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Finite element analysis of vehicle-bridge interaction by an iterative method

Ji-Seong Jo†

POSCO E&C Technical Research Institute, 79-5 Youngcheon, Dongtan, Hwaseong, Gyeonggi-do, Korea

Hyung-Jo Jung‡

Department of Civil and Environmental Engineering, KAIST, 335, Gwahangno, Yuseong-gu, Daejeon 305-701, Korea

Hongjin Kim‡†

School of Architecture & Civil Engineering, Kyungpook National University, 1370 Sangyeok-dong, Buk-gu, Daegu 702-701, Korea

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Abstract. In this paper, a new iterative method for solving vehicle-bridge interaction problems is proposed. Iterative methods have advantages over the non-iterative methods in that it is not necessary to update the system matrix for a given wheel location, and the method can be applied for a new type of car or bridge with few or no modifications. In the proposed method, the necessity of system matrices update is eliminated using the equivalent interaction force acting on the bridge, which is obtained iteratively. Ballast stiffness is included in the interaction forces and the geometric compatibility at the contact points are used as convergence criteria. The bridge is considered as an elastic Bernoulli-Euler beam with surface irregularity and ballast stiffness. The moving vehicle is modeled as a multi-axle mass-spring-damper system having many degrees of freedom depending on the number of axles. The pitching effect, which is the interaction effect between the rear and front wheels when a vehicle begins to enter or leave the bridge, is also considered in the formulation including extended ground boundaries having surface irregularity and ballast stiffness. The applicability of the proposed method is illustrated in the numerical studies.

Keywords: vehicle-bridge interaction; moving vehicle; finite element method; iteration method.

1. Introduction

The vibration of bridges under moving vehicles or trains has long been an interesting subject for

† Researcher, E-mail: meshless@poscoenc.com
‡ Assistant Professor, E-mail: hjung@kaist.ac.kr
‡† Assistant Professor, Corresponding author, E-mail: hjk@knu.ac.kr

many structural engineers in consideration of the design of bridges and railway. The vehicle-bridge interaction analysis is very a complex dynamic problem because there are many factors affecting the dynamic behavior of bridges such as the type, natural frequencies, and damping ratios of bridges, surface roughness, ballast stiffness, the number of vehicles, vehicle characteristics, path, speed, and the position of wheels. A state-of-the-art review of the research conducted on the vibration of bridges under moving vehicles and trains was presented by Au *et al.* (2001).

Methods to obtain dynamic behavior of bridges can be categorized in two aspects. First, they can be categorized by the modeling method of the interaction system. The moving-force model (Smith 1988), the moving-mass model (Ichikawa *et al.* 2000), the moving-sprung-mass model (Smith 1988, Yang and Yau 1997) and the moving-vehicle model (Yang and Fonder 1996, Lou 2005, Yang and Wu 2001) are four developed models for interaction systems. The moving-vehicle model can include the inertia of a vehicle, the bouncing effects of moving mass, and the pitching effect of a car body, and thereby overcomes the drawbacks of the others (Au *et al.* 2001).

Second, the methods can be divided into iterative and non-iterative methods depending on the interaction force calculation. In the non-iterative method, the interaction force between bridge and moving vehicle is obtained once for a given wheel location of moving mass. However, it is necessary to update the system matrices such as mass, stiffness and damping matrices of the system at each time step according to wheel locations. Yang and Yau (1997, 2001) developed a vehicle-bridge interaction element to analyze the interaction system using the finite element method. Lou (2005) proposed a vehicle-track-bridge interaction element considering vehicle's pitching effect. Because the formulation of the non-iterative methods depends on both the bridge and vehicle models, all coefficients must be changed and they are complicated to establish if a new type of car or bridge is introduced (Yang and Fonder 1996).

On the other hand, the iterative method uses the same system matrices regardless of wheel location but estimates the interaction force iteratively for a given wheel location until a convergence criteria is satisfied. Iterative methods have advantages over the non-iterative methods in that it is not necessary to update the system matrix for a given wheel location, and the method can be used for a new type of car or bridge with minor or no modifications. Despite its advantages, only a few iterative methods have been developed recently. Chatterjee *et al.* (1994) analyzed the vibration of a suspension bridge under a moving vehicle considering surface roughness. Interaction forces are used as convergence criteria, and relaxation or Aiken acceleration are also used when convergence was not achieved. Yang and Fonder (1996) proposed a new iterative algorithm using bridge displacements at the wheel position as the convergence criteria. Those methods, however, do not consider ballast stiffness that may be important for railway bridges.

