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Influence of various sources in micropolar thermoelastic medium with voids

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Abstract. The present problem is concerned with the study of deformation of micropolar thermoelastic medium with voids under the influence of various sources acting on the plane surface. The analytic expressions of displacement components, force stress, couple stress, change in volume fraction field and temperature distribution are obtained in the transformed domain for Lord-Shulman (L-S) theory of thermoelasticity after applying the integral transforms. A numerical inversion technique has been applied to obtain the solution in the physical domain. The numerical results are presented graphically. Some useful particular cases have also been deduced.

Keywords: micropolar thermoelastic solid; voids; couple stress; thermoelasticity; integral transform.

1. Introduction

The classical theory of heat conduction predicts infinite speed of heat transportation, if a material conducting heat is subjected to a thermal disturbance, which contradicts the physical facts. Lord and Shulman (1967) incorporated a flux rate term into the Fourier's law of heat conduction and formulated a generalized theory admitting finite speed for thermal signals. Green and Lindsay (1972) have developed a temperature rate dependent thermoelasticity by including temperature rate among the constitutive variables which does not violate the classical Fourier's law of heat conduction when the body under consideration has a centre of symmetry and this theory also predicts a finite speed of heat propagation. Recently Green and Naghdi (1991) established a new thermomechanical theory of deformable media that uses a general entropy balance. The generalized thermoelasticity theories are supposed to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes and or short time intervals, like those occurring in laser units and energy channels.

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Classical theory of elasticity is inadequate to represent the behavior of some modern engineering structures like polycrystalline materials and materials with fibrous or coarse grain. The study of these materials requires incorporating the theory of oriented media. "Micropolar Elasticity" termed by Eringen (1966) is used to describe the deformation of elastic media with oriented particles. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. The force at a point of a surface element of bodies of these materials is completely characterized by a stress vector and a couple stress vector at that point.

The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory has practical utility in investigating various types of geological, biological and synthetic porous materials for which the elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the void volume is included among the kinematic variables and in the limiting case of vanishing this volume, the theory reduces to the classical theory of elasticity.

A non linear theory of elastic materials with voids was developed by Nunziato and Cowin (1979). Later Cowin and Nunziato (1983) developed a theory of linear elastic materials with voids, for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of a beam and small amplitudes acoustic waves. Iesan (1985) derived the basic equations of micropolar elastic materials with voids. Scarpetta (1990) studied the fundamental solutions in micropolar elasticity with voids. Marin (1995, 1996a, 1996b, 1998) discussed different type of problems in micropolar theory of elastic solid with voids. Kumar and Choudhary (2002, 2003) discussed source problems in micropolar elastic medium with voids. Scalia, Pompei and Chirita (2004) studied the spatial behavior in a cylinder made of an isotropic and homogeneous thermoelastic material with voids. Kumar and Ailawalia (2005) studied the response of micropolar elastic half-space with voids due to moving load. Mondal and Acharya (2006) investigated the effect of voids on the propagation of surface waves in a homogeneous micropolar elastic medium with voids. Recently Singh (2007) discussed wave propagation in generalized thermoelastic materials with voids.

In the present problem we have obtained the closed form expressions for two dimensional displacement, stresses, change in volume fraction field and temperature distribution due to mechanical\thermal sources in a micropolar thermoelastic medium with voids for Lord-Shulman (L-S) theory of thermoelasticity. The deformation at any point of the medium due to mechanical\ thermal sources is useful to analyze the deformation field around mining tremors and drilling into the crust of earth. It can also contribute to theoretical consideration of the seismic and volcanic sources since it can account for the deformation field in the entire volume surrounding the source region.

2. Formulation and solution of the problem

We consider a homogeneous micropolar thermoelastic solid with voids in the undeformed state. We take the origin on the plane surface and z-axis normally into the medium, which is represented by $z \geq 0$. A mechanical\thermal source is assumed to be acting at the plane surface $z = 0$ of the rectangular Cartesian co-ordinates. The field equations and constitutive relations for micropolar thermoelastic medium with voids in the absence of body forces, body couples and heat sources can be written by following Cowin and Nunziato (1983) and Lord-Shulman (1967) as

$$(\lambda+\mu)\nabla(\nabla\cdot\vec{u})+(\mu+K)\nabla^2\vec{u}+K(\nabla\times\vec{\phi})+\beta^*\nabla\psi-\nu\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2} \tag{1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla\cdot\vec{\phi})-\gamma\nabla\times(\nabla\times\vec{\phi})+K(\nabla\times\vec{u})-2K\vec{\phi}=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2} \tag{2}$$

$$\alpha^*\nabla^2\psi-\varsigma^*\psi-\omega^*\frac{\partial\psi}{\partial t}-\beta^*\nabla\cdot\vec{u}+mT=\rho\zeta^*\frac{\partial^2\psi}{\partial t^2} \tag{3}$$

and

$$K^*\nabla^2T-mT_0\left(\frac{\partial}{\partial t}+\tau_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\psi=\rho C^*\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T+\nu T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla\cdot\vec{u}) \tag{4}$$

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+K(u_{j,i}-\varepsilon_{ijr}\phi_r)+\beta^*\psi\delta_{ij}-\nu T\delta_{ij} \tag{5}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i} \tag{6}$$

where

$\lambda,\mu,K,\alpha,\beta,\gamma$ are material constants, ρ is the density of micropolar thermoelastic solid with voids, j is microinertia, K^* is the coefficient of thermal conductivity, C^* is the specific heat at constant strain; τ_0 is the thermal relaxation time; $\vec{u}$ is the displacement vector, $\vec{\phi}$ is microrotation vector, ψ is change in volume fraction field, t_{ij} is force stress tensor, m_{ij} is couple stress tensor and $\alpha^*,\beta^*,\varsigma^*,\omega^*$ and ζ^* are material constant due to the presence of voids.

Since we are considering a two-dimensional problem, the components of displacement vector and microrotation vector are

$$\vec{u}=(u_1,0,u_3),\quad \vec{\phi}=(0,\phi_2,0) \tag{7}$$

Using Eq. (7), the field Eqs. (1)-(4) and introducing non-dimensional quantities defined by

$$x'=\frac{\omega}{c_1}x,\quad z'=\frac{\omega}{c_1}z,\quad \phi_2'=\frac{\rho c_1^2}{\nu T_0}j\phi_2,\quad t_{ij}'=\frac{t_{ij}}{\nu T_0},\quad \{u_1',u_3'\}=\frac{\rho\omega c_1}{\nu T_0}\{u_1,u_3\}$$

$$m_{ij}'=\frac{\omega}{c_1\nu T_0}m_{ij},\quad \psi'=\frac{\rho c_1^2}{\nu T_0}\psi,\quad T'=\frac{T}{T_0},\quad t'=\omega t,\quad \tau_0'=\omega\tau_0,\quad a'=\frac{\omega}{c_1}a \tag{8}$$

where

$$c_1^2=\frac{\lambda+2\mu+K}{\rho}\quad\text{and}\quad \omega=\frac{\rho C^*c_1^2}{K^*}$$

in the resulting equations, we obtain the equations in non-dimensional form (after suppressing the primes). Introducing potential functions defined by

$$u_1=\frac{\partial q}{\partial x}+\frac{\partial\Psi}{\partial z},\quad u_3=\frac{\partial q}{\partial z}-\frac{\partial\Psi}{\partial x} \tag{9}$$

in the dimensionless equations, where $q(x,z,t)$ and $\Psi(x,z,t)$ are scalar potential functions, we obtain

$$\nabla^2 q + \frac{\beta^*}{\rho c_1^2}\psi - T = \frac{\partial^2 q}{\partial t^2} \tag{10}$$

$$\frac{(\mu+K)}{\rho c_1^2}\nabla^2\Psi + \frac{K}{\rho c_1^2}\phi_2 = \frac{\partial^2\Psi}{\partial t^2} \tag{11}$$

$$\gamma\nabla^2\phi_2 + \frac{Kc_1^2}{\omega^2}\nabla^2\Psi - \frac{2Kc_1^2}{\omega^2}\phi_2 = \rho j c_1^2\frac{\partial^2\phi_2}{\partial t^2} \tag{12}$$

$$\alpha^*\frac{\omega^2}{c_1^2}\nabla^2\psi - \varsigma^*\psi - \omega^*\omega\frac{\partial\psi}{\partial t} - \beta^*\nabla^2 q + \frac{m\rho c_1^2}{\nu}T = \rho\zeta^*\omega^2\frac{\partial^2\psi}{\partial t^2} \tag{13}$$

and

$$\nabla^2 T - \varepsilon\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2 q - \frac{m\,\nu T_0}{\rho\omega K^*}\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\psi = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T \tag{14}$$

