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Non linear vibrations of stepped beam systems using artificial neural networks

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Abstract. In this study, the nonlinear vibrations of stepped beams having different boundary conditions were investigated. The equations of motions were obtained by using Hamilton's principle and made non dimensional. The stretching effect induced non-linear terms to the equations. Natural frequencies are calculated for different boundary conditions, stepped ratios and stepped locations by Newton-Raphson Method. The corresponding nonlinear correction coefficients are also calculated for the fundamental mode. At the second part, an alternative method is produced for the analysis. The calculated natural frequencies and nonlinear corrections are used for training an artificial neural network (ANN) program which has a multi-layer, feed-forward, back-propagation algorithm. The results of the algorithm produce errors less than 2.5% for linear case and 10.12% for nonlinear case. The errors are much lower for most cases except clamped-clamped end condition. By employing the ANN algorithm, the natural frequencies and nonlinear corrections are easily calculated by little errors, and the computational time is drastically reduced compared with the conventional numerical techniques.

Keywords: stepped beam; nonlinear vibration; perturbation method; artificial neural networks.

1. Introduction

In real life, many engineering problems can be modeled as stepped beams. Examples of these structures include bridges, rails, automotive industries and machine elements. The most important aspect of vibration analysis is to estimate the natural frequencies. If the system is forced with a frequency close to its natural frequencies, the system comes to resonance state and the amplitudes increase dangerously. While computing the natural frequencies of the systems, assuming the systems

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linear makes the calculations easier but the results are usually not reliable. Because no system moves linearly obtained linear results may deceive us. Therefore, nonlinear effects originated from the stretching during the vibration of the beam should be included in the computations as well.

Many studies on beam vibrations, both linear and nonlinear, have previously been performed. Particularly, the nonlinear behavior caused by the immobility of beam-ends has been analyzed by various researchers (Hou and Yuan 1998, McDonald 1991, Pakdemirli and Nayfeh 1994, Öz *et al.* 1998). Qaisi (1997) obtained the nonlinear vibration of beams with simply and clamped supports by using a power series approach and compared the results with existing solutions. Özkaya *et al.* (1997) analyzed mass beam system for different boundary conditions. By considering the effects of stretching, they solved the obtained problem with the method of multiple scales, a perturbation technique. Özkaya (2002) considered a beam-mass system under simply supported end conditions. Studies on stepped beam systems are usually linear. Balasubramanian *et al.* (1990) analyzed vibrations for beams stepped in the middle and acquired natural frequencies for high mode structures. Jang and Bert (1989), obtained the frequency equation for stepped beam under various boundary conditions and computed the smallest natural frequencies for a circular cross-section beam. They compared their results with the results of finite element analysis. In another study, Jang and Bert (1989) obtained natural frequencies for high mode structures using the frequency equation they acquired from their previous study. In a study performed by Naguleswaran (2002), motion equations of three different Euler-Bernoulli stepped beams with all states of boundary conditions were obtained and three natural frequencies were computed using equations of motion. The dynamic stability of a stepped beam carrying mass was studied by Aldraihem and Baz (2002). The stepped beam equations of motion developed a discrete parameter form and a finite element form. Aydogdu and Taskin (2007) explored free vibration of simply supported FG beam and also they found the equations by applying Hamilton's principle. They used Navier type solution method in order to obtain frequencies. Kwon and Park (2002), focused on the effect of the position of the stepped point and thickness ratio on the dynamic characteristics of the system. The equation of motion and boundary conditions were analytically obtained by using Hamilton principle. The exact solutions were compared with the results obtained by FEM. The vibration of beams with up to three step changes in cross section and in which the axial force in portion was contented in Naguleswaran (2003). The frequency equation for classical boundary was expressed and the first three frequency parameters for the three types of beams were displayed. The analysis of stepped beams using finite difference method is studied by using of a single differential equations (Krishnan *et al.* 1998).

At the first part of this study, nonlinear vibration analysis for stepped beams was performed and the contributions of nonlinear terms on natural frequency were investigated. Natural frequencies are calculated for different boundary conditions, stepped ratios and stepped locations by Newton-Raphson Method. Second order non-linear terms of the perturbation series behave as corrections to the linear problem. The corresponding nonlinear correction coefficients are also calculated for the fundamental mode.

At the second part, a different method which is artificial neural network is performed as an alternative to the conventional techniques. Artificial neural network has already been applied at many different studies. Çetinel *et al.* (2002), performed an artificial neural network model to investigate mechanical properties and microstructure evolution in the Tempcore process. Karlık *et al.* (1998), analyzed the vibration of beam-mass systems by using artificial neural networks. Artificial neural networks are also used as examples to obtain natural frequencies of suspension bridges (Çevik *et al.* 2002), to determine natural frequencies and stability regions of axially moving beams (Özkaya and Öz 2002), and for the prediction of wear loss and surface roughness of AA 6351 aluminum alloy (Durmuş *et al.* 2006). In this work, the results of Newton-Raphson Method are used for training an artificial neural network program. That new program is tested with real values which are not used for training procedure. The results of this method are presented together with the others.

2. The system and equation of motion

The considered system is a stepped beam with a single step located at $x = x_s$, where x is the spatial co-ordinate along the beam length. Six different cases of support at the ends of the beam are treated, as shown in Fig. 1.

