Structural Engineering and Mechanics, Vol. 30, No. 4 (2008) 445-466
DOI: http://dx.doi.org/10.12989/sem.2008.30.4.445

A new statistical moment-based structural damage detection method

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(Received September 27, 2007, Accepted August 1, 2008)

Abstract. This paper presents a novel structural damage detection method with a new damage index based on the statistical moments of dynamic responses of a structure under a random excitation. After a brief introduction to statistical moment theory, the principle of the new method is put forward in terms of a single-degree-of-freedom (SDOF) system. The sensitivity of statistical moment to structural damage is discussed for various types of structural responses and different orders of statistical moment. The formulae for statistical moment-based damage detection are derived. The effect of measurement noise on damage detection is ascertained. The new damage index and the proposed statistical moment-based damage detection method are then extended to multi-degree-of-freedom (MDOF) systems with resort to the least-squares method. As numerical studies, the proposed method is applied to both single and multi-story shear buildings. Numerical results show that the fourth-order statistical moment of story drifts is a more sensitive indicator to structural stiffness reduction than the natural frequencies, the second order moment of story drift, and the fourth-order moments of velocity and acceleration responses of the shear building. The fourth-order statistical moment of story drifts can be used to accurately identify both location and severity of structural stiffness reduction of the shear building. Furthermore, a significant advantage of the proposed damage detection method lies in that it is insensitive to measurement noise.

Keywords: damage detection; statistical moment; sensitivity; measurement noise.

1. Introduction

Civil engineering structures begin to deteriorate once they are built and continuously accumulate damage during their service life due to harsh environment such as corrosion, earthquake, and typhoon. Vibration-based structural damage detection methods have therefore attracted considerable attention in recent years for assessment of integrity and safety of complex civil engineering

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structures. Most of currently-used vibration-based structural damage detection methods are built on the idea that the measured modal parameters or the parameters derived from these modal parameters are functions of the physical properties of the structure and, therefore, changes in the physical properties will cause detectable changes in the modal parameters (Doebling *et al.* 1998).

The changes in modal parameters (damage indices) commonly used in vibration-based structural damage detection include natural frequency changes, mode shape changes, mode shape curvature changes, flexibility matrix changes, and modal strain energy changes (Xu *et al.* 2004, Kim *et al.* 2006, Zhao and DeWolf 2007). These damage indices are often estimated experimentally from the structural response time histories measured before and after the changes in physical properties of a structure. However, the structural damage typically is a local phenomenon, and the change of natural frequency has low sensitivity to local damages (Salawu 1997). Furthermore, natural frequency is a global property of the structure and it generally can not provide spatial information about damage location. The damage indices based on mode shape changes or those derivatives can give the spatial information on damage location in theory. However, they may not be effective and reliable in consideration of the number of sensors required and the measurement noise arising from the environment conditions during the test, and they generally do not provide the information regarding damage severity (Pandey *et al.* 1991, Farrar and Jauregui 1998). Alvandi and Cremona (2006) assessed various damage indices and concluded that the modal strain energy is less affected by measurement noise. However, even for the modal strain energy method, a 3% noise level is considered as a high level of noise which already makes it difficult to identify damage location of a structure. Another class of damage detection methods is based on the updating of structural modal matrices to reproduce as closely as possible the measured static or dynamic response of a structure (Link 2001). However, the measured structural responses are always contaminated by measurement noise but model updating exactly reproduces the contaminated structural responses. The results of damage detection from the model updating may thus become unreliable.

In view of the aforementioned studies, it can be seen that although vibration-based damage detection methods and modal updating methods have demonstrated various degrees of success, the damage detection of civil structures still remains a challenging task. The main obstacles are the insensitivity to local/minor structural damage for the methods based on modal properties (particularly modal frequencies) and the high sensitivity to measurement noise for the methods based on derivatives of modal parameters and the model updating methods which might be sensitive to local/minor structural damage. The high sensitivity to measurement noise also inheres in the new group of damage detection methods recently developed in the time domain or the time-frequency domain (Sohn *et al.* 2003, Cho *et al.* 2004, Chen and Xu 2007). Therefore, efficient and effective damage detection methods which are sensitive to local/minor structural damage but insensitive to measurement noise need to be pursued.

The purpose of this study is to explore a new damage detection method which is based on the statistical moments of dynamic responses of a structure under a random excitation with the expectation of being sensitive to minor structural damage and at the same time insensitive to measurement noise. With brief introduction of statistical theory, the principle of the new method is presented through a single-degree-of-freedom (SDOF) system. The sensitivity and noise issues of the proposed method are discussed in relation to various response types and different orders of statistical moment used. The new method is then extended to MDOF system in which the least square optimization technique is used to update stiffness of the system from the measured statistical moments. Numerical studies are presented to demonstrate the feasibility and accuracy of the new method in detecting damage location and evaluating damage severities.

2. Statistical moment theory

To introduce the statistical moment-based damage detection method, a brief review of statistical moment theory is presented here. For a linear structural system, if the excitation is a stationary Gaussian random process, then the structural response is also a stationary Gaussian random process (Meirovitch 1975).

The probability density function (PDF) of a structural response x of Gaussian distribution can be expressed by

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\bar{x})^2}{2\sigma^2}} \tag{1}$$

where $p(x)$ is the PDF of structural response x; σ is the standard deviation of structural response; and $\bar{x}$ is the mean value of structural response. The nth-order statistical moment of structural response can be given in terms of PDF by the following integrals

$$M_n = \int_{-\infty}^{+\infty} (x-\bar{x})^n p(x) dx \quad n = 1, 2, 3, 4, \ldots \tag{2}$$

In general, the odd moments relate to information about the position of the peak of probability density function in relation to the mean value while the even moments indicate the characteristics of the spread of the distribution. Fig. 1 shows variations of probability density function with variance of a zero-mean structural response of Gaussian distribution. It can be seen that for a large variance, the curve tends to be flatter and more spread out. Therefore, the shape of probability density function depends on the value of even statistical moments. Let us focus on even statistical moments up to sixth order for structural damage detection. For a structural response of Gaussian distribution, some relationships exist between the even statistical moments and the variance:

$$M_2 = \int_{-\infty}^{+\infty} (x-\bar{x})^2 p(x) dx = \sigma^2 \tag{3}$$

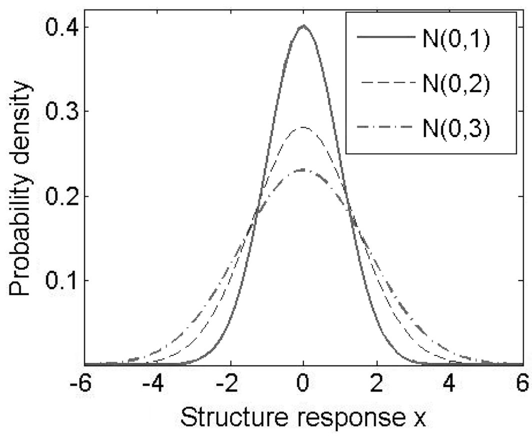


Fig. 1 Variations of Gaussian probability density function with zero mean

$$M_4 = \int_{-\infty}^{+\infty} (x - \bar{x})^4 p(x) dx = 3\sigma^4 \tag{4}$$

$$M_6 = \int_{-\infty}^{+\infty} (x - \bar{x})^6 p(x) dx = 15\sigma^6 \tag{5}$$

The differentiation of Eqs. (3), (4) and (5) with respect to the standard deviation σ then yields

$$\frac{dM_2}{M_2} = \frac{2\sigma \cdot d\sigma}{\sigma^2} = 2\frac{d\sigma}{\sigma} \tag{6}$$

$$\frac{dM_4}{M_4} = \frac{12\sigma^3 \cdot d\sigma}{3\sigma^4} = 4\frac{d\sigma}{\sigma} \tag{7}$$

$$\frac{dM_6}{M_6} = \frac{90\sigma^5 \cdot d\sigma}{15\sigma^6} = 6\frac{d\sigma}{\sigma} \tag{8}$$

