number of calculation steps to the final result is smaller.

The position of the initial value of the relief deviator (intersection of the line 3:1 and the relief ellipse at the CID test or the intersection of the vertical line for the same initial value of average stress and relief ellipse for CIU test) depends on the OCR value.

The Fig. 8 shows the relation of the number of calculation steps and normalized value of failure deviator with a stress failure deviator for a maximum number of calculation steps (100 steps).

For the value of OCR=6/(3-M) at CID test the influence of the number of steps can be neglect. Even at the initial elastic deformation, a relief occurs, because the intersection of the initial relief ellipse is at the same time the value of failure stress deviator, therefore the sample acts as an ideal elastic-plastic. By gradual decrease of the OCR value, the increase deviation is notable. For the CID test, no matter of the OCR value, very good results may be gained with 20 steps (5% of the total load), where its notable that the results are in the 5% of final value.

At Fig. 9, similar as Fig. 8, the calculation results are shown, but for the CIU test. It's notable that there is a significantly lesser sensitivity of failure stress deviator at the number of steps and almost independently of the OCR values. The maximum deviation is 20%, for the minimum number of steps. The calculation with 20 load steps shows results with deviation in limits ≈2%.

The Fig. 10 shows curves q-ε_1 with 100 steps for various OCR for CID test. The decrease of maximum stress deviator values occur with smaller OCR, which is a result of sample tritrturation due to its isotropic relief (increase of pore coefficient), but only for a new load which is not isotropic. In case of a repeated isotropic load, the sample would be stiffer, because the pore coefficient change would occur on the line of isotropic relief and repeated load (gradient κ in e-lnp' diagram). For OCR=6/(3-M) it is notable that the sample acts as ideal elastic-platic.

It's similar to the CIU test (Fig. 11), but now with a significant smaller mutual difference in the final deviator stress values, with a curve form with rapid relief (closer to the ideal elastic-plastic diagram).

Fig. 12 shows the common view of normed failure stress deviators for CID and CIU tests. The deviation at the CIU test are significantly lesser comparing to the CID test and don't reach over 20% of the maximum OCR. The deviations at CID test are greater and they reach ≈70% comparing

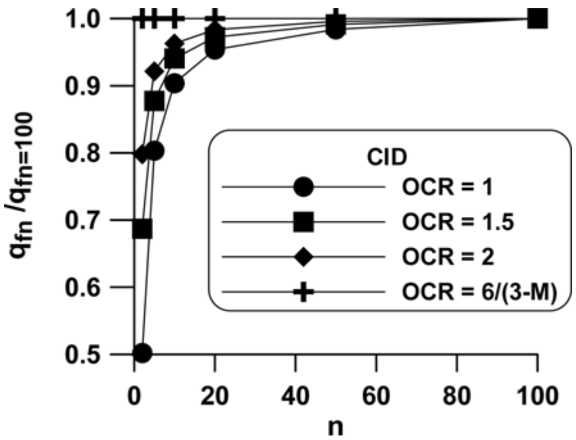

Fig. 8 The diagrams of the influence of the number of steps for various OCR on the value of failure deviator in CID test

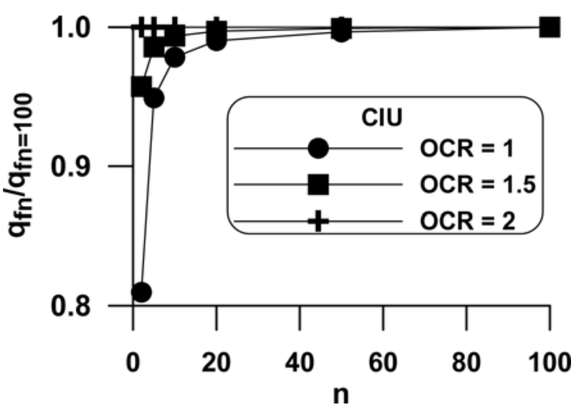

Fig. 9 The diagrams of influence of number of steps for various OCR on the value of failure deviator in CIU test