Applying Laplace transform with respect to time 't' defined by

$$\{\bar{u}_i, \bar{\phi}_2, \bar{\psi}, \bar{T}\}(x,z,p) = \int_0^\infty \{u_i, \phi_2, \psi, T\}(x,z,t)e^{-pt}dt, \quad i = 1,3 \tag{15}$$

and then Fourier transform with respect to 'x' defined by

$$\{\tilde{u}_i, \tilde{\phi}_2, \tilde{\psi}, \tilde{T}\}(\xi,z,p) = \int_{-\infty}^{\infty} \{\bar{u}_i, \bar{\phi}_2, \bar{\psi}, \bar{T}\}(x,z,p)e^{i\xi x}dx, \quad i = 1,3 \tag{16}$$

on Eqs. (10)-(14) and then eliminating $\tilde{T}$ and $\tilde{\phi}_2$ from the resulting expressions, we obtain

$$\left[\frac{d^4}{dz^4} + A\frac{d^2}{dz^2} + B\right]\tilde{\Psi} = 0 \tag{17}$$

and

$$\left[\frac{d^6}{dz^6} + C\frac{d^4}{dz^4} + D\frac{d^2}{dz^2} + E\right]\tilde{q} = 0 \tag{18}$$

where

$$A = a_1a_4 - 2(a_1 + \xi^2) - p^2(a_2 + a_3)$$
$$B = (\xi^2 + a_3p^2)(\xi^2 + 2a_1 + a_2p^2) - a_1a_4\xi^2$$
$$C = -(a_6f_1 + f_3 + f_4)$$
$$D = -(a_6f_2 - f_1f_6 - f_3f_4 - f_5)$$
$$E = -(f_3f_5 - f_2f_6)$$
$$f_1 = a_{11} - a_{10}, \quad f_2 = a_{10}\xi^2 - a_{11}(\xi^2 + p^2)$$

$$f_3 = (\xi^2 + a_7 + pa_8 + p^2 a_9) - a_6 a_{11}, \quad f_4 = 2\xi^2 + (1+\varepsilon)a_{12} + p^2$$

$$f_5 = (\xi^2 + p^2)(\xi^2 + a_{12}), \quad f_6 = a_6(\xi^2 + a_{12}) + a_{12}a_{14}, \quad \varepsilon = \frac{\nu^2 T_0}{\rho\omega K^*}$$

$$a_1 = \frac{Kc_1^2}{\omega^2\gamma}, \quad a_2 = \frac{\rho j c_1^2}{\gamma}, \quad a_3 = \frac{\lambda + 2\mu + K}{\mu + K}, \quad a_4 = \frac{K}{\mu + K}$$

$$a_6 = \frac{\beta^*}{\lambda + 2\mu + K}, \quad a_7 = \frac{\varsigma^* c_1^2}{\omega^2 \alpha^*}, \quad a_8 = \frac{\omega^* c_1^2}{\omega\alpha^*}, \quad a_9 = \frac{\rho\zeta^* c_1^2}{\alpha^*}$$

$$a_{10} = \frac{\beta^* c_1^2}{\omega^2\alpha^*}, \quad a_{11} = \frac{m\rho c_1^4}{\nu\omega^2\alpha^*}, \quad a_{12} = p + \tau_0 p^2, \quad a_{14} = \frac{m\nu T_0}{\rho\omega K^*} \tag{19}$$

The solutions of Eqs. (17) and (88) satisfying the radiation conditions are

$$\tilde{\Psi} = D_1 \exp(-q_1 z) + D_2 \exp(-q_2 z) \tag{20}$$

$$\tilde{\phi}_2 = a_1^* D_1 \exp(-q_1 z) + a_2^* D_2 \exp(-q_2 z) \tag{21}$$

$$\tilde{q} = D_3 \exp(-q_3 z) + D_4 \exp(-q_4 z) + D_5 \exp(-q_5 z) \tag{22}$$

$$\tilde{\psi} = b_1^* D_3 \exp(-q_3 z) + b_2^* D_4 \exp(-q_4 z) + b_3^* D_5 \exp(-q_5 z) \tag{23}$$

$$\tilde{T} = c_1^* D_3 \exp(-q_3 z) + c_2^* D_4 \exp(-q_4 z) + c_3^* D_5 \exp(-q_5 z) \tag{24}$$

where $q_{1,2}^2$ and $q_{3,4,5}^2$ are roots of Eqs. (17) and (18) respectively and

$$a_\Theta^* = \frac{1}{a_4}[q_\Theta^2 - (\xi^2 + a_3 p^2)], \quad b_\Gamma^* = \frac{1}{f_3 - q_\Gamma^2}(f_2 + f_1 q_\Gamma^2)$$
$$c_\Gamma^* = [q_\Gamma^2 - (\xi^2 + p^2) + a_6 b_\Gamma^*], \quad \Theta = 1,2;\ \Gamma = 3,4,5 \tag{25}$$

3. Applications

3.1 Mechanical forces

The boundary conditions at the interface $z = 0$ are

$$t_{33} = -F\Phi(x)\delta(t), \quad t_{31} = 0, \quad m_{32} = 0, \quad T = 0, \quad \frac{\partial\psi}{\partial z} = 0 \tag{26}$$

where $\delta(t)$ is Dirac delta function and $\Phi(x)$ specify the vertical traction distribution function along x-axis, F is the magnitude of force applied.

Applying Integral transforms defined by Eqs. (15) and (16) on the boundary conditions (26) and using Eqs. (5)-(9) and Eqs. (20)-(24), we obtain the expressions for displacement components, force stress, couple stress, change in volume fraction field and temperature distribution for micropolar thermoelastic medium with voids as

$$\tilde{u}_1 = -\frac{F}{\Delta}[q_1\Delta_1 e^{-q_1 z} + q_2\Delta_2 e^{-q_2 z} + i\xi(\Delta_3 e^{-q_3 z} + \Delta_4 e^{-q_4 z} + \Delta_5 e^{-q_5 z})] \tag{27}$$

$$\tilde{u}_3 = \frac{F}{\Delta}[i\xi(\Delta_1 e^{-q_1 z} + \Delta_2 e^{-q_2 z}) - q_3\Delta_3 e^{-q_3 z} - q_4\Delta_4 e^{-q_4 z} - q_5\Delta_5 e^{-q_5 z}] \tag{28}$$

$$\tilde{t}_{31} = \frac{F}{\Delta}[s_1\Delta_1 e^{-q_1 z} + s_2\Delta_2 e^{-q_2 z} + s_3\Delta_3 e^{-q_3 z} + s_4\Delta_4 e^{-q_4 z} + s_5\Delta_5 e^{-q_5 z}] \tag{29}$$

$$\tilde{t}_{33} = \frac{F}{\Delta}[r_1\Delta_1 e^{-q_1 z} + r_2\Delta_2 e^{-q_2 z} + r_3\Delta_3 e^{-q_3 z} + r_4\Delta_4 e^{-q_4 z} + r_5\Delta_5 e^{-q_5 z}] \tag{30}$$

