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On the assessment of modal nonlinear pushover analysis for steel frames with semi-rigid connections

Panam Zarfam† and Massood Mofid‡

Department of Civil Engineering, Sharif University of Technology, Tehran, Iran

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Abstract. Applying nonlinear statistical analysis methods in estimating the performance of structures in earthquakes is strongly considered these days. This is due to the methods' simplicity, timely lower cost and reliable estimation in seismic responses in comparison with time-history nonlinear dynamic analysis. Among nonlinear methods, simplified to be incorporated in the future guidelines, Modal Pushover Analysis, known by the abbreviated name of MPA, simply models nonlinear behavior of structures; and presents a very proper estimation of nonlinear dynamic analysis using lateral load pattern appropriate to the mass. Mostly, two kinds of connecting joints, 'hinge' and 'rigid', are carried out in different type of steel structures. However, it should be highly considered that nominal hinge joints usually experience some percentages of fixity and nominal rigid connections do not employ totally rigid. Therefore, concerning the importance of these structures and the significant flexibility effect of connections on force distribution and elements deformation, these connections can be considered as semi-rigid with various percentages of fixity. Since it seems, the application and implementation of MPA method has not been studied on moment-resistant steel frames with semi rigid connections, this research focuses on this topic and issue. In this regard several rigid and semi-rigid steel bending frames with different percentages of fixity are selected. The structural design is performed based on weak beam and strong column. Followed by that, the MPA method is used as an approximated method and Nonlinear Response History Analysis (NL-RHA) as the exact one. Studying the performance of semi-rigid frames in height shows that MPA technique offers reasonably reliable results in these frames. The methods accuracy seems to decrease, when the number of stories increases and does decrease in correlation with the semi-rigidity percentages. This generally implies that the method can be used as a proper device in seismic estimation of different types of low and mid-rise buildings with semi-rigid connections.

Keywords: Modal Pushover (MPA); semi-rigid connection; non-linear analysis.

1. Introduction

As non-linear dynamic response history is a complex and tedious method in calculating seismic demands, Nonlinear Static Pushover (NSP) and/or incremental nonlinear static analysis- explained in FEMA 273 (1997), ACT-40 (1996) as well as FEMA 356 (2000), are employed. In these methods, the selected structures are gradually imposed through the lateral loads along with height, till the displacement of a specific point (usually roof) reaches the target displacement. The explanations of

† Ph.D. Candidate, E-mail: zarfam@civil.sharif.edu
‡ Professor, Corresponding author, E-mail: mass3001@yahoo.com

MPA can be found in detail, in Krawinkler and Seneviratna articles (Krawinkler and Seneviratna 1998). In these processes, constant distribution of lateral loads is considered. It is also assumed that the final response is determined upon the first mode of the structure. However, these two assumptions become completely approximate, when structure moves into the plastic region. Despite the results correctness, obtained for short and mid-rise structures in the inelastic phase, the constant forces distribution did not consider the values of higher modes. To prevail over this insufficiency, Sasaki *et al.* (1998) and Gupta *et al.* (2000) presented several approaches for the affecting of higher modes on the response of structure. Finally, the MPA method which considered several modes and obtained results of the scientific demands of the buildings, more accurately than others, was presented by Chopra and Goel (2001). This method is based on the dynamic theory of structures; therefore, the significance of this technique is more applicable in high-rise building. The MPA method was primarily checked only for some special structures; however, Chopra and Goel tested it for SAC buildings in 2002 and obtained acceptable results (Goel and Chopra 2002). Then, Chopra and Chintanapakee (2004) studied the accuracy of MPA approach for wide ranges of regular and irregular buildings, as well as a group of ground motions.

According to the experimental test and assessments, it has been cleared that hinge joints do not behave thoroughly hinge and transfer partially the beam moment to the columns. Also rigid connections do not completely behave in rigid form and may show some flexibility. In this regard, AISC guideline has divided building frame connections into three groups- hinge, rigid and semis-rigid, based on allowable stress method; also into two groups- full and partial resistance connections, based on LRFD procedure (AISC 1994). It is of the great importance to note that in full resistant connections, the plastic hinge may get formed in beams. While in partial resistance connection, plastic hinge shall mostly get appeared in the connections. Therefore, it can be understood that four statuses- rigid/semi rigid connections with full and/or partial resistance, are exercised in the moment-resistant steel frames structures.

From the economic point of view as well as, ductility behavior, the semi-rigid connections in the structures act more reasonably and realistically; and this is due to the decreasing in the forces of the elements linked to the connection. However, it is essential to get mentioned, mostly all works in this area has been abandoned and not too many fundamental investigations, neither in Building Codes, nor in research papers, have been offered. According to an extensive investigation on steel structures, the beam-to-column connections are usually one of the most important cause of the general failure of the structures, during the sever earthquakes. This typically reveals the necessity of further study on the semi-rigid connection in partial absorption of the energy, released throughout the earthquakes.

On the other-hand, the performance of the steel structures with semi-rigid connections, are pretty knotty and complicated. Also, it is known that from engineering point of view that all connections according to M-θ curves behave accordingly. Therefore, the necessity of implementation of simple and accurate nonlinear approximate methods is extremely required. In this investigation, first the application of the MPA technique on the moment-resistant frames with semi-rigid connections is fairly introduced. Then, according to the existing procedure, a few trial models are examined. Finally, the comparison of the results between the presented method and non-linear response history analysis (NL-RHA), which are the most significant outcomes of this investigation, is reasonably obtained. In the following, also a simplified/amended MPA method is offered; and comparison of the both methods is presented, as well.

2. Modal Pushover Analysis (MPA)

2.1 General concepts

In MPA, maximum displacement of an appropriate mode of an equivalent single degree of freedom (SDF) structure is calculated, throughout a nonlinear dynamic response history analysis and an arbitrary earthquake record. The target displacement of the main structure in any mode can easily get obtained, throughout the multiplication of this amount by modal incorporating factor. Then, the main structure in any mode is pushed under lateral load, proportional to the shape of considered mode. Finally, the appropriate outcomes are determined; combining the results from pushover analysis responses of the structure in any mode.

2.2 Step-by-step MPA, working procedure

In this paper two types Modal Pushover Analysis are examined. Type (A) of typical MPA method, which has properly been utilized in various references can be summarized in a step-by-step working procedure, as following:

1- Calculating natural frequencies (ω_n) and elastic mode shapes (ϕ_n) of buildings.

