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Dynamic analysis of thin-walled open section beam under moving vehicle by transfer matrix method

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Abstract. Three dimensional coupled bending-torsion dynamic vibrations of thin-walled open section beam subjected to moving vehicle are investigated by transfer matrix method. Through adopting the idea of Newmark-β method, the partial differential equations of structural vibration can be transformed to the differential equations. Then, those differential equations are solved by transfer matrix method. An iterative scheme is proposed to deal with the coupled bending-torsion terms in the governing vibration equations. The accuracy of the presented method is verified through two numerical examples. Finally, with different eccentricities of vehicle, the torsional vibration of thin-walled open section beam and vertical and rolling vibration of truck body are investigated. It can be concluded from the numerical results that the torsional vibration of beam and rolling vibration of vehicle increase with the eccentricity of vehicle. Moreover, it can be observed that the torsional vibration of thin-walled open section beam may have a significant nonlinear influence on vertical vibration of truck body.

Keywords: thin-walled beam; dynamic response; vehicle; transfer matrix method; Newmark-β method.

1. Introduction

The dynamic response of bridge subjected to moving vehicle or train has long been a challenging topic of civil engineering. During the past century, a lot of research works have been continuously contributed to this field. The initial work about this subject was pioneered by Inglis in (Inglis 1934), Then, Hillerborg studied this problem by means of Fourier's transformation method (A. Hillergborg). In recent thirty years, increasingly complex computational models for dynamic analysis of bridge under moving vehicle or train were proposed. On the basis of those models, this subject has been studied intensively and lots of useful conclusions are obtained (Sridharan and Mallik 1979, Wiriyachai *et al.* 1982, Chompooming and Yener 1995, Cheung *et al.* 1999, Yang and Wu 2001, Cheng and Au 2001, Song *et al.* 2003, Xia *et al.* 2003, Lou 2005, Yang and Lin 2005,

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Xia and Zhang 2005). Meantime, it also should be noted that those following works are essentially based on the early theory of Inglis and Hillerborg.

Thin-walled beam with open section has been widely used as structural bending member in bridge engineering. Generally, since the centroid and shear centre of thin-walled open section beam are not coincident, the dynamic vibrations will be coupled between bending and torsion. Timoshenko and Young (1955) firstly developed the theory for coupled vibrations of thin-walled beam. Bishop *et al.* (1989) developed this theory, and gave the solution of the governing differential equations for coupled bending-torsion free vibrations of thin-walled open section beam. Moreover, they announced that omission of warping stiffness will cause considerable errors in the results of natural frequencies and modes of coupled free vibration. Based on the work of Bishop *et al.*, Tanaka and Bercin (1999) studied the triple coupled free vibration of thin-walled beam with nonsymmetrical open section. Recently, Li *et al.* (2004) investigated the free vibration of thin-walled open section beam under axial load by transfer matrix method. For the dynamic analysis of forced vibration of thin-walled beam under moving vehicle, the finite element method can give a general solution (Song *et al.* 2003, Huang *et al.* 1993, 1995). However, development of analytical method based on the classical theory should be imposed more emphasizes, since such method can provide more comprehensive inspections into the nature. To the best knowledge of authors, the work about this subject is limited. In the previous researches, Michaltsos *et al.* (2005) presented the frequency domain solution of coupled bending-torsion vibration of the thin-walled open section beam under moving load based on the classical vibration governing equations.

As a semi-analytical algorithm, transfer matrix method can be efficiently used for one-dimensional periodic structures (although not limited to this case). Once the transfer matrix of a representative element is obtained from the classical governing differential equation, the solution of whole structure can be obtained without requiring great computational efforts (Pestel and Leckie 1993). In those previous literatures, the vibration of one-dimensional structure is studied comprehensively by the transfer matrix method (Li *et al.* 2004, Wang *et al.* 1999, Lee 2000). Although the transfer matrix method can be conveniently used in structural dynamic analysis in frequency domain, the work about applying transfer matrix method into structural dynamic analysis in time domain is comparably rare. Xiang and Zhao (2005) proposed an algorithm by combining transfer matrix method and Newmark-β method to study the vertical bending dynamic vibration of bridge under moving vehicle.

In the present study, the authors intend to study the three-dimensional coupled bending-torsion dynamic vibrations of thin-walled open section beam subjected to moving vehicle following the basic idea proposed by Xiang and Zhao (2005). The classical vibration equations of thin-walled beam are used to model the forced vibrations of bridge. A 7DOF vehicle model, which comprises of 3 rigid bodies, one for the truck body and two for the axle sets, is adopted.

2. Formulation of the transfer matrix

2.1 Governing vibration equations

For generality, a thin-walled beam with nonsymmetrical open section shown in Fig. 1 is considered. The shear center and centriod are denoted by S and C respectively. In the Cartesian coordinate system illustrated in Fig. 1, the x axis coincides with the elastic axis (i.e., loci of the shear center). y_c and z_c are the coordinates of centriod C in S_{yz} plane. Using d'Alembert principle,

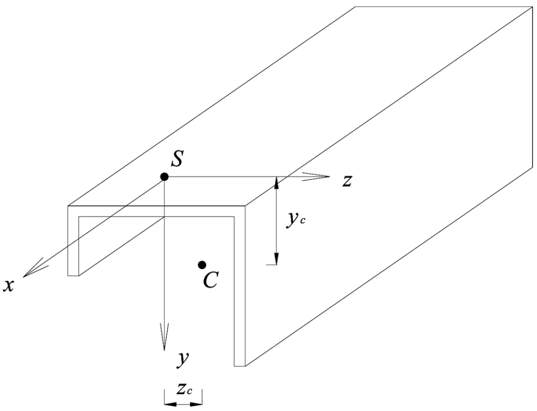


Fig. 1 The thin-walled beam with nonsymmetrical open section

the differential equations for coupled vibrations of thin-walled beam can be written as

$$EI_z \cdot \frac{\partial^4 u}{\partial x^4} + c \cdot \frac{\partial(u - z_c \cdot \theta)}{\partial t} + \overline{m} \cdot \frac{\partial^2 (u - z_c \cdot \theta)}{\partial t^2} = P_y(x, t) \tag{1}$$

$$EI_y \cdot \frac{\partial^4 v}{\partial x^4} + c \cdot \frac{\partial(v - y_c \cdot \theta)}{\partial t} + \overline{m} \cdot \frac{\partial^2 (v - y_c \cdot \theta)}{\partial t^2} = P_z(x, t) \tag{2}$$

$$\begin{aligned} &EI_\omega \cdot \frac{\partial^4 \theta}{\partial x^4} - GI_t \cdot \frac{\partial^2 \theta}{\partial x^2} + c \cdot \frac{\partial \theta}{\partial t} - c \cdot z_c \cdot \frac{\partial u}{\partial t} + c \cdot y_c \cdot \frac{\partial v}{\partial t} \\ &+ \overline{m}_\omega \cdot \frac{\partial^2 \theta}{\partial t^2} - \overline{m} \cdot z_c \cdot \frac{\partial^2 u}{\partial t^2} + \overline{m} \cdot y_c \cdot \frac{\partial^2 v}{\partial t^2} = T_x(x, t) \end{aligned} \tag{3}$$

where u and v represent the displacements of shear centre S in y and z directions, and θ is the torsional rotation angle about x axis. EI_z and EI_y are the flexural rigidities about the neutral axes in S_{xz} and S_{xy} planes, respectively. EI_ω and GI_t are the warping and torsional constants. $\overline{m}$ is the mass per unit length, and $\overline{m}_\omega$ is the polar moment of inertia about the shear center per unit length. c is the damping coefficient. $P_y(x,t)$ and $P_z(x,t)$ are the external forces acted on the beam along y and z directions, and $T_x(x,t)$ represents the applied torque about x axis.