Recently, analytical and experimental evaluation of dynamic effects for existing bridges were performed by Kwasniewski *et al*. using sophisticated FEM 3D model of truck and bridge interaction systems with over half a million elements (Kwasniewski *et al.* 2005, 2006, Li *et al.* 2005). Kwasniewski et al. showed that the actual dynamic responses of a real bridge are very complicated and longitudinal or transverse behavior may be different from numerical results (Kwasniewski *et al.* 2006, Li *et al.* 2005).

In this paper, a new iterative method for solving vehicle-bridge interaction problem is proposed. Ballast stiffness is included in the interaction forces and the geometric compatibility at the contact points is used as a convergence criterion. The bridge is considered as an elastic Bernoulli-Euler beam with surface irregularity and ballast stiffness. The moving vehicle is modeled as a multi-axle mass-spring-damper system having many degrees of freedom depending on the number of axles.

The pitching effect, which is the interaction effect between the rear and front wheels when a vehicle begins to enter or leave the bridge, is also considered in the formulation including extended ground boundaries having surface irregularity and ballast stiffness. The applicability of the proposed method is illustrated in the numerical studies.

2. Equations of motion for vehicle-bridge interaction system

2.1 Modeling of vehicle, surface condition, and bridge

A typical vehicle-bridge interaction system is shown in Fig. 1. The vehicle is modeled as a car body and multi-axle mass-spring-damper system. The car body is modeled as a rigid body with a mass m_c and its transverse velocity is indicated by $v(t)$. The rotational mass moment of inertia J_c about the transverse horizontal axis through its centroid is also included in the modeling of the car body in order to consider the pitching effect. Each axle and car body are interconnected by a spring of stiffness k_i (i = 1, …, n) and a dashpot of damping coefficient c_i (i = 1 …, n) where the mass of each wheel is indicated by $m_{w,\,i}$ (i = 1 …, n). The rigid car motion is described by the vertical displacement z_c and rotation θ_c about its centroid. It is assumed that each wheel set is represented by one vertical DOF, and the vertical displacement of the wheel is described by z_i (i = 1 …, n). To represent surface or railway conditions, the surface irregularity $r(x)$ and ballast stiffness $k_b(x)$ are also included. These two surface conditions are extended to outside of the bridge to ease difficulties due to pitching effects when some axles are on the bridge and the others are outside the bridge.

The bridge is modeled as a simply supported uniform Bernoulli-Euler beam with span length L. It is assumed that the horizontal distance of contact point between vehicle and beam is x_i (i = 1 …, n) from a given origin O. The upward vertical displacements and clockwise rotation of the vehicle and the bridge are taken as positive and they are measured with reference to their respective static equilibrium positions.

