Structural Engineering and Mechanics, Vol. 32, No. 3 (2009) 429-445 429
DOI: http://dx.doi.org/10.12989/sem.2009.32.3.429

Evaluation of thermal stability of quasi-isotropic composite/polymeric cylindrical structures under extreme climatic conditions

Mohamed Gadalla† and Hany El Kadi‡

Mechanical Engineering Department, American University of Sharjah
Sharjah, United Arab Emirates

(Received June 27, 2008, Accepted April 25, 2009)

Abstract. Thermal stability of quasi-isotropic composite and polymeric structures is considered one of the most important criteria in predicting life span of building structures. The outdoor applications of these structures have raised some legitimate concerns about their durability including moisture resistance and thermal stability. Exposure of such quasi-isotropic composite/polymeric structures to various and severe climatic conditions such as heat flux and frigid climate would change the material behavior and thermal viability and may lead to the degradation of material properties and building durability. This paper presents an analytical model for the generalized problem. This model accommodates the non-linearity and the non-homogeneity of the internal heat generated within the structure and the changes, modification to the material constants, and the structural size. The paper also investigates the effect of the incorporation of the temperature and/or material constant sensitive internal heat generation with four encountered climatic conditions on thermal stability of infinite cylindrical quasi-isotropic composite/polymeric structures. This can eventually result in the failure of such structures. Detailed critical analyses for four case studies which consider the population of the internal heat generation, cylindrical size, material constants, and four different climatic conditions are carried out. For each case of the proposed boundary conditions, the critical thermal stability parameter is determined. The results of this paper indicate that the thermal stability parameter is critically dependent on the cylinder size, material constants/selection, the convective heat transfer coefficient, subjected heat flux and other constants accrued from the structure environment.

Keywords: thermal stability; quasi-isotropic composite/polymeric material degradation; non-uniform internal heat generation; cylindrical building structures; material-environment interaction.

1. Introduction

Thermal analysis is increasingly becoming an important tool in determining the reliability of the selection and application of composite materials and polymers in many engineering fields such as electronic packaging, automotive, aerospace, military, marine and civil structures. Prediction of thermal behavior of quasi-isotropic composite/polymeric structures due to environmental effects and internal heat generation after the curing process is considered to be an effective and complementary

† Ph.D., Corresponding author, E-mail: mgadalla@aus.edu
‡ E-mail: hkadi@aus.edu

tool during the design stage. Thermal stability of prescribed parameters are the limiting values below which there will be a steady-state temperature distribution within the structures. Monteverde (2005) investigated the thermal stability of two hot-pressed diborides matrix composites in air at ultra-high temperatures. The two highly-dense composites consisted of HfB_2 or a mixture of ZrB_2 and HfB_2. Both materials were subjected to repeated heating-cooling cycles at 1600ºC and a 20 h exposure at 1450ºC in flowing dry air. The resistance to oxidation was also tested in laboratory air in which the modifications in the microstructure induced by the oxidation were monitored. The author found that the introduction of the SiC particles as sintering aid provided tangible benefits for the resistance to oxidation. Finally, modest weigh gains and limited corrosion depths highlighted a good thermal stability criterion. Ju Wei *et al.* (2005) studied the effect of polymer structure on thermal stability of composite membranes. Novel thermal stable polymer (PPESK) was used as the support material of the thin film composite (TFC) membranes fabricated via interfacial polymerization. They found that the fully aromatic PA/PPESK membranes have better thermal stability than PA/PSF membranes and can work stably under relatively high temperature. The authors also indicated that using stable salt rejection, water flux of fully aromatic PA/PPESK composite membrane will increase linearly when solution temperature rose from 20ºC to 90ºC.

Krawiec and Kaskel (2006) investigated the thermal stability of high surface area silicon carbide materials. The authors first indicated how different precursors, deposition parameters and loadings of SiC/SBA nano-composites influenced the textural properties of mesoporous silicon carbide. Next, they concluded that high surface area silicon carbide showed a higher thermal stability at 1573 K as compared to pure SBA-15 and the thermal stability of mesoporous carbon (CMK-1) was even better than that of porous SiC tested under the same conditions.

Attempting to investigate the cause of discoloration of the central region of a tall stack of plywood panels which was being cured in an ambient environment, Squire (1967) reported that no damage to the panels will be ensured if a dimensionless parameter, $q_o \beta r_o^2 / K$, does not exceed the value of 2.0; where q_o and β are material constants and K is the thermal conductivity of the material. Earlier, Landau (1959) had stated in a more lucid way, that an infinite slab would fail to exhibit a steady-state temperature distribution, if a dimensionless number, $q_o \beta L^2 / K$, exceeds the value of 0.88; where L is the half-thickness of the Landau's slab. Both investigators recognized the fact that thermal instability or thermal viability will result if the system fails to dissipate all the heat conducted into and generated from within the system - the latter, being closely related to the chemical reactions in the generic material, can and will, sometimes, continue long after the process of manufacture had been completed. In this regard, it is interesting to note that the Great Coulee Dam, one of the largest concrete structure in the United States with 9,155,942 m^3 (website), was poured with pre-chilled concrete just to compensate for the heat release from the exothermic reaction within the concrete long after the pouring process.

Both investigations cited above (Squire 1967, and Landau and Lifshitz 1959) introduce the internal heat generation source into their analysis through an exponential function which only depends on temperature. At the system's boundaries both investigations prescribed a constant temperature equal to that of the system's environment. Under these conditions, their analyses yielded simplistic results. In retrospect, the former assumption is a good approximation for more or less explosive combustion reactions, the assumption of a uniform heat source population and the use of constant temperature on the boundaries are, perhaps, plausible simplifications.

In a recent work Gadalla and El Kadi (2005) investigated the thermal stability of a composite slab-structure subjected to harsh environmental conditions. Their formulation was tested for three cases with different temperature boundary conditions. A critical thermal stability parameter was obtained for each of the cases. This parameter was shown to be function of the material properties as well as the position within the composite slab. The extension of this study to the case of a composite cylinder is considered in the current work.