To obtain the time-domain solutions of Eqs. (1) to (3), the incremental form of Eqs. (1) to (3) for time step i can be written as

$$EI_z \cdot \frac{\partial^4 \Delta u_i}{\partial x^4} + c \cdot \frac{\partial(\Delta u_i - z_c \cdot \Delta\theta_i)}{\partial t} + \overline{m} \cdot \frac{\partial^2 (\Delta u_i - z_c \cdot \Delta\theta_i)}{\partial t^2} = \Delta P_{yi} \tag{4}$$

$$EI_y \cdot \frac{\partial^4 \Delta v_i}{\partial x^4} + c \cdot \frac{\partial(\Delta v_i + y_c \cdot \Delta\theta_i)}{\partial t} + \overline{m} \cdot \frac{\partial^2 (\Delta v_i + y_c \cdot \Delta\theta_i)}{\partial t^2} = \Delta P_{zi} \tag{5}$$

$$\begin{aligned} &EI_\omega \cdot \frac{\partial^4 \Delta\theta_i}{\partial x^4} - GI_t \cdot \frac{\partial^2 \Delta\theta_i}{\partial x^2} + c \cdot \frac{\partial \Delta\theta_i}{\partial t} - c \cdot z_c \cdot \frac{\partial \Delta u_i}{\partial t} + c \cdot y_c \cdot \frac{\partial \Delta v_i}{\partial t} \\ &+ \overline{m}_\omega \cdot \frac{\partial^2 \Delta\theta_i}{\partial t^2} - \overline{m} \cdot z_c \cdot \frac{\partial^2 \Delta u_i}{\partial t^2} + \overline{m} \cdot y_c \cdot \frac{\partial^2 \Delta v_i}{\partial t^2} = \Delta T_{xi} \end{aligned} \tag{6}$$

It can be observed that there exist coupled terms in Eqs. (4) to (6). Therefore, it is almost impossible to obtain the analytical solutions for Eqs. (4) to (6) directly. To overcome this dilemma, those coupled terms are moved to the right hands, and Eqs. (4) to (6) will be rewritten as

$$EI_z \cdot \frac{\partial^4 \Delta u_i}{\partial x^4} + c \cdot \frac{\partial \Delta u_i}{\partial t} + \overline{m} \cdot \frac{\partial^2 \Delta u_i}{\partial t^2} = \Delta P_{yi} + z_c \cdot c \cdot \frac{\partial \Delta \theta_i}{\partial t} + z_c \cdot \overline{m} \cdot \frac{\partial^2 \Delta \theta_i}{\partial t^2} \tag{7}$$

$$EI_y \cdot \frac{\partial^4 \Delta v_i}{\partial x^4} + c \cdot \frac{\partial \Delta v_i}{\partial t} + \overline{m} \cdot \frac{\partial^2 \Delta v_i}{\partial t^2} = \Delta P_{zi} - y_c \cdot c \cdot \frac{\partial \Delta \theta_i}{\partial t} - y_c \cdot \overline{m} \cdot \frac{\partial^2 \Delta \theta_i}{\partial t^2} \tag{8}$$

$$EI_\omega \cdot \frac{\partial^4 \Delta \theta_i}{\partial x^4} - GI_t \cdot \frac{\partial^2 \Delta \theta_i}{\partial x^2} + c \cdot \frac{\partial \Delta \theta_i}{\partial t} + \overline{m}_\omega \cdot \frac{\partial^2 \Delta \theta_i}{\partial t^2}$$

$$= \Delta T_{xi} + c \cdot z_c \cdot \frac{\partial \Delta u_i}{\partial t} - c \cdot y_c \cdot \frac{\partial \Delta v_i}{\partial t} + \overline{m} \cdot z_c \cdot \frac{\partial^2 \Delta u_i}{\partial t^2} - \overline{m} \cdot y_c \cdot \frac{\partial^2 \Delta v_i}{\partial t^2} \tag{9}$$

Then, those three coupled equations can be solved by the iterative scheme described as following.

First, the values of $\frac{\partial \Delta u_i}{\partial t}, \frac{\partial^2 \Delta u_i}{\partial t^2}, \frac{\partial \Delta v_i}{\partial t}, \frac{\partial^2 \Delta v_i}{\partial t^2}, \frac{\partial \Delta \theta_i}{\partial t}$ and $\frac{\partial^2 \Delta \theta_i}{\partial t^2}$ at the right hands of Eqs. (7) to (9) are set to be zero, and those three equations are uncoupled and can be solved by the following presented time-domain transfer matrix algorithm. Then, substituting the values of $\frac{\partial \Delta u_i}{\partial t}, \frac{\partial^2 \Delta u_i}{\partial t^2}, \frac{\partial \Delta v_i}{\partial t}, \frac{\partial^2 \Delta v_i}{\partial t^2}, \frac{\partial \Delta \theta_i}{\partial t}$ and $\frac{\partial^2 \Delta \theta_i}{\partial t^2}$ obtained from the previous step into the right hands of Eqs. (7) to (9), the new results of the displacements, velocities and accelerations can be computed with a similar procedure. This process will be repeated until a satisfactory convergence is reached.

2.2 Time-domain transfer matrix method

Using the idea of Newmark-β method (1959), the incremental acceleration and velocity at time step i can be given as

$$\frac{\partial^2 \Delta y_i}{\partial t^2} = \frac{\Delta y_i}{\beta \Delta t^2} - \frac{\dot{y}_i}{\beta \Delta t} - \frac{\ddot{y}_i}{2\beta} \tag{10}$$

$$\frac{\partial \Delta y_i}{\partial t} = \frac{\Delta y_i}{2\beta \Delta t^2} - \frac{\dot{y}_i}{2\beta} + \left(1 - \frac{1}{4\beta}\right) \Delta t \ddot{y}_i \tag{11}$$

in which y, $\dot{y}$ and $\ddot{y}$ can represent the displacement, velocity and acceleration of u, v, or θ, respectively. Newmark suggested that the value of parameter β would be in the range of $\frac{1}{6} \le \beta \le \frac{1}{2}$. For $\beta = \frac{1}{4}$ this method is proven to be unconditionally stable and provides satisfactory accuracy.