$$\tilde{\phi}_2 = \frac{F}{\Delta}[a_1^*\Delta_1 e^{-q_1 z} + a_2^*\Delta_2 e^{-q_2 z}] \tag{31}$$

$$\tilde{m}_{32} = -\frac{F\omega^2\gamma}{\rho c_1^4\Delta}[a_1^* q_1\Delta_1 e^{-q_1 z} + a_2^* q_2\Delta_2 e^{-q_2 z}] \tag{32}$$

$$\tilde{\psi} = \frac{F}{\Delta}[b_3^*\Delta_3 e^{-q_3 z} + b_4^*\Delta_4 e^{-q_4 z} + b_5^*\Delta_5 e^{-q_5 z}] \tag{33}$$

$$\tilde{T} = \frac{F}{\Delta}[c_3^*\Delta_3 e^{-q_3 z} + c_4^*\Delta_4 e^{-q_4 z} + c_5^*\Delta_5 e^{-q_5 z}] \tag{34}$$

where

$$\Delta = g_{11}(Gs_3 - Hr_3) - g_{12}(Gs_4 - Hr_4) + g_{13}(Gs_5 - Hr_5)$$

$$\Delta_{1,2} = \pm a_{2,1}^* q_{2,1} F\tilde{\Phi}(\xi)(s_3 g_{11} - s_4 g_{12} + s_5 g_{13})$$

$$\Delta_3 = F\tilde{\Phi}(\xi)Hg_{11}, \quad \Delta_4 = -F\tilde{\Phi}(\xi)Hg_{12}, \quad \Delta_5 = F\tilde{\Phi}(\xi)Hg_{13}$$

$$r_{1,2} = \frac{i\xi\lambda}{\rho c_1^2}q_{1,2}, \quad r_\Gamma = q_\Gamma^2 - \frac{\xi^2\lambda}{\rho c_1^2} + \frac{\beta^*}{\rho c_1^2}b_\Gamma^* - c_\Gamma^*$$

$$s_{1,2} = \mu\xi^2 + (\mu + K)q_{1,2}^2 - Ka_{1,2}^*, \quad s_\Gamma = (2\mu + K)i\xi q_\Gamma$$

$$g_{11} = c_4^* b_5^* q_5 - c_5^* b_4^* q_4, \quad g_{12} = c_3^* b_5^* q_5 - c_5^* b_3^* q_3$$

$$g_{13} = c_3^* b_4^* q_4 - c_4^* b_3^* q_3, \quad G = a_1^* q_1 r_2 - a_2^* q_2 r_1$$

$$H = a_1^* q_1 s_2 - a_2^* q_2 s_1 \tag{35}$$

3.2 Thermal sources

The boundary conditions at the interface $z = 0$ are

$$t_{31} = 0, \quad t_{33} = 0, \quad m_{32} = 0, \quad T = F\eta(x)\delta(t), \quad \frac{\partial\psi}{\partial z} = 0 \tag{36}$$

The expressions for displacement, microrotation, force stress, tangential couple stress, change in volume fraction field and temperature distribution are given by Eqs. (27)-(34) with Δ_Ω replaced by $\Delta_\Omega^0(\varepsilon = 1, 2, 3, 4, 5)$ in Eq. (35) where

$$\Delta_{1,2}^{0} = \mp\frac{\overset{*}{a}_{2,1}q_{2,1}}{T_0}\tilde{\eta}(\xi)(r_3\overset{*}{g}_{11} - r_4\overset{*}{g}_{12} + r_5\overset{*}{g}_{13})$$

$$\Delta_3^0 = -\frac{1}{T_0}\tilde{\eta}(\xi)(G\overset{*}{g}_{11} - \overset{*}{g}_{21}H), \quad \Delta_4^0 = \frac{1}{T_0}\tilde{\eta}(\xi)(G\overset{*}{g}_{12} - \overset{*}{g}_{22}H)$$

$$\Delta_5^0 = -\frac{1}{T_0}\tilde{\eta}(\xi)(G\overset{*}{g}_{13} - \overset{*}{g}_{23}H), \quad \overset{*}{g}_{11} = s_4\overset{*}{b}_5q_5 - s_5\overset{*}{b}_4q_4, \quad \overset{*}{g}_{12} = s_3\overset{*}{b}_5q_5 - s_5\overset{*}{b}_3q_3$$

$$\overset{*}{g}_{13} = s_3\overset{*}{b}_4q_4 - s_4\overset{*}{b}_3q_3, \quad \overset{*}{g}_{21} = r_4\overset{*}{b}_5q_5 - r_5\overset{*}{b}_4q_4, \quad \overset{*}{g}_{22} = r_3\overset{*}{b}_5q_5 - r_5\overset{*}{b}_3q_3$$

$$\overset{*}{g}_{23} = r_3\overset{*}{b}_4q_4 - r_4\overset{*}{b}_3q_3 \tag{37}$$

3.3 Concentrated force/thermal point source

In order to determine displacements and stresses due to concentrated normal force described as Dirac delta function we use $\{\Phi(x), \eta(x)\} = \delta(x)$. The Fourier transform of $\Phi(x)$ with respect to pair (x, ξ) will be

$$\{\tilde{\Phi}(\xi), \tilde{\eta}(\xi)\} = 1 \tag{38}$$

3.1.2 Uniformly distributed force/source

The solution due to distributed force/source applied on the half space is obtained by setting

$$\{\Phi(x), \eta(x)\} = \begin{bmatrix} 1 & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{bmatrix}$$

in Eqs. (26) and (36). The Fourier transform with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width 2a applied at the plane surface $z = 0$ in dimensionless form after suppressing the primes becomes

$$\{\tilde{\Phi}(\xi), \tilde{\eta}(\xi)\} = \left[2\,\mathrm{Sin}\left(\frac{\xi c_1 a}{\omega}\right)/\xi\right], \quad \xi \neq 0 \tag{39}$$

3.1.3 Linearly distributed force/source

The solution due to linearly distributed force/source is obtained by substituting

$$\{\Phi(x), \eta(x)\} = \begin{bmatrix} 1 - \frac{|x|}{a} & \text{if } |x| \le a \\ 0 & \text{if } |x| > a \end{bmatrix}$$

The Fourier transform in case of linearly distributed force/source applied at the plane surface $z = 0$ of the system in dimensionless form are

$$\{\tilde{\Phi}(\xi), \tilde{\eta}(\xi)\} = \frac{2\left[1 - \mathrm{Cos}\left(\frac{\xi c_1 a}{\omega}\right)\right]}{\frac{\xi^2 c_1 a}{\omega}} \tag{40}$$

The expressions for displacement, microrotation, force stress, couple stress, change in volume fraction field and temperature distribuition may be obtained as given by Eqs. (27)-(34), by replacing the value of $\tilde{\Phi}(\xi)$ and $\tilde{\eta}(\xi)$ from Eqs. (38)-(40) in case of concentrated, uniformly distributed and linearly distributed mechanical and thermal sources respectively.

