Structural Engineering and Mechanics, Vol. 3, No. 5 (1995) 429-443
DOI: http://dx.doi.org/10.12989/sem.1995.3.5.429

Mechanics of kinking and buckling of plastic board drains

Madhira R. Madhav†

Department of Civil Engineering, Indian Institute of Technology, Kanpur 208016, India

Yeong Mog Park‡

Department of Civil Engineering, Yeungnam University, Gyongsan 713-749, Korea

Norihiko Miura†

Department of Civil Engineering, Saga University, Saga, Japan

Abstract. The deformational response of plastic board drains installed to accelerate consolidation of soft soils, is examined as a problem of downdrag. The drain is modelled as a beam-column in which the axial load increases nonlinearly with depth. The soil response is represented by the Winkler medium whose coefficient of subgrade modulus increases linearly with depth. The governing equations for the drain-soil system are derived and solved as an eigenvalue problem. The critical buckling loads and the shape of the drain are obtained as functions of the normalized subgrade modulus of the soil at the top, the parameters signifying the variation of axial load along the length of the drain and the increase of subgrade modulus with depth. The derived deformed shapes of the drain are consistent with the observed ones.

Key words: buckling; consolidation; downdrag; flexural stiffness; kinking; mechanics; modulus of subgrade reaction; plastic board drains.

1. Introduction

Vertical drains offer themselves as one of the most cost effective choices for improving the behavior of soft compressible soils. Consequent to their installation, the length of the drainage path in the soil is reduced at least by an order of magnitude, and the direction of flow by ninety degrees into a more favorable flow direction. As a result, the time for consolidation decreases significantly by two to three orders of magnitude compared to that for the untreated ground. The adventage of prefabricated plastic board drains, referred to as PD (Hansbo 1981), has made it possible to install them even in difficult environments, e.g., in water, newly reclaimed ground, etc., at low cost, with minimal disturbance, and in very short time. The depth of installation of PD is increasing and presently is more than 20 m (Tanaka 1990, Crawford, *et al.* 1992).