2- Shaping of the capacity curves, in the form of base-shear versus the roof-displacement ($V_{bn} - u_{rn}$) for the n^{th} mode, under force distribution of $\mathbf{s}_n^* = \mathbf{m}\phi_n$

3- Identifying the capacity curves as bi-linear curves, with the strain-hardening ratio α_n, (Fig. 1(a)).

4- Converting idealized capacity curve to SDF one, using force-displacement relation: $F_{sn}/L_n - D_n$ (Fig. 1(b)).

5- Calculating maximum displacement D_n in the n^{th} mode of non-linear SDF system, throughout the solution of equation $\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + F_{sn}/L_n = -\ddot{u}_g(t)$ or from non-elastic and/or design spectrum for each record.

6- Pushing the structure until the roof-displacement reaches the target displacement (u_{rno}), obtained by $u_{rno} = \Gamma_n\phi_{rn}D_n$ equation for each record and then determining appropriate responses throughout the pushover analysis. In this formula, Γ_n is modal participation factor and ϕ_{rn} is value of the n^{th} mode, in roof level.

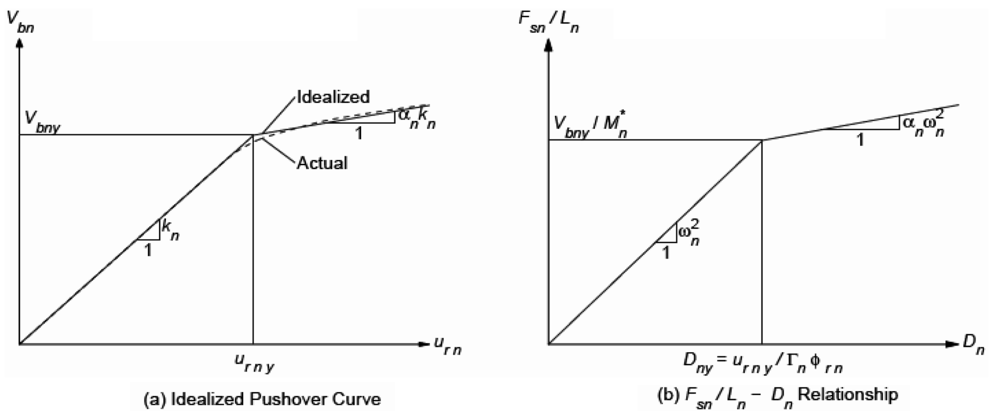

Fig. 1 (a) Idealized capacity curve, (b) Force-displacement relation of the equivalent SDF structure

7- Repeating the steps 3 to 6 for the numbers of appropriate modes, sufficient for reaching the proper accuracy. It should get noted the experience shows that for regular steel structures, the first 2 or 3 modes would definitely be sufficient.
8- Do the erythematic averaging of final responses in each mode for a group of ground motions.
9- Determination of maximum modal responses through the SRSS method.
However, in Type (B), some slight modifications from step 5 should get employed, as followings:
5*- $\hat{D}_n$ is obtained for each mode through erythematic averaging of maximum deformation values of the equivalent single degree of freedom system for a group of ground motions.
6*- The median value of target displacement get identified, concerning $u_{rno} = \Gamma_n \phi_{rn} \hat{D}_n$ formula.
The rest shall be the same, not including step 8.

3. Structural systems, ground motions and response statistics

3.1 Presenting the structural system

3.1.1 Rigid frames

Different types of example framing systems are selected, as exhibited in Fig. 2. All illustrated test structures are steel moment-resisting frames with IPB and INP normal European standard, rigid diaphragm and variable fixity. Furthermore, each frame consists of three spans of five meters and storey height of three meters. The primary design is based on UBC code (2005), according to stiff soil (NEHRP site, class D) and high risk region. In consistent with strong columns-weak beam philosophy, it expected that all plastic hinges get formed in end-beams and supports. Drifts are also carefully monitored, according to the code requirements.

3.1.2 Semi rigid frames

As it was mentioned in the previous section the frames have been loaded and optimally designed, based on UBC (2005) and AISC (2006) codes (AISC 2006). All designs are base on full fixity of

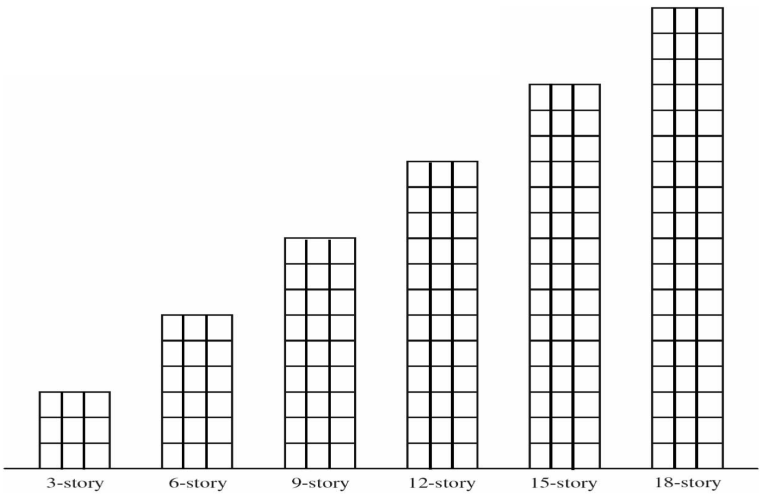

Fig. 2 The number of stories of the applied three span frames

Table 1 The features of semi-rigid connections, used in various frames

		Section	IPE 300	IPE 330	IPE 360	IPE 400	IPE 450
Beam		L (cm)	500	500	500	500	500
		Ix (cm^4)	8356	11770	16270	23130	33740
		My (t.cm)	1337	1712	2169	2776	3600
Connection's Rigidity	R = 90%	K_θ (t.cm/rad)	613334	863923	1194225	1697751	2476529
		$M_p = 0.75$	1003	1284	1627	2082	2700
		α (%)	10	10	10	10	10
	R = 75%	K_θ (t.cm/rad)	204445	287974	398075	565917	825510
		$M_p = 0.75$	1003	1284	1627	2082	2700
		α (%)	10	10	10	10	10
	R = 50%	K_θ (t.cm/rad)	68148	95991	132692	188639	275170
		$M_p = 0.75$	1003	1284	1627	2082	2700
		α (%)	10	10	10	10	10

the connections. However, in this study beam-line theory is used to determine the connection stiffness (Kishi and Chen 1986), obtaining 90%, 75% and 50% fixity of the connections. It should get also mentioned that in order to model the semi-rigid connection, the parameters such as primary stiffness, stiffness after yielding and the yield moment values have to be carefully determined. The formula, used for stiffness determination is

$$K_\theta = \frac{2REI}{(100-R)L} \tag{1}$$

Where, EI is beam elastic module, R for connection fixity percentage and L is the beam length. The R values are 90%, 75% & 50%, respectively; which shall get used in the evaluation of K_θ, in Eq. (1). Also, other parameters needed in semi-rigid connection modeling are given in Table 1.