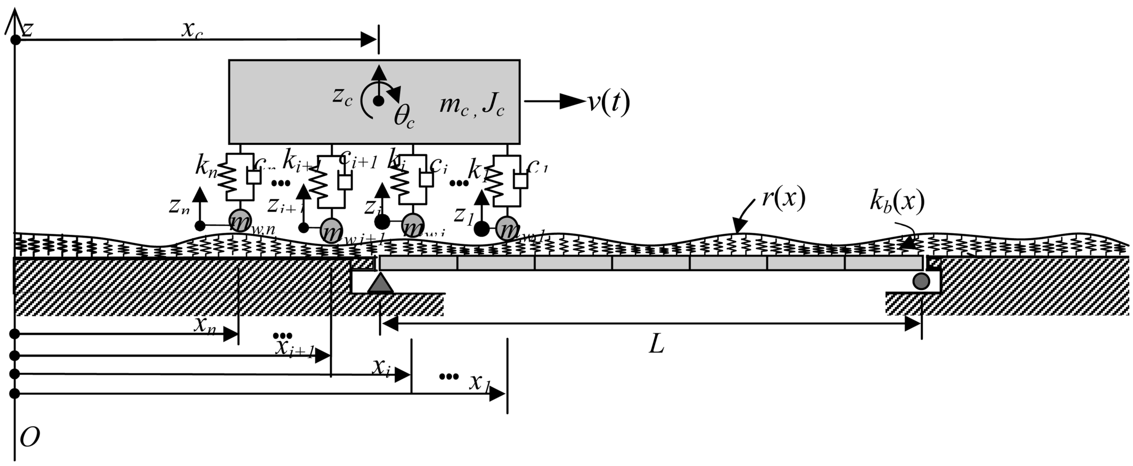


Fig. 1 Model of vehicle-bridge interaction system

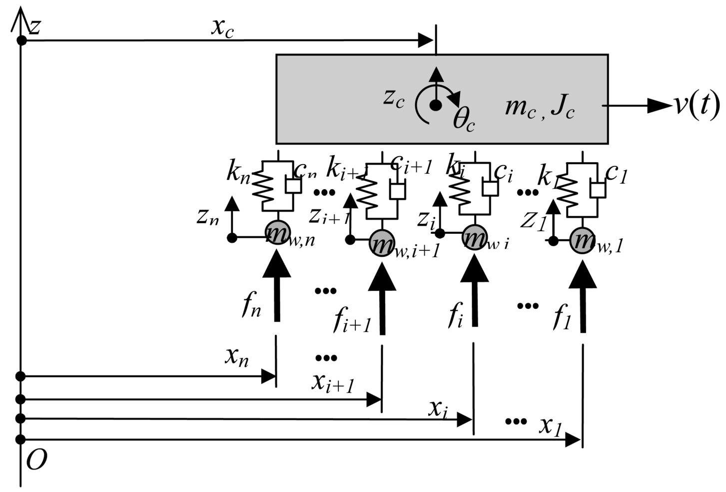


Fig. 2 Interaction forces of the vehicle-bridge system

2.2 Equations of motion for the vehicle and bridge

The equations of motion for the vehicle (Fig. 2) can be written as

$$J_c\ddot{\theta}_c + \sum_{i=1}^{n} c_i[\dot{z}_c - \dot{z}_i + \dot{\theta}_c(x_c - x_i)](x_c - x_i) + \sum_{i=1}^{n} k_i[z_c - z_i + \theta_c(x_c - x_i)](x_c - x_i) = 0 \tag{1}$$

$$m_c\ddot{z}_c + \sum_{i=1}^{n} c_i[\dot{z}_c - \dot{z}_i + \dot{\theta}_c(x_c - x_i)] + \sum_{i=1}^{n} k_i[z_c - z_i + \theta_c(x_c - x_i)] = -m_c g \tag{2}$$

$$m_i\ddot{z}_i - c_i[\dot{z}_c - \dot{z}_i + \dot{\theta}_c(x_c - x_i)] - k_i[z_c - z_i + \theta_c(x_c - x_i)] = f_i \tag{3}$$

where g is gravitational acceleration and f_i is interaction force at i-th wheel position. Eqs. (1) to (3) can be expressed in the matrix form as

$$\mathbf{M_v}\ddot{\mathbf{z}} + \mathbf{C_v}\dot{\mathbf{z}} + \mathbf{K_v}\mathbf{z} = \mathbf{f} \tag{4}$$

where $\mathbf{M_v}$, $\mathbf{C_v}$, and $\mathbf{K_v}$ are

$$\mathbf{M_v} = \begin{bmatrix} J_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_{w,1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{w,i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{w,i+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{w,n} \end{bmatrix} \tag{5}$$

$$\mathbf{C_v} = \begin{bmatrix} \sum\limits_{i=1}^{n} c_i(x_c-x_i)^2 & \sum\limits_{i=1}^{n} c_i(x_c-x_i) & -c_1(x_c-x_1) & \cdots & -c_i(x_c-x_i) & -c_{i+1}(x_c-x_{i+1}) & \cdots & -c_n(x_c-x_n) \\ \sum\limits_{i=1}^{n} c_i(x_c-x_i) & \sum\limits_{i=1}^{n} c_i & -c_1 & \cdots & -c_i & -c_{i+1} & \cdots & -c_n \\ -c_1(x_c-x_1) & -c_1 & c_1 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 & 0 & 0 & 0 \\ -c_i(x_c-x_i) & -c_i & 0 & 0 & c_i & 0 & 0 & 0 \\ -c_{i+1}(x_c-x_{i+1}) & -c_{i+1} & 0 & 0 & 0 & c_{i+1} & 0 & 0 \\ \vdots & \vdots & 0 & 0 & 0 & 0 & \ddots & 0 \\ -c_n(x_c-x_n) & -c_n & 0 & 0 & 0 & 0 & 0 & c_n \end{bmatrix} \tag{6}$$