† Professor
‡ Assistant Professor

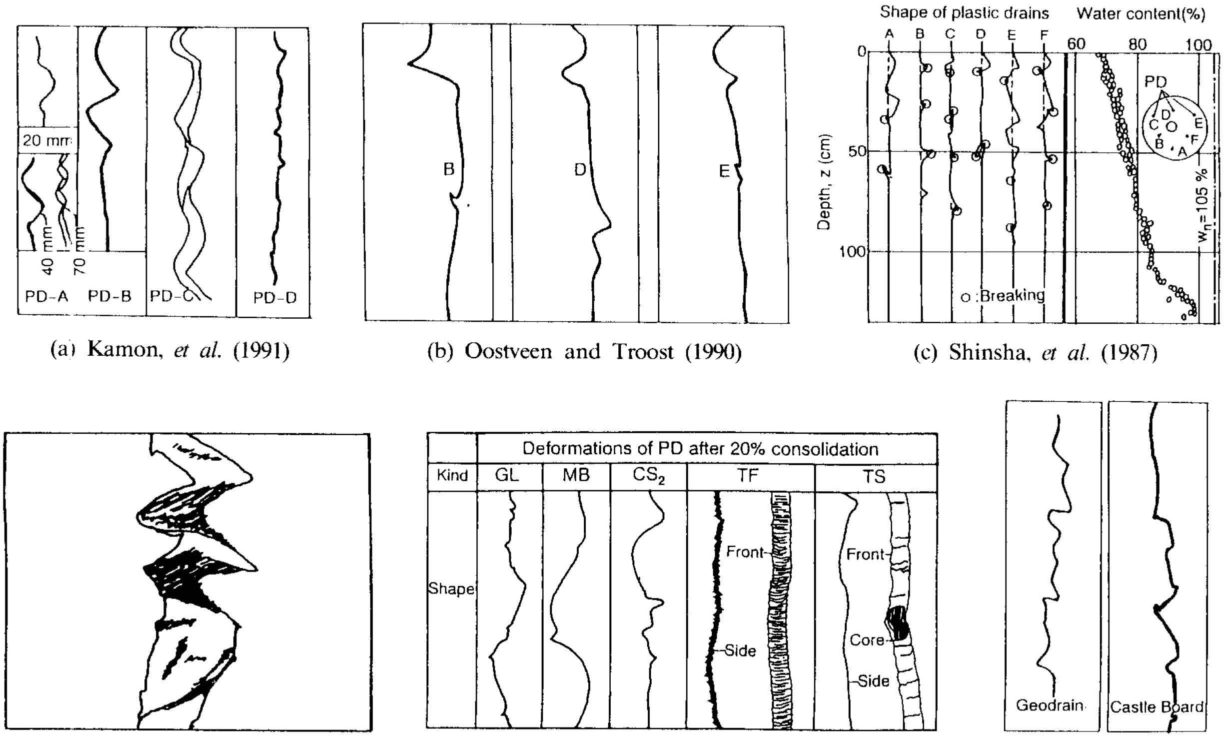
The degree of disturbance caused by the installation of relatively small sized PD, is controlled by the size of the mandrel and of the detachable shoe located at the mandrel tip, and is small compared to that of sand drains. Plastic drains are therefore appropriate also in the case of sensitive clays and soils having anisotropy with respect to permeability. The PD are composed of a thin or flat plastic core wrapped over by a filter sleeve made of nonwoven geosynthetic. Many studies (Ali 1991, Jamiolkowski, *et al.* 1983, Koda, *et al.* 1989, Pradhan, *et al.* 1991, Rixner, *et al.* 1986) report on the design and reliability of PD especially with regard to their hydraulic, mechanical and durability characteristics. The most relevant characteristics for the long term performance of PD are the integrity and the hydraulic properties-the discharge capacity, q_w, of the core, and permeability, k_f, of the filter sleeve. Jamiolkowski, *et al.* (1983) summarized many studies investigating the above parameters. Because of the small size of the strip shaped PD, their discharge capacity is sensitive to the lateral confining stress, migration of and clogging due to fine particles of the soil, folding/kinking, material degradation, etc., (Miura, *et al.* 1993) and therefore time dependent.

2. Observed shape of plastic drains

Kamon, *et al.* (1991) and Miura, *et al.* (1991) measured the shape of the PD after the deformation from laboratory scale model tests. Fig. 1 depicts the deformed shape of a PD in Ariake clay after undergoing an axial strain of nearly 40% under an applied stress of 294 kPa for 1030 days. The observed shapes of plastic board drains after the settlement of the soft soil of the order of 15 to 20% of its thickness, reported by various authors, are depicted in Fig. 2. Two distinct types of deformed shape can be noted:

(1) Buckling-uniform change in curvature,

Fig. 1 Deformed shape of PD after consolidation ($\varepsilon_l = 40\%$) using large scale steel consolidation apparatus.



(a) Kamon, *et al.* (1991) (b) Oostveen and Troost (1990) (c) Shinsha, *et al.* (1987)

(d) Kremer, *et al.* (1983) (e) Miura, *et al.* (1991) (f) Jimbo (1991)

Fig. 2 Different deformed shapes of PD after consolidation.

(2) Kinking-sharp change in curvature.

Depending on the type of drain and the soil in which it is installed, both buckling and kinking are observed throughout the length of the drain. While it is difficult to identify the specific causes for buckling and kinking, the following different shapes of the deformed drains can be noted:

(i) Buckling (Kamon, *et al.* 1991: PD.-A and C)

(ii) Buckling with mild kinking (Jimbo 1991: Geodrains, Miura, *et al.* 1991: MB, TS).

(iii) Buckling with sharp kinking (Kamon, *et al.* 1991: PD.-B and D, Jimbo 1991: CB, Kremer, *et al.* 1983, Miura, *et al.* 1991: GL, CS_2, Oostveen and Troost 1990).