Although a uniform source distribution in the cylinder is a reasonable simplification, it would seem much more pragmatic to incorporate in the analysis a spatial dependence simulating the source distribution in the medium. This is especially important in composite materials where the manufacturing process may, in some cases, produce a less-than-uniform structure. The selection of a spatially-dependent function is more restricted for the case of a cylinder than for a plane slab (Liu and Ainsworth 1984) aiming at mathematical amenability. For this reason, a one-term power function with an arbitrary constant power m was used in the current study; this power can be positive, zero, or negative. Squire's case (1967) of uniformly populated heat sources is thus a special case of the present study if m is set to zero.

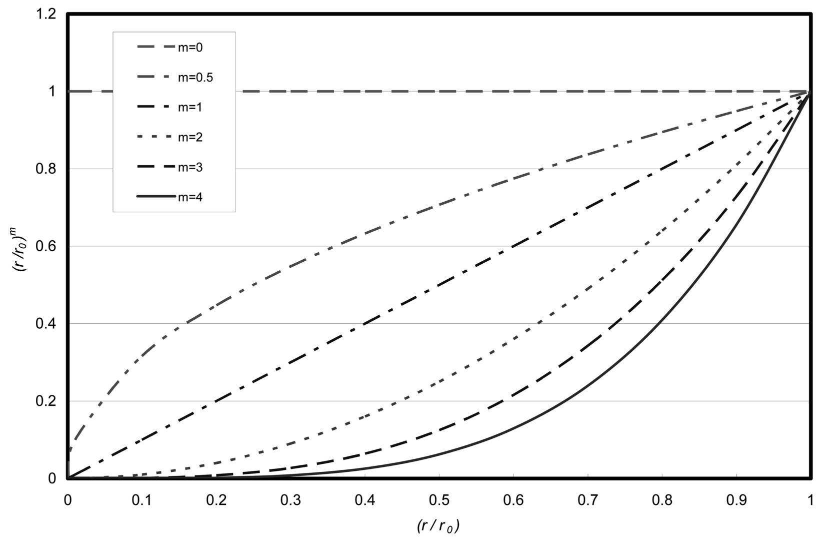
Mathematical simplicity and feasibility aside, one might note that over-designed structures, mismatched construction materials and other practicable environmental boundary conditions such insulated or nearly-insulated boundaries are all among the consequences of thermal instability. It is, therefore, wise to re-examine and re-design disproportionate structural dimensions, re-examine insulated boundaries and replace materials of low conductivity (except for cryogenic and refrigeration equipments). For this reason, this paper analyzes and predicts the magnitudes of the critical parameter, Γ_c for five cases of boundary conditions, and for several values of the arbitrary power coefficient (m).

In the case of hollow cylinders with convective boundaries, this critical parameter is no longer a constant as was the case in (Squire 1967, and Landau and Lifshitz 1959). It now depends on several factors: for instance, the material constant (m) and the Biot number ($Bi = hr_o/K$, where h is the convective heat transfer coefficient, r_0 is the outer radius of the cylinder and K is the thermal conductivity) (Holman 2002) in the case of solid cylinder with convective boundaries or the material constant, the radii ratio ($\zeta = r_i/r_o$, and the Biot number in the case of a hollow cylinder with convective boundaries. Graphically, the thermal stability critical parameter (Γ_c) may be thought of as the altitude of a surface $\Gamma_c = \Gamma_c(m, \zeta, Bi)$ below (or between) which the cylinder will be thermally stable and performance-worthy. It was found (Gadalla 1992) that an insulated outer surface and/or $m = -2$ can sustain no internal heat generation, however small.

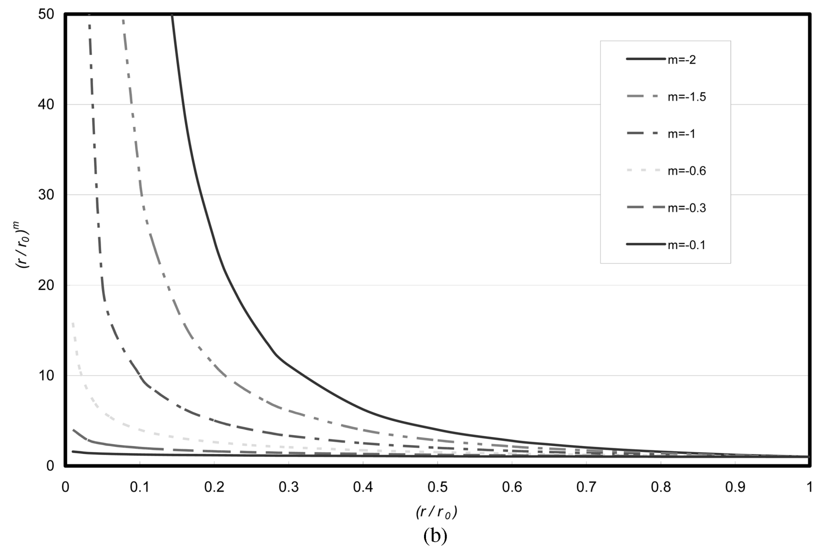
In as much as the bounds of the thermal stability of a cylindrical structure system is eminently determined by the boundary conditions to which the system is subjected, an adverse deviation from the "designed-for" conditions can significantly alter the life span of the system, not mentioning the weathering and creeping of the material with time. It might not be too soon to recommend that the critical parameter Γ_c be considered for inclusion in the building code and the specifications of structural design.

2. Problem formulation and solution

Consider an infinite hollow cylinder having an inner radius of r_i and an outer radius of r_o with a constant thermal conductivity (K). If this cylinder is seeded with an internal heat source, the rate of internal heat generation can be assumed to be governed by the law



(a)



(b)

Fig. 1 (a) Effect of the value of m on the term $(r/r_o)^m$ for the various ratios r/r_o for values of $m \geq 0$, (b) Effect of the value of m on the term $(r/r_o)^m$ for the various ratios r/r_o for values of $m < 0$

$$q = q_o\left(\frac{r}{r_o}\right)^m e^{\beta(T-T_o)} \tag{1}$$

where q_o, m, and β are material constants, and T_0 is the ambient temperature. A special case for this equation is due to Squire (1967) and can be obtained by setting $m = 0$. A heat balance yields the following governing equation for the temperature distribution in the cylinder

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{q_o}{K}\left(\frac{r}{r_o}\right)^m e^{\beta(T-T_o)} = 0 \tag{2}$$

The effect of the value of m on the term $(r/r_o)^m$ for the various ratios r/r_o is shown in Figs. 1(a) and 1(b); Fig. 1(a) shows this variation for $m \geq 0$ while Fig. 1(b) shows the variation for $m < 0$. The Squire case ($m = 0$) is also shown.