Substituting Eqs. (10) and (11) into Eqs. (7) to (9) yields

$$EI_z \cdot \frac{d^4 \Delta u_i}{dx^4} + \left(\frac{c}{2\beta \Delta t} + \frac{\overline{m}}{\beta \Delta t^2}\right) \Delta u_i = \Delta P_{yi} + z_c \cdot c \cdot \frac{\partial \Delta \theta_i}{\partial t} + z_c \cdot \overline{m} \cdot \frac{\partial^2 \Delta \theta_i}{\partial t^2}$$

$$+c\left[\frac{\dot{u}_i}{2\beta}-\left(1-\frac{1}{4\beta}\right)\Delta t\ddot{u}_i\right]+\overline{m}\left(\frac{\dot{u}_i}{\beta\Delta t}+\frac{\ddot{u}_i}{2\beta}\right) \tag{12}$$

$$EI_y\cdot\frac{d^4\Delta v_i}{dx^4}+\left(\frac{c}{2\beta\Delta t}+\frac{\overline{m}}{\beta\Delta t^2}\right)\Delta v_i = \Delta P_{zi}-y_c\cdot c\cdot\frac{\partial\Delta\theta_i}{\partial t}-y_c\cdot\overline{m}\cdot\frac{\partial^2\Delta\theta_i}{\partial t^2}$$

$$+c\left[\frac{\dot{v}_i}{2\beta}-\left(1-\frac{1}{4\beta}\right)\Delta t\ddot{v}_i\right]+\overline{m}\left(\frac{\dot{v}_i}{\beta\Delta t}+\frac{\ddot{v}_i}{2\beta}\right) \tag{13}$$

$$EI_\omega\cdot\frac{d^4\Delta\theta_i}{dx^4}-GI_t\cdot\frac{d^2\Delta\theta_i}{dx^2}+\left(\frac{c}{2\beta\Delta t}+\frac{\overline{m}_\omega}{\beta\Delta t^2}\right)\Delta\theta_i= T_{xi}+c\cdot z_c\cdot\frac{\partial\Delta u_i}{\partial t}-c\cdot y_c\cdot\frac{\partial\Delta v_i}{\partial t}$$

$$+\overline{m}\cdot z_c\cdot\frac{\partial^2\Delta u_i}{\partial t^2}-\overline{m}\cdot y_c\cdot\frac{\partial^2\Delta v_i}{\partial t^2}+c\left[\frac{\dot{\theta}_i}{2\beta}-\left(1-\frac{1}{4\beta}\right)\Delta t\ddot{\theta}_i\right]+\overline{m}_\omega\left(\frac{\dot{\theta}_i}{\beta\Delta t}+\frac{\ddot{\theta}_i}{2\beta}\right) \tag{14}$$

Thus, Eqs. (7) to (9) are transformed to differential equations, which can be solved by the transfer matrix method conveniently. The solution of Eq. (12) or (13) by the transfer matrix method can be found in the study of Xiang and Zhao (2005). In this paper, the solution of Eq. (14) will be presented.

The characteristic equation of Eq. (14) can be expressed as

$$EI_\omega\cdot r^4-GI_t\cdot r^2+\left(\frac{c}{2\beta\Delta t}+\frac{\overline{m}_\omega}{\beta\Delta t^2}\right) = 0 \tag{15}$$

Generally, the value of the torsional constant GI_t is far less than that of the warping constant EI_ω, therefore, the four characteristic roots of Eq. (15) can be written as

$$\pm k_1 \pm k_2 i \tag{16}$$

Then, the solution of Eq. (14) is given as

$$\Delta\theta_i = c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) + c_4 f_4(x) + \sum_{j=1}^{4} D_j(x) f_j(x) \tag{17}$$

in which $c_1 \sim c_4$ are a set of coefficients, $\sum_{j=1}^{4} D_j(x) f_j(x)$ is the particular solution of Eq. (14), and the expressions of $f_j(x)$ are given as

$$\begin{aligned} f_1(x) &= \cosh k_1x\cdot\cos k_2x, & f_2(x) &= \cosh k_1x\cdot\sin k_2x \\ f_3(x) &= \sinh k_1x\cdot\cos k_2x, & f_4(x) &= \sinh k_1x\cdot\sin k_2x \end{aligned} \tag{18}$$

By adopting the method of variation of parameters, the solutions of $D_j(x)$ can be determined (Zill and Cullen 2001). The analytical expressions of $D_j(x)$ when the right hand of Eq. (14) is a concentrated load or a linear varying distributing load are given in the appendix.

From Eq. (17), and considering $\sum_{j=1}^{4} D'_j(x) f_j(x) = \sum_{j=1}^{4} D'_j(x) f'_j(x) = \sum_{j=1}^{4} D'_j(x) f''_j(x) = 0$, the analytical expressions of incremental torsional rotation ratio $\Delta\theta'_i$, incremental bimoment ΔB_i, and incremental torque ΔT_i are obtained

$$\begin{aligned}\Delta\theta'_i = \frac{d\Delta\theta_i}{dx} &= c_1[k_1 f_3(x) - k_2 f_2(x)] + c_2[k_1 f_4(x) + k_2 f_1(x)] \\ &+ c_3[k_1 f_1(x) - k_2 f_4(x)] + c_4[k_1 f_2(x) + k_2 f_3(x)]\end{aligned} \tag{19}$$

$$\Delta B_i = -EI_\omega \cdot \frac{d^2\Delta\theta_i}{dx^2} = -EI_\omega \cdot \{c_1[k_1^2 f_1(x) - k_2^2 f_1(x) - 2k_1k_2 f_4(x)] + c_2[k_1^2 f_2(x) - k_2^2 f_2(x) + 2k_1k_2 f_3(x)] + c_3[k_1^2 f_3(x) - k_2^2 f_3(x) - 2k_1k_2 f_2(x)] + c_4[k_1^2 f_4(x) - k_2^2 f_4(x) + 2k_1k_2 f_1(x)] \tag{20}$$

$$\Delta T_i = -EI_\omega \cdot \frac{d^3\Delta\theta_i}{dx^3} + GI_t \cdot \frac{d\Delta\theta_i}{dx} = -EI_\omega \cdot \{c_1[-k_1^3 f_3(x) - k_2^3 f_2(x) + 3k_1^2 k_2 f_2(x) + 3k_1k_2^2 f_3(x)] + c_2[-k_1^3 f_4(x) + k_2^3 f_1(x) - 3k_1^2 k_2 f_1(x) + 3k_1k_2^2 f_4(x)] + c_3[-k_1^3 f_1(x) - k_2^3 f_4(x) + 3k_1^2 k_2 f_4(x) + 3k_1k_2^2 f_1(x)] + c_4[-k_1^3 f_2(x) + k_2^3 f_3(x) - 3k_1^2 k_2 f_3(x) + 3k_1k_2^2 f_2(x)]\} + GI_t \cdot \{c_1[k_1 f_3(x) - k_2 f_2(x)] + c_2[k_1 f_4(x) + k_2 f_1(x)] + c_3[k_1 f_1(x) - k_2 f_4(x)] + c_4[k_1 f_2(x) + k_2 f_3(x)]\} \tag{21}$$