The above expressions reveal that the relative changes in higher even statistical moment possesses higher sensitivity to the relative change in the standard deviation of a structural response. A more general approach to calculate the statistical moments of the measured structural response whenever it fits the Gaussian distribution or not is to use summation-type relationships as follows (Martin 1989)

$$M_1 = \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{9}$$

$$M_2 = \frac{1}{N}\sum_{i=1}^{N} x_i^2 - \bar{x}^2 \tag{10}$$

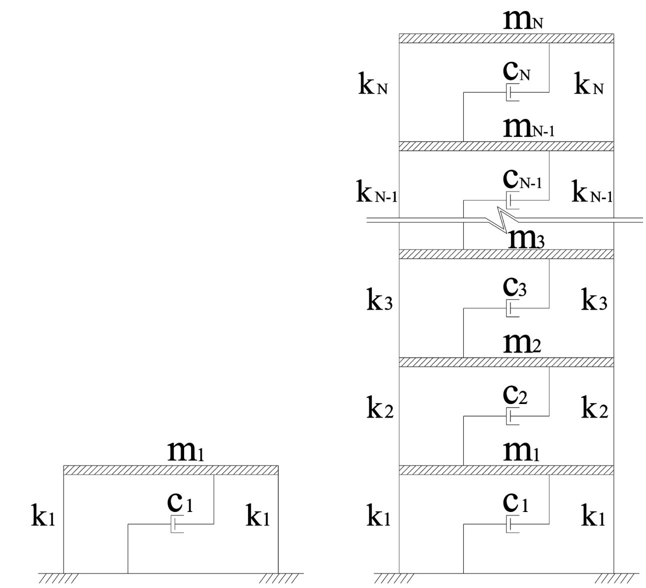
$$M_4 = \frac{1}{N}\sum_{i=1}^{N} x_i^4 - \frac{4}{N}\bar{x}\sum_{i=1}^{N} x_i^3 + \frac{6}{N}\bar{x}^2\sum_{i=1}^{N} x_i^2 - 3\bar{x}^4 \tag{11}$$

$$M_6 = \frac{1}{N}\sum_{i=1}^{N} x_i^6 - \frac{6}{N}\bar{x}\sum_{i=1}^{N} x_i^5 + \frac{15}{N}\bar{x}^2\sum_{i=1}^{N} x_i^4 - \frac{20}{N}\bar{x}^3\sum_{i=1}^{N} x_i^3 + \frac{15}{N}\bar{x}^4\sum_{i=1}^{N} x_i^2 - 5\bar{x}^6 \tag{12}$$

where N is the data point in a structural response time history recorded.

3. Statistical moment-based damage detection method for SDOF system

Structural damage such as stiffness losses in a structure will cause changes in both statistical moments and probability density function of the structure under random excitation. Therefore, the changes in statistical moments, particularly higher even statistical moments, may be sensitive to structural damage. In this regard, the principle of statistical moment-based damage detection method is put forward in terms of a SDOF system in this section. The sensitivity of statistical moments to structural damage is first discussed for different types of structural responses and different orders of statistical moments. The formulae for statistical moment-based damage detection are then derived. The effect of measurement noise on damage detection is finally ascertained.



(a) Single-story shear building (b) N-story shear building

Fig. 2 Building models

3.1 Statistical moments

Let us consider single-story shear building under zero-mean white-noise ground acceleration of Gaussian distribution as shown in Fig. 2(a). Considering only linear-elastic structural system, the equation of motion of the shear building can be expressed as

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g \tag{13}$$

or

$$\ddot{x} + 2\xi\omega_o\dot{x} + \omega_o^2 x = -\ddot{x}_g \tag{14}$$

where m, c and k are respectively the mass, damping coefficient and stiffness coefficient of the building; x, $\dot{x}$ and $\ddot{x}$ are respectively the relative displacement, velocity and acceleration responses of the building to the ground; $\ddot{x}_g$ is the white noise ground acceleration; ξ is the damping ratio of the building; and ω_o is the circular natural frequency of the building and it is equal to $\sqrt{k/m}$. If a structure is a linear system, the power spectrum $S(\omega)$ and the variance σ^2 of the structural response can be obtained by

$$S(\omega) = |H(\omega)|^2 S_f(\omega) \tag{15}$$

$$\sigma^2 = \int_{-\infty}^{+\infty} S(\omega)d\omega \tag{16}$$

where $S_f(\omega)$ is the power spectrum of ground excitation; and $H(\omega)$ stands for the frequency response function (FRF). If the ground excitation is an ideal white noise, $S_f(\omega)$ is then a constant

S_0 over the whole frequency zone from $-\infty$ to $+\infty$. For a SDOF system, the module of the displacement FRF, $H_d(\omega)$, the velocity FRF, $H_v(\omega)$, and the acceleration FRF, $H_a(\omega)$, can be obtained as follows

$$|H_d(\omega)| = \frac{1}{m\sqrt{(\omega_0^2-\omega^2)^2+(2\xi\omega_0\omega)^2}} \tag{17}$$

$$|H_v(\omega)| = \frac{\omega}{m\sqrt{(\omega_0^2-\omega^2)^2+(2\xi\omega_0\omega)^2}} \tag{18}$$

$$|H_a(\omega)| = \frac{\omega^2}{m\sqrt{(\omega_0^2-\omega^2)^2+(2\xi\omega_0\omega)^2}} \tag{19}$$

Substituting Eq. (17) to Eq. (15) and then Eq. (16) leads to the variance or second-order moment of displacement response σ_d^2.

$$M_2^{dis} = \sigma_d^2 = \int_{-\infty}^{+\infty} |H_d(\omega)|^2 S_f(\omega)d\omega = \frac{\pi S_0}{2\xi\sqrt{mk^3}} \tag{20}$$

In a similar way, the variance or second-order moment of velocity, σ_v^2 and acceleration, σ_a^2 can be obtained as follows

$$M_2^v = \sigma_v^2 = \frac{\pi S_0}{2\xi\sqrt{m^3 k}} \tag{21}$$

$$M_2^a = \sigma_a^2 = \frac{\pi S_0(1-4\xi^2)\sqrt{k}}{2\xi\sqrt{m^5}} \tag{22}$$

The fourth-order and sixth-order moments of displacement, velocity and acceleration can then be given as

$$M_4^{dis} = \frac{3\pi^2 S_0^2}{4m\xi^2 k^3} \tag{23}$$

$$M_4^v = \frac{3\pi^2 S_0^2}{4m^3\xi^2 k} \tag{24}$$

$$M_4^a = \frac{3\pi^2 S_0^2}{4m^5\xi^2}(1-4\xi^2)^2 k \tag{25}$$

$$M_6^{dis} = \frac{15\pi^3 S_0^3}{8\xi^3\sqrt{m^3 k^9}} \tag{26}$$

$$M_6^v = \frac{15\pi^3 S_0^3}{8\xi^3\sqrt{m^9 k^3}} \tag{27}$$

$$M_6^a = \frac{15\pi^3 S_0^3}{8\xi^3\sqrt{m^{15}}}(1-4\xi^2)^3\sqrt{k^3} \tag{28}$$