Introducing the new variables

$$\theta = T - T_o; \quad \zeta = \frac{r}{r_o}$$

Eq. (2) can be written as

$$\frac{d^2\theta}{d\zeta^2} + \frac{1}{\zeta}\frac{d\theta}{d\zeta} + \frac{q_o r_o^2}{K}\zeta^m e^{\beta\theta} = 0 \tag{3}$$

To reduce Eq. (3) into a more amenable form, the following transformation is introduced

$$\beta\theta(\zeta) = W(\zeta) - m\ln\zeta \tag{4}$$

The transformed equation can be written in the form

$$\frac{1}{\zeta}\frac{d}{d\zeta}(\zeta W') = -\Gamma e^w \tag{5}$$

where

$$\Gamma = \frac{\beta q_o r_o^2}{K} \tag{6}$$

and the prime denotes differentiation with respect to ζ.

Differentiating Eq. (5) with respect to ζ yields

$$\frac{1}{\zeta}(\zeta W')'' - \frac{1}{\zeta^2}(\zeta W')' = W'(-\Gamma e^w) \tag{7}$$

Substituting Eq. (5) into the R.H.S. of Eq. (7), we obtain

$$\zeta(\zeta W')'' - (\zeta W')' = (\zeta W')(\zeta W')' \tag{8}$$

Multiplying throughout by ζ^2, Eq. (8) takes the form of a total differential, i.e.

$$[\zeta(\zeta W')']' = 2(\zeta W')' + \frac{1}{2}[(\zeta W')^2]' \tag{9}$$

whose first primitive is

$$\zeta(\zeta W')' = 2(\zeta W') + \frac{1}{2}(\zeta W')^2 + 2C_1^2 \tag{10}$$

where $2C_1^2$ is the constant of integration.

For brevity, we denote

$$U(\zeta) = \zeta W' \tag{11}$$

Eq. (10) now reads

$$\zeta\frac{dU}{d\zeta} - 2U = \frac{U^2}{2} + 2C_1^2 \tag{12}$$

Separation of variables in Eq. (12) yields

$$\frac{2dU}{U^2 + 4U + 4C_1^2} = \frac{d\zeta}{\zeta} \tag{13}$$

The outcome of the analysis now depends on the relative magnitude of the constant C_1. For reasons we shall soon see, we choose $C_1^2 < 1$. Consequently, Eq. (13) can be written in the following form

$$-2\int\frac{dU}{\gamma^2 - (U+2)^2} = \ln(C_2\zeta) \tag{14}$$

where $\gamma = 2\sqrt{1 - C_1^2}$ and C_2 is a constant of integration. Evaluation of the integral in Eq. (14) gives

$$U(\zeta) = -2 + 2\gamma\frac{1 - C_2^\gamma\zeta^\gamma}{1 + C_2^\gamma\zeta^\gamma} \tag{15}$$

Using Eq. (11), Eq. (15) becomes after integration

$$W(\zeta) = \ln\frac{C_3\zeta^\gamma}{\zeta^2(1 + C_2^\gamma\zeta^\gamma)^2} \tag{16}$$

where C_3 is a constant of integration.

Subsequent substitution of Eq. (16) into Eq. (4) yields

$$\beta\theta(\zeta) = \ln C_3 + \gamma\ln\zeta - (2+m)\ln\zeta - 2\ln(1 + C_2^\gamma\zeta^\gamma) \tag{17}$$

To obtain a general form suitable for the case studies under consideration, some algebraic manipulations are required; first, differentiating Eq. (17), we get

$$\frac{d\beta\theta(\zeta)}{d\zeta} = \frac{\gamma - 2 - m}{\zeta} - 2\gamma\frac{C_2^\gamma\zeta^{\gamma-1}}{1 + C_2^\gamma\zeta^\gamma} \tag{18}$$

Then, taking the exponential function of both sides of Eq. (17), we obtain

$$e^{\beta\theta(\zeta)} = \frac{C_3\zeta^\gamma}{\zeta^{2+m}(1 + C_2^\gamma\zeta^\gamma)^2} \tag{19}$$

Eq. (19) includes three arbitrary constants: γ, C_2, and C_3 - recall that $(\gamma/2)^2 = 1 - C_1^2$. Since there are only two boundary conditions, one for each of the cylindrical surfaces at $\zeta > 0$ and $\zeta = 1$, one more condition is needed for the determination of these three constants. This needed condition can be obtained, however, by substituting Eq. (17) into Eq. (3), yielding

$$\Gamma C_3 = 2\gamma^2 C_2^\gamma \tag{20}$$

In the case of a solid cylindrical composite structure, to obtain a finite value for the temperature along the cylinder axis, the value of γ must be set at $(2 + m)$ (see Eq. 17). Substituting, we get

$$\beta\theta(\zeta) = \ln C_3 - \ln(1 + C_2^{2+m}\zeta^{2+m})^2 \tag{21}$$

Differentiating with respect to ζ, we get

$$\frac{d(\beta\theta(\zeta))}{d\zeta} = -2(2+m)\frac{C_2^{2+m}\zeta^{1+m}}{1 + C_2^{2+m}\zeta^{2+m}} \tag{22}$$

For a solid Squire cylinder (a solid infinite cylinder in which the rate of internal heat generation is independent of the heat source location in the cylinder, but is an exponential function of the temperature differential; i.e., $m = 0$), Eqs. (21) and (22) are simplified as

$$\beta\theta(\zeta) = \ln\frac{C_3}{(1 + C_2^2\zeta^2)^2} \tag{23}$$

$$\frac{d\beta\theta(\zeta)}{d\zeta} = -4\frac{C_2^2\zeta}{1 + C_2^2\zeta^2} \tag{24}$$

For a solid cylinder with $m > 0$, the required condition becomes

$$\Gamma C_3 = 2(2+m)^2 C_2^{2+m} \tag{25}$$

For a Squire cylinder where $m = 0$, this condition is reduced to

$$\Gamma C_3 = 8C_2^2 \tag{26}$$

Eq. (17) will now be used to develop different solutions for five different cases for solid and hollow cylinders subjected to a variety of boundary conditions as shown in Fig. 2. Each boundary condition for hollow cylinders is supplemented with Eq. (20) for a full determination of the constants, γ, C_2 and C_3. While Eq. (21) will be used to determine the constants C_2 and C_3 for solid cylinders.