Defining the state vector and coefficient vector as

$$\{z(x)\} = \{\Delta\theta_i \quad \Delta\theta_i' \quad \Delta B_i \quad \Delta T_i \quad 1\}^T \tag{22}$$

$$\{C\} = \{c_1 \quad c_2 \quad c_3 \quad c_4 \quad 1\} \tag{23}$$

Then, Eqs. (17), (19) to (21) can be expressed in the matrix form as

$$\{Z(x)\} = [H(x)]\{C\} \tag{24}$$

For a given element, assuming that the coordinates of the left and right ends are 0 and L, substituting $x = 0$ and $x = L$ into Eq. (24) yields

$$\{Z(0)\} = [H(0)]\{C\} \tag{25}$$

$$\{Z(L)\} = [H(L)]\{C\} \tag{26}$$

From Eqs. (25) and (26), it can be obtained that

$$\{Z(L)\} = [H(L)][H(0)]^{-1}\{Z(0)\} = [T]\{Z(0)\} \tag{27}$$

in which $[T]$ is the element transfer matrix, and the analytical expressions of its' nontrivial elements are given as

$$T_{11} = f_1(L) - \frac{(k_1^2 - k_2^2)f_4(L)}{2k_1k_2}$$

$$T_{12} = \frac{k_1[GI_t - EI_\omega(k_1^2 - 3k_2^2)]f_2(L) - k_2[GI_t - EI_\omega(k_2^2 - 3k_1^2)]f_3(L)}{2EI_\omega k_1k_2(k_1^2 + k_2^2)}$$

$$T_{13} = \frac{f_4(L)}{2EI_\omega k_1k_2}, \qquad T_{14} = \frac{k_1 f_2(L) - k_2 f_3(L)}{2EI_\omega k_1k_2(k_1^2 + k_2^2)}, \qquad T_{15} = \sum_{j=1}^{4} D_j(x) f_j(x)$$

$$T_{21} = \frac{(k_1^2 - k_2^2)[k_1 f_2(L) - k_2 f_3(L)]}{2k_1k_2}, \qquad T_{22} = f_1(L) + \frac{[GI_t - EI_\omega(k_1^2 - k_2^2)]f_4(L)}{2EI_\omega k_1k_2}$$

$$T_{23} = -\frac{k_1 f_2(L) + k_2 f_3(L)}{2EI_\omega k_1k_2}, \qquad T_{24} = -\frac{f_4(L)}{2EI_\omega k_1k_2}, \qquad T_{25} = \sum_{j=1}^{4} D_j(x) f_j'(x)$$

$$T_{31} = \frac{EI_\omega(k_1^2 + k_2^2)^2 f_4(L)}{2k_1k_2}$$

$$T_{32} = \frac{k_1[EI_\omega(k_1^2-k_2^2)-GI_t]f_2(L)-k_2[EI_\omega(k_1^2+k_2^2)+GI_t]f_3(L)}{2k_1k_2}$$

$$T_{33} = f_1(L)+\frac{(k_1^2-k_2^2)f_4(L)}{2k_1k_2}, \qquad T_{34} = -\frac{k_1f_2(L)+k_2f_3(L)}{2k_1k_2}, \qquad T_{35} = -EI_\omega\sum_{j=1}^{4}D_j(x)f_j''(x)$$

$$T_{41} = \frac{(k_1^2+k_2^2)\{k_1[EI_\omega(k_1^2+k_2^2)-GI_t]f_2(L)+k_2[EI_\omega(k_2^2+k_1^2)+GI_t]f_3(L)\}}{2k_1k_2}$$

$$T_{42} = \frac{[G_t^2-EI_\omega GI_t(k_1^2-k_2^2)+EI_\omega{}^2(k_1^2+k_2^2)^2]f_4(L)}{2EI_\omega k_1k_2}$$

$$T_{43} = \frac{k_1[EI_\omega(k_1^2-3k_2^2)-GI_t]f_2(L)+k_2[EI_\omega(3k_1^2-k_2^2)-GI_t]f_3(L)}{2EI_\omega k_1k_2}$$

$$T_{44} = f_1(L)+\frac{[EI_\omega(k_1^2-k_2^2)-GI_t]f_4(L)}{2EI_\omega k_1k_2}$$

$$T_{45} = -EI_\omega\sum_{j=1}^{4}D_j(x)f_j'''(x)+GI_t\sum_{j=1}^{4}D_j(x)f_j'(x), \qquad T_{55} = 1 \tag{28}$$

Once the transfer matrices for all elements are known, multiplying those matrices successively, the global transfer matrix for whole structure is obtained

$$\{Z\}_{n+1} = [T]_n\{Z\}_{n-1} = [T]_n[T]_{n-1}\{Z\}_{n-2} = \cdots = \prod_{m=1}^{n}[T]_m\{Z\}_0 \tag{29}$$

i.e.,

$$\{Z\}_{n+1} = [T]_G\{Z\}_0 \tag{30}$$

where $\{Z\}_m$ ($m = 1 \sim n+1$) is the state vector of the *mth* node, $[T]_G$ is the global transfer matrix, and $[T]_m$ ($m = 1 \sim n$) element transfer matrix of the *mth* element. Once the global transfer matrix is obtained, the boundary condition should be introduced into Eq. (30). For example, the boundary conditions for the torsion of simply supported thin-walled beam are specified as $\theta = 0$ and $B = 0$. Then, the value of $\{Z\}_0$ is solved. Finally, substituting $\{Z\}_0$ into Eq. (29), the results of all state vectors can be computed successively.

3. Vehicle model

To investigate the dynamic responses of vehicle, a lot of 2D or 3D vehicle models, which consider such vehicle motions as bouncing, swaying, pitching, yawing, rolling, sliding, and so on are proposed during the past three decades. The vehicle model adopted in the present study is illustrated in Fig. 2. In this model, the vehicle consists of three rigid bodies: one for the truck body, and two for the axle sets. The truck body is modeled as a rigid body with mass *mc* and mass moment of inertia J_{cx} and J_{cz} about x and z axis through its centroid. Therefore, 3 DOFs are assigned to the truck body with respect to its gravity centre: the vertical displacement (y_c), the rotation about z axis (pitching angle ϕ_{cz}), and the rotation about x axis (rolling angle ϕ_{cx}). Each axle

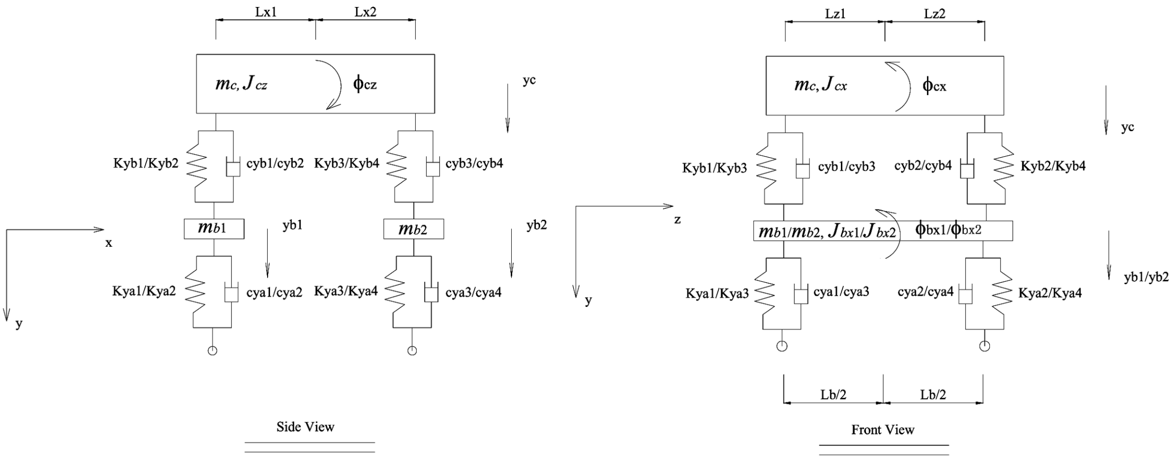


Fig. 2 The vehicle model

set is assumed to be a rigid body with mass m_{b1} or m_{b2} and mass moment of inertia J_{bx1} or J_{bz2} about x axis through its centroid. Then, the motions of axle set can be described by the vertical displacement (y_{b1} or y_{b2}) and the rotation about the x axis (ϕ_{bx1} or ϕ_{bx2}). It is always assumed that the wheels keep in contact with the bridge. Using d'Alembert principle, the motion equations of vehicle can be finally obtained. To obtain the dynamic responses of vehicle, following the procedure presented by Xiang and Zhao (2005), the motion equations of vehicle vibration can be transferred as algebraic equations with the idea of Newmark-β method. The vertical interactive force P_y^i between the ith tire and bridge is given as

