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Technical Note

Dynamic response of a beam supported with damper under moving load

Murat Reis† and Yaşar Pala

Uludağ University, Engineering Faculty, Mechanical Engineering Department, Görükle,16059, Bursa, Turkey

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1. Introduction

Dynamic response of uniform beams and rods under moving loads has received considerable attention within the framework of Euler-Bernoulli beam theory. In the case of a concentrated force moving with a constant velocity along the beam, neglecting damping forces, Timoshenko found a solution and gave an expression for the critical velocity (Timoshenko 1927). Stanisic and Hardin also studied the same problem for simply supported beam carrying a moving mass (Stanisic and Hardin 1969). Esmail Zadeh and Ghorashi investigated the behavior of a finite beam carrying moving point masses (Esmail Zadeh and Ghorashi 1995). They have also dealt with the vibration analysis of beams due to partially distributed masses moving with constant velocity.

In order to reduce the amplitude of vibrating beam, dampers can be used. The behavior of the system in this case is practically very important and needs to be analyzed in detail. Gürgöze considered a somewhat similar problem and derived eigencharacteristics of a clamped free beam carrying a tip mass and damped by a single viscous damper (Gürgöze 1998). In this paper, as a different technique, the response of a simply supported finite beam which is subjected to viscous damping and carries a moving force is investigated by transforming singular moving force and singular damping force to a continuously distributed force field by means of the property of Dirac delta distribution function. This enables us to study with eigenvalues of hinged-hinged finite beam.

2. Analysis

The system under consideration is shown in Fig. 1. The uniform beam is simply supported, moving velocity V and moving load F are constants. Linearly viscous damper is located at $x = \eta L$, where $0 < \eta < 1$.

† Ph.D., Corresponding author, E-mail: reis@uludag.edu.tr

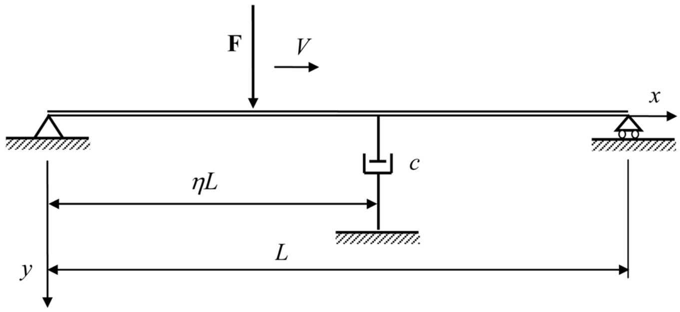

Fig. 1 Simply Supported Damped Finite Beam Carrying Moving Load F

According to the Euler-Bernoulli beam theory, the governing partial differential equation describing the transverse vibration of the beam carrying the time-varying force $F(x, t)$ per unit length is (Stanisic and Hardin 1969)

$$EI\frac{\partial^4 y}{\partial x^4} + m\frac{\partial^2 y}{\partial t^2} = f(x,t) \tag{1}$$

In the present work, in contrast to the approach in a former paper (Gürgöze 1998), we don't divide the domain of solution into two parts. Instead, both damping force and the moving load are considered as distributed loads. Through this approach, matching conditions at $x = \eta L$ don't have to be used. The calculation of eigenvalues in this situation becomes very simple. We assume that the displacement $y(x, t)$ in the forced vibration of the beam has the form

$$y(x,t) = \sum_{n=1}^{\infty} a_n(t)X_n(x) \tag{2}$$

Here, $a_n(t)$'s are unknown functions to be determined, $X_n(x)$'s are the eigenfunctions of the beam in free vibration. Since the beam is hinged at both ends, eigenfunctions must be taken as

$$X_n(x) = \sin\frac{n\pi}{L}x \qquad n = 1, 2, 3, \ldots \tag{3}$$

In the present case, substituting Eq. (3) into Eq. (2) yields

$$y(x,t) = \sum_{n=1}^{\infty} a_n(t)\sin\bar{k}_n x \tag{4}$$

Before we replace Eq. (4) into Eq. (1), moving force F and the viscous damping force must be expanded into Fourier sinus series to avoid the matching conditions at $x = \eta L$. It is a simple matter to show that the moving force is expressed as

$$F = F(x,t) = F\delta(x - Vt) = \frac{2F}{L}\sum_{n=1}^{\infty} \sin\frac{n\pi Vt}{L}\sin\bar{k}_n x \tag{5}$$

Here, $\delta(x - Vt)$ is Dirac delta function. For the damping force $F_d = c\dot{y}(\eta L, t)$ at the point $x = \eta L$, we assume

$$F_d(\eta L, t) = \sum_{n=1}^{\infty} \frac{2c\dot{a}_n\sin(\bar{k}_n\eta L)}{L}\sin\bar{k}_n x \tag{6}$$

We take care that the force F_d is expressed in term of unknown $\dot{a}_n(t)$'s. Inserting Eq. (10), Eq. (5) and Eq. (6) into Eq. (1) and arranging terms, we obtain

$$\ddot{a}_n + D_n\dot{a}_n + K_n a_n = F_0 \sin\omega_F t \qquad n = 1, 2, \ldots \tag{7}$$

Here, we have introduced the following abbreviations

$$D_n = \frac{2c\sin\bar{k}_n(\eta L)}{mL}, \quad K_n = \frac{\bar{k}_n^4}{b^2}, \quad F_0 = \frac{2F}{EILb^2} = \frac{2F}{mL}, \quad \omega_F = \frac{n\pi V}{L} \tag{8}$$

The solution of homogenous part of Eq. (7) is Pala (2006)

$$(a_n)_h = b_1 e^{s_1 t} + b_2 e^{s_2 t} \tag{9}$$

For the proper solution of Eq. (7), let us assume

$$(a_n)_p = \gamma_n \sin\omega_F t + \xi_n \cos\omega_F t \tag{10}$$

Substitution of Eq. (10) into Eq. (7) gives

$$\gamma_n = \frac{(K_n - \omega_F^2)F_0}{[(\omega_F^2 - K_n)^2 + D_n^2\omega_F^2]}, \quad \xi_n = \frac{-D_n\omega_F F_0}{[(\omega_F^2 - K_n)^2 + D_n^2\omega_F^2]} \tag{11}$$

Combining Eq. (10) and Eq. (9) to have the general solution of Eq. (7) gives

$$a_n(t) = (a_n)_h + (a_n)_p = b_1 e^{s_1 t} + b_2 e^{s_2 t} + \gamma_n \sin\omega_F t + \xi_n \cos\omega_F t \tag{12}$$

Applying initial conditions $y(x, 0) = 0$, $\dot{y}(x, 0) = 0$ yield.

$$b_1 = \frac{\xi_n s_2 - \gamma_n\omega_F}{s_1 - s_2}, \quad b_2 = \frac{\gamma_n\omega_F - \xi_n s_1}{s_1 - s_2} \tag{13}$$

The problem is now completely solved.

In this work, the effect of moving loads on the damped beams has been investigated by using Fourier sinus series approach. According to the present method, matching conditions do not constitute a problem in the solution, and frequency analysis is readily carried out. The method may also be extended to the case of several damping and linear springs.

As an extension of the method, one can also investigate the effect of curvature on the response of the beam without making much change since the curved shape can be easily expanded into Fourier sinus or cosines series. When other methods are used, one cannot readily obtain analytical results, and is forced into using numerical techniques.

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