$$\mathbf{C_v} = \begin{bmatrix} \sum\limits_{i=1}^{n} k_i(x_c-x_i)^2 & \sum\limits_{i=1}^{n} k_i(x_c-x_i) & -k_1(x_c-x_1) & \cdots & -k_i(x_c-x_i) & -k_{i+1}(x_c-x_{i+1}) & \cdots & -k_n(x_c-x_n) \\ \sum\limits_{i=1}^{n} k_i(x_c-x_i) & \sum\limits_{i=1}^{n} k_i & -k_1 & \cdots & -k_i & -k_{i+1} & \cdots & -k_n \\ -k_1(x_c-x_1) & -k_1 & k_1 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 & 0 & 0 & 0 \\ -k_i(x_c-x_i) & -k_i & 0 & 0 & k_i & 0 & 0 & 0 \\ -k_{i+1}(x_c-x_{i+1}) & -k_{i+1} & 0 & 0 & 0 & k_{i+1} & 0 & 0 \\ \vdots & \vdots & 0 & 0 & 0 & 0 & \ddots & 0 \\ -k_n(x_c-x_n) & -k_n & 0 & 0 & 0 & 0 & 0 & k_n \end{bmatrix} \tag{7}$$

are the mass, damping and stiffness matrices of the vehicle part, respectively, and

$$\mathbf{z} = \begin{bmatrix} \theta_c \\ z_c \\ z_1 \\ \vdots \\ z_i \\ z_{i+1} \\ \vdots \\ z_n \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} 0 \\ -m_v g \\ f_1 - m_{w,1} g \\ \vdots \\ f_i - m_{w,i} g \\ f_{i+1} - m_{w,i+1} g \\ \vdots \\ f_n - m_{w,n} g \end{bmatrix} \tag{8}$$

are displacement and interaction force vectors, respectively.

Using the interaction force vector, **f**, given in Eq. (8), the equation of motion for the bridge can be written as

$$\mathbf{M}_b\ddot{\mathbf{u}}+\mathbf{C}_b\dot{\mathbf{u}}+\mathbf{K}_b\mathbf{u} = \mathbf{Lf} \tag{9}$$

where $\mathbf{M}_b$, $\mathbf{C}_b$ and $\mathbf{K}_b$ are the mass, damping and stiffness matrices of the bridge, respectively, $\mathbf{u}$ is a displacement vector of the bridge, and $\mathbf{L}$ is an interaction force location matrix (Yang and Wu 2001).

3. Interaction force and convergence criterion

3.1 Interaction force

The interaction force at the i-th wheel position can be written as (Yang and Yau 1997)

$$f_i = k_b(x_i)[\mathbf{N}_{x_i}\mathbf{u}_{x_i}+r(x_i)-z_i] \tag{10}$$

where $\mathbf{N}_{x_i}$ and $\mathbf{u}_{x_i}$ are the interpolation function and displacement vector for the bridge element contacting with the i-th wheel. The Eq. (10) can be rearranged as

$$z_i = \mathbf{N}_{x_i}\mathbf{u}_{x_i}+r(x_i)-\frac{f_i}{k_b(x_i)} \tag{11}$$

Differentiating the above Eq. (11) with respect to time yields

$$\dot{z}_i = \mathbf{N}_{x_i}\dot{\mathbf{u}}_{x_i}-\frac{\dot{f}_i}{k_b(x_i)} \tag{12}$$

$$\ddot{z}_i = \mathbf{N}_{x_i}\ddot{\mathbf{u}}_{x_i}-\frac{\ddot{f}_i}{k_b(x_i)} \tag{13}$$

Using Eq. (3) and Eqs. (11) to (13), the interaction force can be obtained as

$$f_i = m_i\left[\mathbf{N}_{x_i}\ddot{\mathbf{u}}_{x_i}-\frac{\ddot{f}_i}{k_b(x_i)}+g\right]+c_i\left[\mathbf{N}_{x_i}\dot{\mathbf{u}}_{x_i}-\frac{\dot{f}_i}{k_b(x_i)}-\dot{z}_c+\dot{\theta}_c(x_i-x_c)\right]$$
$$+k_i\left[\mathbf{N}_{x_i}\mathbf{u}_{x_i}+r(x_i)-\frac{f_i}{k_b(x_i)}-z_c+\theta_c(x_i-x_c)\right] \tag{14}$$

For the iterative scheme, the interaction force and its time derivatives in the left side of Eq. (14) are approximated as following

$$f_{i,t+\Delta t}^{k+1} = f_{i,t+\Delta t}^{k} \tag{15}$$

$$\dot{f}_{i,t+\Delta t}^{k+1} = \frac{f_{i,t+\Delta t}^{k}-f_{i,t}}{\Delta t} \tag{16}$$

$$\ddot{f}_{i,t+\Delta t}^{k+1} = \frac{\dot{f}_{i,t+\Delta t}^{k}-\dot{f}_{i,t}}{\Delta t} \tag{17}$$

where Δt is a time step and k is an iteration number.