3.2 Ground motions

In order to fulfill nonlinear dynamic response history analysis on the presented models as well as on the equivalent single degree of freedom structures, similar to each mode, a set of 15 records with large intensity and small distance (LMSR) are defined, given in Table 2. In averaging, the selected records should be of the same region; therefore, all records are selected from California State with large intensity (M ≈ 6.5-6.9 Richter scale) and small distances (R ≈ 13-30 km), registered on stiff soil (NEHRP site, class D).

3.3 Response statistics

The dynamic responses of any structural system to 15 different ground motions are determined, throughout the Nonlinear Response History Analysis (NL-RHA), and MPA technique using non-linear finite element program (Allahabadi and Powell 1988). The exact maximum value of structural response (r), calculated through NL-RHA, is presented as $r_{\text{NL-RHA}}$ and similar results from

Table 2 The records used in LMSR set

NO	EVENT	Moment magnitude	STATION	R (km)	PGA (cm/s^2)
1	Imperial Valley 1979	6.5	Chihuahua	28.7	249
2	Imperial Valley 1979	6.5	Cucapah	23.6	303
3	Imperial Valley 1979	6.5	El Centro Array #12	18.2	140
4	Imperial Valley 1979	6.5	Parachute Test Site	14.2	109
5	Loma Prieta 1989	6.9	Gilroy Array #3	14.4	360
6	Loma Prieta 1989	6.9	Gilroy Array #4	16.1	208
7	Loma Prieta 1989	6.9	Hollister City Hall	28.2	211
8	Loma Prieta 1989	6.9	Hollister Diff. Array	25.8	274
9	Loma Prieta 1989	6.9	Saratoga - W Valley Coll	13.7	250
10	Northridge 1994	6.7	Canoga Park - Topanga Canyon	15.8	412
11	Northridge 1994	6.7	Canyon Country - W Lost Cany	13.0	473
12	Northridge 1994	6.7	Northridge - Saticoy St	29.5	361
13	Super station Hills 1987	6.7	El Centro Imp. Co. Cent	13.9	253
14	Super station Hills 1987	6.7	Westmorland Fire Station	13.3	207
15	Super station Hills 1987	6.7	Wildlife Liquef. Array	24.4	203

approximated MPA method is also shown as r_{MPA}. Furthermore, the response ratio for each ground motion, $r^*_{\text{MPA}} = r_{\text{MPA}} \div r_{\text{NL-RHA}}$ which indicates the bias of MPA, is determined.

Comparing this response ratio with unity, the average response is under estimated, if the response ratio is less than 1, and over estimated otherwise. In this research Median values are shown as $\hat{x}$ and are defined in the erythematic average n (=15) of the observed values of (x_i) r_{MPA} , $r_{\text{NL-RHA}}$ and r^*_{MPA} . The dispersion (δ) calculated by r^*_{MPA} , is defined in the form of observed standard deviation of natural logarithm, as following

$$\hat{x} = \exp\left[\frac{\sum_{i=1}^{n} \ln x_i}{n}\right], \quad \delta = \left[\frac{\sum_{i=1}^{n} (\ln x_i - \ln\hat{x})^2}{n-1}\right]^{1/2} \tag{2}$$

4. Evaluation of results accuracy

The aim of this section is evaluating the accuracy and correctness of MPA method on moment-resisting steel with rigid and semi-rigid connections and different fixity percentages. In this regard the median values of drift of a specific story calculated from MPA, as well as NL-RHA methods, are properly compared. Response statistics of bias and dispersion in demand estimation are analyzed and presented, using MPA method. The results of the inter-story drift ratio do indicate structural damage severity.

4.1 The results of nonlinear response history analysis

Fig. 3 shows the median values of drift of the particular story, determined by NL-RHA for all 24 rigid and semi-rigid frames. It is observed that the drift of the storey does increase heterogeneously, for high-rise frames. Besides, as the fixity percentages of the semi-rigid connection decrease, the displacement of the story increases, as well.

In order to show the effects of semi-rigid connections on the story drift demands, Fig. 4 represents the ratio of rigid frame storey drift to semi-rigid one. The difference between this ratio and unity shows the effects of the presence of semi-rigid connections. Regarding the charts, the taller frames with semi-rigid connections show the maximum effects on the upper stories; and the drift of the storey in these semi-rigid frames with fixed- moments of 90%, 75% and 50% have increasing values of about 10%, 20% and 40%, respectively, in comparison with full fixity rigid frames.