3.2 Sensitivity analysis

Based on Eqs. (21) to (28), the following sensitivity equations can be derived:

$$\frac{dM_2^{dis}}{M_2^{dis}} = -\frac{3}{2}\frac{dk}{k}, \quad \frac{dM_2^{v}}{M_2^{v}} = -\frac{1}{2}\frac{dk}{k}, \quad \frac{dM_2^{a}}{M_2^{a}} = \frac{1}{2}\frac{dk}{k} \tag{29}$$

$$\frac{dM_4^{dis}}{M_4^{dis}} = -3\frac{dk}{k}, \quad \frac{dM_4^{v}}{M_4^{v}} = -\frac{dk}{k}, \quad \frac{dM_4^{a}}{M_4^{a}} = \frac{dk}{k} \tag{30}$$

$$\frac{dM_6^{dis}}{M_6^{dis}} = -\frac{9}{2}\frac{dk}{k}, \quad \frac{dM_6^{v}}{M_6^{v}} = -\frac{3}{2}\frac{dk}{k}, \quad \frac{dM_6^{a}}{M_6^{a}} = \frac{3}{2}\frac{dk}{k} \tag{31}$$

From the above equations, it can be observed that the relative changes of the statistical moments of displacement and velocity are negatively proportional to the relative change of stiffness while the relative change of the statistical moment of acceleration is positively proportional to the relative change of stiffness. The ratio of the relative change of the second-order, fourth-order and six-order moments of displacement to the relative change of stiffness are always three times of the counterparts of velocity and acceleration positively or negatively. This result reflects that the relative change of the statistical moment of displacement is two times more sensitive to the relative change of stiffness than those of velocity and acceleration. Thus, only the statistical moment of displacement is examined in this study. Furthermore, it can be observed that the relative change of higher order moment of displacement is more sensitive to the relative change of stiffness.

3.3 Damage detection

Based on Eqs. (20), (23) and (26), the stiffness of the structure can be obtained from the second-order, fourth-order, and sixth-order statistical moments of displacement response, respectively.

$$k = \sqrt[3]{\frac{\pi^2 S_0^2}{4m\xi^2 (M_2^{dis})^2}} \tag{32}$$

$$k = \sqrt[3]{\frac{3\pi^2 S_0^2}{4m\xi^2 M_2^{dis}}} \tag{33}$$

$$k = \sqrt[9]{\frac{225\pi^6 S_0^6}{64\xi^6 m^3 (M_6^{dis})^2}} \tag{34}$$

The statistical moment-based damage detection on a SDOF system can be therefore carried out in the following steps: (1) measure or compute the displacement response of the SDOF system under ground acceleration which is zero-mean Gaussian white noise random process; (2) calculate the even statistical moments, such as $\hat{M}_4^u$, using Eqs. (3) to (5) or Eqs. (10) to (12); (3) calculate the structural stiffness of the undamaged structure $\hat{k}^u$ using Eqs. (32) to (34); (4) use the same procedure to calculate the structural stiffness of the damaged structure $\hat{k}^d$ under another random excitation; and (5) the structural damage severity μ can then be identified by

$$\mu = \frac{\hat{k}^d - \hat{k}^u}{\hat{k}^u} \times 100\% \tag{35}$$

In the above, the symbol '^' represents 'estimated' in contrast with 'theoretical'. The superscript '*u*' and '*d*' stand for 'undamaged' and 'damaged', respectively.

3.4 Effect of measurement noise

There are many sources causing measurement noise to pollute desirable signals during either laboratory tests or field measurements of civil structures. The type and intensity of measurement noise depend on the type and size of a structure, the measurement system, and the environment surrounding the structure. In this numerical study, the measurement noise is assumed to be white noise, indicating that the measurement noise is caused by many sources of equal importance. The measurement noise intensity is defined as the ratio of the root mean square (RMS) of measurement noise ε to the RMS of displacement response x.

$$\alpha = \frac{RMS(\varepsilon)}{RMS(x)} \times 100\% \tag{36}$$

The effect of measurement noise on damage detection is measured in terms of the noise effect ratio γ.

$$\gamma = \frac{\hat{k}_n - \hat{k}}{\hat{k}} \times 100\% \tag{37}$$

where $\hat{k}_n$ is the identified structural stiffness with considering the effect of measurement noise while $\hat{k}$ is the counterpart without considering the effect of measurement noise. By defining the structural response with measurement noise as y, there is a relationship

$$y = y + \varepsilon \tag{38}$$

where x is the actual structural response; ε is the measurement noise independent of x. Then, by taking a noise intensity of 15%, for example, one may have

$$\sigma_y^2 = \sigma_x^2 + \sigma_\varepsilon^2 = 1.0225\,\sigma_x^2 \tag{39}$$

As a result, the statistical moments of the structural responses with measurement noise can be obtained as

$$M_{2y} = 1.0225 M_{2x} \tag{40}$$

$$M_{4y} = 3(\sigma_y^2)^2 = 3 \times (1.0225)^2 \sigma_x^4 = (1.0225)^2 M_{4x} \tag{41}$$

$$M_{6y} = 15(\sigma_y^2)^3 = 15 \times (1.0225)^3 \sigma_x^6 = (1.0225)^3 M_{6x} \tag{42}$$

where M_{2y}, M_{4y} and M_{6y} are respectively the second-order, fourth-order and sixth-order moment of y; M_{2x}, M_{4x} and M_{6x} are the counterparts of x, respectively. Using Eqs. (32) to (34) leads to

$$k^n = \left\{ \begin{array}{l} \sqrt[3]{\dfrac{\pi^2 S_0^2}{4m\xi^2 M_{2y}^2}} = \dfrac{1}{\sqrt[3]{1.0225^2}} \sqrt[3]{\dfrac{\pi^2 S_0^2}{4m\xi^2 M_{2x}^2}} \\ \sqrt[3]{\dfrac{3\pi^2 S_0^2}{4m\xi^2 M_{4y}}} = \dfrac{1}{\sqrt[3]{1.0225^2}} \sqrt[3]{\dfrac{3\pi^2 S_0^2}{4m\xi^2 M_{4x}}} \\ \sqrt[3]{\dfrac{225\pi^6 S_0^6}{64\xi^6 m^3 M_{6y}^2}} = \dfrac{1}{\sqrt[3]{1.0225^2}} \sqrt[9]{\dfrac{225\pi^6 S_0^6}{64\xi^6 m^3 M_{6x}^2}} \end{array} \right\} = 0.9853k \tag{43}$$

where k^n is the theoretically identified stiffness with considering measurement noise; and k is the corresponding stiffness without considering measurement noise. It can be seen that the theoretical noise effect ratio is only 1.47% even at the noise intensity of 15%. This result indicates that the statistical moment of displacement response is not sensitive to measurement noise.