When the i-th wheel is outside of the bridge, the Eq. (14) becomes

$$f_i = m_i\left[-\frac{\ddot{f}_i}{k_b(x_i)}+g\right]+c_i\left[-\frac{\dot{f}_i}{k_b(x_i)}-\dot{z}_c+\dot{\theta}_c(x_i-x_c)\right]+k_i\left[r(x_i)-\frac{f_i}{k_b(x_i)}-z_c+\theta_c(x_i-x_c)\right] \quad (18)$$

where the conditions $\mathbf{u} = \dot{\mathbf{u}} = \ddot{\mathbf{u}} = 0$ are used. If $r(x) = 0$ and $k_b(x) = \infty$, Eq. (18) represents the case of the condition with no surface irregularity and ballast stiffness.

3.2 Convergence criterion

The convergence of the solutions at a given time t is achieved using the geometric compatibility at the wheel position. The wheel displacement z_i from vehicle part should be equal to the bridge displacement including surface irregularity and ballast deformation at the wheel position.

The displacement for bridge part, $\bar{z}_i$, can be obtained as

$$\bar{z}_i = \mathbf{N}_{x_i}\mathbf{u}_{x_i}+r(x_i)-\frac{f_i}{k_b(x_i)} \quad (19)$$

where $\mathbf{N}_{x_i}\mathbf{u}_{x_i}$ is the displacement of the bridge, $r(x_i)$ is the surface irregularity and $-f_i/k_b(x_i)$ is ballast deformation at the i-th wheel position. Then, the geometric compatibility can be written as

$$z_i = \bar{z}_i \quad (20)$$

Using Eq. (20), the convergence criteria can be introduced as

$$\text{Norm }(\mathbf{z}_w-\bar{\mathbf{z}}_w)\le\varepsilon \quad (21)$$

where

$$\mathbf{z}_w = \begin{bmatrix} z_1 \\ \vdots \\ z_i \\ z_{i+1} \\ \vdots \\ z_n \end{bmatrix}, \quad \bar{\mathbf{z}}_w = \begin{bmatrix} \bar{z}_1 \\ \vdots \\ \bar{z}_i \\ \bar{z}_{i+1} \\ \vdots \\ \bar{z}_n \end{bmatrix}$$

and ε is a given tolerance. The tolerance, ε, can be taken as a value sufficient to obtain the solution with reasonable accuracy (e.g. 1.0×10^{-5}~1.0×10^{-8}).

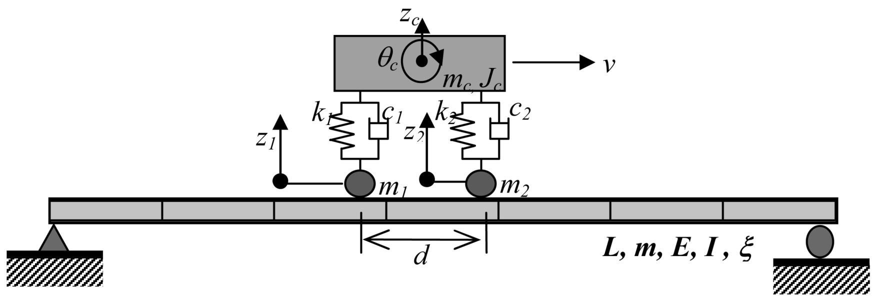
4. Numerical examples

Two numerical examples are considered to verify the proposed method. The first case is a simple 2D beam subjected to a moving vehicle without surface irregularity and ballast stiffness (Yang and Wu 2001) and the second one is a simple 2D beam with surface irregularity and ballast stiffness. As indicated by Kwasniewski *et al.* (2006) and Li *et al.* (2005), a simple 2D beam is sometimes very effective to capture major physical phenomena though it can not show the exact behavior of a real bridge.