(iv) Kinking (Sinsha, *et al.* 1987, see Fig. 1).

Kinking may possibly result from either a local defect in the core of the drain or due to a soft pocket in the soil. While Sinsha, *et al.* (1987) and Oostveen and Troost (1990) report some instances of buckling and kinking only in the top half to two-thirds length of the drain, in most instances the drains exhibit both types of deformation throughout their length.

Lawrence and Koerner (1988) postulate different possible drain configurations due to large settlements as Fig. 3 (i) uniform bending, (ii) sinusoidal bending, (iii) local bending, (iv) local kinking, (v) multiple kinking. Ali (1991) reports the flow behavior of kinked strip drains. Holtz and Christopher (1987), Oostveen and Troost (1990), Kamon, *et al.* and Miura, *et al.* (1993) compared the discharge capacities of straight and buckled drain configurations and related these to the bending rigidity and the structure of the core and the stiffness of the filter jacket.

Thus the mechanical characteristics, such as the tensile strength of the core and the filter,

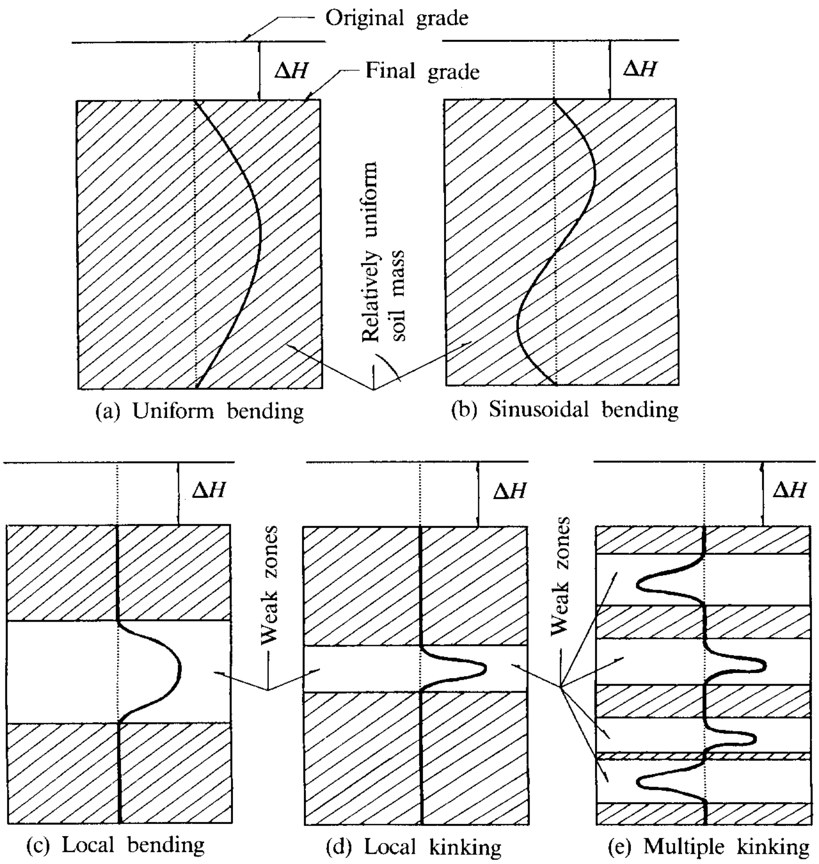


Fig. 3 Various possible drain configurations due to soil settlement (after Lawrence and Koerner 1988).

and the buckling strength of the core are important, in view of the stresses the drains are subjected to during their installation and performance later. Some guidelines (Kremer, *et al.* 1983, Jamiolkowsi, *et al.* 1983, Van Zanten 1986) are available prescribing the tensile strength σ_t to be greater than 5 kN/m, and strain at failure ε_f to be less than 10%, for PD. It is also desirable that the core and the filter strengths and stiffnesses should be comparable and compatible. However, the effect of large settlements of the consolidating soil layers, usually 15% to 25% of the total layer thickness, on the drain folding, kinking, or buckling, is not fully investigated.