3. Case studies

Case A - Solid cylindrical structure ($m \neq 0$), with constant boundary temperature (T_0). The sole boundary condition for this case as shown in Fig. 2(a) is

$$T(r_o) = T_o$$

This condition can also be written as

$$\theta(1) = 0 \tag{27}$$

Substituting for the boundary condition in Eq. (21), we get

$$C_3 = (1 + C_2^{2+m})^2 \tag{28}$$

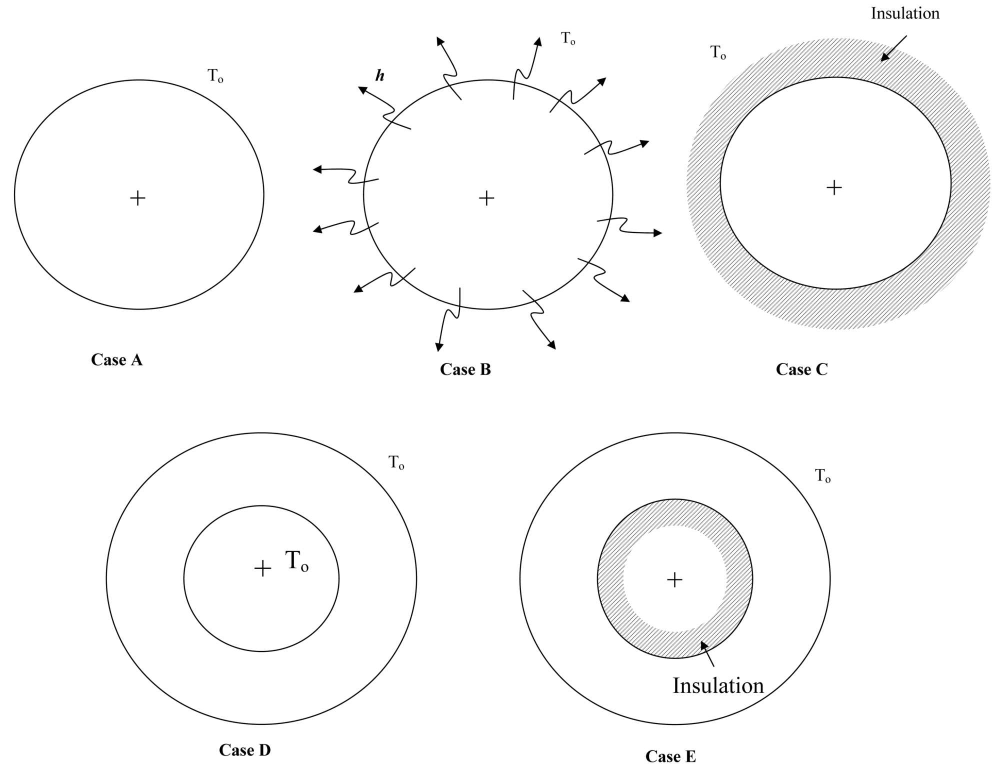


Fig. 2 Schematic representation of analyzed case studies and their corresponding boundary conditions

To solve for C2 and C3 we need the aid of Eq. (25) namely $C_3 = 2(2+m)^2 C_2^{2+m}/\Gamma$. Eliminating C_3 between Eqs. (25) and (28), we obtain

$$\Gamma = \frac{2(2+m)^2 C_2^{2+m}}{(1+C_2^{2+m})^2} \tag{29}$$

To determine the critical value of the parameter Γ, differentiate Γ in Eq. (29) with respect to C_2^{2+m}, i.e.

$$\frac{d\Gamma}{dC_2^{2+m}} = 2(2+m)^2 \frac{1-C_2^{2+m}}{(1+C_2^{2+m})^3}$$

Setting $d\Gamma/dC_2^{2+m}$ to zero, we obtain

$$C_{2c}^{2+m} = 1 \tag{30}$$

where the subscript c denotes criticality.

Consequently, Eqs. (28) and (29) give

$$C_{3c} = 4 \tag{31}$$

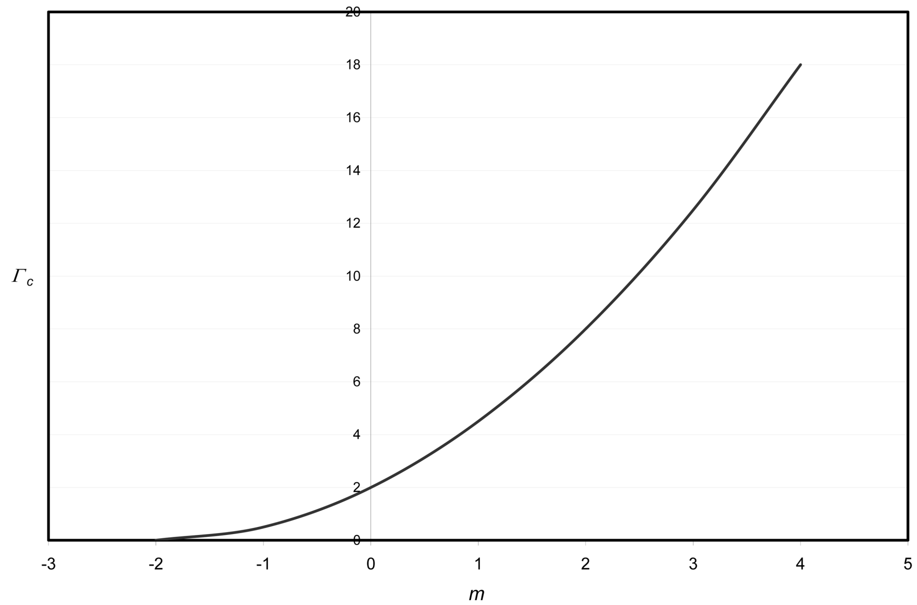


Fig. 3 Case A: Variation of the critical stability parameter (Γ_c) with the power coefficient (m)

Substituting for the value of C_{2c} from Eq. (30) into Eq. (29), we obtain the critical value of the parameter Γ to be

$$\Gamma_c = 2\left(1+\frac{m}{2}\right)^2 \tag{32}$$

or for simplification

$$\sqrt{\Gamma_c/2} = 1+\frac{m}{2} \tag{33}$$

Fig. 3 shows the value of the critical thermal stability parameter Γ_c as a function of the power coefficient, m. As shown, the value of Γ_c systematically increases with an increase of m. The figure also shows that for m = 0 (Squire's case), the critical thermal-stability parameter, $\Gamma_c = 2$, which is the same result obtained by Gadalla (1992).