4. Statistical moment-based damage detection method for MDOF system

In this section, the statistical moment-based damage detection method is extended to a MDOF system. Let us consider an N-story linear shear building subjected to ground acceleration $\ddot{x}_g$ as shown in Fig. 2(b). The lumped mass, horizontal stiffness coefficient, and structural damping coefficients of ith story of the building are denoted as m_i, k_i and c_i, respectively, where $i = 1, 2, \ldots, N$. The equation of motion in the matrix form for this shear building can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{MI}\ddot{x}_g(t) \tag{44}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass matrix, damping matrix and stiffness matrix of the building structure, respectively; $\ddot{\mathbf{x}}(t), \dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ are the acceleration, velocity and displacement response vectors relative to the ground, respectively; and $\mathbf{I}$ is the column vector with all its elements equal to unity. The ground acceleration $\ddot{x}_g$ is taken as zero-mean white noise ground excitation whose power spectral density is a constant S_0. By adopting the Rayleigh damping assumption, Eq. (44) can be decoupled through the following transformation:

$$\mathbf{x} = \mathbf{\Phi z} \tag{45}$$

where $\mathbf{\Phi}$ is the mass-normalized modal matrix of the system. The uncoupled equations of motion of the system can then be expressed as

$$\ddot{z}_i(t) + 2\xi_i\omega_i\dot{z}_i(t) + \omega_i^2 z_i(t) = \Gamma_i \ddot{x}_g(t) \qquad i = 1, 2, 3, \ldots, N \tag{46}$$

$$\Gamma_i = \sum_{k=1}^{N} m_k \phi_{ki} \tag{47}$$

where Γ_i is the ith participation factor; ξ_i is the ith modal damping ratio; ω_i is the ith circular natural frequency; and ϕ_{ki} is the kth component of the ith mode shape. By using the mode superposition method and taking the Fourier transform, the Fourier transform of the displacement response x_i of the ith floor can be obtained

$$X_i(\omega) = \sqrt{S_o}\sum_{k=1}^{N}\alpha_{ik}(\omega)m_k \tag{48}$$

$$\alpha_{ik}(\omega) = \sum_{j=1}^{N}\frac{\phi_{ij}\cdot\phi_{kj}}{\omega_j^2-\omega^2+2i\omega\omega_j\xi_j} \tag{49}$$

Because story drifts are directly related to horizontal stiffness reduction, the statistical moments of story drifts other than floor displacements are considered in this section. The Fourier transform of the ith story drift can be obtained by

$$\Delta X_i(\omega) = X_i(\omega)-X_{i-1}(\omega) = \sqrt{S_o}\sum_{k=1}^{N}\Delta\alpha_{ik}(\omega)m_k \quad i = 1,2,3,\ldots,N \tag{50}$$

$$\Delta\alpha_{ik}(\omega) = \alpha_{ik}(\omega)-\alpha_{(i-1)k}(\omega), \quad \text{and} \quad \alpha_{0k}(\omega) = 0 \tag{51}$$

The power spectral density (PSD) function of the ith story drift Δx_i can be expressed as

$$S_{\Delta x_i}(\omega) = S_o\left(\sum_{k=1}^{N}\Delta\alpha_{ik}(\omega)m_k\right)\left(\sum_{k=1}^{N}\Delta\alpha_{ik}(\omega)^*\cdot m_k\right) \tag{52}$$

where $\Delta\alpha_{ik}(\omega)^*$ is the conjugate of $\Delta\alpha_{ik}(\omega)$. Therefore, the variance of the ith story drift can be calculated by

$$\sigma_{\Delta x_i}^2 = \int_{-\infty}^{+\infty}S_{\Delta x_i}(\omega)d\omega = \int_{-\infty}^{+\infty}S_o\left(\sum_{k=1}^{N}\Delta\alpha_{ik}(\omega)m_k\right)\left(\sum_{k=1}^{N}\Delta\alpha_{ik}(\omega)^*\cdot m_k\right)d\omega \tag{53}$$

Since the ith story drift is a stationary random process, its statistical moments can be computed by

$$M_{2i} = \sigma_{\Delta x_i}^2, \quad M_{4i} = 3\sigma_{\Delta x_i}^4, \quad M_{6i} = 15\sigma_{\Delta x_i}^6, \quad i = 1,2,\ldots,N \tag{54}$$

The second-order, fourth-order, and sixth-order statistical moment vectors can be expressed as

$$\mathbf{M}_2 = [M_{21}, M_{22}, \ldots, M_{2N}] \tag{55}$$

$$\mathbf{M}_4 = [M_{41}, M_{42}, \ldots, M_{4N}] \tag{56}$$

$$\mathbf{M}_6 = [M_{61}, M_{62}, \ldots, M_{6N}] \tag{57}$$

Based on the above derivation, it can be seen that if the mass matrix and the damping ratios of the system are kept invariant, the statistical moments of story drifts are the function of the horizontal stiffness. Therefore, the ith-order moment vector of story drift, denoted as $\mathbf{M}_i(i = 2, 4, 6)$, can be theoretically calculated for the given stiffness vector $\mathbf{k} = [k_1, k_2, \ldots, k_N]$. For damage detection, the *ith*-order statistical moment vector shall be estimated from the measured displacement responses based on Eqs. (3) to (5) or Eqs. (10) to (12), denoted as $\hat{\mathbf{M}}_i$. The residual vector between $\mathbf{M}_i$ and $\hat{\mathbf{M}}_i$ can be written as

$$\mathbf{F}(\mathbf{k}) = \mathbf{M}_i(\mathbf{k}) - \hat{\mathbf{M}}_i \tag{58}$$

Ideally, if the given stiffness vector $\mathbf{k}$ is equal to the actual value, the 2-norm of the residual vector, $\|\mathbf{F}(\mathbf{k})\|^2$, will be zero. Practically, the optimal stiffness vector can be identified by the least-squares method, that is, giving $\mathbf{k}$ an initial value and minimizing $\|\mathbf{F}(\mathbf{k})\|^2$. Thus, the statistical moment-based damage detection method for a MDOF system can be carried out in the three stages: (1) the statistical moments of story drifts are estimated by the measured displacements for undamaged and damaged building respectively; (2) the structural stiffness vector is identified by the least-squares method for the undamaged and damaged building respectively; and (3) the structural damage including damage existence, location and severity can be detected by comparing the identified stiffness vector $\mathbf{k}^u$ for the undamaged building with the identified stiffness vector $\mathbf{k}^d$ for the damaged building. To investigate the effect of measurement noise on the statistical moment-based damage detection, the measurement noise can be added to the measured displacement response vector. The contaminated displacement response vector is then used to estimate the contaminated statistical moment vector to obtain the stiffness vector. The noise effect is finally assessed in a similar way to the SDOF system.