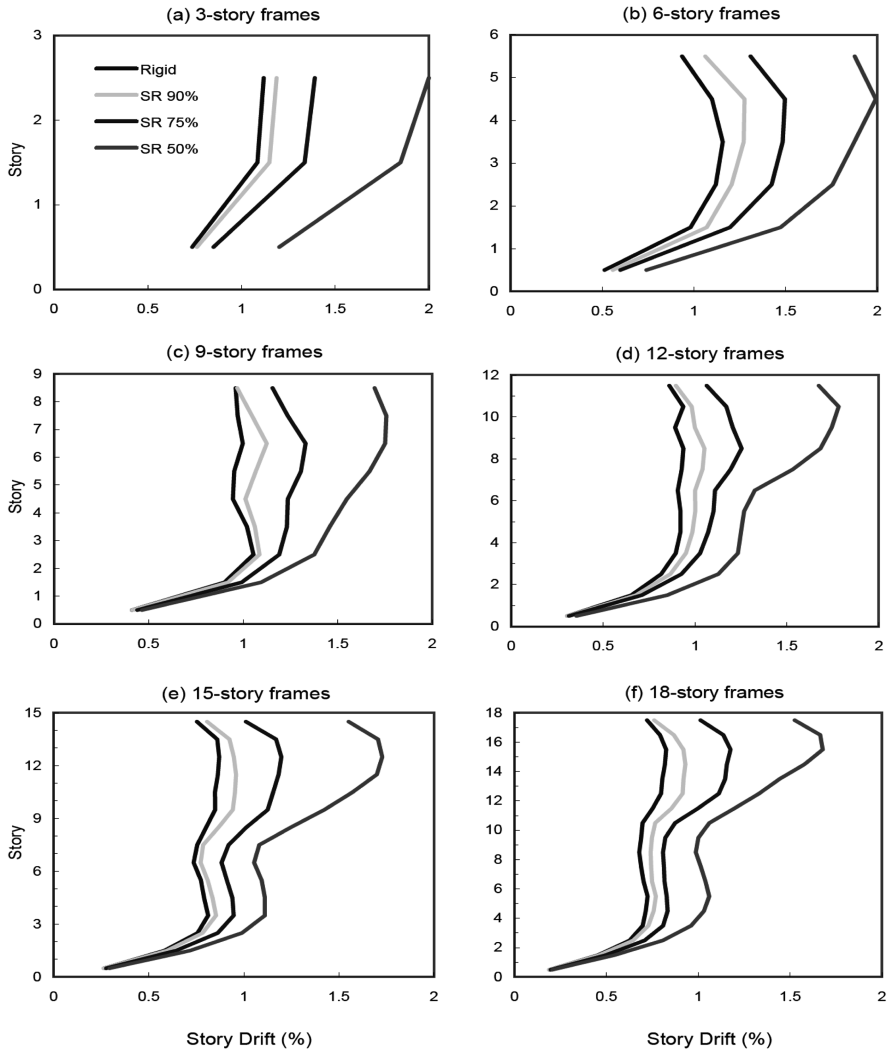

Fig. 3 Median drift of the story, calculated by NL-RHA method

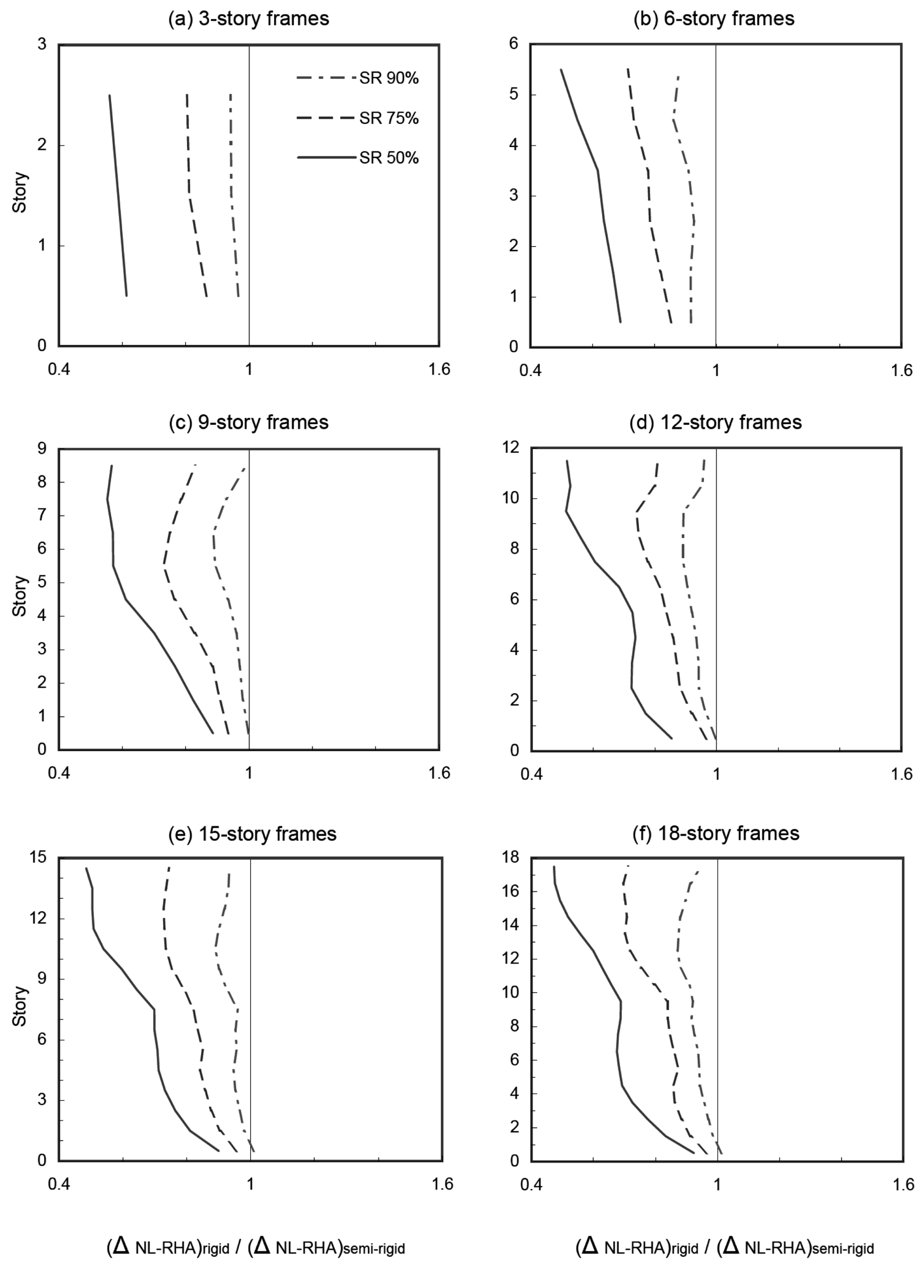

Fig. 4 The ratio of median drift of the story for rigid frames to semi-rigid ones

4.2 Comparison of the results

MPA method is fulfilled for all 24 frames and 15 motions, and the ratios of some modes are considered. The combined drift values of the storey are calculated regarding 1, 2 and 3 modes, as well. Figs. 5 and 6 shows the average drift values of the storey for 24 cases of rigid and semi-rigid connection frames, with moments fixity of 90%, 75% and 50%, along with NL-RHA results