Case B - Solid cylindrical structure with convective heat transfer

In this case as shown in Fig. 2(b), the boundary condition must satisfy the equation

$$-K\frac{dT}{dr}\bigg|_{r=r_o} = h[T(r_o)-T_0] \tag{34}$$

This can also be rewritten as

$$-\frac{d\theta}{d\zeta}(1) = Bi\,\theta(1) \tag{35}$$

Substitution of Eq. (21) into Eq. (36) yields

$$\frac{2(2+m)C_2^{2+m}}{1+C_2^{2+m}} = Bi\ln\frac{C_3}{(1+C_2^{2+m})^2} \tag{36}$$

Eliminating C_3 between Eqs. (37) and (26)

$$\frac{\Gamma}{2(2+m)^2} = \frac{C_2^{2+m}}{(1+C_2^{2+m})^2}e^{\frac{-2(2+m)}{Bi}\frac{C_2^{2+m}}{(1+C_2^{2+m})}} \tag{37}$$

To obtain the critical value of the parameter Γ, differentiate it with respect to C_2^{2+m}, and subsequently equating the differential term to zero, we obtain

$$C_{2c}^{2(2+m)} + \frac{2(2+m)}{B_i}C_{2c}^{2+m} - 1 = 0 \tag{38}$$

Solving

$$C_{2c}^{2+m} = -\frac{2+m}{Bi}\sqrt{1+\frac{(2+m)^2}{Bi^2}} \tag{39}$$

Substitution of Eq. (39) into Eq. (37) yields

$$\sqrt{\frac{\Gamma_c}{2}} = (2+m)\frac{\sqrt{-w+\sqrt{1+w^2}}}{1-w+\sqrt{1+w^2}}e^{-w\frac{-w+\sqrt{1+w^2}}{1-w+\sqrt{1+w^2}}} \tag{40}$$

where $w = (2+m)/B_i$. Eq. (41) reduces to the solution for a squire cylinder with convective

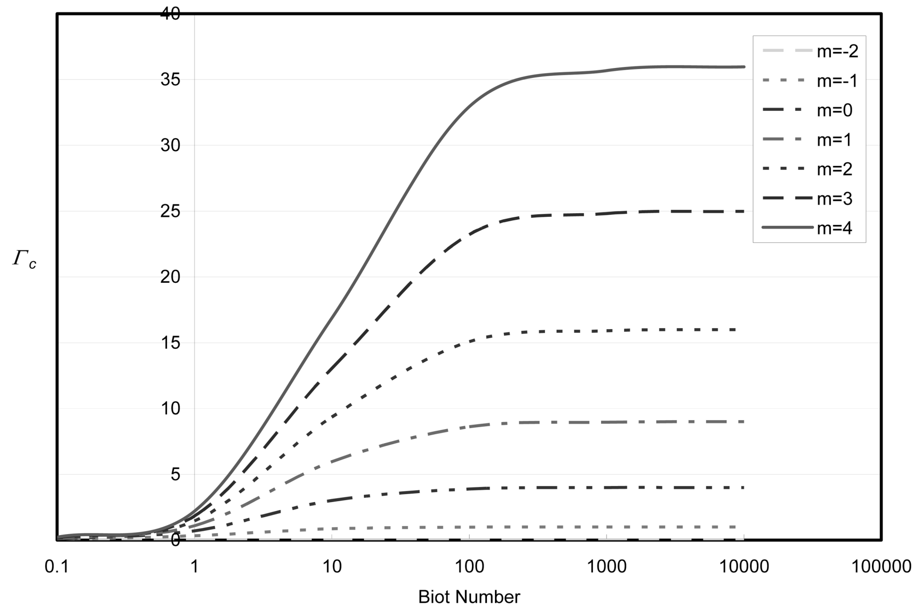


Fig. 4 Case B: Variation of the critical stability parameter (Γ_c) with Biot number for different values of the power coefficient (m)

boundary, if $m = 0$; it becomes $\sqrt{\Gamma_c/2} = 0$ if $m = -2$.

Eq. (41) is graphically depicted in Fig. 4 for $m = -2$ to 4. The figure shows the values of the critical thermal stability parameter Γ_c as a function of Biot number. The figure shows that an increase in the power coefficient m results in an increase in the value of Γ_c. The figure also shows that $\Gamma_c = 0$ at $m = -2$ as indicated in Holman (2002).

Case C - Solid cylindrical structure with insulated surface

According to Eq. (21), the temperature distribution for a solid cylinder with insulated surfaces is

$$\beta\theta(\zeta) = \ln C_3 - \ln(1 + C_2^{2+m}\zeta^{2+m})^2$$

For this case the boundary condition on the outer insulated cylindrical surface as indicated in Fig. 2(c) is

$$\frac{d}{d\zeta}(\theta(1)) = 0 \tag{41}$$

Or, from Eq. (21)

$$0 = -2(2+m)\frac{C_2^{2+m}}{1 + C_2^{2+m}} \tag{42}$$

To obtain a non-trivial solution, $(2 + m) \neq 0$, i.e.

$$C_2^{2+m} = 0 \tag{43}$$

Consequently, the temperature distribution in the cylinder is

$$\beta\theta(\zeta) = \ln C_3 \tag{44}$$

The constant C_3 is now determined by the use of Eq. (26), which, using Eq. (44), states that

$$\Gamma C_3 = 0 \tag{45}$$

Since we must have a finite temperature in the cylinder, we require that C_3 be different from zero; hence we conclude that

$$\Gamma_c = \Gamma = 0 \tag{46}$$

Case D - Hollow cylindrical stricture with constant temperature T_0 on both boundaries