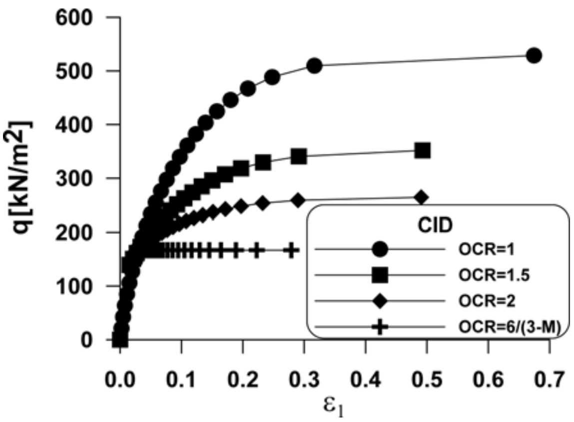

Fig. 10 The stress-deformation diagrams q-ε_1 for various OCR at CID test

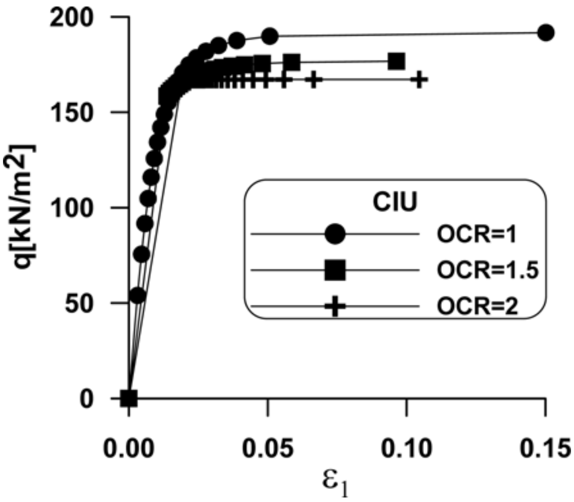

Fig. 11 The stress-deformation diagrams q-ε_1 for various OCR at CIU test

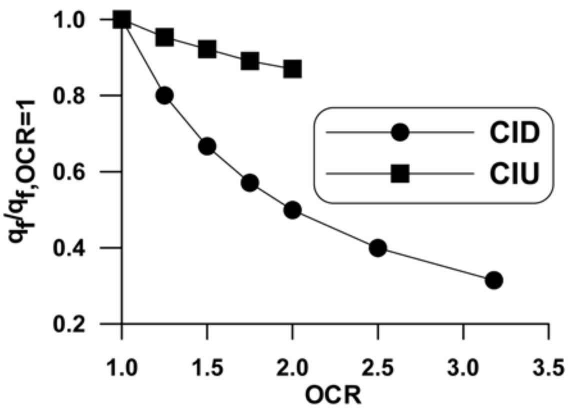

Fig. 12 The common view for normed failure stress deviators for CID and CIU tests

to the minimum OCR value.

The increment load size has a great influence on the final load-deformation soil diagram, especially in cases where the dominant plastic deformations take place form the start of load applying till the sample failure. It has been shown that, regardless to the drained or undrained conditions, for the stress path, where the vertical axis of the initial relief ellipse intersects the line of critical conditions in the point of final failure stress deviator value, the increment load apply amount may be neglect.

In that case failure occurs immediately after the initial elastic deformation and the material performs as an ideal elastic-plastic one.

In generally, it may be said that the undrained soil state is less sensitive to the number- of calculation steps. On the presented examples, the maximum deviator stress deviation in the undrained test was 20% comparing to the value of failure deviator for 100 steps, while for the drained sample the maximum deviation was 50%. For the value of increment of 5% of the total load (20 steps), satisfactory good results for CID test can be gained, while for the CIU test the necessary number of steps for the same accuracy is even less. The influence of the load history, expressed threw the OCR, is significant due to the change of position of the initial load in the q-p' diagram, and in that way significant to the influence to the final shape of the stress-load diagram, and to accomplish results with satisfactory accuracy.

The presented procedure for the soil response calculation, where the material unlinear soil on an acceptable accurate way, is involved. The calculation results may be used independently or the calculation results can be the input values for the existing commercial programs, where for the input values are used, for example, the hyperbolic curves of Duncan-Chang-s model (Hard Soil in Plaxis program) or parameters of Cam Clay model (in Geo Sigma program or Soft Soil in Plaxisu). The work presents the results of the test simulation calculation in the three-axis device for the standard tests, although the modelling in all stress directions is possible, with a condition of knowing the soil parameters of the accepted model, including the pore coefficient. Its also possible to lead direct compares of the calculation results by the shown model and the results of the gained commercial programs. In that case it is necessary to know the mutual differences in the assumptions of the analysed models.

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