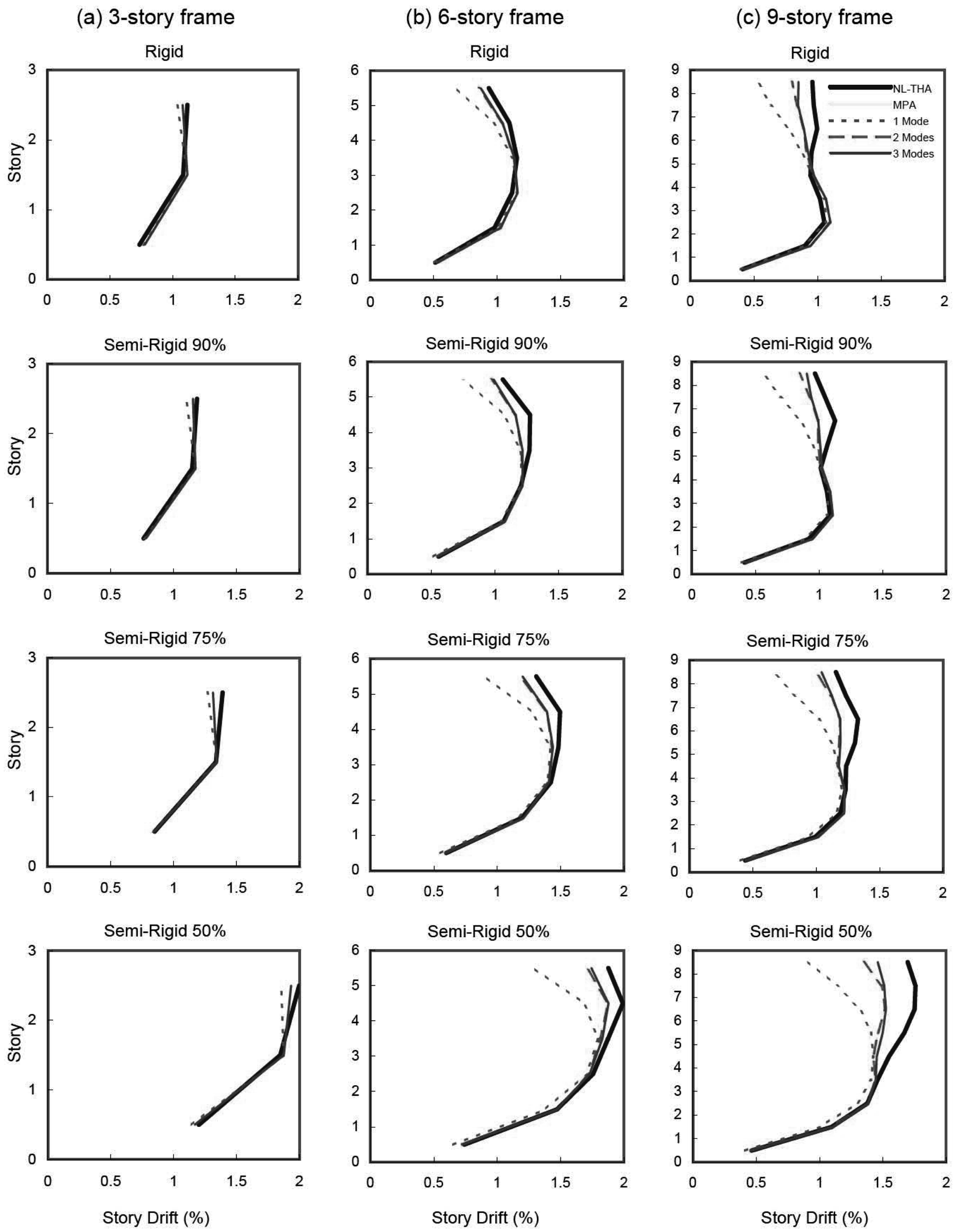

Fig. 5 Median drift of the stories, determined by MPA methods- concerning mode numbers and NL-RHA for rigid and semi-rigid frames of 90%, 75% and 50% moment fixity