The boundary conditions for this case as shown in Fig. 2(d) are:

$T(r_i) = T_o$ and $T(r_o) = T_o$; in other words, using the dimensionless variables introduced earlier

$$\theta(\zeta) = 0 \quad \text{and} \quad \theta(1) = 0 \tag{47}$$

Substituting for the two boundary conditions in Eq. (17) results in two relations between the constants C_2 and C_3, i.e.

$$C_3 = \zeta^{2+m-\gamma}(1 + C_2^{\gamma}\zeta^{\gamma})^2 \tag{48}$$

$$C_3 = (1 + C_2^{\gamma})^2 \tag{49}$$

Eliminating C_3 between Eqs. (49) and (20), we get

$$(1 + C_2^{\gamma})^2 = \frac{2\gamma^2}{\Gamma} C_2^{\gamma} \tag{50}$$

Solving for C_2^{γ}, we obtain

$$C_2^{\gamma} = \frac{\gamma^2}{\Gamma} - 1 + \sqrt{\frac{\gamma^2}{\Gamma}\left(\frac{\gamma^2}{\Gamma} - 2\right)} \tag{51}$$

To be certain that the value of C_2^{γ} obtained from the previous equation is real, set the contents of the radical to zero. This results in

$$\gamma_c^2 = 2\Gamma_c \tag{52}$$

Substituting in Eq. (51)

$$(C_2^{\gamma})_c = 1 \tag{53}$$

Substituting in Eq. (52), the value of the critical parameter C_3 is

$$(C_3)_c = 4 \tag{54}$$

Substituting from Eqs. (52)-(54) into Eq. (49) yields

$$4 = \zeta^{2+m-\sqrt{2\Gamma_c}}(1 + \zeta^{\sqrt{2\Gamma_c}})^2$$

or

$$\zeta^{\sqrt{\frac{\Gamma_c}{2}}} = \frac{1 - \sqrt{1 - \zeta^{2+m}}}{\zeta^{1+\frac{m}{2}}} \tag{55}$$

Rearranging, the critical parameter can be given explicitly as

$$\sqrt{\frac{\Gamma_c}{2}} = \frac{1}{\ln\zeta}\ln\left[\frac{1 - \sqrt{1 - \zeta^{2+m}}}{\zeta^{1+\frac{m}{2}}}\right] \tag{56}$$

Eq. (56) is depicted in Fig. 5 as a group of curves for Γ_c as a function of ζ for various values of the power exponent m (a material constant). For a specific value of ζ, an increase in m results in a higher value of the critical thermal stability parameter Γ_c. This increase in the thermal stability of the material is mainly due to the lower value of the internal heat generation rate developed within the cylinder. The figure also shows that increasing ζ (decreasing the cylindrical thickness for the

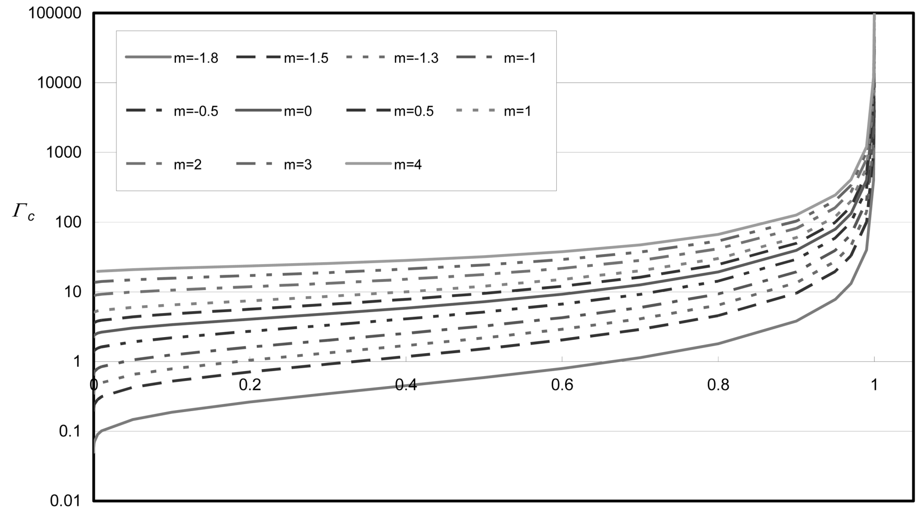


Fig. 5 Case D: Variation of the critical stability parameter (Γ_c) with the radius parameter (ζ) for different values of the power coefficient (m)

same value of the inside radius) for the same value of m results in a gradual increase in the value of Γ_c until the value of ζ is close to unity. At this point, the value of Γ_c increases asymptotically.

It can be shown for a hollow polymeric cylinder ($m = 0$ and $\zeta > 0$), Eq. (56) becomes

$$\left.\sqrt{\frac{\Gamma_c}{2}}\right|_{m=0} = \frac{1}{\ln\zeta}\ln\left[\frac{1-\sqrt{1-\zeta^2}}{\zeta}\right] \tag{57}$$

For the case of a solid composite cylinder ($m \neq 0$ and $\zeta \to 0$), the equation becomes

$$\left.\sqrt{\frac{\Gamma_c}{2}}\right|_{\zeta\to 0} = \lim_{\zeta\to 0}\frac{\dfrac{\partial}{\partial\zeta}\ln\left[\dfrac{2}{\zeta^{2+m}} - \sqrt{\dfrac{1}{\zeta^{2+m}} - 1}\right]}{\dfrac{\partial}{\partial\zeta}\ln\zeta}$$

or

$$\left.\sqrt{\frac{\Gamma_c}{2}}\right|_{\zeta\to 0} = \left.\left(1+\frac{m}{2}\right)\frac{(1-\zeta^{2+m})^{1/2}}{(1-\zeta^{2+m})^{3/2}}\right|_{\zeta\to 0}$$

Simplifying

$$\left.\sqrt{\frac{\Gamma_c}{2}}\right|_{\zeta\to 0} = 1+\frac{m}{2} \tag{58}$$

Eq. (57) presents the critical thermal-stability parameter for hollow polymeric cylinder and its critical relationship with the cylinder size ratio ζ (depicted graphically in Fig. 5 for $m = 0$). The magnitude of the critical thermal-stability parameter Γ_c for this case is seen to rise almost vertically from 2 at $\zeta = 0$ as indicted in Eq. (58), slowly attains a magnitude of 7.22 at $\zeta = 0.5$, and steeply rise again as $\zeta \to 1$ The results also indicate that a hollow polymeric cylinder can tolerate much greater internal heat generation rate than a solid polymeric cylinder of the same outer radius. This added capability is mainly due to the second boundary condition at the inner radius providing an additional venue of heat dissipation from the cylinder to the atmospheric surroundings, unless this venue is blocked with insulation or poor ventilation process. Finally Eq. (58) assures the critical thermal-stability parameter obtained by Eq. (33) for a solid cylindrical composites subjected to a constant temperature.