4.1 Simple beam subjected to moving vehicle without surface irregularity and ballast stiffness (Yang and Wu 2001)

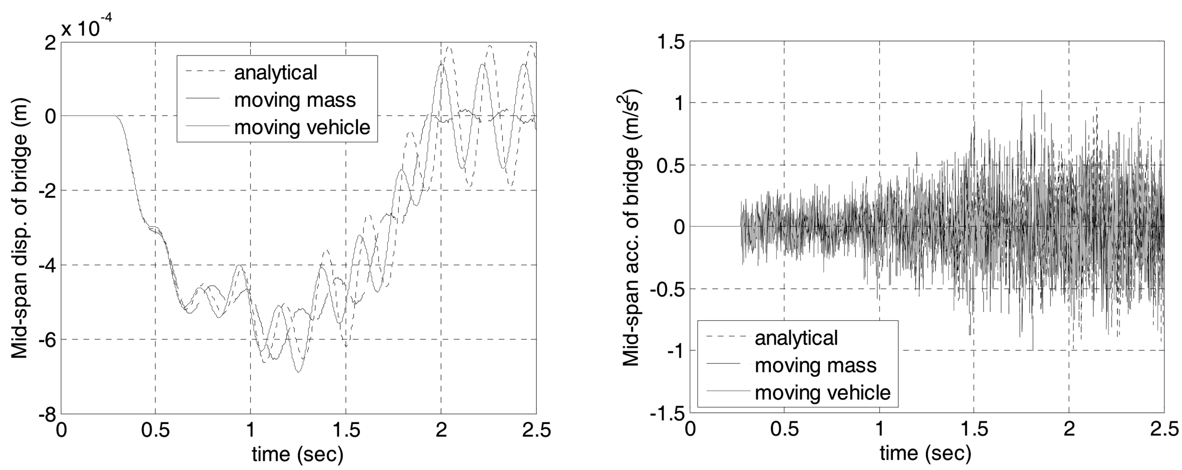
Fig. 3 shows a simply supported beam subjected to a moving vehicle, where the vehicle mass m_v is supported by dashpot-springs with spring constant k_i (i = 1, 2) and dashpots with damping coefficient c_i (i = 1, 2), which are further supported by a wheel mass of m_i (i = 1, 2). For illustration, the effect of the dashpot damping and wheel mass are neglected.

The displacement and acceleration of the midpoint of the bridge are presented in Fig. 4 and the pitching and acceleration of the vehicle are plotted in Fig. 5. The analytical solution is obtained using only the 1st mode of the bridge and the moving-force model presented in Smith (1988). For



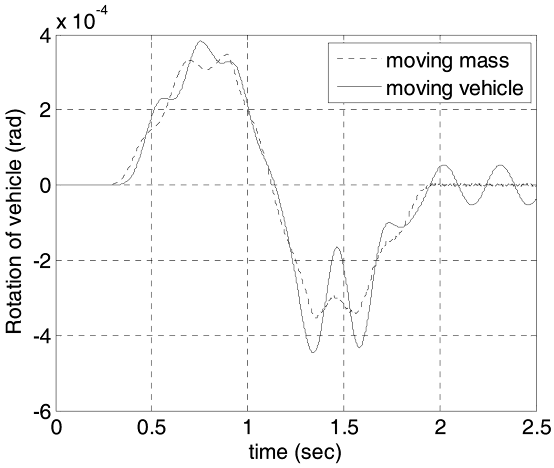
v = 27.78 m/s	L = 30 m
$k_{1,2}$ = 41,350 kN/m	E = 2.943x10^7 kN/m^2
$c_{1,2}$ = 0.0 kN s/m	I = 8.65 m^4
m_c = 540 ton	ξ = 0.0
J_c = 13,800 ton m^2	m = 36 ton/m
$m_{1,2}$ = 0.00 ton	$r(x)$ = $k_b(x)$ = 0
d = 17.5 m	No. of Elements = 20

Fig. 3 Interaction forces of the vehicle-bridge system

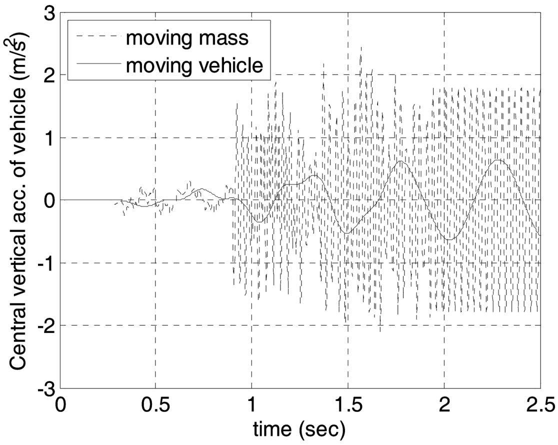


(a) Displacement (b) Acceleration

Fig. 4 Mid-span displacement and acceleration of the bridge



(a) Pitching



(b) Acceleration

Fig. 5 Pitching and acceleration of the vehicle

Table 1 Maximum and r.m.s response of the bridge

Model	Analytical		Moving mass		Moving vehicle	
	Disp. (m)	Acc. (m/sec^2)	Disp. (m)	Acc. (m/sec^2)	Disp. (m)	Acc. (m/sec^2)
Max. value	6.641e-004	9.929e-001	6.546e-004	1.128e+000	6.884e-004	8.761e-001
r.m.s value	2.476e-004	2.250e-001	2.321e-004	2.505e-001	2.422e-004	2.062-001