$$P_y^1 = \frac{m_{b1}(g-\ddot{y}_{b1})}{2} - \frac{J_{bx1}\ddot{\phi}_{bx1}}{L_b} + \frac{L_{x2}}{L_{x1}+L_{x2}}\frac{L_{z2}}{L_{z1}+L_{z2}}m_c(g-\ddot{y}_c) + \frac{1}{2}\frac{J_{cz}\ddot{\phi}_{cz}}{L_{x1}+L_{x2}} - \frac{1}{2}\frac{J_{cx}\ddot{\phi}_{cx}}{L_{z1}+L_{z2}} \tag{29}$$

$$P_y^2 = \frac{m_{b1}(g-\ddot{y}_{b1})}{2} - \frac{J_{bx1}\ddot{\phi}_{bx1}}{L_b} + \frac{L_{x2}}{L_{x1}+L_{x2}}\frac{L_{z1}}{L_{z1}+L_{z2}}m_c(g-\ddot{y}_c) + \frac{1}{2}\frac{J_{cz}\ddot{\phi}_{cz}}{L_{x1}+L_{x2}} + \frac{1}{2}\frac{J_{cx}\ddot{\phi}_{cx}}{L_{z1}+L_{z2}} \tag{30}$$

$$P_y^3 = \frac{m_{b2}(g-\ddot{y}_{b2})}{2} - \frac{J_{bx2}\ddot{\phi}_{bx2}}{L_b} + \frac{L_{x1}}{L_{x1}+L_{x2}}\frac{L_{z2}}{L_{z1}+L_{z2}}m_c(g-\ddot{y}_c) - \frac{1}{2}\frac{J_{cz}\ddot{\phi}_{cz}}{L_{x1}+L_{x2}} - \frac{1}{2}\frac{J_{cx}\ddot{\phi}_{cx}}{L_{z1}+L_{z2}} \tag{31}$$

$$P_y^4 = \frac{m_{b2}(g-\ddot{y}_{b2})}{2} + \frac{J_{bx2}\ddot{\phi}_{bx2}}{L_b} + \frac{L_{x1}}{L_{x1}+L_{x2}}\frac{L_{z1}}{L_{z1}+L_{z2}}m_c(g-\ddot{y}_c) - \frac{1}{2}\frac{J_{cz}\ddot{\phi}_{cz}}{L_{x1}+L_{x2}} + \frac{1}{2}\frac{J_{cx}\ddot{\phi}_{cx}}{L_{z1}+L_{z2}} \tag{32}$$

in which the dot stands for differentiation with respect to time t, and g is gravity acceleration.

4. Example studies

4.1 Example 1: A simply supported thin-walled beam under moving force

The coupled lateral-torsion vibration of a simply supported thin-walled beam with open section subjected to moving constant force, as illustrated in Fig. 3, is investigated by Michaltsos *et al.*

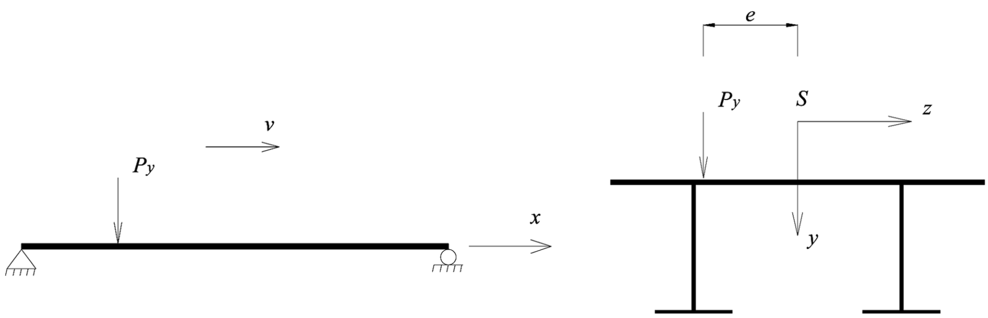


Fig. 3 A simply supported thin-walled beam under moving force

(2005). In this paper, this example is adopted to verify the accuracy of the presented algorithm.

The span of beam is 50 m, and the structural properties are given as following: $E = 2.1 \times 10^{10}$ Pa, $I_z = 0.227$ m^4, $I_y = 1.654$ m^4, $I_\omega = 0.88$ m^6, $I_t = 6 \times 10^5$ m^4, $\overline{m} = 2512$ kg/m, $\overline{m}_\omega = 1476.6$ kgm^2/m, $y_c = 1.376$ m, and $z_c = 0$. The damping coefficient of beam is assumed to be zero. In the study of Michaltsos, the moving force model is used, and two levels of the magnitude of moving force Py are considered, i.e., P_y equals 1500 N or 50000 N, respectively (Michaltsos *et al.* 2005). For the presented vehicle model, the following data are assumed: $m_c = 0.5\ P_y/g$, $m_{b1} = m_{b2} = 0.25P_y/g$, $J_{cx} = J_{cz} = J_{bx1} = J_{bx2} = 2.0$ kgm^2, $L_{x1} = L_{x2} = L_{z1} = L_{z2} = 0.1$ m, $L_b = 0.2$ m, $K_{ya1} = K_{ya2} = K_{ya3} = K_{ya4} = K_{yb1} = K_{yb2} = K_{yb3} = K_{yb4} = 200$ N/m, and $c_{ya1} = c_{ya2} = c_{ya3} = c_{ya4} = c_{yb1} = c_{yb2} = c_{yb3} = c_{yb4} = 0$. In this study, the eccentricity e of 2 m is adopted. The moving velocity of load v_l is 20 m/s for P_y equals 1500 N, and 10 m/s for Py equals 50000N. The results of the vertical displacements of centroid, transverse displacements of deck, and rotational angle about shear centre at midspan are plotted in Figs. 4-5. To verify the accuracy of the presented procedure, the vertical displacement is compared with the analytical solution given by Krylov (1905), and the transverse displacement and rotational angle are compared with those presented by Michaltsos *et al.* (2005).

From Figs. 4-5, it can be observed that the results of this paper agree with those previous researches well. The discrepancies may be due to the different theories adopted in programming and the inertia of vehicle considered in the present study.

4.2 Example 2: A simply supported beam bridge under a moving suspended rigid body

To verify the accuracy of dynamic response of vehicle, a simply supported beam subjected to moving suspend rigid body as illustrated in Fig. 6 is studied. The properties of beam are assumed to be $L = 30$ m, $E = 2.943 \times 10^4$ MPa, $I = 8.65$ m^4, $\overline{m} = 3.6 \times 10$ kg/m, and $c = 0$. For the moving rigid body, the following data are adopted: $m_c = 5.4 \times 10^5$ kg , $J_{cx} = 1.38 \times 10^7$ $kg\ m^2$, $m_{b1} = m_{b2} = J_{bx1} = J_{bx2} = 0$, $K_{yb1} = K_{yb2} = K_{yb3} = K_{yb4} = 2.0675 \times 104$ kN/m, $c_{yb1} = c_{yb2} = c_{yb3} = c_{yb4} = 0$, $K_{ya1} = K_{ya2} = K_{ya3} = K_{ya4} = 4.135 \times 10^{12}$ kN/m, $c_{ya1} = c_{ya2} = c_{ya3} = c_{ya4} = 0$, and $L_{x1} = L_{x2} = 8.75$ m.