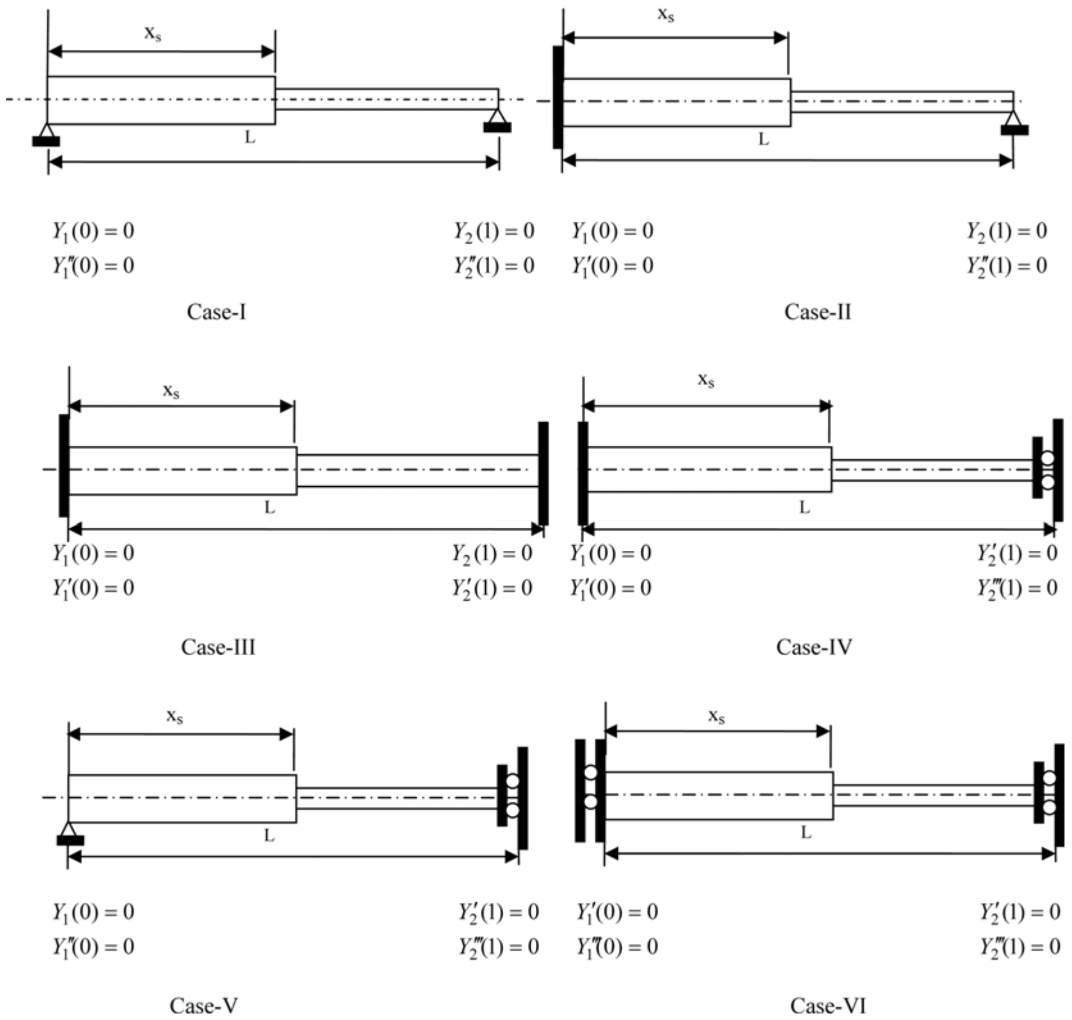

Fig. 1 The support end conditions for six different cases

Non-linear coupled integro-differential equations are obtained using Hamilton's principle (Özkaya and Tekin 2007)

$$\ddot{w}_1 + w_1^{iv} = \frac{1}{2[\eta + (1-\eta)/\alpha^2]}\left[\int_0^\eta w_1'^2 dx + \alpha^2 \int_\eta^1 w_2'^2 dx\right] w_1'' \tag{1}$$

$$\ddot{w}_2 + \alpha^2 w_2^{iv} = \frac{1}{2\alpha^2[\eta + (1-\eta)/\alpha^2]}\left[\int_0^\eta w_1'^2 dx + \alpha^2 \int_\eta^1 w_2'^2 dx\right] w_2'' \tag{2}$$

where w_1 and w_2 are dimensionless left and right transverse displacements, α is defined as the ratio of the diameter of the second portion to the diameter of the first portion $(\alpha = r_2/r_1)$. The dot denotes differentiations with respect to the non-dimensional time t and the prime denotes differentiations with respect to spatial variable x. The equations are made dimensionless through the definitions

$$x = x^*/L,\ w_{1,2} = w_{1,2}^*/R_{1,2}, \quad \eta = x_s/L, \quad t = (1/L^2)(EI_1/\rho A_1)^{1/2} t^* \tag{3}$$

where L is the length of the beam, $R_{1,2}$ is the radius of gyration of the beam cross-section with respect to the neutral axis, η is the non-dimensional position parameter ($0 \le \eta \le 1$), E is the Young's Modulus, A is the cross-sectional area and ρ is the density of the beam. I is the moment of inertia of the beam cross section with respect to the neutral axis.

The end boundary conditions are given in Fig. 1. The intermediate boundary conditions are

$$w_1 = \alpha w_2, \quad w_1' = \alpha w_2', \quad w_1'' - \alpha^5 w_2'' = 0, \quad w_1''' - \alpha^5 w_2''' = 0 \quad \text{at} \quad x = \eta \tag{4}$$

3. Conventional analysis

In this section, we search for the approximate solutions of Eqs. (1) and (2) with the associated boundary conditions. We apply the method of multiple scales (a perturbation technique) to the partial differential system and boundary conditions directly. This direct treatment of partial differential systems (the direct perturbation method) has some advantages over the more common method of discrediting the partial differential system and then applying perturbation (the discretization perturbation method). In our case, however, both methods may yield identical results, since we are not considering a higher order perturbation scheme. Due to the absence of quadratic non-linearities, we assume expansions of the forms

$$w_1(x,t;\varepsilon) = \varepsilon w_{11}(x,T_0,T_2) + \varepsilon^3 w_{13}(x,T_0,T_2) + \ldots \tag{5}$$

$$w_2(x,t;\varepsilon) = \varepsilon w_{21}(x,T_0,T_2) + \varepsilon^3 w_{23}(x,T_0,T_2) + \ldots \tag{6}$$

where ε is a small book-keeping parameter artificially inserted into the equations. This parameter can be taken as 1 at the end upon keeping in mind, however, that deflections are small. We therefore investigate a weakly non-linear system. $T_0 = t$ and $T_2 = \varepsilon^2 t$ are the fast and slow time scales one obtains, to order ε,

$$(\dot{}) = D_0 + \varepsilon^2 D_2, \quad (\ddot{}) = D_0^2 + 2\varepsilon^2 D_0 D_2, \quad D_n = \frac{\partial}{\partial T_n} \tag{7}$$

$$D_0^2 w_{11} + w_{11}^{iv} = 0; \quad D_0^2 w_{21} + \alpha^2 w_{21}^{iv} = 0 \tag{8}$$

$$w_{11} = \alpha w_{21}, \quad w'_{11} = \alpha w'_{21}, \quad w''_{11} = \alpha^5 w''_{21}, \quad w'''_{11} = \alpha^5 w'''_{21} \quad \text{at} \quad x = \eta \tag{9}$$

$$w_{11} = w''_{11} = 0 \quad \text{at} \quad x = 0, \quad w_{21} = w''_{21} = 0 \quad \text{at} \quad x = 1 \tag{10}$$