Table 2 Maximum and r.m.s response of the vehicle

Model	Moving mass		Moving vehicle	
	Disp. (m)	Acc. (m/sec^2)	Disp. (m)	Acc. (m/sec^2)
Max. value	3.653e-003	6.009e-001	4.465e-003	9.884e-002
r.m.s value	1.889e-004	8.865e-001	3.286e-002	2.999e-001

the comparison with a non-iterative method, the results obtained with a moving-mass model of Ichikawa *et al.* (2000), which are denoted as moving mass, are presented along with analytical solutions. The results of the proposed method are denoted as moving vehicle. It can be seen in Figs. 4 and 5 that the proposed method matches well with the analytical solution.

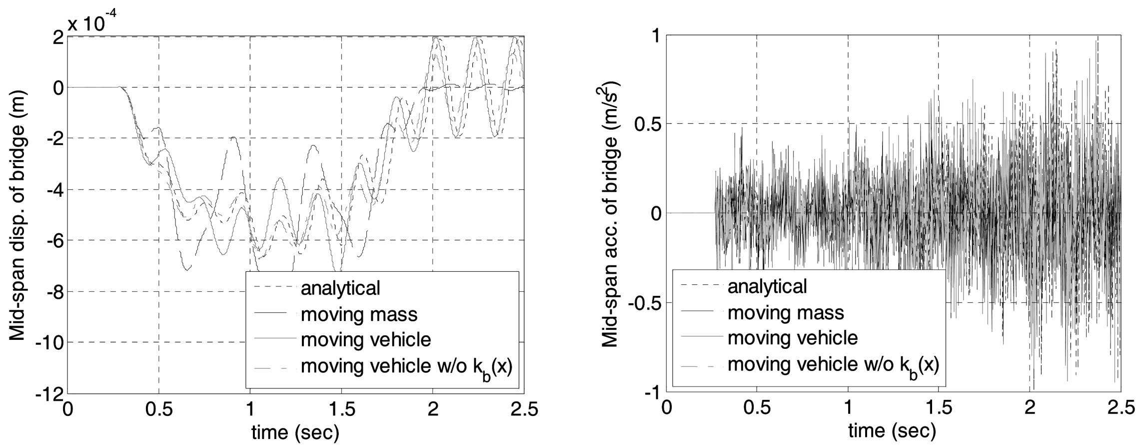
The maximum and r.m.s. displacement of the bridge and those of a vehicle are summarized in Tables 1 and 2. In this example, the maximum displacement of the bridge and the vehicle body are obtained when a moving vehicle model is used. On the other hand, the maximum acceleration of the bridge and the vehicle body are obtained when the moving mass model is used. This is because the moving mass model can not consider suspension systems, and thereby causes the overestimation of the vertical acceleration on the vehicle. Since the maximum or r.m.s value of acceleration have been taken as a common measurement of the passenger's riding comfort (Yang and Wu 2001), the analysis with the moving vehicle model needs to be performed for more accurate results.

4.2 Simple beam subjected to moving vehicle with surface irregularity and ballast stiffness

The surface irregularity and ballast stiffness are added in to the same bridge and vehicle interaction system shown in Fig. 3 in the second example. The following surface irregularity function proposed in Rohman and Duaij (1996) is adopted