The moving velocity of vehicle is 27.78 m/s (100 km/h). In this example, only the vertical vibration of bridge and vehicle is concerned. Yang and Wu (2001) studied the dynamic response of this example also. The time history responses of midpoint vertical displacement of beam, and vertical acceleration of rigid body are plotted in Figs. 7-8, respectively. As can be seen, the results presented in this paper agree with those of Yang and Wu very well, and the results comparison demonstrates the accuracy of the method presented by authors.

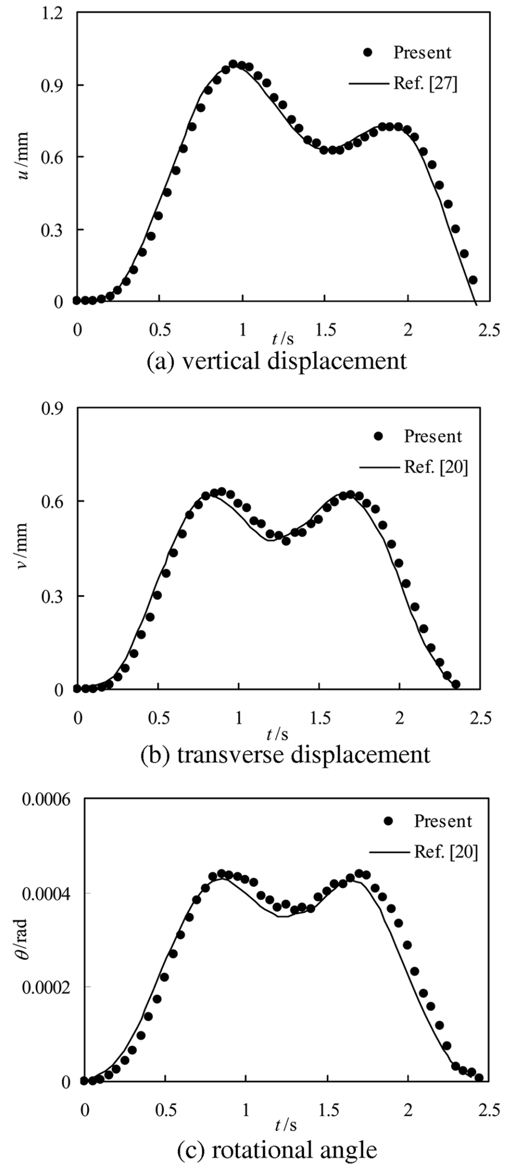


Fig. 4 The dynamic displacements of bridge at midspan when P_y = 1500 N, e = 2 m, and v_l = 20 m/s

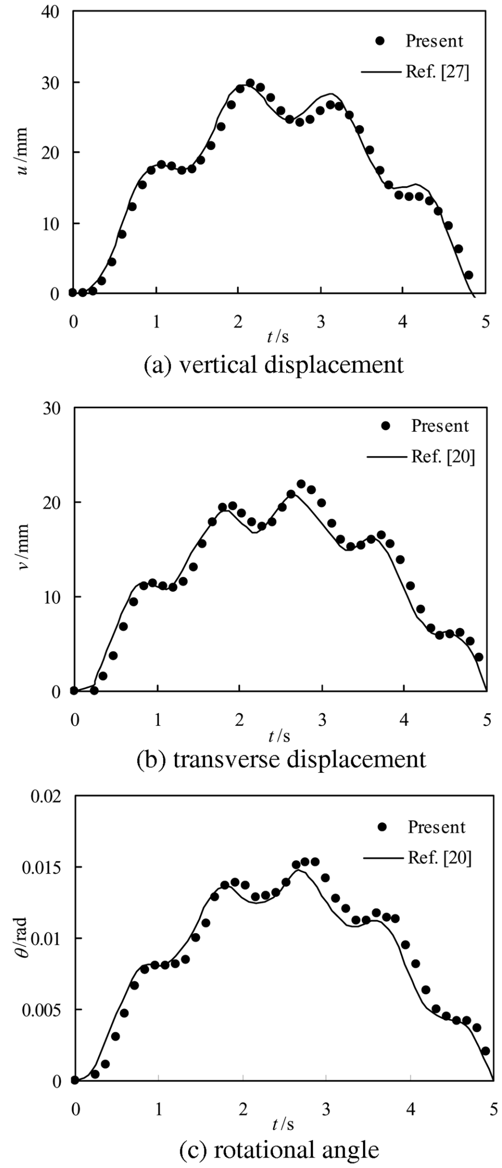


Fig. 5 The dynamic displacements of bridge at midspan when P_y = 50000 N, e = 2 m, and v_l = 10 m/s