and, to order ε^3

$$D_0^2 w_{13} + w_{13}^{iv} = -2D_0 D_2 w_{11} + \frac{1}{2[\eta + (1-\eta)/\alpha^2]}\left[\int_0^\eta w_{11}'^2 dx + \alpha^2 \int_\eta^1 w_{21}'^2 dx\right] w''_{11} \tag{11}$$

$$D_0^2 w_{23} + \alpha^2 w_{23}^{iv} = -2D_0 D_2 w_{21} + \frac{1}{2\alpha^2[\eta + (1-\eta)/\alpha^2]}\left[\int_0^\eta w_{11}'^2 dx + \alpha^2 \int_\eta^1 w_{21}'^2 dx\right] w''_{21} \tag{12}$$

$$w_{13} = \alpha w_{23}, \quad w'_{13} = \alpha w'_{23}, \quad w''_{23} = \alpha^5 w''_{23}, \quad w'''_{13} = \alpha^5 w'''_{23} \quad \text{at} \quad x = \eta \tag{13}$$

$$w_{13} = w''_{13} = 0 \quad \text{at} \quad x = 0, \quad w_{23} = w''_{23} = 0 \quad \text{at} \quad x = 1 \tag{14}$$

Eqs. (10) and (14) are the boundary conditions corresponding to Case I. Boundary conditions for other cases can be written similarly.

The problem at order ε is linear. We assume a solution of the form

$$w_{11} = [A(T_2)e^{i\omega T_0} + cc]Y_1(x), \quad w_{21} = [A(T_2)e^{i\omega T_0} + cc]Y_2(x) \tag{15}$$

where cc represents the complex conjugate of the preceding terms. Substituting Eq. (15) into Eqs. (8)-(10), one will have

Y_1 and Y_2 are displacement functions of x.

$$Y_1^{iv} - \omega^2 Y_1 = 0, \quad Y_2^{iv} - \frac{1}{\alpha^2}\omega^2 Y_2 = 0 \tag{16}$$

$$Y_1 = \alpha Y_2, \quad Y'_1 = \alpha Y'_2, \quad Y''_1 = \alpha^5 Y''_2, \quad Y'''_1 = \alpha^5 Y'''_2 \quad \text{at} \quad x = \eta \tag{17}$$

Solving Eqs. (15)-(17) exactly for different end conditions yields the mode shapes Y_i and natural frequencies ω. The transcendental equations were numerically solved for the first five modes. Using the method of multiple scales by Eqs. (11)-(14), the nonlinear amplitude dependent frequencies are calculated approximately as follows

$$\omega_{n1} = \omega + \theta' = \omega + \lambda a_0^2 \tag{18}$$

where a_0 is the amplitude of vibration. To this order of approximation, then, the non-linear frequencies have a parabolic relation with the maximum amplitude of vibration. λ can be defined as the non-linear correction coefficient. It can be also said, λ is a measure of the effect of stretching. The non-linearities are of hardening type.

$$\lambda = (3/16)(\Lambda b^2/\omega) \tag{19}$$

where

$$b = \int_0^\eta Y_1'^2 dx + \alpha^2 \int_\eta^1 Y_2'^2, \quad \Lambda = \frac{1}{[\eta + (1-\eta)/\alpha^2]} \tag{20}$$

Note that the arbitrary coefficients C of the mode shapes are to be calculated from the following normalization condition

$$\int_0^\eta Y_1^2 dx + \alpha^4 \int_\eta^1 Y_2^2 dx = 1, \quad f = \int_0^\eta F_1 Y_1 dx + \alpha_1^4 \int_\eta^1 F_2 Y_2 dx \tag{21}$$

Obviously, the natural frequencies and correction coefficient λ can be obtained by deriving the equations above. However, the conventional methods generally include long derivations and complicated calculations. Furthermore, each frequency should be analyzed separately in conventional methods. But intelligent calculating techniques give us opportunities to investigate all the frequencies together in a very short time.

4. Artificial neural networks

Nowadays, scientists and especially engineers are trying to develop intelligent machines. Artificial neural systems are present-day examples of such machines that have great potential to further improve the quality of our life. In this work, Artificial Neural Network Method is presented as an alternative to the conventional techniques.

It is well known that people and animals are much better and faster at recognizing images than most advanced computers. Although computers outperform both biological and artificial neural systems for tasks based on precise and fast arithmetic operations, artificial neural systems represent the promising new generation of information processing networks. Advances have been made in applying such systems for problems found intractable or difficult for traditional computation. Neural networks can supplement the enormous processing power of digital computers with the ability to make sensible decisions and to learn by ordinary experience, as humans do.

Network computation is performed by a dense mesh of computing nodes and connections. They operate collectively and simultaneously on most or all data and inputs. The basic processing elements of neural networks are called *artificial neurons*, or simply *neurons*. Often they are simply called *nodes*. Neurons perform as summing and non-linear mapping junctions. In some cases, they can be considered as threshold units that fire when their total input exceeds certain bias levels. Neurons usually operate in parallel and are configured in regular architectures. They are often organised in layers, and feedback connections both within the layer and toward adjacent layers are allowed. Connection strength is expressed by a numerical value called *weight*, which can be modified.

Among the artificial neural networks, the elementary multilayer perceptrons (MLP) with sigmoidal transfer function have been successfully applied to solve some difficult and diverse problems as non-linear discriminant function classifiers. The feedforward network learns from the input data by the supervision of the output data creating single linear discriminant functions by each sigmoid hidden unit and combines them. Thus, this piecewise linear discriminant function works as a non-linear discriminator.

Training the network in a supervised manner with a highly popular algorithm known as the error back-propagation (BP) has become very popular. BP is an optimization technique for implementing gradient descent in weight space for multilayer feed forward networks.

The basic idea of the technique is to efficiently compute partial derivatives of an approximating function $F(w;x)$ realised by the network with respect to all the elements of the adjustable weight vector w for a given value of input vector x and output vector y. The weights are adjusted to fit linear piecewise discriminant functions to feature space for the best class separability. The difference between the network output and the supervisor output is minimized according to a predefined error function (performance criterion) such as mean square error (MSE) etc. The neural network system has been applied with multilayer perceptron and BP algorithm by supervised training (Çetinel *et al.* 2002).

In this part of the work, artificial neural network method is used to determine the natural frequencies and the contribution of the non-linear terms to the fundamental frequency at different end conditions of stepped beam systems. The results will be compared with those of the Newton Raphson Method.