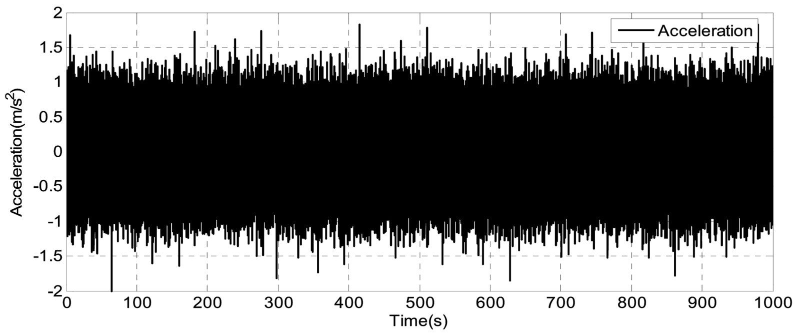
5. Numerical example of SDOF system

5.1 Numerical model

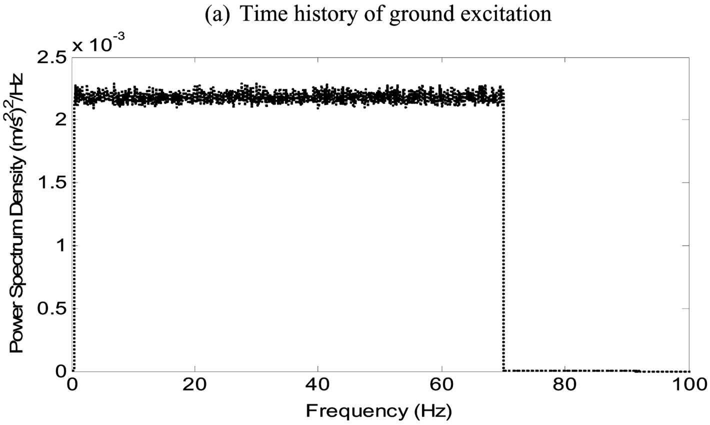
A three-story shear building model used by Zhao *et al.* (2005) will be utilized as an example building model in next section to demonstrate the effectiveness of the statistical moment-based damage detection method for MDOF systems. To be consistent with this building model, the mass and horizontal stiffness of single-story shear building model (see Fig. 2), which will be discussed in this section, are taken as 230.2 kg and 5.46×10^5 N/m, respectively. The damping ratio is 1%. The ground excitation is taken as a zero-mean white-noise stationary process. The duration of the ground excitation time history is 1000s with the sampling frequency of 256 Hz. The ground excitation time history is generated using the method of digital simulation of a random process developed by Shinozuka and Jan (1972). The excitation time history generated is ergodic regardless of the number of frequency intervals. This makes the method directly applicable to a time domain analysis in which the ensemble average can be evaluated in terms of the temporal average. Note that the simulated process is of Gaussian distribution by virtue of the central limit theorem. Fig. 3 presents the attributes of a simulated band-limited white Gaussian excitation, which includes its time history, power spectrum density and probability density distribution. It can be seen that the intensity of the power density function (PDF) is $2.18 \times 10^{-3}(\mathrm{m/s^2})^2/\mathrm{Hz}$ within a frequency band from 0.5 Hz to 70 Hz.

5.2 Sensitivity of PDF to structural damage

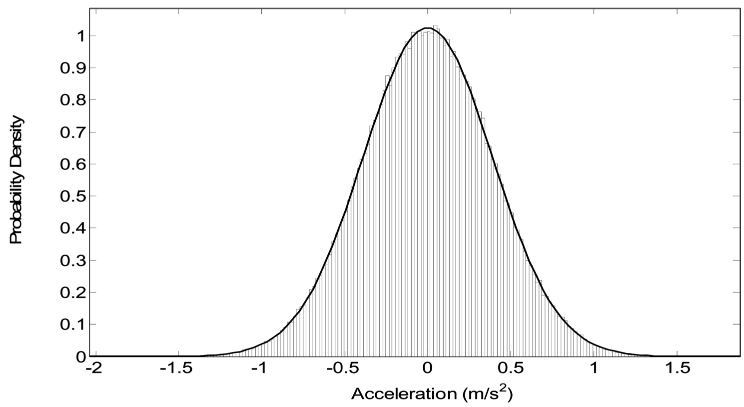
The sensitivity of PDF of different responses of the building to structural damage severity is numerically investigated. The displacement, velocity and acceleration responses of the building with different stiffness k_0, $0.98k_0$, $0.95k_0$, $0.90k_0$, $0.80k_0$ and $0.70k_0$ are computed for the same ground excitation. These stiffness coefficients represent different damage severities, that is, 0%, 2%, 5%,



(a) Time history of ground excitation



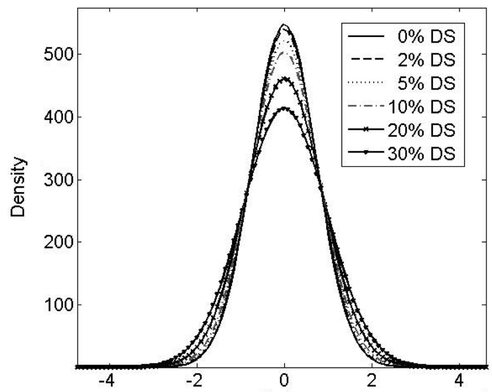
(b) Power spectrum density of ground excitation



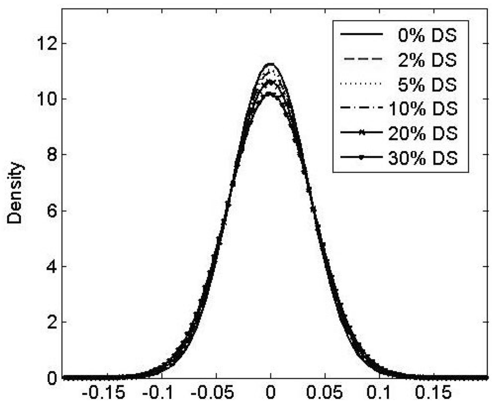
(c) Probability density distribution of ground excitation

Fig. 3 Simulated ground excitation

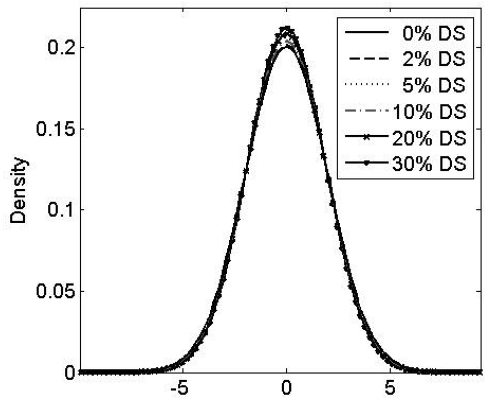
10%, 20% and 30%, correspondingly. The reduction in horizontal stiffness of a shear building structure can be seen as the real damage of either bracing system or shear wall of a building structure. The displacement, velocity and acceleration response time histories are then used to compute their PDF curves. The PDF curves are finally fitted by the Gaussian PDF curves and shown in Figs. 4(a), (b) and (c) for displacement, velocity, and acceleration responses, respectively, in which DS stands for damage severity.



(a) Displacement ($\times 10^{-3}$ m)



(b) Velocity (m/s)



(c) Acceleration (m/s^2)

Fig. 4 Probability density functions of different responses for different stiffness values

It can be seen that the PDF curves of displacement and velocity responses become flatter with the increase of structural damage severity but the PDF curves of acceleration responses show the contrary phenomenon. With reference to Fig. 1, one may conclude that the statistical moments of displacement and velocity responses get bigger with the decrease of structural stiffness but those of acceleration responses become smaller. Furthermore, the PDF curves of displacement responses present more apparent and consistent changes with respect to different damage severities than those of velocity and acceleration responses. Accordingly, the PDF curve of displacement response is more sensitive to structural damage severity than those of velocity and acceleration responses. These results are all consistent with the previous conclusions drawn from theoretical analysis given in Section 3.2. The displacement responses will be therefore utilized hereinafter in order to effectively conduct damage detection.