3. Flexural stiffness of PD

The flexural or bending stiffness of the PD is determined by the simple bending test. A length of the drain is stretched and clamped at both ends over a clear span, L, of 320 mm. The deflected shape of the drain under its own weight is noted. A force F is applied through weights suspended from the center ensuring that the load is distributed over the full width of the drain, and displacements δ are measured at the center and the quarter points. The flexural stiffness of the drain EI_d is then calculated from the relation

$$EI_d = FL^3 C_x / \delta \quad (1)$$

Table 1 Flexural stiffness of PD

Kind	Flexural stiffness EI (Nm^2)	Moment of inertia, (cm^4)	C.S.A.C A_c (cm^2)	Young's modulus, E (kPa)	
				Bending test	D.T.T.
CS	0.401	0.0069883	1.585	646780.5	573813.8
CS_2	0.622	0.0067160	1.258	826445.0	926145.8
GL	0.422	0.0051766	1.156	723096.5	815206.2
MW	0.403	0.0059959	1.740	667316.5	672117.8
TS	0.343	0.0081664	1.533	415546.5	420011.5

*C.S.A.C.: Cross-sectional area of core
**D.T.T.: Direct tension test (Kinjo Rubber Co., LTD.)

where C_x is a coefficient that depends on the distance, x of the point where the deflection is measured. Values of C_x for x/L equal to 0.25 and 0.5 are respectively 1/384 and 1/192. The flexural stiffnesses of different drains, measured in the linear elastic range, are presented in Table 1. The values of EI_d given in Table 1 under bending test are the. average values for the range loads tested (1.0 to 6.0 N). The stiffnesses of the drains tested fall within a narrow range of 0.40 to 0.73 Nm^2 with Super Tefnal (TS) and the Geodrain (GL) giving the smallest and the highest values respectively. The cross-sectional shape and area of the cores of the drains are measured. Assuming that only the cores of the drains provide the bending stiffness, i.e. the filter sleeve has no bending stiffness, a justifiable assumption, the Young's modulus of the drain material is computed from the flexural stiffness and the moment of inertia, I_d, of the core. The Young's modulus of the drain is also evaluated from the direct tension test on the drains. The values of the Young's moduli from both bending and direct tension tests compare well (Fig. 4).

4. Mechanics of buckling and kinking

Soft compressible normally consolidated clays settle by significantly large amounts (usually 15% to 25% of their thickness). The settlements of the top layers are much more than those in the lower half of the thickness. In a highly compressible soil, the compression of the layer near the top is very high and decreases with depth (Fig. 5) even in the one-dimensional case, since the initial effective overburden stress increases with depth while the stress increment is constant with depth. Also the drains discharge from the top and the consolidation of the soil progresses from the top to the bottom. For a typical case of a N.C, soil of 30 m thickness with $C_c/(1+e_0)$ equal to 0.2, and with a landfill of 2.0 m on top, the settlement (Fig. 5) of the top 1 m thickness is 34% compared to 3% of the lowest 1 m thickness. As the drain is anchored into the freely draining granular layer laid on top of the soft clay, it moves down along with the deforming soil. Since the drain is highly flexible, it will bend, buckle or fold to accommodate the reduced distance between the two ends. A possible mechanism for the PD-soft soil system is buckling due to downdrag or negative skin friction. As the soil settles and moves down, it drags the drain because of the stresses mobilized at the drain/soil interface. If the downdrag force is greater than the critical buckling load or if the deformations are excessively large, the drain buckles. Since the stiffness of the drain is very small, if a defect or a flaw exists locally, it will form a kink there. Unlike conventional buckling phenomenon in columns and piles, the PD buckles with the load on it increasing with the depth due to downdrag forces.