4.1 Architecture

The multilayer perceptron architecture used in this study is shown at Fig. 2. Two different computer programs are used for calculations. One of them is a special C based software and the other is MATLAB-NNTOOLBOX.

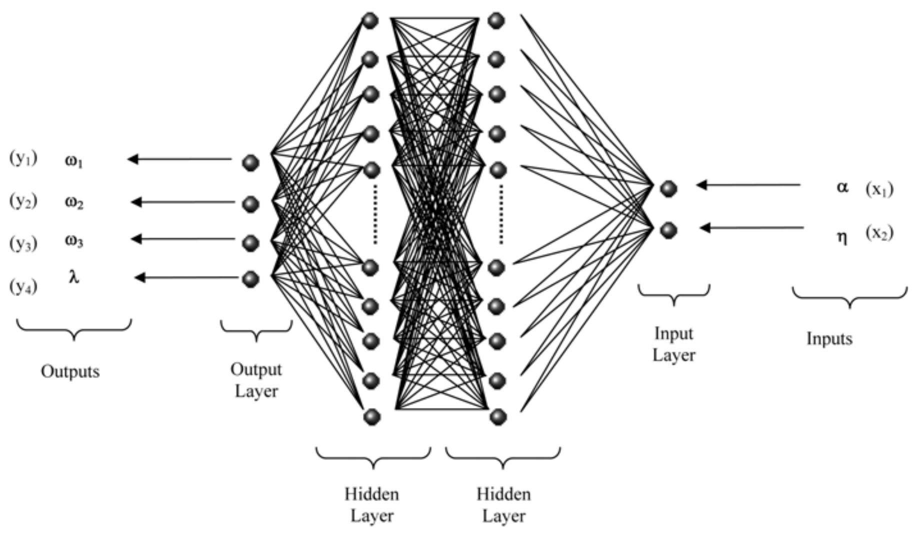

Fig. 2 The network architecture

Table 1 Best architectures of six different end conditions

End Conditions	Input Variables	Output Variables	Hidden Layer	Node Number of Hidden Layers	Iteration Number	The Program Used
pinned-pinned		ω_1	2	15-15	20000	C
clamped-pinned			2	15-15	20000	C
clamped-clamped	α	ω_2	2	14-14	20000	C
pinned-sliding	η	ω_3	2	20-20	400	Matlab
clamped-sliding			2	19-19	300	Matlab
sliding-sliding		λ	2	17-17	400	Matlab

As it is shown in Fig. 2, two input nodes for input variables and four output nodes for output variables are used in this work. The input variables are α, η and the output variables are ω_1, ω_2, ω_3 and λ, as well.

4.2 Training and testing

Training phase is the main stage of the neural network applications (Çetinel *et al.* 2002, Karlık *et al.* 1998, Özkaya and Pakdemirli 1999). Some input and corresponding output values are needed to carry out this procedure. In this work, the programs are trained by using the input and output values which are obtained from the conventional method, Newton Raphson. 85% of N.R. values are used for training while 15% of them are left for testing. A lot of training iterations are made by trial end error and finally minimum errors are obtained in the architectures of end conditions as shown in Table 1. Learning rate and momentum value are taken 0.9 and 0.7 respectively in all applications. Finally, the A.N.N. programs are tested by the real values which are not used for training (Çevik *et al.* 2002).

5. Numerical results

In this section, numerical results are presented for all cases used in this work. In Table 2, the comparison between N.R. and A.N.N. learning values are presented. As seen in the table, the learning values give little errors in most cases which means the A.N.N. programs work fine. After all, the final procedure is now to test the programs with N.R. values which are not used for training. In Table 3, the A.N.N. test results are presented by using the testing phase of the algorithms shown at Table 1. λ values in Tables 2 and 3 are the non-linear correction coefficients for fundamental frequencies. The results are good enough to decide that, these artificial neural network programs can be used for calculations of nonlinear stepped beam frequencies.

Fig. 3 and Fig. 4 are graphical displays of N.R. and A.N.N. test results of pinned-pinned end condition. In Fig. 5, the scatter diagram between N.R. ve A.N.N. results of ω_1 is shown. Similar comparisons are made for the other cases. Figs. 6, 7, 8 are the test results of “clamped-pinned” end conditions while Figs. 9, 10, 11 are of “pinned-sliding”.

If the results of conventional numerical methods and artificial neural networks are compared, it is clearly seen that the ω_1-ω_2-ω_3-λ values can be easily obtained without using any numerical calculating method. Artificial neural network method gives fast and good results with low errors for

Table 2 The comparison of the N.R. and A.N.N. learning results for different end conditions