5.3 Sensitivity of statistical moments to structural damage

As mentioned above, changes in the structural stiffness values are clearly illustrated in the PDF of displacement responses which would take the form of well known Gaussian bell shaped curve for a

Table 1 Change ratios of natural frequency and statistical moments (SDOF system)

Scenario	$\frac{\omega_0^d - \omega_0^u}{\omega_0^u}$ (%)	$\frac{M_2^d - M_2^u}{M_2^u}$ (%)	$\frac{M_4^d - M_4^u}{M_4^u}$ (%)	$\frac{M_6^d - M_6^u}{M_6^u}$ (%)
1	−1.00	3.08	6.25	9.52
2	−2.53	8.00	16.64	25.96
3	−5.13	17.12	37.17	60.66
4	−10.55	39.75	95.31	172.96
5	−16.33	70.75	191.55	397.80

linear SDOF structural system under the ground excitation of zero-mean white noise. The theoretical second-order, fourth-order and sixth-order statistical moments which represent the characteristics of distribution are now computed for the undamaged building and the damaged building with the stiffness reduction of 2%, 5%, 10%, 20% and 30% (Scenario 1 to Scenario 5) respectively using Eqs. (20), (23), and (26), respectively. The corresponding circular frequencies of the undamaged building and the damaged building are also computed. The theoretical circular frequency and the theoretical second-order, fourth-order and sixth-order statistical moments of the undamaged building are donated as ω_0^u, M_2^u, M_4^u and M_6^u, respectively. The counterparts of the damaged building are donated as ω_0^d, M_2^d, M_4^d and M_6^d, respectively. The change ratios of these values to those of the undamaged building are listed in Table 1. It can be seen that statistical moments are more sensitive to structural damage than circular natural frequency. Furthermore, it can be seen that higher-order statistical moment is more sensitive to structural damage than lower-order moment. This has also been theoretically interpreted in Section 3.2. It seems that the higher-order statistical moment would be a good index for structural damage detection. However, the statistical moments are random variables in the actual numerical calculation due to the effects of limited time duration and the transitory unstable dynamic responses at the initial stage. It is, therefore, necessary to investigate the stability of higher statistical moment value. To this end, 20 ground acceleration time histories are generated randomly and then exerted on the undamaged structure. The statistical moments of displacement response are computed. The mean value and standard deviation of 20 statistical moments and then the coefficient of variation δ are calculated.

$$\delta = \frac{std(\hat{M}_i^u)}{mean(\hat{M}_i^u)} \times 100\% \tag{62}$$

where $std(\hat{M}_i^u)$ is the standard deviation of the *i*th statistical moment and $mean(\hat{M}_i^u)$ is its mean value. The results show that the coefficients of variance of the second-order, fourth-order and sixth-order moments are 8.9%, 16.9% and 24.2%, respectively. The coefficient of variance is larger for higher statistical moment. Namely, higher statistical moment is less stable. Therefore, as far as a damage index is concerned, the fourth-order moment may be a good choice which represents a compromise measure between sensitivity and stability. In the following study, the fourth-order moments are adopted for damage detection. In addition, the stability of the identified structural stiffness using the mean value of 20 fourth-order moments is also investigated. The mean value of corresponding structural stiffness identified is 549880 N/m which only has a bias of 0.71% and the coefficients of variance 0.42%, comparing with the true value. This promising result paves the way for the following damage detection.

Table 2 Damage detection results and theoretical values (SDOF system)

Scenario	$\hat{M}_4^d$ (10^{-13} m^4)	M_4^d (10^{-13} m^4)	$\hat{k}^d$ (N/m)	k^d (N/m)	$\frac{\hat{k}^d - \hat{k}^u}{\hat{k}^u}$ (%)	$\frac{k^d - k^u}{k^u}$ (%)
1	2.5516	2.5530	535218	535080	−2.20	−2
2	2.1644	2.2310	524683	518700	−4.12	−5
3	3.2873	3.3646	495871	491400	−9.38	−10
4	3.8272	3.8841	438702	436800	−19.83	−20
5	7.2024	7.3511	385054	382200	−29.64	−30

5.4 Damage detection results

In this section, the damage detection is carried out without considering the effect of measurement noise. The identified stiffness from the fourth-order moment of displacement response of the undamaged building, $\hat{k}^u$, is 547244 N/m, which is very close to the theoretical stiffness k^u of 546000 N/m. The identified stiffness $\hat{k}^d$ from the fourth-order moments of displacement responses of the damaged building, $\hat{M}_4^d$, is tabulated in Table 2 for Scenario 1 to Scenario 5. In the table, $\hat{M}_4^d$ stands for the estimated fourth-order moment of the damaged building using Eq. (11) while M_4^d is the theoretical value derived by Eq. (23) using the theoretical stiffness k^d. The maximum difference between the identified and theoretical stiffness among all the five cases is 0.88% only. The identified damage severities for the five damage cases are 2.20%, 4.12%, 9.38%, 19.83% and 29.64%, which correspond to the theoretical values, 2%, 5%, 10%, 20%, and 30% respectively. It can be seen that even for the damage severity of 2%, the proposed statistical moment-based damage detection method produces a satisfactory result if measurement noise is not considered.

5.5 Effect of measurement noise

Random white measurement noises are now introduced into the structural displacement responses to investigate the effect of measurement noise on damage detection. Five noise intensities are considered and they are 1%, 2%, 5%, 10% and 15%, respectively. Table 3 displays the noise effect ratio γ obtained for the aforementioned five damage cases and five noise intensities. As shown in the table, the noise effect ratio is only related to noise intensity and has almost nothing to do with damage severity. The measurement noise has only small effects on the damage detection. Even when the noise intensity is as high as15%, the absolute γ values for the five damage cases are only 1.88%, 1.63%, 1.72%, 1.39% and 1.65%.

Table 3 Noise effect ratio γ (SDOF system)

α	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
1%	−0.01%	−0.01%	−0.01%	−0.01%	−0.01%
2%	−0.03%	−0.03%	−0.03%	−0.03%	−0.03%
5%	−0.21%	−0.19%	−0.19%	−0.13%	−0.17%
10%	−0.90%	−0.80%	−0.76%	−0.57%	−0.71%
15%	−1.88%	−1.63%	−1.72%	−1.39%	−1.65%

It can be concluded that the fourth statistical moment is a sensitive measure but it is insensitive to measurement noise. By using the fourth moment as a damage index, the proposed statistical moment-based damage detection method can provide not only reliable damage detection results but explicit estimation of noise effects on damage detection results.

6. Numerical example of MDOF system

In this section, the robustness of the statistical moment-based damage detection method is numerically demonstrated based on the three-story shear building model mentioned before. Various damage cases with different damage severities and locations are investigated by making use of the inherent relationship between the fourth-order moments of structural responses and structural properties. Random white measurement noises are also introduced into the structural responses to investigate the effect of measurement noise on the damage detection quality.

6.1 Numerical model

The mass and horizontal stiffness coefficients of the three-story shear building model are respectively 230.2 kg and 5.46×10^5 N/m for the first story, and 230.4 kg and 5.04×10^5 N/m for the second and third story (Zhao *et al.* 2005). The mass of each floor is assumed to be invariant. The first and second modal damping ratios are both taken as 1% and the third modal damping ratio is taken as 1.24% according to the Rayleigh damping assumption. The ground acceleration is taken as zero-mean white noise simulated by the aforementioned method. The duration of ground acceleration time history generated is 1000s with the sampling frequency of 256 Hz.

Listed in Table 4 are the values of the fourth-order moments for all the three stories of the undamaged building, $\hat{M}^u_{4i}(i = 1, 2, 3)$, obtained using Eq. (11). The identified stiffness values of the undamaged building, $\hat{k}^u_i(i = 1, 2, 3)$ using the corresponding values of the fourth-order moments and the least-squares method are also listed in Table 4. It can be seen that the identified horizontal stiffness coefficients of the undamaged building $\hat{k}^u_i$ are very close to the theoretical values k^u_i. This lays down a good foundation for the coming damage detection.