4. Particular cases

Case 4.1: Neglecting voids effect (i.e., $\omega^* = \varsigma^* = \beta^* = \zeta^* = 0$), we obtain the corresponding expressions for displacements, microrotation, stresses and temperature distribution as

$$\tilde{u}_1 = -\frac{F}{\Delta^*}[q_1\Delta_1^* e^{-q_1 z} + q_2\Delta_2^* e^{-q_2 z} + i\xi(\Delta_3^* e^{-q_3' z} + \Delta_4^* e^{-q_4' z})] \tag{41}$$

$$\tilde{u}_3 = \frac{F}{\Delta^*}[i\xi(\Delta_1^* e^{-q_1 z} + \Delta_2^* e^{-q_2 z}) - q_3'\Delta_3^* e^{-q_3' z} - q_4'\Delta_4^* e^{-q_4' z}] \tag{42}$$

$$\tilde{t}_{31} = \frac{F}{\Delta^*}[s_1\Delta_1^* e^{-q_1 z} + s_2\Delta_2^* e^{-q_2 z} + s_3^*\Delta_3^* e^{-q_3' z} + s_4^*\Delta_4^* e^{-q_4' z}] \tag{43}$$

$$\tilde{t}_{33} = \frac{F}{\Delta^*}[r_1\Delta_1^* e^{-q_1 z} + r_2\Delta_2^* e^{-q_2 z} + r_3^*\Delta_3^* e^{-q_3' z} + r_4^*\Delta_4^* e^{-q_4' z}] \tag{44}$$

$$\tilde{\phi}_2 = \frac{F}{\Delta^*}[a_1^*\Delta_1^* e^{-q_1 z} + a_2^*\Delta_2^* e^{-q_2 z}] \tag{45}$$

$$\tilde{m}_{32} = -\frac{F\omega^2\gamma}{\rho c_1^4\Delta^*}[a_1^* q_1\Delta_1^* e^{-q_1 z} + a_2^* q_2\Delta_2^* e^{-q_2 z}] \tag{46}$$

$$\tilde{T} = \frac{F}{\Delta^*}[c_3^*\Delta_3^* e^{-q_3' z} + c_4^*\Delta_4^* e^{-q_4' z}] \tag{47}$$

where

$$\Delta^* = G(s_3^* b_4^* - s_4^* b_3^*) - H(r_3^* b_4^* - r_4^* b_3^*)$$

$$\Delta_{1,2}^* = \pm a_2^* q_2 \tilde{\Phi}(\xi)(s_3^* b_4^* - s_4^* b_3^*), \quad \Delta_{3,4}^* = \pm H b_{4,3}^* \tilde{\Phi}(\xi)$$

$$r_{3,4}^* = q_{3,4}'^2 - \frac{\xi^2\lambda}{\rho c_1^2} - c_{3,4}^*, \quad c_{3,4}^* = [q_{3,4}'^2 - (\xi^2 + p^2)]$$

$$q_{3,4}'^2 = \frac{(C^{*2} - 4D^*)^{1/2}}{2.}, \quad C^* = -[2\xi^2 + p^2 + a_{12}(1+\varepsilon)]$$

$$D^* = (\xi^2 + p^2)(\xi^2 + a_{12}) + \varepsilon a_{12}\xi^2, \quad s_{3,4}^* = (2\mu + K)i\xi q_{3,4}' \tag{48}$$

4.1a: The expressions for displacements, microrotation, force stress, couple stress and temperature distribution can be obtained for a concentrated, uniformly and linearly distributed force by replacing $\tilde{\Phi}(\xi)$ from Eqs. (38)-(40) respectively, in Eqs. (41)-(47).

4.1b: The expressions for displacement, microrotation, force stress, couple stress and temperature distribution for thermal source are given by Eqs. (41)-(47) with Δ_{Ξ}^{*} replaced by $\Delta_{\Xi}^{*0}(\Xi = 1,2,3,4)$ in Eq. (48) where

$$\Delta_{1,2}^{*0} = \mp\frac{\overset{*}{a}_{2,1}q_{2,1}}{T_0}\tilde{\eta}(\xi)(r_3^*s_4^* - r_4^*s_3^*)$$

$$\Delta_3^{*0} = -\frac{1}{T_0}\tilde{\eta}(\xi)(Gs_4^* - Hr_4^*), \quad \Delta_4^{*0} = \frac{1}{T_0}\tilde{\eta}(\xi)(Gs_3^* - Hr_3^*) \tag{49}$$

The expressions for displacements, stresses and temperature distribution can be obtained for a thermal point source, uniformly and linearly distributed thermal sources by replacing $\tilde{\eta}(\xi)$ from Eqs. (38)-(40) respectively, in Eqs. (41)-(47) and using Eq. (49).

Case 4.2: Neglecting thermal effect, the expressions for displacements, microrotation, stresses and change in volume fraction field are obtained as

$$\tilde{u}_1 = -\frac{F}{\Delta^{**}}[q_1\Delta_1^{**}e^{-q_1z} + q_2\Delta_2^{**}e^{-q_2z} + i\xi(\Delta_3^{**}e^{-q_3''z} + \Delta_4^{**}e^{-q_4''z})] \tag{50}$$

$$\tilde{u}_3 = \frac{F}{\Delta^{**}}[i\xi(\Delta_1^{**}e^{-q_1z} + \Delta_2^{**}e^{-q_2z}) - q_3''\Delta_3^{**}e^{-q_3''z} - q_4''\Delta_4^{**}e^{-q_4''z}] \tag{51}$$

$$\tilde{t}_{31} = \frac{F}{\Delta^{**}}[s_1\Delta_1^{**}e^{-q_1z} + s_2\Delta_2^{**}e^{-q_2z} + s_3^{**}\Delta_3^{**}e^{-q_3''z} + s_4^{**}\Delta_4^{**}e^{-q_4''z}] \tag{52}$$

$$\tilde{t}_{33} = \frac{F}{\Delta^{**}}[r_1\Delta_1^{**}e^{-q_1z} + r_2\Delta_2^{**}e^{-q_2z} + r_3^{**}\Delta_3^{**}e^{-q_3''z} + r_4^{**}\Delta_4^{**}e^{-q_4''z}] \tag{53}$$

$$\tilde{\phi}_2 = \frac{F}{\Delta^{**}}[a_1^*\Delta_1^{**}e^{-q_1z} + a_2^*\Delta_2^{**}e^{-q_2z}] \tag{54}$$

$$\tilde{m}_{32} = -\frac{F\omega^2\gamma}{\rho c_1^4\Delta^{**}}[a_1^*q_1\Delta_1^{**}e^{-q_1z} + a_2^*q_2\Delta_2^{**}e^{-q_2z}] \tag{55}$$

$$\tilde{\psi} = \frac{F}{\Delta^{**}}[b_3^{**}\Delta_3^{**}e^{-q_3''z} + b_4^{**}\Delta_4^{**}e^{-q_4''z}] \tag{56}$$

where

$$\Delta^{**} = G(s_3^{**}b_4^{**}q_4'' - s_4^{**}b_3^{**}q_3) - H(r_3^{**}b_4^{**}q_4'' - r_4^{**}b_3^{**}q_3)$$

$$\Delta_{1,2}^{**} = \pm\tilde{\Phi}(\xi)a_{2,1}^*q_{2,1}^*(s_3^{**}b_4^{**}q_4'' - s_4^{**}b_3^{**}q_3)$$

$$\Delta_{3,4}^{**} = \pm\tilde{\Phi}(\xi)Hb_{4,3}^{**}q_{4,3}\,, \quad q_{3,4}^{''2} = \frac{(C^{**2} - 4.D^{**})^{1/2}}{2.}$$

$$C^{**} = a_6a_{10} - (2\xi^2 + p^2 + a_7 + a_8p + a_9p^2)$$

$$D^{**} = (\xi^2 + p^2)(\xi^2 + a_7 + a_8p + a_9p^2) - a_6a_{10}\xi^2$$

$$b_{3,4}^{**} = \frac{-1}{a_6}[q_{3,4}^{''2} - (\xi^2 + p^2)]$$

$$r_{3,4}^{*} = q_{3,4}^{''2} - \frac{\xi^2\lambda}{\rho c_1^2} + \frac{\beta^*}{\rho c_1^2}b_{3,4}^{**}\,, \quad s_{3,4}^{**} = (2\mu + K)i\xi q_{3,4}^{''} \tag{57}$$

4.2a: Again the expressions for displacements, microrotation, force stress, couple stress and change in volume fraction field can be obtained for a concentrated, uniformly and linearly distributed force by replacing $\tilde{\Phi}(\xi)$ from Eqs. (38)-(40) respectively, in Eqs. (50)-(56).