Cases	α	η	ω_1 (N.R.)	ω_1 (A.N.N)	ω_2 (N.R.)	ω_2 (A.N.N)	ω_3 (N.R.)	ω_3 (A.N.N)	λ	λ (A.N.N)
pinned-pinned	0.5	0.1	4.9049	4.799275	19.3844	19.4404	43.3230	42.710724	15.2674	15.306901
		0.5	4.6769	4.659126	29.5223	29.6536	59.8612	59.570136	7.2454	7.251928
		0.9	9.4535	9.366956	34.5078	34.3543	74.4401	74.921956	1.6180	1.587802
	0.9	0.1	8.8859	8.886823	35.5846	35.7368	80.2162	80.698914	2.5791	2.599078
		0.5	9.2635	9.200878	37.6235	37.1706	83.8807	84.445147	2.2121	2.226920
		0.9	9.8592	9.802246	39.3275	39.0904	88.1718	89.006242	1.8148	1.779893
	2	0.1	18.9070	18.90865	69.0156	68.9720	148.8800	149.052043	0.2022	0.163558
		0.5	9.3538	9.328313	59.0446	61.0179	119.7220	113.702509	0.9056	1.006909
		0.9	9.8097	9.780810	38.7689	38.5465	86.6459	86.962662	1.9408	2.000266
clamped-pinned	0.5	0.1	9.3276	9.220178	30.0416	30.1718	62.1994	62.110469	16.6018	16.587237
		0.5	11.9969	12.04917	33.6891	33.8794	75.7010	74.596501	7.2862	7.200019
		0.9	14.5190	14.71221	42.8347	42.3436	87.5893	87.324596	1.7264	1.734991
	0.9	0.1	14.6177	14.61933	46.7747	46.5274	96.5512	96.470267	2.3014	2.257310
		0.5	14.7321	14.68430	47.7120	47.4153	98.2320	97.629965	2.1469	2.200421
		0.9	15.3964	15.42029	49.7297	49.3804	103.3875	102.456796	1.5881	1.616960
	2	0.1	22.6597	22.69916	83.4091	82.9746	175.7170	174.619557	0.1697	0.170596
		0.5	14.6995	14.80557	77.7005	76.6113	133.5500	131.461840	0.5161	0.571863
		0.9	15.2629	15.28217	48.9488	49.0272	101.6930	101.029334	1.6614	1.541801
clamped-clamped	0.5	0.1	13.5175	13.80214	37.0158	38.4802	71.9733	75.055378	12.9252	12.896000
		0.5	14.6697	14.95998	44.4760	43.9640	83.2323	86.869998	4.4488	4.542408
		0.9	17.2833	17.57773	51.8736	53.0546	99.9791	100.565566	1.1458	1.050207
	0.9	0.1	21.1471	21.18938	57.5678	58.4598	111.7192	112.046948	1.7829	1.797976
		0.5	21.1232	21.10266	58.7287	60.1365	114.2058	114.856648	1.5231	1.652412
		0.9	21.2421	21.62837	59.6741	61.2885	118.2298	120.106549	1.2576	1.230474
	2	0.1	34.5665	34.74141	103.7470	103.70716	203.4260	202.654460	0.1432	0.035793
		0.5	29.3393	29.41209	88.9519	89.975156	166.4651	168.609824	0.5561	0.607948
		0.9	27.0350	27.20769	74.0317	74.945418	143.9460	143.737976	1.6156	1.636695
pinned-sliding	0.5	0.1	1.2317	1.231668	10.9700	10.969980	59.0180	59.018010	3.9692	3.969211
		0.5	1.1171	1.136483	13.8031	13.971591	46.9627	47.098066	3.1046	3.036862
		0.9	1.3660	1.366098	19.0304	19.030372	58.5800	58.579835	1.1308	1.130882
	0.9	0.1	2.2209	2.220861	20.0028	20.002799	55.6488	55.648872	0.6463	0.646348
		0.5	2.2481	2.248094	21.2715	21.271501	58.1381	58.138153	0.6413	0.641331
		0.9	2.3952	2.395192	21.7081	21.708104	60.6400	60.640096	0.5053	0.505288
	2	0.1	4.8788	4.878804	40.6631	40.663104	104.4240	104.424037	0.0454	0.045380
		0.5	2.8559	2.855899	34.0779	34.077926	76.4774	76.477394	0.0876	0.087550
		0.9	2.1934	2.193402	22.0876	22.087589	64.6080	64.608065	0.2355	0.235500
clamped-sliding	0.5	0.1	3.3931	3.393125	18.2323	18.232286	44.7156	44.715684	3.2897	3.289707
		0.5	6.7728	6.749638	18.6831	18.699849	55.7544	57.180761	2.8059	2.631543
		0.9	4.6592	4.659159	26.7071	26.707084	71.8354	71.835351	0.7555	0.755501
	0.9	0.1	5.3406	5.340547	28.4698	28.469813	69.4804	69.480317	0.4575	0.457452
		0.5	5.6652	5.665223	28.8143	28.814324	70.6385	70.638453	0.4328	0.432759
		0.9	5.5395	5.539488	29.5684	29.568367	73.4666	73.466632	0.3586	0.358642
	2	0.1	6.5284	6.528404	48.6283	48.628308	125.9639	125.963950	0.0383	0.038300
		0.5	5.4699	5.547954	41.2870	37.129768	98.3819	100.016254	0.0393	0.039222
		0.9	4.6792	4.679200	30.6594	30.659384	78.8433	78.843276	0.1397	0.139700
sliding-sliding	0.5	0.1	4.7074	4.707426	20.2310	20.230987	47.2719	47.272027	9.2642	9.264162
		0.5	8.7794	8.779345	24.5332	24.533172	61.9125	61.912449	0.8925	0.892503
		0.9	7.5372	7.537184	36.0812	36.081160	86.3529	86.352954	0.2229	0.222900
	0.9	0.1	9.0167	9.016717	36.1233	36.123333	81.2516	81.251681	2.3766	2.376627
		0.5	9.4501	9.450060	37.2304	37.230382	84.5397	84.539828	0.0201	0.020124
		0.9	9.6174	9.617369	38.7063	38.706321	87.5284	87.528311	0.1621	0.162140
	2	0.1	15.0743	15.07430	72.1624	72.162447	172.7058	172.705907	0.4001	0.400099
		0.5	17.5587	17.55869	49.0663	49.066343	123.8250	123.824899	0.0583	0.058300
		0.9	9.4148	9.414788	40.4620	40.461975	94.5438	94.543762	0.0339	0.033900

Table 3 The comparison of the N.R. and A.N.N. test results for different end conditions