6.2 Damage detection results: Damage severities

The main purpose of this section is to demonstrate the effectiveness of the statistical moment-based damage detection method for identifying damage severities of the three-story building. Five single-damage cases with different damage severities in the first story are considered. The details of the five damage cases are listed in Table 5 in which the theoretical stiffness values k^u_1, k^u_2 and k^u_3 of the undamaged building are presented in Table 4. The theoretical damage severities are actually 2%,

Table 4 Identified statistical moments and stiffness of undamaged building (MDOF system)

Story	$\hat{M}^u_{4i}$ (m^4)	$\hat{k}^u_i$ (N/m)	k^u_i (N/m)
1	2.3732×10^{-12}	552514	546000
2	1.3744×10^{-12}	510059	504000
3	1.7623×10^{-13}	509807	504000

Table 5 Five single-damage Scenarios 1-5 with stiffness k_i^u (MDOF system)

Scenario	Story 1	Story 2	Story 3
1	$0.98k_1^u$	k_2^u	k_3^u
2	$0.95k_1^u$	k_2^u	k_3^u
3	$0.90k_1^u$	k_2^u	k_3^u
4	$0.80k_1^u$	k_2^u	k_3^u
5	$0.70k_1^u$	k_2^u	k_3^u

Table 6 Identified damage severities μ (%) for Scenarios 1-5 (MDOF system)

Scenario	Story 1	Story 2	Story 3
1	−2.04	−0.04	−0.01
2	−5.53	−0.57	−0.50
3	−11.03	−1.16	−1.05
4	−20.20	−0.26	−0.23
5	−30.42	−0.61	−0.56

5%, 10%, 20% and 30%, respectively, in the first story of the building.

For each damage case, the fourth-order moments of story drift are computed for each story of the damaged building and then used to identify stiffness coefficients of the damaged building using the least-squares method. With reference to the identified stiffness coefficients of the undamaged building (see Table 4), the damage severities of each story are finally calculated for each damage case. The results are listed in Table 6, in which the result from Scenarios 2 is also plotted in Fig. 5. In comparison with the actual damage severities shown in Table 5, it can be seen that the identified damage severities are quite close to the actual damage severities.

6.3 Damage detection results: Damage locations

To demonstrate the accuracy of the statistical moment-based damage detection method for

Table 7 Eleven more damage Scenarios 6-16 with stiffness k_i^u (MDOF system)

Scenario	Story 1	Story 2	Story 3
6	k_1^u	$0.95k_2^u$	k_3^u
7	k_1^u	$0.90k_2^u$	k_3^u
8	k_1^u	k_2^u	$0.90k_3^u$
9	$0.98k_1^u$	$0.98k_2^u$	k_3^u
10	$0.90k_1^u$	$0.95k_2^u$	k_3^u
11	$0.90k_1^u$	$0.90k_2^u$	k_3^u
12	k_1^u	$0.95k_2^u$	$0.90k_3^u$
13	k_1^u	$0.80k_2^u$	$0.90k_3^u$
14	k_1^u	$0.70k_2^u$	$0.90k_3^u$
15	$0.90k_1^u$	k_2^u	$0.70k_3^u$
16	$0.95k_1^u$	$0.90k_2^u$	$0.80k_3^u$

Table 8 Identified damage severities μ (%) for Scenarios 6-16 (MDOF system)

Scenario	Story 1	Story 2	Story 3
6	−1.03	−6.00	−0.96
7	−1.05	−10.96	−0.69
8	−1.22	−1.23	−11.00
9	−3.29	−3.31	−1.21
10	−9.89	−4.89	0.03
11	−11.09	−11.12	−1.08
12	−0.34	−5.33	−10.27
13	−0.64	−20.51	−10.56
14	−0.46	−30.33	−10.37
15	−10.47	−0.52	−30.37
16	−5.90	−10.87	−20.70

identifying damage locations of the three-story building, 11 damage scenarios with combination of various damage severities and locations are considered. The details of damage cases are presented in Table 7 in which the theoretical stiffness values of the undamaged building k_1^u, k_2^u and k_3^u are presented in Table 4. Scenarios 6 and 7 simulate single damage in the second story with damage severities of 5% and 10%, respectively. Scenario 8 has single damage in the third story with damage severity of 10%. Scenarios 9 to 15 simulate two damages at different stories with the same or different damage severities. In Scenario 16, the three stories all have damage but with different severities.

By using the same procedure as used in Section 6.2, the damage severities μ are identified for each damage scenario and listed in Table 8. The identification results for Scenarios 7, 14 and 16 are also plotted in Fig. 5. It can be seen that not only the damage severities can be found but also the damage locations are detected at the same time except for the small damage scenarios of 2% stiffness reduction in which the predicted damage severities are relatively less accurate.

6.4 Effect of measurement noise

To assess the effect of measurement noise on the damage detection of the three-story building, white noises are added to the story drifts of the undamaged building. Two noise intensities, α = 5% and 15%, are considered respectively. The fourth-order moments and horizontal stiffness coefficients of the building are then calculated. The results are listed in Table 9. Compared with the results presented in Table 4 for the scenarios without considering measurement noise, it can be seen that the effects of measurement noise on the fourth-order moments and horizontal stiffness coefficients

Table 9 Identified statistical moments and stiffness of undamaged building with noise (MDOF system)

Story	α = 5%		α = 15%	
	$\hat{M}_{4i}^{un}$ (m^4)	$\hat{k}_i^{un}$ (N/m)	$\hat{M}_{4i}^{un}$ (m^4)	$\hat{k}_i^{un}$ (N/m)
1	2.3858×10^{-12}	553110	2.4781×10^{-12}	544726
2	1.3809×10^{-12}	510682	1.4380×10^{-12}	502638
3	1.7703×10^{-13}	510295	1.8438×10^{-13}	502336

Table 10 Identified damage severities μ^n (%) for Scenarios 2, 7, 11, 14 and 16 with noise intensity $\alpha = 5\%$ (MDOF system)

Scenario	Story 1	Story 2	Story 3
2	−5.74	−0.79	−0.71
7	−1.26	−11.16	−1.02
11	−11.27	−11.32	−1.28
14	−0.65	−30.47	−10.54
16	−6.14	−11.11	−20.89

Table 11 Identified damage severities μ^n (%) for Scenarios 2, 7, 11, 14 and 16 with noise intensity $\alpha = 15\%$ (MDOF system)

Scenario	Story 1	Story 2	Story 3
2	−5.43	−0.42	−0.35
7	−1.11	−10.96	−0.96
11	−11.25	−11.23	−1.20
14	−0.60	−30.39	−10.43
16	−6.19	−11.10	−20.90