Case 4.3: Neglecting micropolarity effect, the expressions for displacements, force stress, change in volume fraction field and temperature distribution are obtained as

$$\tilde{u}_1 = -\frac{F}{\Delta^{***}}[q_1'\Delta_1^{***}e^{-q_1'z} + i\xi(\Delta_3^{***}e^{-q_3z} + \Delta_4^{***}e^{-q_4z} + \Delta_5^{***}e^{-q_5z})] \tag{58}$$

$$\tilde{u}_3 = \frac{F}{\Delta^{***}}[i\xi\Delta_1^{***}e^{-q_1'z} - q_3\Delta_3^{***}e^{-q_3z} - q_4\Delta_4^{***}e^{-q_4z} - q_5\Delta_5^{***}e^{-q_5z}] \tag{59}$$

$$\tilde{t}_{31} = \frac{F}{\Delta^{***}}[s_1^*\Delta_1^{***}e^{-q_1'z} + s_3^*\Delta_3^{***}e^{-q_3z} + s_4^*\Delta_4^{***}e^{-q_4z} + s_5^*\Delta_5^{***}e^{-q_5z}] \tag{60}$$

$$\tilde{t}_{33} = \frac{F}{\Delta^{***}}[r_1^*\Delta_1^{***}e^{-q_1'z} + r_3\Delta_3^{***}e^{-q_3z} + r_4\Delta_4^{***}e^{-q_4z} + r_5\Delta_5^{***}e^{-q_5z}] \tag{61}$$

$$\tilde{\psi} = \frac{F}{\Delta^{***}}[b_3^*\Delta_3^{***}e^{-q_3z} + b_4^*\Delta_4^{***}e^{-q_4z} + b_5^*\Delta_5^{***}e^{-q_5z}] \tag{62}$$

$$\tilde{T} = \frac{F}{\Delta^{***}}[c_3^*\Delta_3^{***}e^{-q_3z} + c_4^*\Delta_4^{***}e^{-q_4z} + c_5^*\Delta_5^{***}e^{-q_5z}] \tag{63}$$

where

$$\Delta^{***} = g_{11}(r_1^*s_3^* - r_3s_1^*) - g_{12}(r_1^*s_4^* - r_4s_1^*) + g_{13}(r_1^*s_5^* - r_5s_1^*)$$

$$\Delta_1^{***} = -\tilde{\Phi}(\xi)(s_3^*g_{11} - s_4^*g_{12} + s_5^*g_{13}), \quad \Delta_3^{***} = \tilde{\Phi}(\xi)s_1^*g_{11}$$

$$\Delta_4^{***} = -\tilde{\Phi}(\xi)s_1^*g_{12}, \quad \Delta_5^{***} = \tilde{\Phi}(\xi)s_1^*g_{13}\,, \quad r_1^* = \frac{i\xi\lambda}{\rho c_1^2}q_1'$$

$$s_1^* = \mu\xi^2 + \mu q_1'^2\,, \quad s_\Gamma^* = 2\mu i\xi q_\Gamma\,, \quad q_1'^2 = \xi^2 + \frac{\rho c_1^2}{\mu}p^2\,, \quad \Gamma = 3, 4, 5 \tag{64}$$

4.3a: The expressions for displacements, force stress, change in volume fraction field and temperature distribution can be obtained for a concentrated, uniformly and linearly distributed force by replacing $\tilde{\Phi}(\xi)$ from Eqs. (38)-(40) respectively, in Eqs. (58)-(63).

4.3b: The expressions for displacement, force stress, change in volume fraction field and temperature distribution for thermal source are given by Eqs. (58)-(63) with Δ_Ξ^{***} replaced by $\Delta_\Xi^{***0}(\Xi = 1, 3, 4, 5)$ in Eq. (64) where

$$\Delta_1^{***0} = \frac{1}{T_0}\tilde{\eta}(\xi)[r_3(s_4^* b_5^* q_5 - s_5^* b_4^* q_4) - r_4(s_3^* b_5^* q_5 - s_5^* b_3^* q_3) + r_5(s_3^* b_4^* q_4 - s_4^* b_3^* q_3)]$$

$$\Delta_3^{***0} = -\frac{1}{T_0}\tilde{\eta}(\xi)[r_1^*(s_4^* b_5^* q_5 - s_5^* b_4^* q_4) - s_1^*(r_4 b_5^* q_5 - r_5 b_4^* q_4)]$$

$$\Delta_4^{***0} = \frac{1}{T_0}\tilde{\eta}(\xi)[r_1^*(s_3^* b_5^* q_5 - s_5^* b_3^* q_3) - s_1^*(r_3 b_5^* q_5 - r_5 b_3^* q_3)]$$

$$\Delta_5^{***0} = -\frac{1}{T_0}\tilde{\eta}(\xi)[r_1^*(s_3^* b_4^* q_4 - s_4^* b_3^* q_3) - s_1^*(r_3 b_4^* q_4 - r_4 b_3^* q_3)] \quad (65)$$

The expressions for displacements, stresses and temperature distribution can be obtained for a thermal point source, uniformly and linearly distributed thermal sources by replacing $\tilde{\eta}(\xi)$ from Eqs. (38)-(40) respectively, in Eqs. (58)-(63) and using Eq. (65).

5. Inversion of the transform

The transformed displacements, microrotation and stresses are functions of y, the parameters of Laplace and Fourier transforms p and ξ respectively, and hence are of the form $\tilde{f}(\xi, y, p)$. To get the function in the physical domain, first we first invert the Fourier transform and then Laplace transform by using the method applied by Sharma and Kumar (1997).

6. Numerical results and discussions

We take the case of magnesium crystal (Eringen 1984) like material (micropolar elastic solid) subjected to mechanical and thermal disturbances for numerical calculations. The physical constants used are

$$\rho = 1.74 \text{ gm/cm}^3, \quad j = 0.2 \times 10^{-15} \text{ cm}^2, \quad \lambda = 9.4 \times 10^{11} \text{ dyne/cm}^2$$

$$\mu = 4.0 \times 10^{11} \text{ dyne/cm}^2, \quad K = 1.0 \times 10^{11} \text{ dyne/cm}^2, \quad \gamma = 0.779 \times 10^{-4} \text{ dyne}$$

The thermal parameters are given as

$$T_0 = 23^{\circ}\text{C}, \quad K^* = 0.6 \times 10^{-2} \text{ cal/cm sec }^{\circ}\text{C}, \quad C^* = 0.23 \text{ cal/gm }^{\circ}\text{C}$$

The void parameters are taken as

$$\alpha^* = 3.668\times10^{-4}\ \text{dyne},\quad \beta^* = 1.13849\times10^{11}\ \text{dyne/cm}^2,\quad \varsigma^* = 1.475\times10^{11}\ \text{dyne/cm}^2$$
$$\omega^* = 0.0787\times10^{-2}\ \text{dyne sec/cm}^2,\quad \zeta^* = 1.753\times10^{-15}\ \text{cm}^2$$

The variations of normal displacement u_3, normal force stress t_{33}, tangential m_{32} couple stress, Volume fraction field ψ and temperature distribution T with distance 'x' at the plane z = 1.0 for at t = 0.1 and 0.5 Lord-Shulman (L-S) theory have been shown for (a) micropolar thermoelastic solid with voids (MTESWV) by solid line (———) at t = 0.1 and dashed line (- - - -) at t = 0.5 (b) micropolar thermoelastic solid (MTES) by solid line with centered symbol (*—*—*) at t = 0.1 and dashed line with centered symbol (*- -* - -*) at t = 0.5 and (c) thermoelastic solid with voids (TESWV) by solid line with centered symbol dashed line with centered symbol (⊙—⊙—⊙) at t = 0.1 and dashed line with centered symbol (⊙----⊙----⊙) at t = 0.5 respectively. These distributions are shown in Figs. 1-20 for non-dimensional thermal relaxation time τ_0 = 1.0 and for isothermal boundary.