Cases	α	η	ω_1 (N.R.)	ω_1 (A.N.N)	ω_2 (N.R.)	ω_2 (A.N.N)	ω_3 (N.R.)	ω_3 (A.N.N)	λ	λ (A.N.N)
pinned-pinned	0.3	0.7	2.6497	2.674393	29.7208	28.893938	53.4021	55.039697	9.5891	9.145854
	0.4	0.7	4.3843	4.278635	29.7737	29.803662	68.0416	67.318730	6.8705	6.860221
	0.5	0.4	4.5199	4.459633	23.9542	24.000733	62.4896	60.651755	9.1012	8.951545
	0.6	0.6	6.6270	6.678246	33.5778	33.910694	69.1784	69.263667	4.1553	4.117201
	0.7	0.5	7.3431	7.419132	34.4100	34.607218	71.4214	71.705283	3.7663	3.750467
	0.8	0.3	7.9807	7.925639	33.3166	33.265706	76.9485	76.421883	3.5252	3.504584
	0.9	0.3	8.9744	8.901823	36.6632	36.899408	82.9488	82.601080	2.5096	2.432027
	2	0.5	9.3538	9.328313	59.0446	61.017910	119.7220	113.702509	0.9056	1.006909
	3	0.8	8.7401	8.852147	33.9454	35.898761	85.8298	88.747890	1.4010	1.499750
clamped-pinned	0.3	0.7	7.6644	7.653704	39.197554	40.3986	56.6249	64.508018	7.2697	7.266599
	0.4	0.8	9.4995	9.367896	35.453305	35.4895	90.9798	92.475576	7.6675	7.341685
	0.5	0.6	10.5991	10.714077	40.523716	41.1604	70.1527	70.258254	6.3422	6.455244
	0.6	0.4	13.5598	13.656601	34.431333	34.6424	78.8078	76.577062	6.7625	6.608462
	0.7	0.3	14.2120	14.340521	39.163129	38.9081	80.2940	79.039651	5.0723	5.180102
	0.8	0.3	14.5037	14.679575	42.935922	42.7614	89.3354	88.016428	3.2670	3.367610
	0.9	0.4	14.7772	14.635280	46.982523	47.0983	97.9224	99.398885	2.2302	2.164292
	2	0.8	14.6087	14.619028	49.884031	48.5432	108.0390	109.458782	1.4031	1.356997
	3	0.8	12.9824	12.768877	45.096617	44.1506	103.6590	105.346338	1.0062	1.219294
clamped-clamped	0.3	0.8	9.8698	11.500045	35.8462	40.856709	94.6421	95.011813	5.4128	4.538698
	0.4	0.6	11.5791	11.804513	47.9022	46.920379	70.1226	71.158748	4.5260	4.362970
	0.5	0.3	18.2284	17.838703	37.5075	39.501483	69.3587	65.836949	11.705	13.760195
	0.6	0.4	17.6339	17.752749	43.6361	43.847382	91.8607	92.512218	4.2429	4.414219
	0.7	0.3	19.6060	19.313728	47.2998	49.479541	93.9033	93.043961	3.4871	3.670403
	0.8	0.5	19.6133	19.713209	56.0103	57.065118	106.2870	106.020223	1.9545	1.967197
	0.9	0.4	21.1252	20.923606	58.2570	58.772726	113.2033	119.643580	1.6679	1.793535
	2	0.7	36.4569	38.681812	75.0150	76.553842	138.7171	150.420236	1.4630	1.522726
	3	0.7	43.4287	49.559113	99.4166	96.514367	141.8880	174.692818	2.8022	2.719522
pinned-sliding	0.3	0.8	0.4654	0.458339	22.1092	21.581094	43.8381	41.938412	2.9124	2.918261
	0.4	0.5	0.8073	0.869536	10.7697	11.359392	43.4957	44.713823	4.7422	4.414106
	0.5	0.5	1.1171	1.136483	13.8031	13.971591	46.9627	47.098066	3.1046	3.036862
	0.6	0.4	1.4402	1.451543	14.3960	14.468685	47.2523	46.711449	2.2398	2.208662
	0.7	0.6	1.7377	1.740536	20.2570	20.259903	51.0193	51.260167	1.2234	1.219515
	0.8	0.4	1.9833	1.963999	19.1062	18.941525	54.7841	55.020910	0.9552	0.981091
	0.9	0.3	2.2263	2.261956	20.4161	20.723005	57.6069	57.375657	0.6588	0.633501
	2	0.6	2.5050	2.065824	34.3304	38.845741	74.0340	74.111252	0.1024	0.108488
	3	0.7	1.6797	1.656700	31.6503	28.034565	97.4706	86.224467	0.0477	0.041019
clamped-sliding	0.3	0.8	5.2577	5.406993	30.2836	29.339633	46.9338	48.481632	0.6916	0.709386
	0.5	0.5	6.7728	6.749638	18.6831	18.699849	55.7544	57.180761	2.8059	2.631543
	0.6	0.6	6.1514	6.166496	24.3495	24.619875	58.8908	58.592515	1.0269	1.030703
	0.7	0.4	6.0597	6.062006	24.1190	24.147876	60.7130	61.617817	0.9882	0.996718
	0.8	0.6	5.6908	5.723901	28.1165	27.918902	66.8598	65.998263	0.5788	0.557021
	0.9	0.6	5.6125	5.680783	29.2438	29.097054	71.0209	67.837302	0.4289	0.387607
	2	0.5	5.4699	5.547954	41.2870	37.129768	98.3819	100.016254	0.0393	0.039222
sliding-sliding	0.3	0.8	8.3118	7.558375	37.4137	36.588600	53.2760	58.418514	0.1148	0.122031
	0.4	0.8	8.1731	7.917926	39.0242	38.652114	65.5805	66.437544	0.1363	0.133230
	0.5	0.3	6.1190	5.995633	27.2267	28.132444	54.1841	54.596032	9.8268	9.187962
	0.6	0.5	8.9907	8.911563	27.7182	27.669883	70.2824	69.751113	0.1171	0.104241
	0.7	0.8	8.6535	8.633113	37.8676	37.933345	84.5838	84.746673	0.1305	0.128524
	0.8	0.4	9.0454	8.937551	33.8685	33.953643	77.4082	76.882164	0.7560	0.752103
	0.9	0.3	9.3296	9.329608	36.6407	36.640718	81.9840	81.984095	1.6493	1.649252
	2	0.2	16.3348	13.800480	77.2033	91.799678	152.0331	171.160318	0.4328	0.376037
	3	0.4	28.5641	28.564090	54.2447	54.244640	182.0946	182.094497	0.1229	0.122900

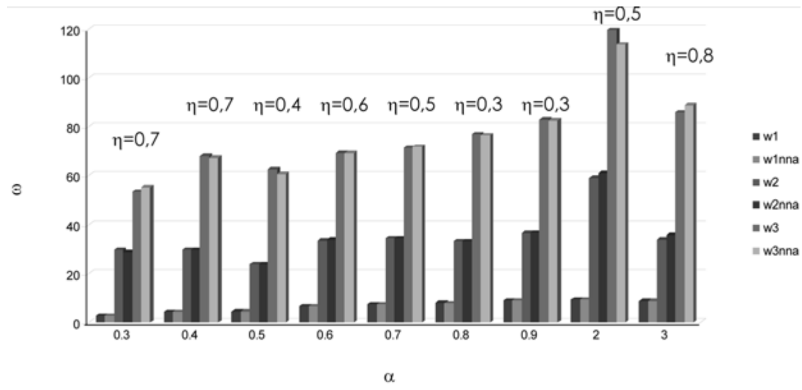

Fig. 3 The comparison between N.R. and A.N.N. test results of ω_1-ω_2-ω_3 for different α-η values (pinned-pinned)