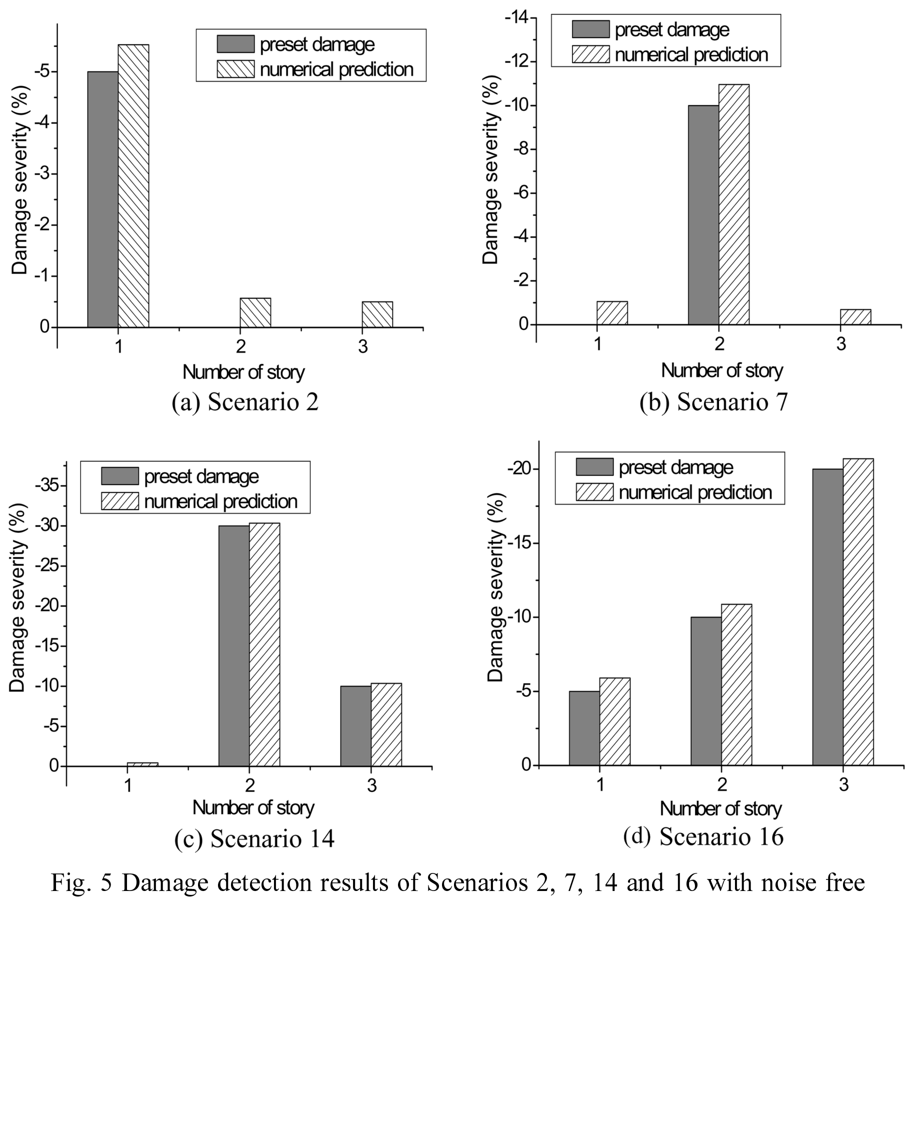
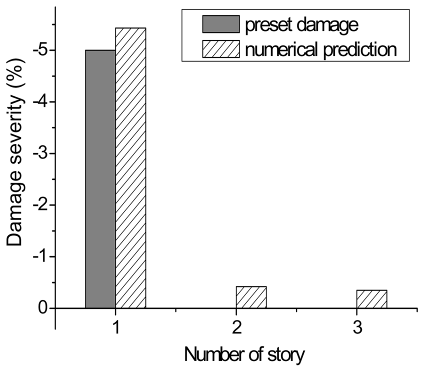
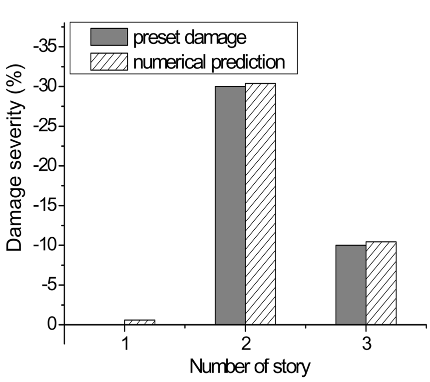


Fig. 5 Damage detection results of Scenarios 2, 7, 14 and 16 with noise free

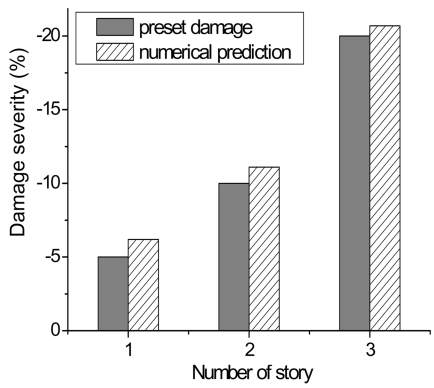


(a) Scenario 2 (noise 15%)

(b) Scenario 7 (noise 15%)



(c) Scenario 14 (noise 15%)



(d) Scenario 16 (noise 15%)

Fig. 6 Damage detection results of Scenarios 2, 7, 14 and 16 with measurement noise

are very minimal when noise intensity is changed from 5% to 15%. The white noises of the aforementioned intensities are then added to the story drifts of the damaged building case by case. Five scenarios listed in Table 5 or 7 are considered for the three story building. The identified results with 5% and 15% measurement noise intensities are listed in Tables 10 and 11, respectively. The four of the five scenarios with 15% measurement noise intensities are also plotted in Fig. 6. Compared with the counterpart results without measurement noise, it can be seen that even the measurement noise intensity is as high as 15%, it has only very little impact on the identified damage results: the damage severities and damage locations can still be properly identified. The robustness and reliability of the proposed method are demonstrated again.

7. Conclusions

A new structural damage detection method has been proposed in this study based on the statistical moments of dynamic responses of a building structure under ground motion. The basic equations for sensitivity analysis and damage detection have been derived. The proposed method has also been applied to a single-story shear building as well as a three-story shear building with various damage severities and damage locations and with/without measurement noise. It was found that the relative change of the statistical moment of story drifts is two times more sensitive to the relative change of building stiffness than those of velocity and acceleration. The relative change of second-order moment of story drifts is more sensitive to the relative change of natural frequency of the building. Furthermore, the relative changes of fourth-order and sixth-order moments of story drifts are more sensitive to the relative change of the second-order moment of story drift, and the fourth-order moment is more stable than the sixth-order moment. Therefore, the fourth-order moment of story drift of a shear building has been proposed as a new damage index. It was also found from the numerical examples that the fourth-order moments of story drifts could be used to accurately identify both damage location and severity of the shear building. A significant advantage of the proposed damage detection method lies in that it is insensitive to measurement noise. Even when the measurement noise intensity is as high as 15%, the method based on the fourth-order moments of structural responses still presents highly reliable results on detecting damage severity and damage location.

Nevertheless, at least two steps should be accomplished in the near future before the proposed method is applied to real structures. A well-planed experimental investigation on the proposed method shall be first conducted to confirm the feasibility and accuracy of the proposed method, in which some issues such as structural damping assumption, measurement noise, and degrees of structural responses can be further investigated. Secondly, the basic theory of the statistical moment-based damage detection method presented in this paper shall be extended to more complicate structures under more unrestricted excitations.

Acknowledgements

The work presented in this paper was financially supported by The Hong Kong Polytechnic University through a postgraduate scholarship to the first author and a Niche Area Program in Performance-based Health Monitoring of Large Civil Engineering Structures to the second author. The authors are grateful to Dr. X.J. Hong and Dr. B. Chen of The Hong Kong Polytechnic University for their beneficial discussions.

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