7. Discussions for various cases

7.1 Mechanical forces

7.1.1 Concentrated force

The variations of normal displacement are similar in nature for a particular medium at different times. These variations are less oscillatory for MTES. The values of normal displacement for MTESWV and TESWV increase initially and then oscillate with increase in horizontal distance. The increase is more sharp at t = 0.1. Also these variations for MTESWV and TESWV are of comparable magnitude in the range $2.0 \le x \le 10.0$. These variations of normal displacement are shown in Fig. 1.

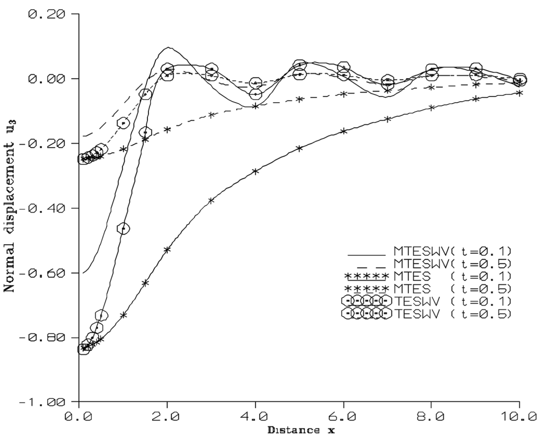

Fig. 1 Variation of Normal displacement u_3 with distance x (Concentrated force: Isothermal boundary)

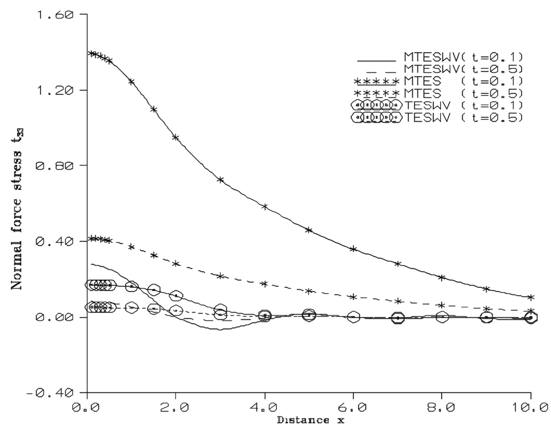

Fig. 2 Variation of Normal force stress t_{33} with distance x (Concentrated force: Isothermal boundary)

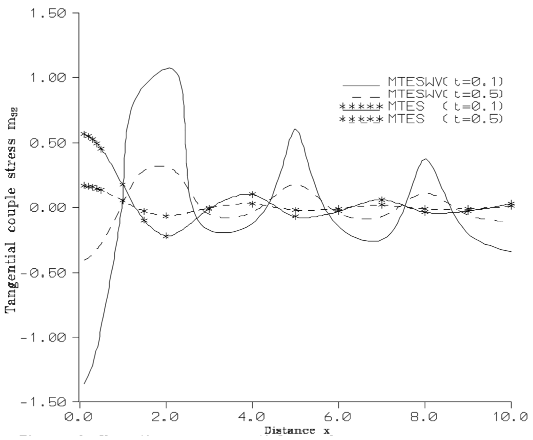

Fig. 3 Variation of Tangential couple stress m_{32} with distance x (Concentrated force: Isothermal boundary)

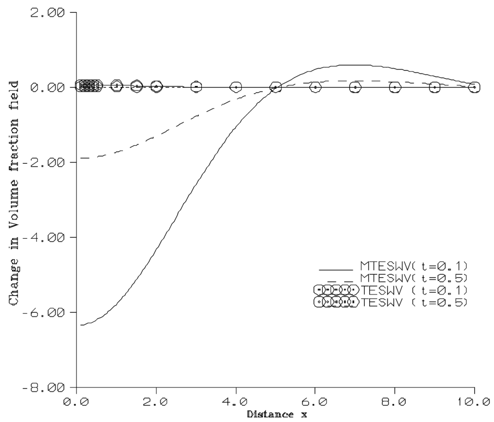

Fig. 4 Variation of change in Volume fraction field V with distance x (Concentrated force: Isothermal boundary)

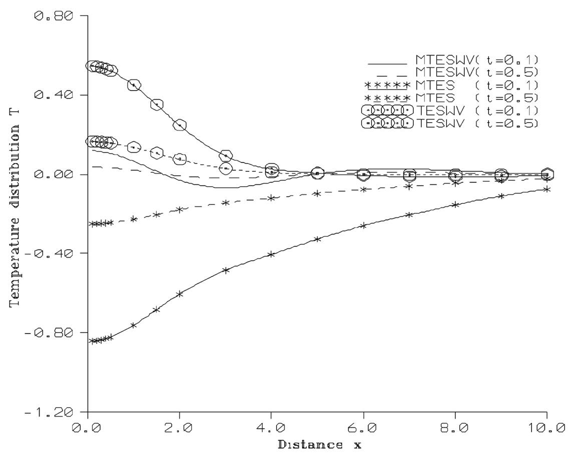

Fig. 5 Variation of Temperature distribution T with distance x (Concentrated force: Isothermal boundary)

The values of normal force stress for MTESWV and TESWV lie in a very short range as compared to the values for MTES. The values of normal force stress for a medium with voids (MTESWV and TESWV) are very close to each other in the range $4.0 \leq x \leq 10.0$. These variations of normal force stress for different mediums are shown in Fig. 2.

It is observed form Fig. 3 that the variations of tangential couple stress are oscillatory in nature for both the mediums (MTESWV and MTES). These variations are more oscillatory for MTESWV. In the initial range, the values of tangential couple stress increase sharply for MTESWV whereas the behaviour is opposite in nature for MTES.

Fig. 4 depicts that the change in volume fraction field is very less for TESWV i.e., significant micropolarity effect is observed. Near the point of application of source, the value of temperature distribution at a particular time is maximum for TESWV and minimum for MTES. These values of temperature distribution for different medium decrease in magnitude with increase in horizontal distance. The variations of temperature distribution are shown in Fig. 5.

7.1.2 Uniformly distributed force

When uniformly distributed force is applied on the boundary of the surface then the variations of normal displacement and normal force stress are more oscillatory in nature for MTESWV in comparison to the variations obtained for MTES and TESWV. With increase in time the magnitude of oscillations of normal displacement and normal force stress decrease. These variations of normal displacement and normal force stress are shown in Figs. 6 and 7 respectively.

The variations of tangential couple stress, change in volume fraction field and temperature distribution are similar in nature to the variations obtained in case of concentrated force with difference in magnitude. These variations of tangential couple stress, change in volume fraction field and temperature distribution are shown in Figs. 8, 9 and 10 respectively.

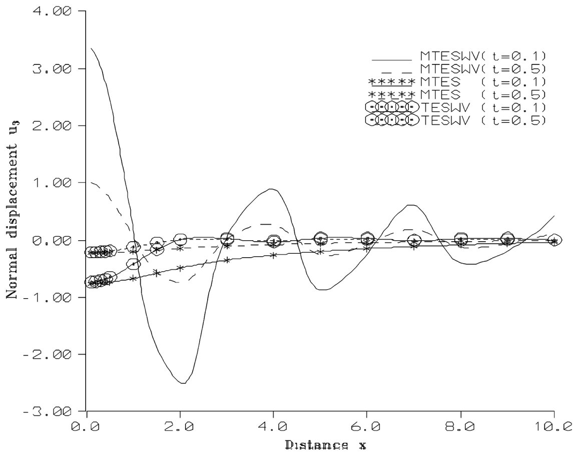

Fig. 6 Variation of Normal displacement u_3 with distance x (Uniformly distributed force: Isothermal boundary)