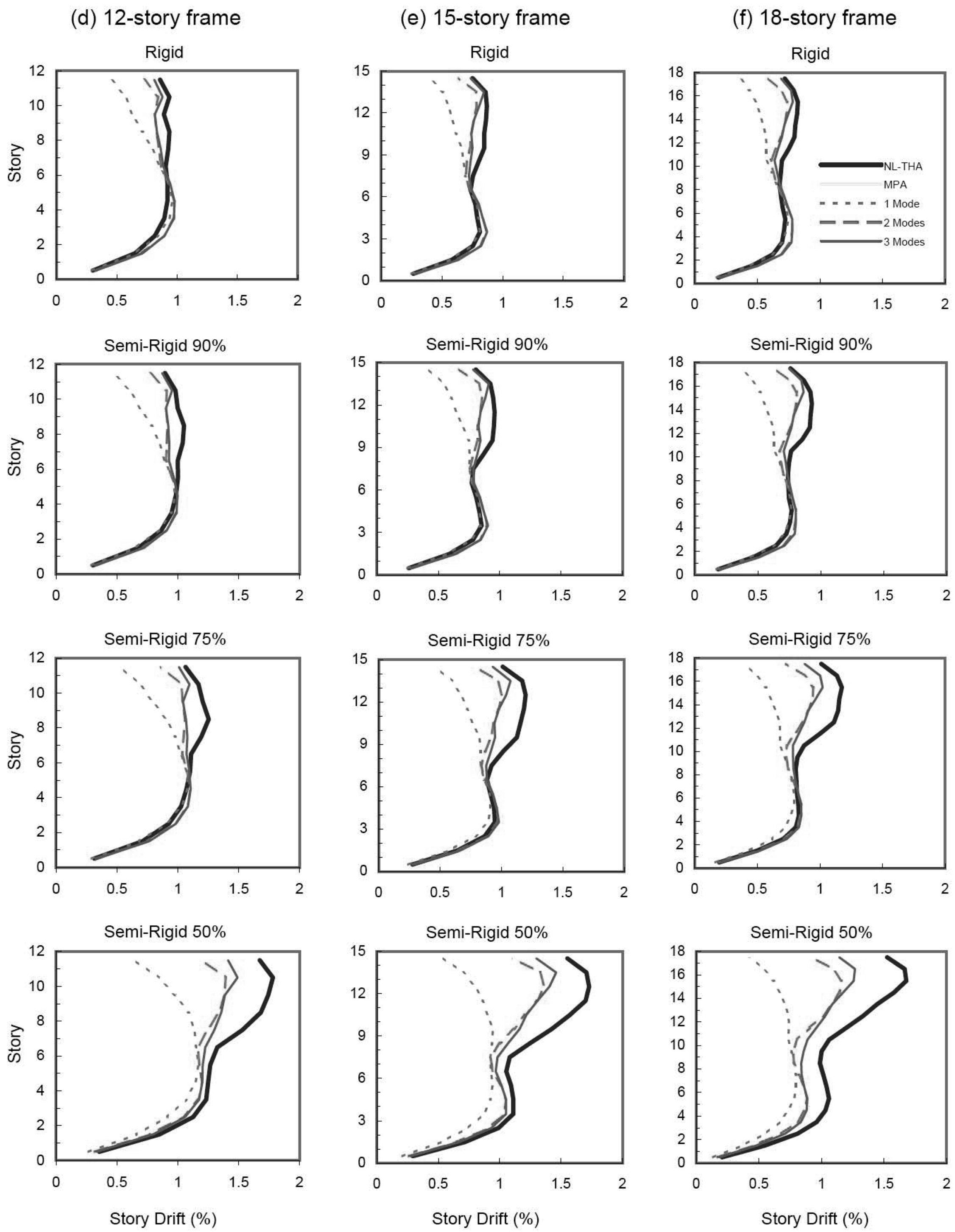

Fig. 6 Median drift of the stories, determined by MPA methods concerning mode numbers and NL-RHA for rigid and semi-rigid frames of 90%, 75% & 50% moment fixity

obtained in Fig. 3. The point to be remarked in these Figures is that the first mode is not sufficient by itself for calculating the drift of the storey. However, when modes 2 and 3 come into account, the outcomes significantly get improved. Also the MPA method represents reliable results for short and mid-rise stories of all frames. However, the accuracy of this method decreases in the upper stories of high-rise frames.

4.3 Bias and dispersion of MPA method

Fig. 7 shows the median ratio of drift, concerning 3 modes for rigid and semi-rigid frames. These results which indicate the bias of the method, offer the conclusions as follows:

- MPA method has the least bias for 3 storey frames and show relatively higher bias in case of

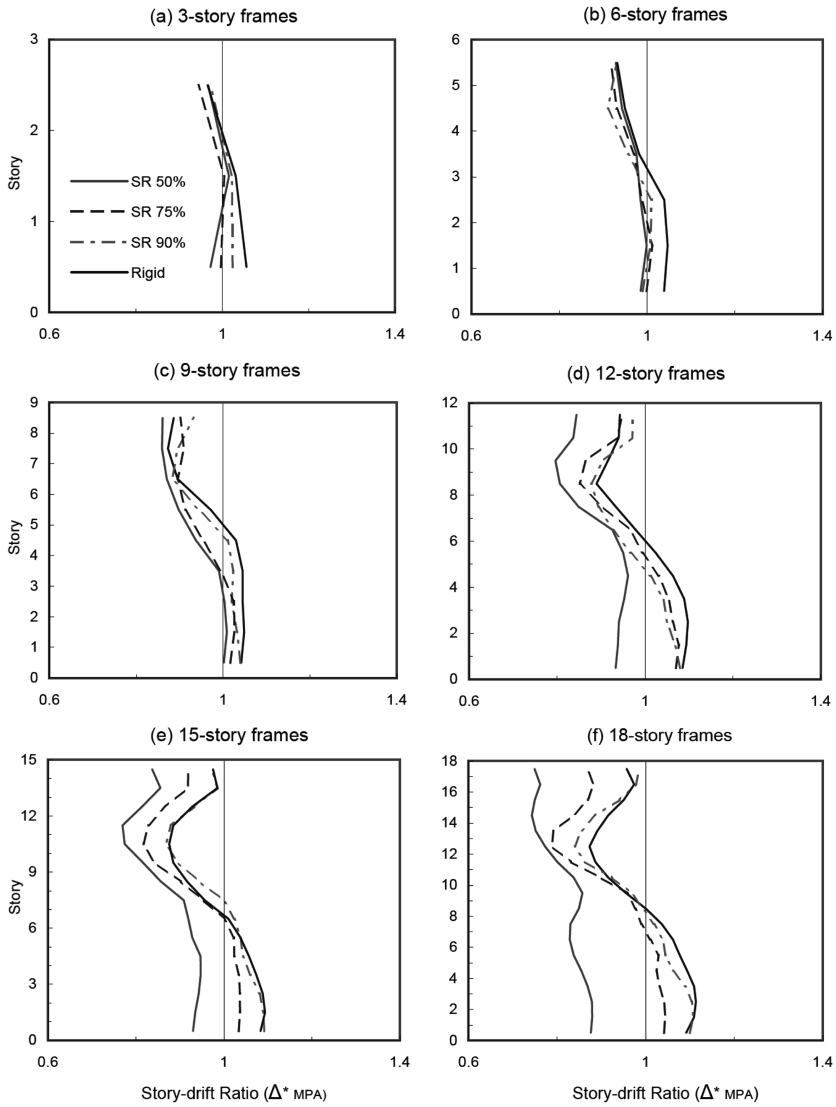

Fig. 7 Median ratio of $\Delta^{*}_{\mathrm{MPA}}$, the storey drift for rigid and semi-rigid frames

increasing the storey numbers.

- MPA methods estimates correctly and accurately the seismic demands for 3 and 6 rigid and semi-rigid storey frames; the bias values of the method for these frames are less than 5 and 10 percents, respectively.
- MPA method estimates the drifts of less than 25% for 9, 12, 15 and 18 storey frames although upper storey frames of 15 and 18 storey semi-rigid connections are underestimated with the bias of almost 30%.
- In the frame with constant number of stories, the bias of MPA method for the frames with lower fixed moment percentages intend to increase in comparison with the ones with higher moment's fixity.