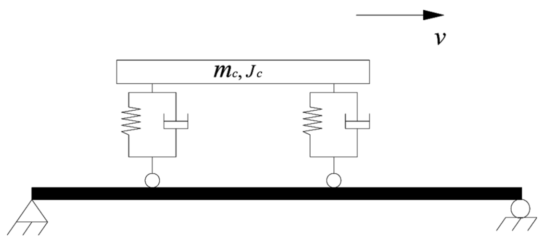


Fig. 6 A simply supported beam subjected to a moving suspended rigid body

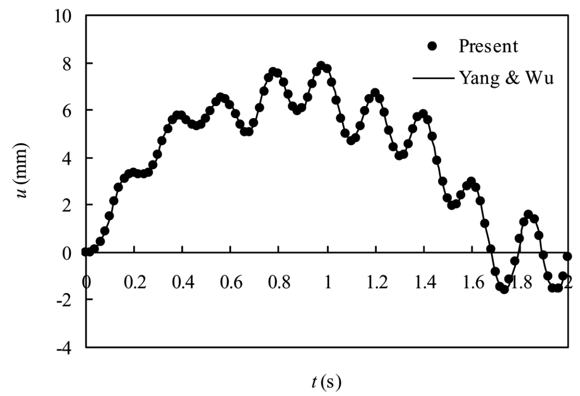


Fig. 7 Dynamic midspan vertical displacement of beam

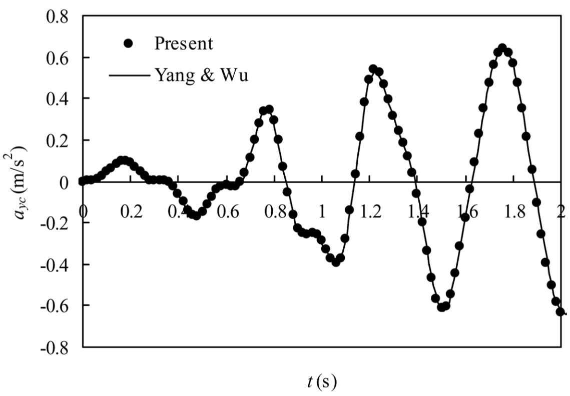


Fig. 8 Vertical acceleration of rigid body

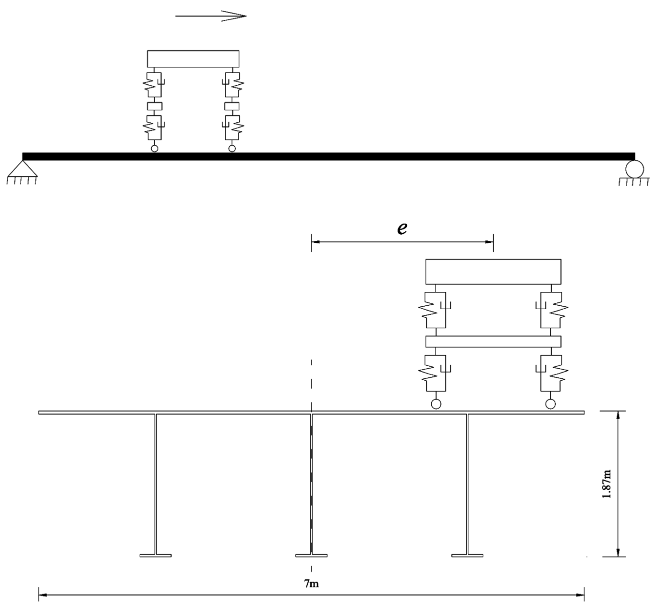
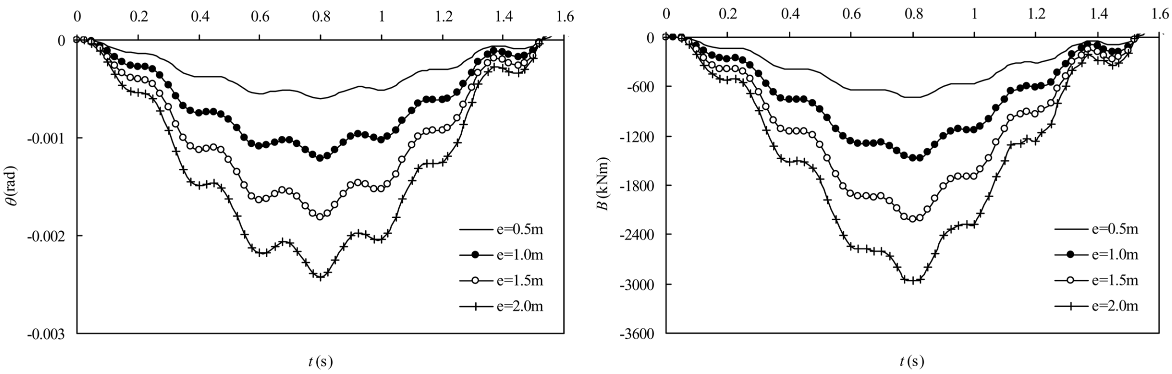


Fig. 9 A simply supported thin-walled beam with open section under moving vehicle

4.3 Example 3: A simply supported thin-walled beam under moving vehicle

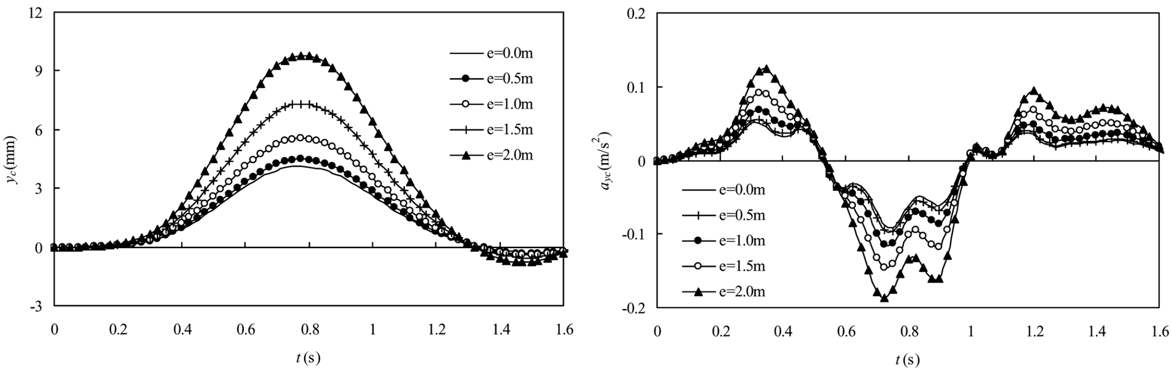
As shown in Fig. 9, a simply supported thin-walled beam with following properties, $L = 30$ m, $E = 2.0 \times 10^{11}$ Pa, $I_z = 0.1773$ m^4, $I_y = 1.528$ m^4, $I = 0.5197$ m^6, $I_t = 1.74 \times 10^4$ m^4, $\overline{m} = 3349.6$ kg/m, $\overline{m}_\omega = 3.54 \times 10^4$ kgm^2/m, $y_c = 0.678$ m, $z_c = 0$, and $c = 0$, is considered. The parameters for the vehicle are assumed to be: $m_c = 21000$ kg, $m_{b1} = m_{b2} = 200$ kg, $J_{cx} = 32000$ kgm^2, $J_{cz} = 74000$ kgm^2, $J_{bx1} = J_{bx2} = 300$ kgm^2, $L_{x1} = L_{x2} = 2.1$ m, $L_{z1} = L_{z2} = 0.9$ m, $L_b = 1.8$ m, $K_{ya1} = K_{ya2} = K_{ya3} = K_{ya4} = 6.4 \times 10^5$ m/N, $c_{ya1} = c_{ya2} = c_{ya3} = c_{ya4} = 7.5 \times 10^3$ Ns/m, $K_{yb1} = K_{yb2} = K_{yb3} = K_{yb4} = 1.2 \times 10^6$ N/m, and $c_{yb1} = c_{yb2} = c_{yb3} = c_{yb4} = 2.1 \times 10^4$ Ns/m. The moving velocity of vehicle is 80 km/h (22.22 m/s).

In this example, the authors intend to investigate the influences of eccentricity of vehicle e on the dynamic responses of beam and vehicle. For dynamic vibration of beam, the torsion vibration is the most direct response induced by eccentricity of vehicle. In Fig. 10, the time history of rotational angle and bimoment of beam at midspan with different eccentricity of vehicle are plotted. For dynamic vibration of vehicle, due to the rotation of beam, the rolling vibration of truck body will be