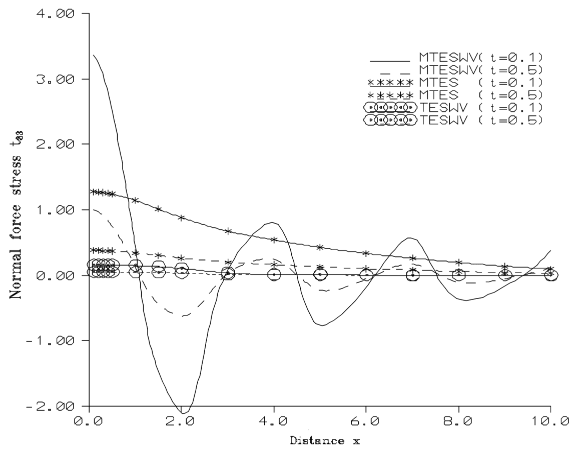

Fig. 7 Variation of Normal force stress t_{33} with distance x (Uniformly distributed force: Isothermal boundary)

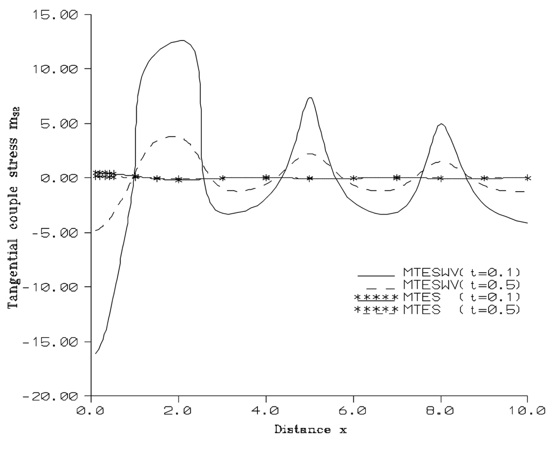

Fig. 8 Variation of Tangential couple stress m_{32} with distance x (Uniformly distributed force: Isothermal boundary)

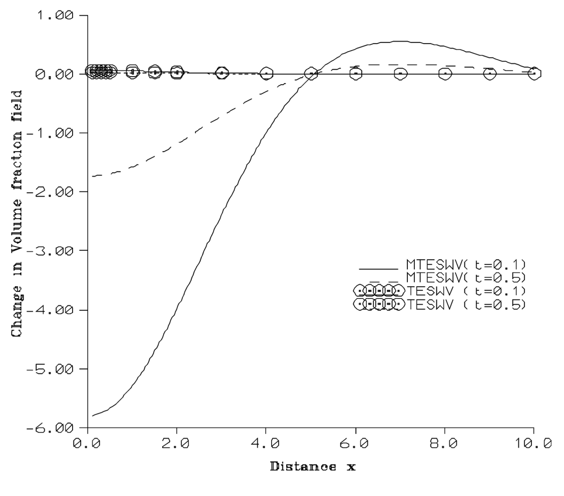

Fig. 9 Variation of change in Volume fraction field V with distance x (Uniformly distributed force: Isothermal boundary)

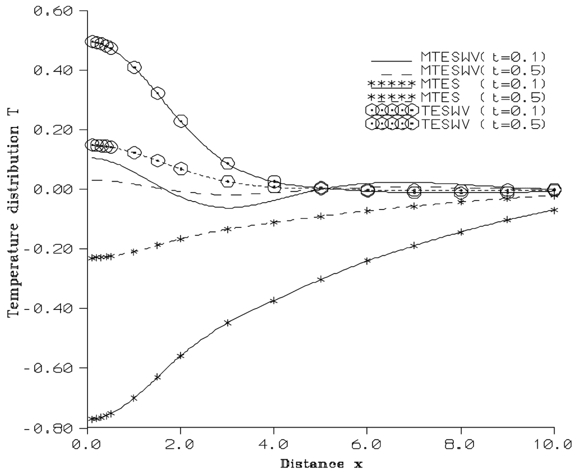

Fig. 10 Variation of Temperature distribution T with distance x (Uniformly distributed force: Isothermal boundary)

7.2 Thermal sources

7.2.1 Thermal point source

The values of normal displacement for MTES and TESWV increase sharply and then oscillate with horizontal distance. However, these variations are opposite in nature for MTESWV. Also the magnitude of sharpness decrease with increase in time. These variations of normal displacement for different mediums are shown in Fig. 11.

While the variations of normal force stress are oscillatory in nature for MTESWV and TESWV in the entire range, the values of normal force stress goes on decreasing for MTES. These variations of normal force stress are depicted in Fig. 12.

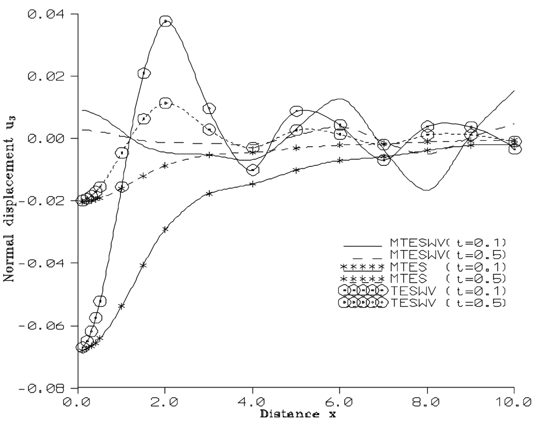

Fig. 11 Variation of Normal displacement u_3 with distance x (Thermal point source: Isothermal boundary)

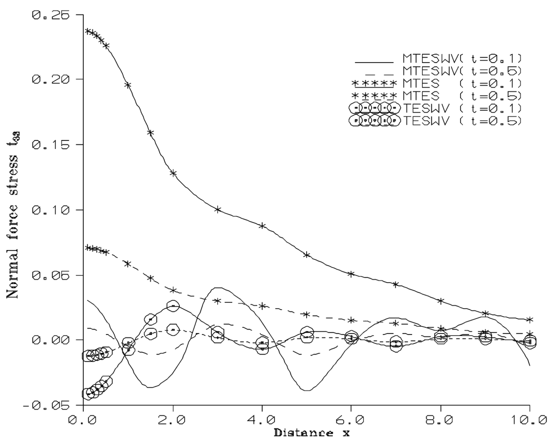

Fig. 12 Variation of Normal force stress t_{33} with distance x (Thermal point source: Isothermal boundary)

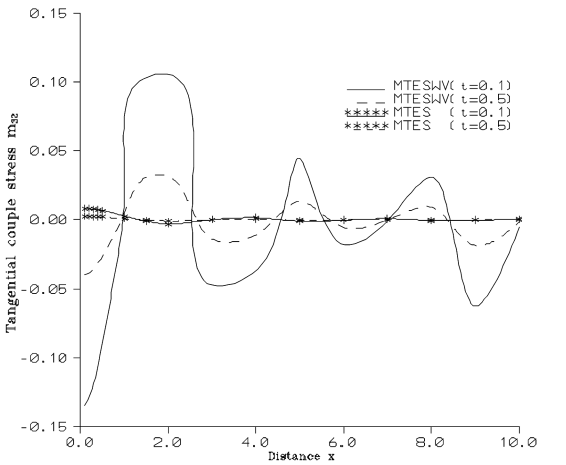

Fig. 13 Variation of Tangential couple stress m_{32} with distance x (Thermal point source: Isothermal boundary)

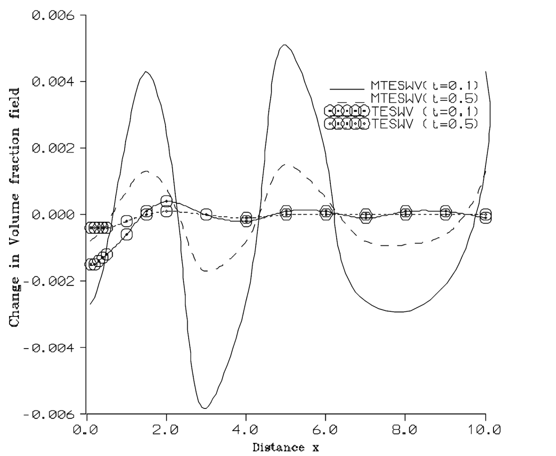

Fig. 14 Variation of change in Volume fraction field V with distance x (Thermal point source: Isothermal boundary)