Case E - Hollow cylindrical structure with inner boundary insulated and outer boundary at a constant temperature

The boundary conditions for the case of a hollow cylinder with an insulated inner boundary and an outer boundary kept at a constant temperature T_0 as shown in Fig. 2(e) are

$$\frac{d\theta}{d\zeta}(\zeta) = 0, \quad \theta(1) = 0 \tag{59}$$

Substituting for the two boundary conditions in Eqs. (17) and (18) results in two relations between the constants C_2 and C_3, i.e.

$$0 = \frac{\gamma-(2+m)}{\zeta} - \frac{2\gamma C_2^{\gamma}\zeta^{\gamma-1}}{1+C_2^{\gamma}\zeta^{\gamma}} \tag{60}$$

$$C_3 = (1+C_2^{\gamma})^2 \tag{61}$$

Since the present case and the previous case (case D) have the same constant temperature condition on the outer boundary at which $\zeta = 1$, we may make good use of the partial results of case *D*, namely,

$$\gamma_c^2 = 2\Gamma_c\,, \quad C_{2c}^{\gamma_c} = 1 \quad \text{and} \quad C_{3c} = 4$$

Substitution for these variables into Eq. (61) gives

$$\frac{\sqrt{2\Gamma_c}-(2+m)}{\zeta} = \frac{2\sqrt{2\Gamma_c}\zeta^{\sqrt{2\Gamma_c}-1}}{1+\zeta^{\sqrt{2\Gamma_c}}} \tag{62}$$

Solving for $\zeta^{\sqrt{2\Gamma_c}}$, results in

$$\zeta^{\sqrt{2\Gamma_c}} = \frac{\sqrt{2\Gamma_c}-(2+m)}{\sqrt{2\Gamma_c}+(2+m)} \tag{63}$$

Taking logarithm of both sides of Eq. (63)

$$\ln\zeta = \frac{1}{\sqrt{2\Gamma_c}}\ln\left[\frac{\sqrt{2\Gamma_c}-(2+m)}{\sqrt{2\Gamma_c}+(2+m)}\right] \tag{64}$$

which can be written as

$$\sqrt{\frac{\Gamma_c}{2}} = \frac{1}{2\ln\zeta}\ln\left[\frac{\sqrt{\frac{\Gamma_c}{2}}-\left(1+\frac{m}{2}\right)}{\sqrt{\frac{\Gamma_c}{2}}+\left(1+\frac{m}{2}\right)}\right] \tag{65}$$

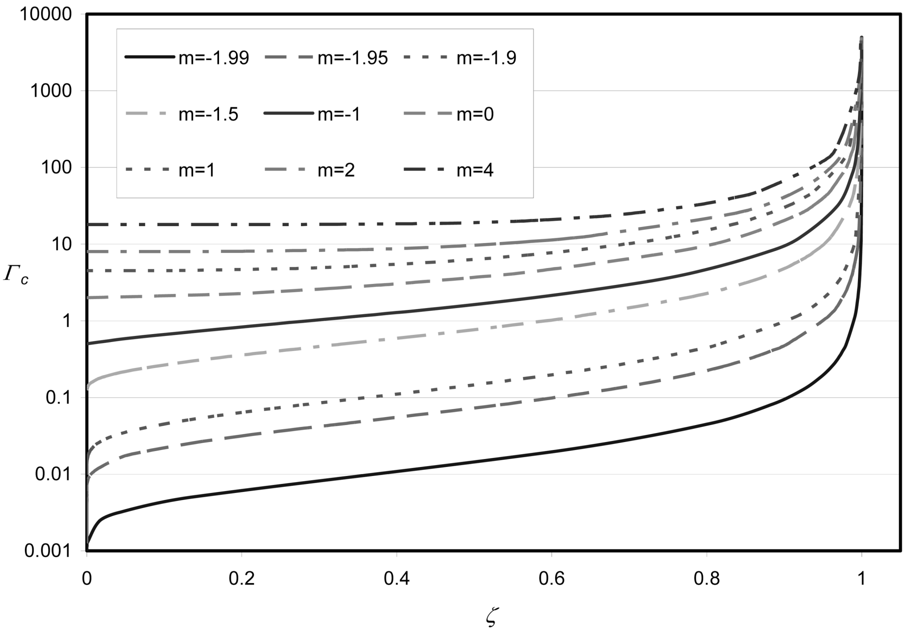


Fig. 6 Case E: Variation of the critical stability parameter (Γ_c) with the radius parameter (ζ) for different values of the power coefficient (m)

The graphical representation of Eq. (65) is depicted in Fig. 6 where Γ_c is plotted as a function of ζ for various values of the power exponent m (material constant). As was the case in case study D, for a specific value of ζ, an increase in m results in a higher value of the critical thermal-stability parameter Γ_c. The figure also shows that increasing ζ for the same value of m results in a vertical increase in the value of Γ_c at $\zeta = 0$ followed by a gradually increase of Γ_c for higher values of ζ and then a steep rise in Γ_c as ζ is close to unity. At this point, the value of Γ_c increases asymptotically.

Had the inner boundary of the cylindrical composite/polymeric structures in case D been insulated as described in case E, the critical thermal-stability parameter would have been reduced to almost half of that of case D (for example, for $\zeta = 0.5$, $\Gamma_c = 3.7$). This is due to the blockage in the rate of energy dissipated form the inner surface of the composite/polymeric structures.