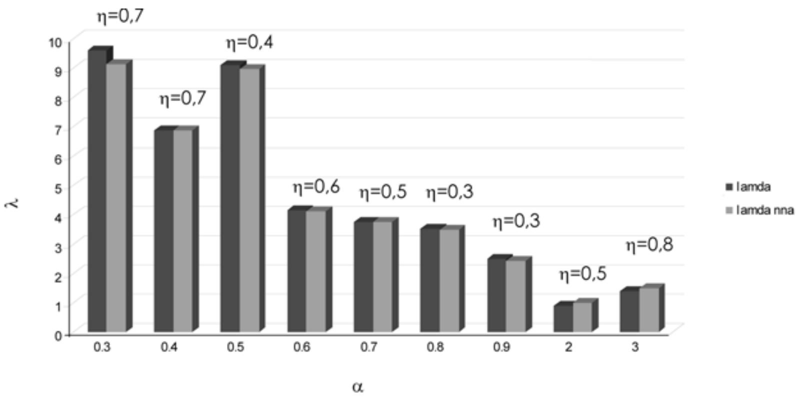

Fig. 4 Test results of λ for different α-η values: Obtained from conventional method and artificial neural networks (pinned-pinned)

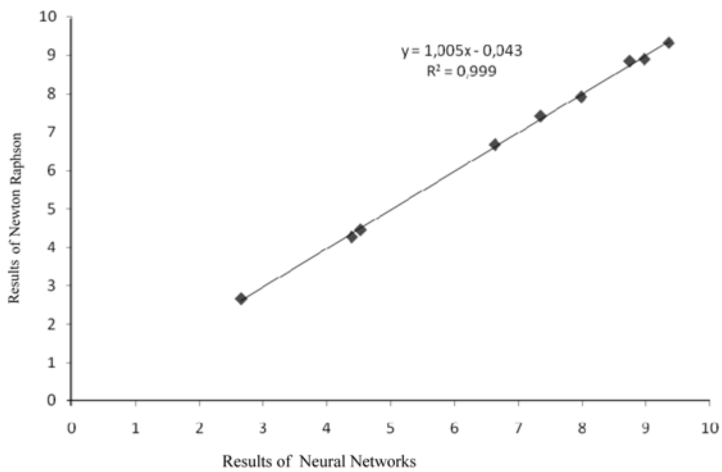

Fig. 5 Scatter Diagrams of ω_1 test values (R^2 : coefficient of determination) (pinned-pinned)

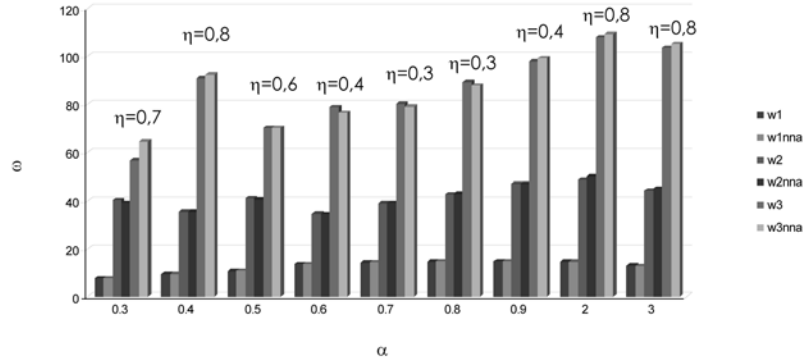

Fig. 6 The comparison between N.R. and A.N.N. test results of ω_1-ω_2-ω_3 for different α-η values (clamped-pinned)

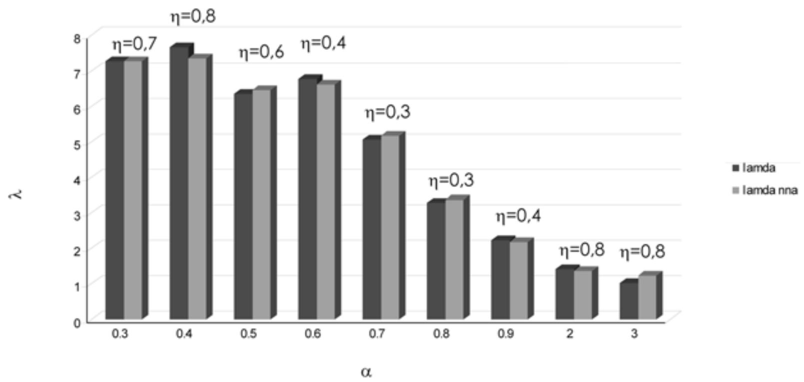

Fig. 7 Test results of λ for different α-η values: Obtained from conventional method and artificial neural networks (clamped-pinned)

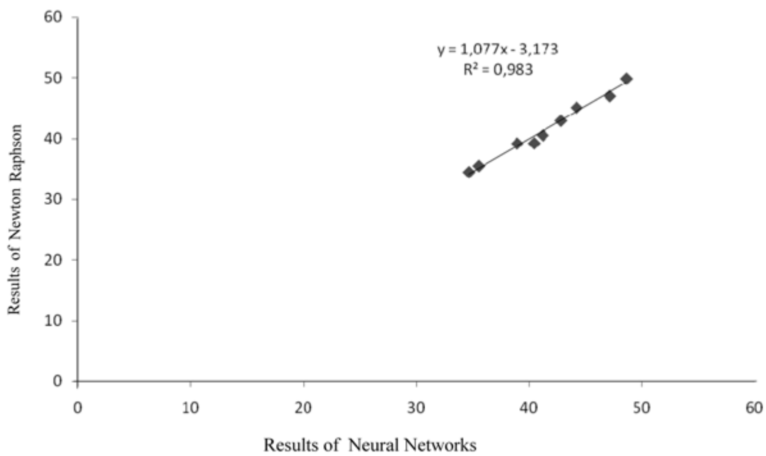

Fig. 8 Scatter Diagrams of ω_2 test values (R^2: coefficient of determination) (clamped-pinned)

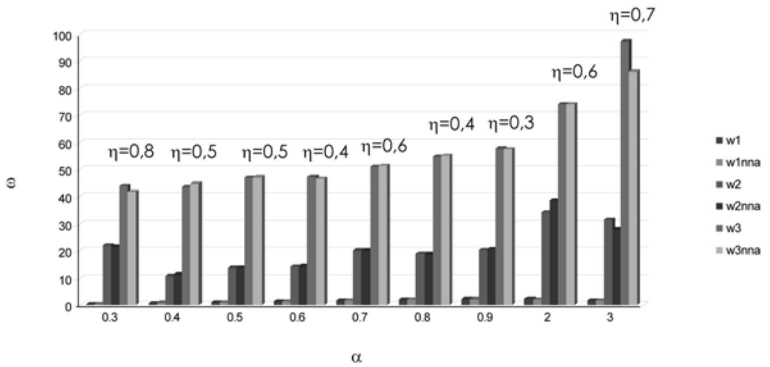

Fig. 9 The comparison between N.R. and A.N.N. test results of ω_1-ω_2-ω_3 for different α-η values (pinned-sliding)