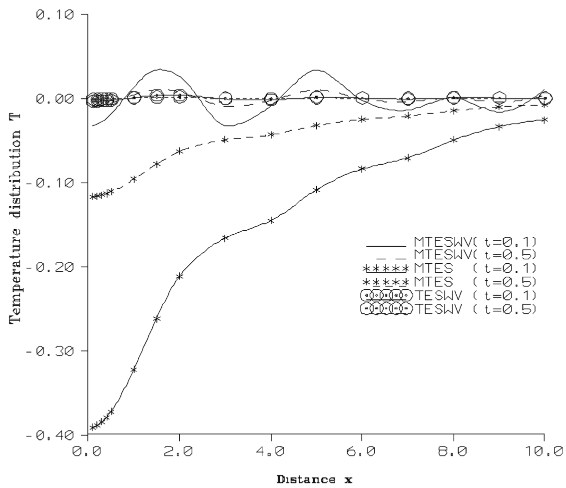

Fig. 15 Variation of Temperature distribution T with distance x (Thermal point source: Isothermal boundary)

It is observed from Fig. 13 that the variations of tangential couple stress are similar as discussed in case of mechanical force. The values of change in volume fraction field for MTESWV at t = 0.1 are highly oscillatory in nature and the magnitude of these oscillations decrease with increase in time. These variations, on neglecting micropolarity effect i.e., for TESWV are less oscillatory in nature and hence these values lie in a short range. These variations may be observed in Fig. 14.

It is interesting to observe from Fig. 15 that the variations of temperature distribution for MTES are similar to the variations obtained in case of mechanical forces. But the values of temperature distribution for other mediums i.e., MTESWV and TESWV lie in a short range, which is in contrast to the variations obtained on application of mechanical forces.

7.2.2 Uniformly distributed thermal source

The variations of all the quantities are similar in nature with difference in magnitude to the variations obtained on application of thermal point source. These variations of normal displacement, normal force stress, tangential couple stress, change in volume fraction field and temperature distribution are shown in Figs. 16-20 respectively.

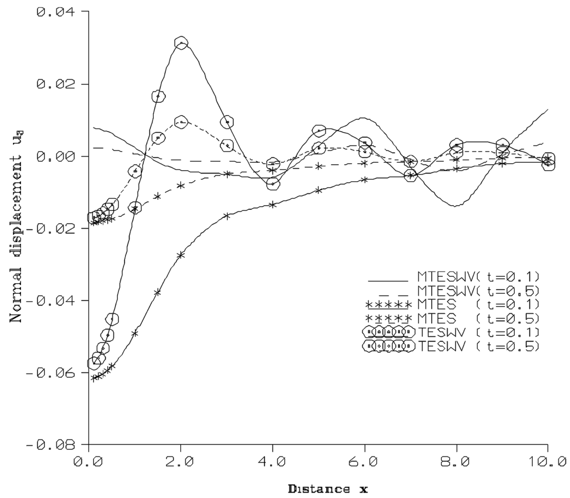

Fig. 16 Variation of Normal displacement u_3 with distance x (Uniformly distributed thermal source : Isothermal boundary)

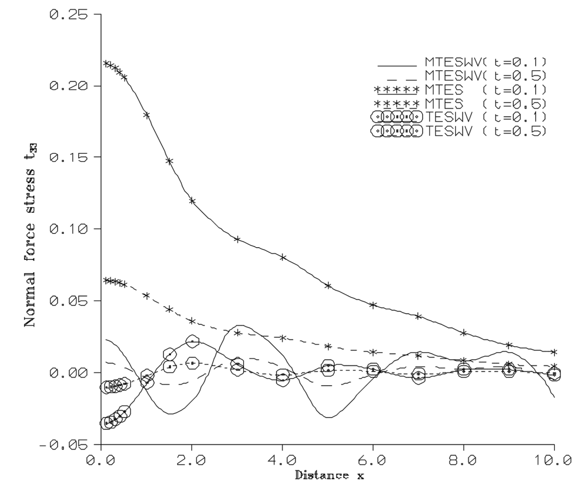

Fig. 17 Variation of Normal force stress t_{33} with distance x (Uniformly distributed thermal source : Isothermal boundary)

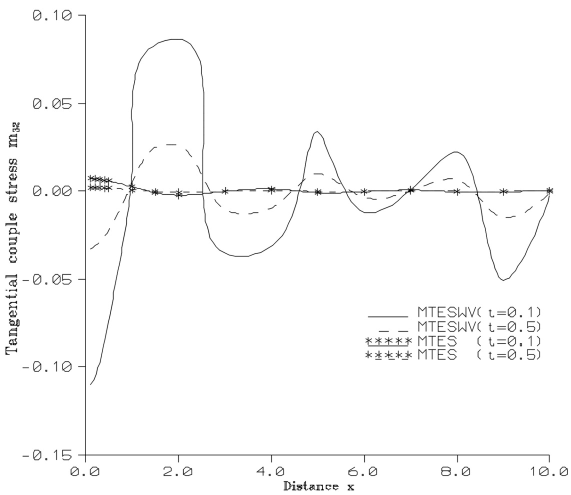

Fig. 18 Variation of Tangential couple stress m_{32} with distance x (Uniformly distributed thermal source : Isothermal boundary)

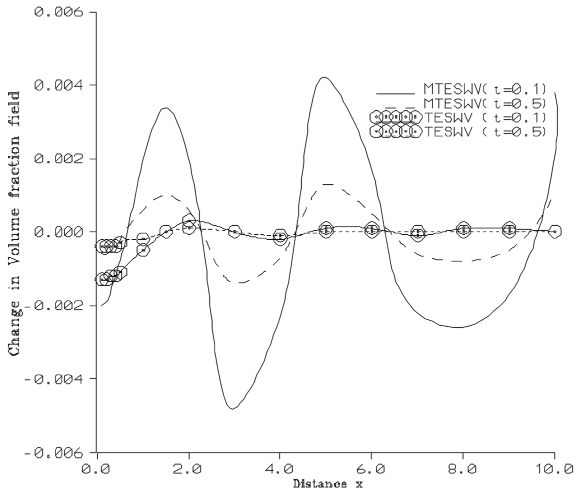

Fig. 19 Variation of change in Volume fraction field V with distance x (Uniformly distributed thermal source : Isothermal boundary)

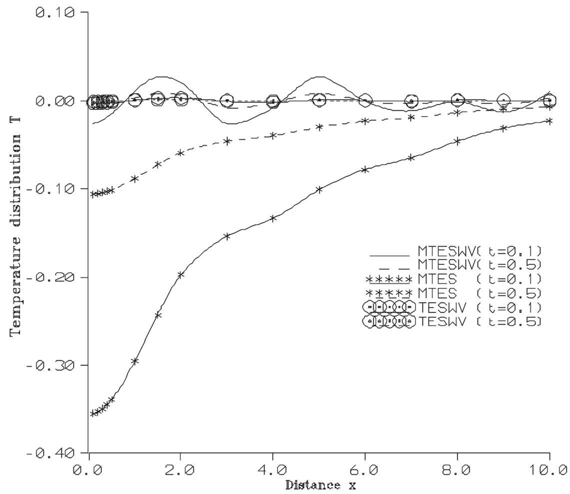

Fig. 20 Variation of Temperature distribution T with distance x (Uniformly distributed thermal source : Isothermal boundary)

8. Conclusions

Significant micropolarity and voids effect are observed on all the quantities. The variations of change in volume fraction field and temperature distribution are similar in nature for concentrated force and uniformly distributed force. Also the variations of tangential couple stress are similar for both mechanical and thermal sources.

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