Eq. (65) shows that for $\zeta > 0$, a value of $m = -2$ will result in a critical value of

$$\sqrt{\frac{\Gamma_c}{2}} = 0$$

This shows the necessity of limiting the value of the coefficient m such that the maximum value obtained for ζ is not greater than unity. To obtain the limiting value of the coefficient m, let

$$\frac{\sqrt{\frac{\Gamma_c}{2}} - \left(1 + \frac{m}{2}\right)}{\sqrt{\frac{\Gamma_c}{2}} + \left(1 + \frac{m}{2}\right)} \leq 1$$

or

$$\sqrt{\frac{\Gamma_c}{2}} + \left(1 + \frac{m}{2}\right) \geq \sqrt{\frac{\Gamma_c}{2}} - \left(1 + \frac{m}{2}\right)$$

Thus: $m \geq -2$.
Therefore, we conclude for $m \leq -2$, $\sqrt{\frac{\Gamma_c}{2}} = 0$ If $m = -2$, then $\zeta^{\sqrt{2\Gamma_c}} = 1$.

4. Observations

When the temperature of the cylindrical surface of quasi isotropic composite/polymeric materials is subjected a temperature equal to the ambient temperature T_0, the convective heat transfer coefficient is infinitely high. The Squire boundary condition is therefore the most efficient mechanism of dissipating energy from the cylindrical structures. On the other hand the insulated outer surface boundary would in turn reduce the amount of energy dissipated to the environment to zero, thus the coefficient of convective heat transfer is zero. When the magnitude of the value of the convective heat transfer coefficient is between the two limits, the cylindrical composite/polymeric structures will be exposed to convective heat transfer with Bi number greater than zero and less than infinity. Therefore the environment has a great impact on thermal stability on cylindrical structures. To control the life span of the structure, the critical value of thermal stability parameter could be controlled by choosing some selected material properties or by resizing the structure for specified material properties. This can be achieved during the design stage as well as the repairing process.

5. Conclusions

Analytical modeling and solution for thermal stability of cylindrical quasi-isotropic composite and/or polymeric structures under different configurations and boundary condition is developed. The analytical model represents a class of closed form solution that can be a very effective tool during the design stage and/or during the repair of a structure due to any malfunction. Since these structures are subjected to different environmental boundary conditions depending on the engineering application, thermal instability or thermal viability of these structures may result if the structures fail to dissipate all of the internal energy generated long after the manufacturing process or due to an internal heat generation source. Based on the analytical model and the case studies presented, it is concluded that:

1. The critical thermal stability parameter of the quasi-isotropic composite structure is not linearly related to the material constant or the cross sectional properties.
2. The thermal behavior of cylindrical structures with altered sections and/or materials due to modification, maintenance or repairing can differ drastically from the original structures under the same environment.
3. The critical thermal stability parameters of quasi-isotropic composites with positive material constant m are higher than for polymeric structures of the same size.
4. The critical thermal stability parameter of polymeric structures represents the upper limit of composites structures with negative material constants. This is due to the higher capability of polymers in dissipating the heat generated within the structure to the environment.

5. Hollow cylindrical quasi-isotropic composites/polymeric structures can tolerate much greater internal heat generation rate than solid structures of the same outer radius.
6. Any inadvertent changes in the size and/or the material of the cylindrical structures from the designed-for conditions can alter the life span of these structures. Additionally, the environment in which a cylindrical structure is used can significantly alter its life-span.
7. An effective tool for selecting the desired material constants and the trade-off between the material, structural cross section and/or environment was developed.

References

Abbassi, A. and Shahnazari, M.R. (2004), "Numerical modeling of mold filling and curing in non-isothermal RTM process", *Appl. Therm. Eng.*, **24**, 2453-2465.

Blest, D.C., Duffy, B.R., McKee, S. and Zulkifle, A.K. (1999), "Curing simulation of thermoset composites", *Compos. Part A-Appl. S.,* **30**, 1289-1309.

Blest, D.C., McKee, S., Zulkifle, A.K. and Marshall, P. (1999), "Curing simulation by autoclave resin infusion", *Compos. Sci. Technol.,* **59**, 2297-2313.

Costa, V.A.F. and Sousa, A.C.M. (2003), "Modeling of flow and thermo-kinetics during the cure of thick laminated composites", *Int. J. Therm. Sci.*, **42**, 15-22.

David Rouison, Sain M. and Couturier, M. (2004), "Resin transfer molding of natural fiber reinforced composites cure simulation", *Compos. Sci. Technol.*, **64**, 629-644.

Gadalla, M. (1992), "Bounds of Thermal stability of cylindrical structures with non-uniform internal heat generation", In: Proceedings of the 27th Intersociety of Energy Conversion Engineering Conference, San Diego, California.

Gadalla, M. and El Kadi, H. (2005), "Thermal stability of composite slab-structures in harsh environment", *Compos. Struct.*, **71**, 447-452.

Holman, J.P. (2002), *Heat Transfer*. International Edition: McGraw Hill.

Krawiec, P. and Kaskel, S. (2006), "Thermal stability of high surface area silicon carbide materials", *J. Solid State Chem.*, **179**, 2281-2289.

Landau, L.D. and Lifshitz, E.M. (1959), *Fluid Mechanics*. Reading: Addison-Wesley, 191.

Liu, C.K. and Ainsworth, O.R. (1984), "Bonds of stability in plane conducting systems with non-uniform internal heat generation", In: SECTAM XII, Callaway Gardens GA, May.

Monteverde, F. (2005), "The thermal stability in air of hot-pressed diboride matrix composites for uses at ultra-high temperatures", *Corros. Sci.*, **47**, 2020-2033.

Squire, W. (1967), "A mathematical analysis of self-ignition", In: Noble D. editor. Application of Undergraduate Mathematics in Engineering. Macmillan, 184.

Wei, J., Jian, X., Wu, C., Zhang, S. and Yan, C. (2005), "Influence of polymer structure on thermal stability of composite membranes", *J. Membrane Sci.*, **256**, 116-121.

WWW.informat.io/title=Great_Coulee_Dam.

Yang, H.C. and Colton Jonathon, S. (1995), "Thermal analysis of thermoplastic composites during processing", *Polym. Compos.*, **16**, 198-203.