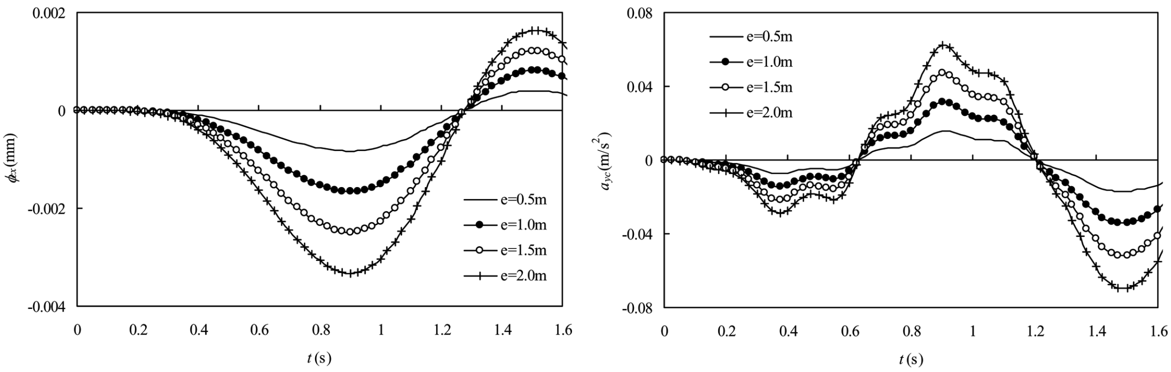
(a) rotational angle (b) bimoment

Fig. 10 The dynamic torsional responses of bridge at midspan



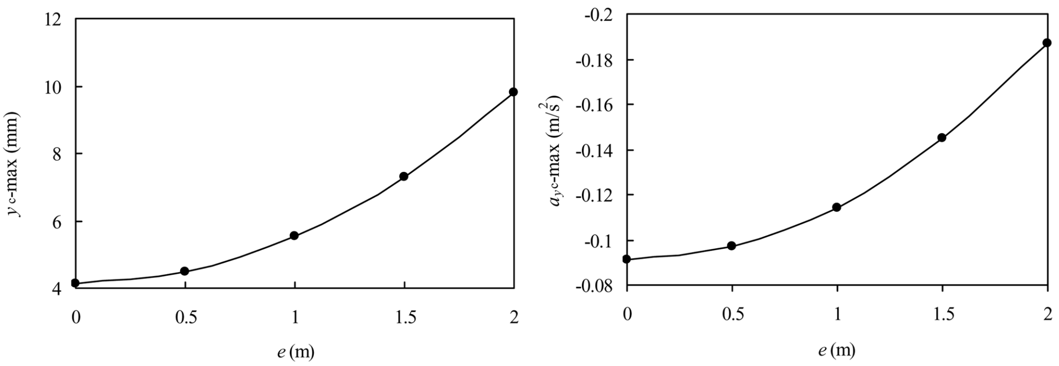
(a) deflection (b) acceleration

Fig. 11 The vertical vibration of truck body



(a) rolling angle (b) acceleration

Fig. 12 The rolling vibration of truck body



(a) deflection (b) acceleration

Fig. 13 The relations between absolute maximum vertical dynamic response of truck body and e

activated. Meantime, the rotation of beam produces an additional vertical deflection of deck at contact points between tire and bridge, which will influence the vertical responses of truck body. Therefore, the dynamic vertical deflection and acceleration, rolling angle and acceleration of truck body are given in Figs. 11 and 12.

From Fig. 10, it can be found that the dynamic torsional vibrations of beam increases almost linearly with the eccentricity of vehicle. Generally, the major cause for vehicle-induced vibration of bridge is the gravity of vehicle. With increment of eccentricity of vehicle, the torque applied on the beam increase linearly. Therefore, a near linear relationship between torsional vibration and eccentricity of vehicle can be predicted. Since the rolling vibration of truck body is mainly activated by torsion of beam, a similar tendency is also observed for rolling vibration of truck body in Fig. 12. However, for the vertical vibration of truck body, an obvious nonlinear increase of dynamic responses with the eccentricity of vehicle is observed. To give a more clear inspection about this phenomenon, the relations between absolute maximum vertical dynamic deflection and acceleration of truck body and eccentricity of vehicle e are plotted in Fig. 13. It can be found that the maximum responses of vertical vibration of truck body trend to increase more quickly with increment of eccentricity of vehicle. As mentioned above, an additional vertical deflection of deck is caused by rotation of beam, and this additional vertical deflection is a production of rotational angle of beam and transverse distance between tire and vertical symmetric axis. Since the rotational angle of beam is an almost linear function of eccentricity of vehicle, the additional vertical deflection of deck will be approximate second times function of eccentricity of vehicle. Therefore, a nonlinear increase of vertical vibration of truck body with eccentricity of vehicle can be observed.

5. Conclusions

In this paper, an algorithm, which combines transfer matrix method with Newmark-β method, is presented for dynamic analysis of thin-walled open section beam subjected to moving vehicle. An iterative scheme is proposed to deal with the coupled bending-torsion terms in the governing vibration equations. Through two numerical examples, the accuracy of the algorithm is demonstrated.

Considering that the memory requirement of this algorithm is low, and only matrix multiplication operation is involved in solution, the proposed algorithm may be more efficient compared with the finite element method.

The effect of eccentricity of vehicle on vibrations of thin-walled open section beam and vehicle are studied. From the results, it can be found that eccentricity of vehicle has an almost linear influence on the torsional vibration of beam and rolling vibration of truck body. Moreover, it can be concluded that the torsional vibration of thin-walled open section beam may have a significant nonlinear influence on vertical vibration of truck body. This phenomenon should be imposed further researches in the following studies.

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Appendix

When the right hand of Eq. (14) is a concentrated load, which is given as $P \cdot \delta(x - x_0)$, the analytical expressions of $D_j(x)$ are given as

$$D_1(x) = P \cdot \frac{-k_1 f_2(x_0) + k_2 f_3(x_0)}{2EI_\omega k_1 k_2 (k_1^2 + k_2^2)}, \qquad D_2(x) = P \cdot \frac{k_1 f_1(x_0) + k_2 f_4(x_0)}{2EI_\omega k_1 k_2 (k_1^2 + k_2^2)}$$

$$D_3(x) = P \cdot \frac{k_1 f_4(x_0) - k_2 f_1(x_0)}{2EI_\omega k_1 k_2 (k_1^2 + k_2^2)}, \qquad D_4(x) = P \cdot \frac{-k_1 f_3(x_0) - k_2 f_2(x_0)}{2EI_\omega k_1 k_2 (k_1^2 + k_2^2)}$$

When the right hand of Eq. (14) is a linear varying distributing load, which is given as $P_0 + ax$, the analytical expressions of $D_j(x)$ are given as

$$D_1(x) = \frac{1}{2EI_\omega k_1 k_2 (k_1^2 + k_2^2)^3} [-2P_0 k_1 k_2 (k_1^2 + k_2^2) + 2k_1 k_2 (k_1^2 + k_2^2)(P_0 + ax) f_1(x) +$$
$$a k_1 (k_1^2 - 3k_2^2) f_2(x) + a k_2 (k_2^2 - 3k_1^2) f_3(x) - (k_1^4 - k_2^4)(P_0 + ax) f_4(x)]$$

$$D_2(x) = \frac{1}{2EI_\omega k_1 k_2 (k_1^2 + k_2^2)^3} [a k_1 (k_1^2\, 3k_2^2) + 2k_1 k_2 (k_1^2 + k_2^2)(P_0 + ax) f_2(x) -$$
$$a k_1 (k_1^2 - 3k_2^2) f_1(x) + a k_2 (k_2^2 - 3k_1^2) f_4(x) + (k_1^4 - k_2^4)(P_0 + ax) f_3(x)]$$

$$D_3(x) = \frac{1}{2EI_\omega k_1 k_2 (k_1^2 + k_2^2)^3} [a k_2 (k_2^2 - 3k_1^2) + (k_1^4 - k_2^4)(P_0 + ax) f_2(x) -$$
$$a k_2 (k_2^2 - 3k_1^2) f_1(x)\; a (k_1^2 - 3k_2^2) f_4(x) - 2k_1 k_2 (k_1^2 + k_2^2)(P_0 + ax) f_3(x)]$$

$$D_4(x) = \frac{1}{2EI_\omega k_1 k_2 (k_1^2 + k_2^2)^3} [P_0 (k_1^4 - k_2^4) - (k_1^4 - k_2^4)(P_0 + ax) f_1(x) -$$
$$a k_2 (k_2^2 - 3k_1^2) f_2(x) + a (k_1^2 - 3k_2^2) f_3(x) - 2k_1 k_2 (k_1^2 + k_2^2)(P_0 + ax) f_4(x)]$$