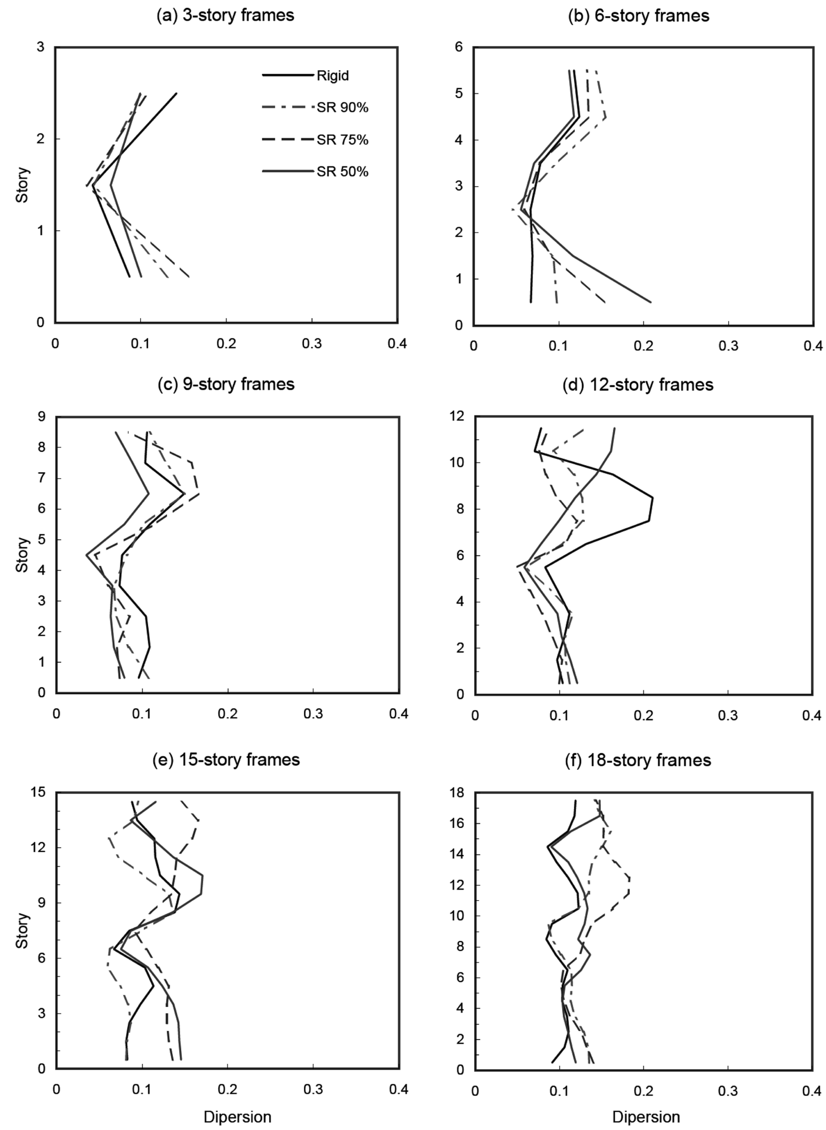

Fig. 8 The ratio dispersion of drift of the storey (Δ^*_{MPA}) for rigid and semi-rigid frames

Fig. 8 shows the ratio dispersion of drift (Δ^*_{MPA}), plotted all along the height of 24 frames. The least dispersion is less than 5%, occurred in short frames. However, it intends to increase in high-rise frames although the intention is not permanent. The dispersion is less than 25% for high-rise frames. In general, MPA method dispersion is to increase with the height increasing, comparing with NL_RHA method, in special rigid or semi-rigid frames as the higher mode ratios become more important.

In order to show how the bias and dispersion interpret the correctness and accuracy of MPA method, MPA results of the storey drift (Δ_{MPA}) are depicted against the accurate values ($\Delta_{NL\text{-}RHA}$). The results for the drifts in the upper, lower and median stories of 12 storey semi-rigid frames with moment fixity percentages of 90, 70 and 50 are represented. In this Figure, any point is for a ground motion and the dots on the diameter show $\Delta_{NL\text{-}RHA} = \Delta_{MPA}$. The point over the line indicate that MPA has over estimations for any record; the points bellow the line show the under estimations of MPA. Median value and ratio dispersion of the drift Δ^*_{MPA} are given in this Figure as well.