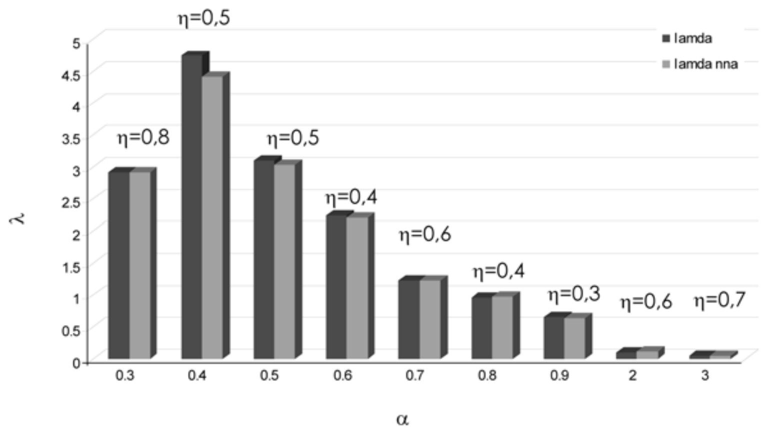

Fig. 10 Test results of λ for different α-η values: Obtained from conventional method and artificial neural networks (pinned-sliding)

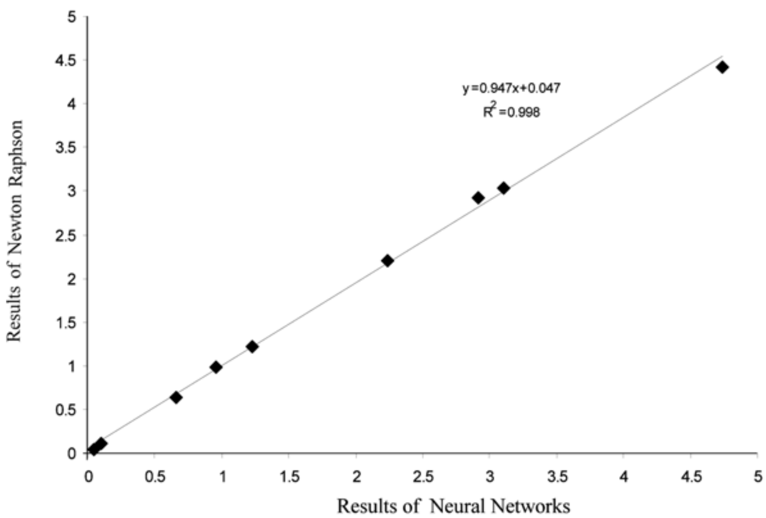

Fig. 11 Scatter Diagrams of λ test values (R^2: coefficient of determination) (pinned-sliding)

Table 4 % error and correlation values of learning for all cases

End Condition		ω_1	ω_2	ω_3	λ
pinned-pinned	% Error	0,69%	0,59%	0,56%	2,23%
	R^2	0,999	0,999	0,999	0,999
clamped-pinned	% Error	0,58%	0,64%	0,77%	5,72%
	R^2	0,999	0,999	0,999	0,999
clamped-clamped	% Error	1,30%	2,06%	2,50%	10,12%
	R^2	0,999	0,998	0,999	0,999
pinned-sliding	% Error	0,00%	0,00%	0,00%	0,00%
	R^2	1,000	1,000	1,000	1,000
clamped-sliding	% Error	0,00%	0,00%	0,00%	0,00%
	R^2	1,000	1,000	1,000	1,000
sliding-sliding	% Error	0,00%	0,00%	0,00%	0,00%
	R^2	1,000	1,000	1,000	1,000

Table 5 % error and correlation values of testing for all cases

End Condition		ω_1	ω_2	ω_3	λ
pinned-pinned	% Error	1,06%	1,61%	1,90%	3,30%
	R^2	0,999	0,995	0,988	0,999
clamped-pinned	% Error	0,90%	1,27%	2,89%	4,55%
	R^2	0,997	0,983	0,968	0,996
clamped-clamped	% Error	4,94%	3,51%	5,12%	6,87%
	R^2	0,989	0,992	0,965	0,978
pinned-sliding	% Error	3,71%	4,06%	2,39%	4,17%
	R^2	0,955	0,940	0,977	0,998
clamped-sliding	% Error	0,96%	2,24%	2,18%	3,37%
	R^2	0,997	0,985	0,988	0,998
sliding-sliding	% Error	3,56%	2,89%	2,88%	4,58%
	R^2	0,986	0,982	0,985	0,999

almost all cases. Only the errors at clamped-clamped end condition are found a little high. In Table 4 and 5, %error and correlation values of learning and testing are shown, respectively.

6. Conclusions

In this work, the vibrations of a stepped beam under six different supporting conditions were treated. The non-linear equations of motion including stretching due to immovable end conditions were derived. Linear and non-linear analyses were investigated. Approximate solutions can be searched by applying the method of multiple scales directly to the partial differential equations. The first term lead to the linear problem. The natural frequencies were calculated for different stepped ratios, step location and end conditions. The second terms provide the non-linear corrections to the linear problem. The exact values of linear frequencies were calculated using the Newton-Raphson method. The correction coefficients to the linear frequencies in the case of nonlinearities were also calculated from the formulas given in a previous paper (Özkaya and Tekin 2007). For each end condition, step location, step ratio and frequency, numerical analysis should be repeated, a lengthy process which requires the convergence of iterations. When the initial guesses are not close enough, the algorithm may diverge also. Some key values obtained using the conventional analyses were then used in training an ANN algorithm. After half an hour of training for the linear and non- linear cases, the frequencies become available almost instantly. The results of the algorithm produce errors less than 2.5% for linear case and 10.12% for nonlinear case. The errors are much lower for most cases except clamped-clamped end condition. By employing the ANN algorithm, computational time is drastically reduced compared with the conventional numerical techniques (Özkaya and Öz 2002, Durmuş *et al.* 2006). These errors are considerably low. ANN algorithms cannot, of course, replace totally the conventional numerical techniques, since they need some key values for training. However, for involved problems in structural vibrations where excessive iterations are needed for convergence, they can be implemented as an efficient supplementary tool, reducing drastically the computational cost.

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