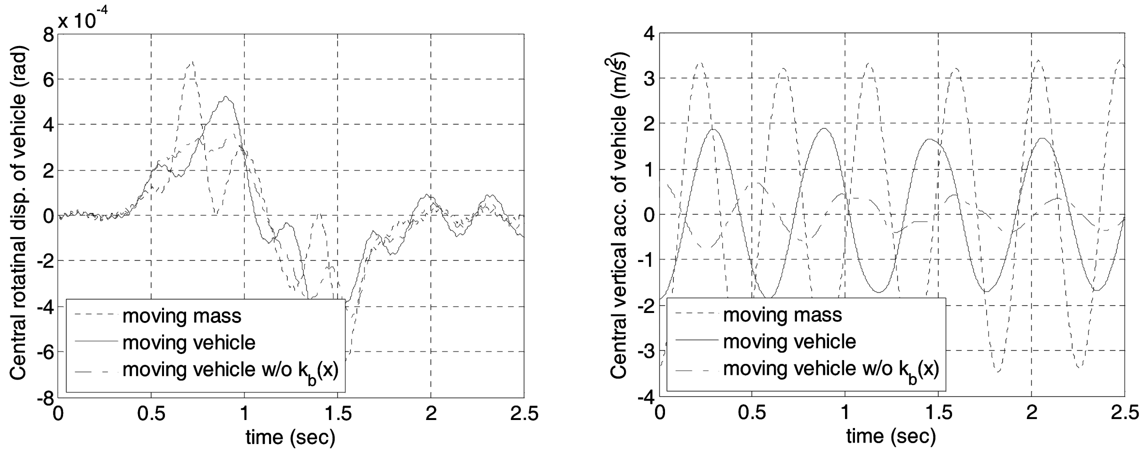
$$r(x) = a\left(1-\cos\frac{2\pi x}{l_1}\right) \tag{17}$$

where a is the irregularity amplitude in a wave length l_1. $a = 5.0\times10^{-3}$m and $l_1 = 1.0$ m are used in this example. The ballast stiffness is taken as a constant value of $k_b(x) = 124{,}050$ kN/m. For comparison, a moving vehicle model without ballast stiffness (that is, $k_b(x) = \infty$) is also solved. The vehicle is assumed to move from the outside of the bridge.



(a) Displacement (b) Acceleration

Fig. 6 Mid-span displacement and acceleration of the bridge



(a) Pitching (b) Acceleration

Fig. 7 Pitching and acceleration of the vehicle

Table 3 Maximum and r.m.s response of the bridge

Model	Analytical		Moving mass		Moving vehicle, $k_b(x) = \infty$		Moving vehicle, $k_b(x) = 124{,}050$ kN/m	
	Disp. (m)	Acc. (m/sec^2)	Disp. (m)	Acc. (m/sec^2)	Disp. (m)	Acc. (m/sec^2)	Disp. (m)	Acc. (m/sec^2)
Max. value	6.641e-004	9.929e-001	8.119e-004	6.909e-001	6.727e-004	9.679e-001	7.367e-004	7.647e-001
r.m.s value	2.476e-004	2.250e-001	2.694e-004	1.830e-001	2.414e-004	1.835e-001	2.536e-004	2.284e-001

Table 4 Maximum and r.m.s response of the vehicle

Model	Moving mass		Moving vehicle, $k_b(x) = \infty$		Moving vehicle, $k_b(x) = 124{,}050$ kN/m	
	Disp. (m)	Acc. (m/sec^2)	Disp. (m)	Acc. (m/sec^2)	Disp. (m)	Acc. (m/sec^2)
Max. value	6.810e-004	3.470e+000	4.553e-004	7.657e-001	5.373e-004	1.895e+000
r.m.s value	2.245e-004	2.343e+000	1.990e-004	3.678e-001	2.247e-004	1.271e+000

The displacement and acceleration of the midpoint of the bridge are presented in Fig. 6 and the pitching and acceleration of the vehicle are plotted in Fig. 7. It can be notice that ballast stiffness can affect the dynamic response of the vehicle-bridge interaction significantly. The maximum and r.m.s. displacement of the bridge and those of the vehicle are summarized in Tables 3 and 4. Similar to the first example, the maximum displacements of the bridge and vehicle body are obtained when the moving mass model is used, while the maximum accelerations of the bridge obtained is in the analytical solution and the maximum vertical acceleration of the vehicle body is obtained in the moving mass model.

5. Conclusions

In this paper, a new iterative method for analyzing vehicle-bridge interaction system is proposed. Iterative methods have advantages over the non- iterative methods in that it is not necessary to update the system matrix for a given wheel location, and the method can be used for a new type of car or bridge with minor or no modifications. In the proposed method, the interaction force term contains the effect of ballast stiffness which is ignored in the previous iterative methods. Further, the pitching effect, which is the interaction effect between the rear and front wheels when a vehicle begins to enter or leave the bridge, is also considered in the formulation. This effect could be easily formulated and solved in general ways using extended ground boundaries and the proposed interaction forces containing the effects of surface roughness and ballast stiffness.

The versatility and applicability of the proposed method have been demonstrated in the numerical examples. The first example shows that the proposed method matches well with the analytical solution. The second example shows that the ballast stiffness can not be ignored for the dynamic response of the vehicle-bridge interaction.

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