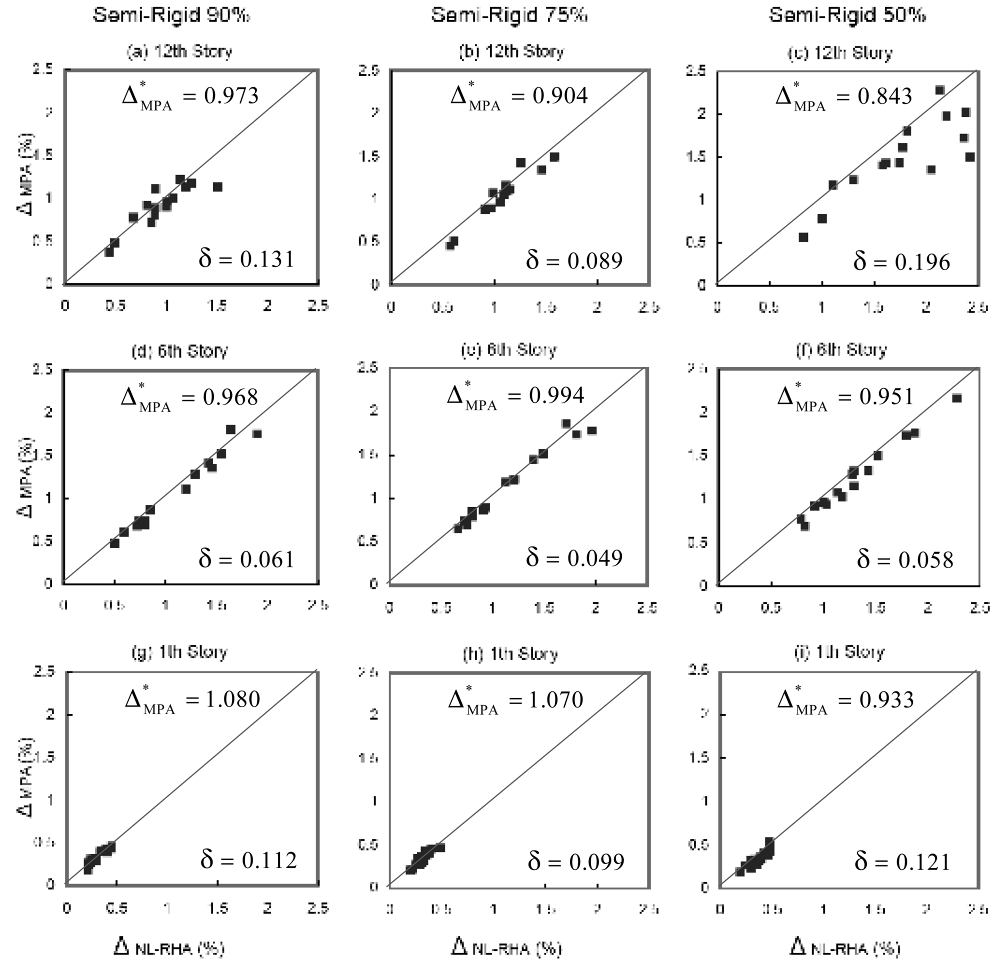

Fig. 9 The chart of MPA method results of lateral drift Δ_{MPA} against accurate values gained by nonlinear response history analysis method $\Delta_{NL\text{-}RHA}$ for upper, median and lower stories of 12 storey semi-rigid frames

The point observed in part ‘c’ of this Figure is drift of the highest storey in 12-storey semi-rigid frame with 50% moment fixity and has the great bias of 15%, as well as large dispersion of 0.2%, because the points are distributed in wide ranges of over and under estimations. Therefore, in this case, MPA method is not accurate for some motions.

Comparing this case with part ‘b’ for 12 storey semi-rigid frame with 75% moment fixity, it is observed that the bias is still great, around 10%, but dispersion is smaller and has reached 0.09, showing that MPA estimation is more accurate than the pervious case. This comparison shows that an approximate method like MPA has greater accuracy for specific motions where the bias and dispersion are both small.

Surveying the Fig. 9, the parts ‘d’ to ‘i’, shows that MPA can well estimate the drift of the storey in the middle height and first storey for most of the motions because the bias and dispersion are both small. The parts ‘b’ and ‘c’ confirm the idea that MPA predictions in upper stories of high-rise frames, over 12 stories, are uncertain.

4.4 Comparing 2 types of MPA method

In Fig. 10, the median ratio of drift Δ^*_{MPA} MPA for 12 storey rigid and semi-rigid frames of 90, 70 & 50 percent moment fixity are presented. These results show that in considering the ratios of several modes, the relative median displacement, obtained by type (B) method, is not totally equal to those of type (A) ones. Comparing the median ration of drift- obtained by types (A) and (B)

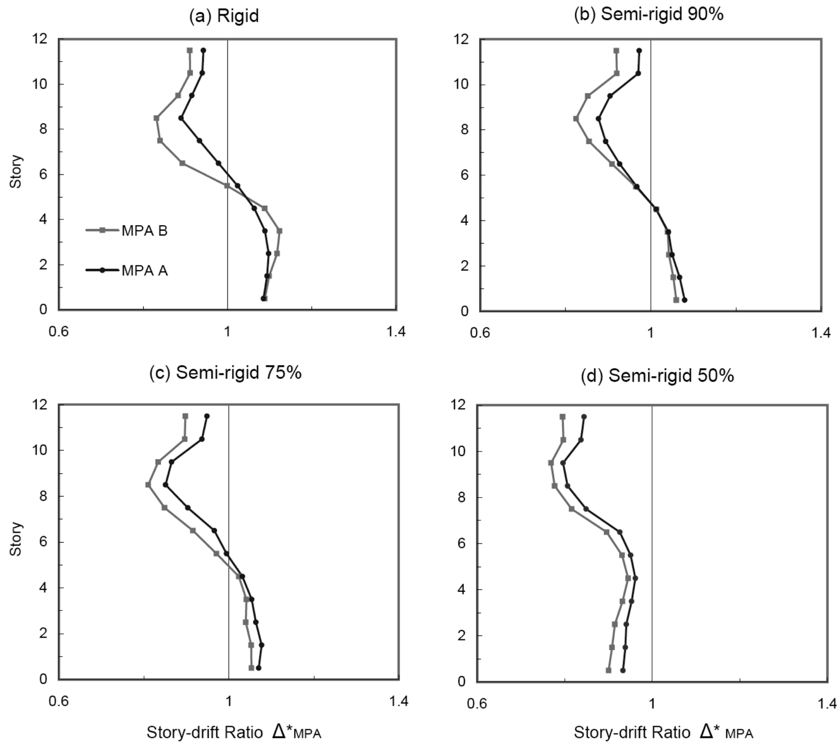

Fig. 10 Comparing median ratio of drift of the storey, Δ^*_{MPA}, determined by A&B types of MPA method for 12 storey rigid & semi rigid frames of 90%, 75% and 50% fixed moments

methods with unity, it is observed that the bias caused by over estimation in type (B) method can add 10% to 15% to the innate bias, existed in type (A) method, in all frames.

5. Conclusions

- MPA method offers reliable results for moment-resistant steel frames with semi-rigid connections, where its accuracy in the lower and middle stories will be higher than that of upper ones.
- The amount of drift in semi-rigid connections with different fixity, get increased incrementally, with respect to fully rigid connections.
- The accurate results of drifts of the storey, determined by NL-RHA method for rigid and semi-rigid frames show that as the moment fixity percentages of semi-rigid connections decrease, the storey drift increase, as well.
- If the sufficient 2 or 3 modes are considered, the storey drift, determined by MPA method, is close to the accurate results of NL-RHA. Therefore, the first mode which is the base of nonlinear statistical analysis methods will not have proper estimations of seismic demand by itself.
- The bias in MPA method for the frames with higher period, caused by two factors: higher height and lower moment fixity of connection intend to increase; but the intention is not permanent.
- The dispersion for any kind of rigid or semi-rigid frame separately intends to increase with increasing in the storey numbers, because the ratio of higher modes becomes more important.
- MPA method will have greater accuracy in the estimation of seismic demands, if the bias and dispersion are both small; where one of these two is great, the estimations will be uncertain.
- The MPA method bias decreases in the estimation of storey drift with increasing the higher mode number ratios. The MPA method intends to under estimation in drift of higher stories for all frames even with considering 3 modes. This under estimation never disappears, even if all modes of the structure are considered.
- In the following, a simpler type of MPA method is presented, offering reliable results in practical activities; and few differences are observed in the results in comparison with the main method.

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