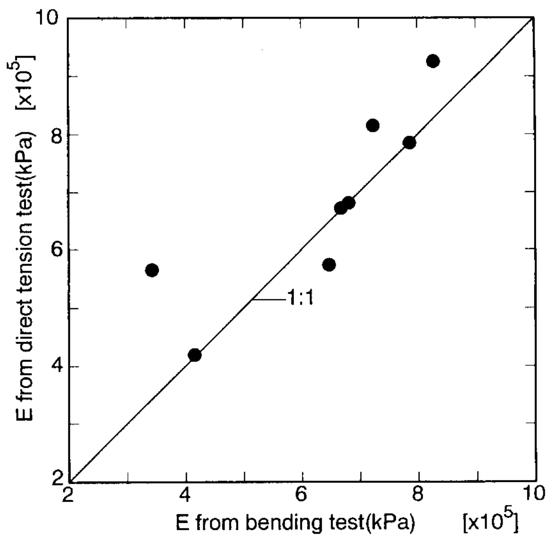


Fig. 4 E values from bending and direct tension tests.

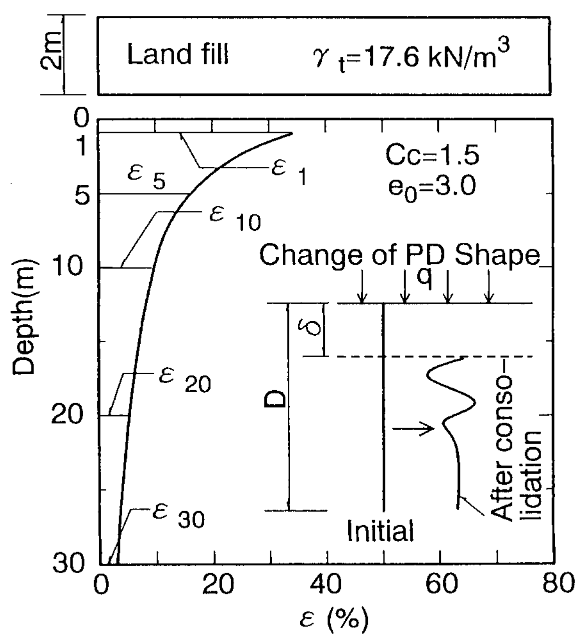
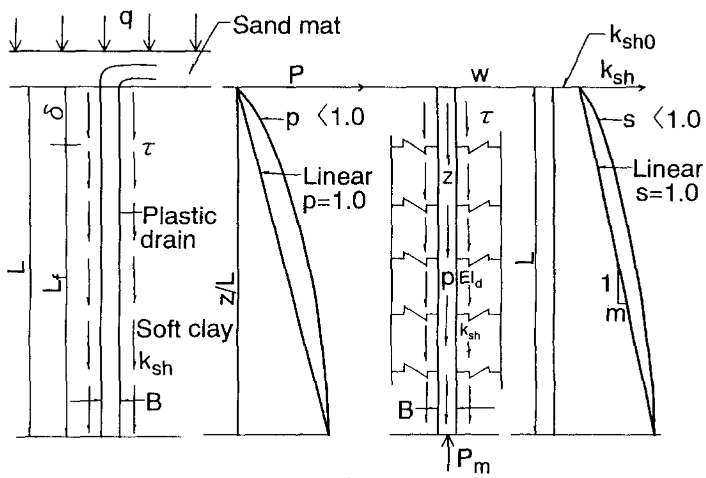


Fig. 5 Distribution of ε_1 with depth.

Buckling of piles acted upon by superstructure loads and treating soil as a Winkler medium, has been a subject of study (Davisson 1963), for some time. Toakley (1965) and Reddy and Valsangkar (1970) extend the analysis for cases where the axial load and the horizontal coefficient of subgrade reaction vary with depth. Madhav and Davis (1975) treat the soil as an elastic continuum and obtain the buckling loads for piles. Kerr (1988) presents a simple nonlinear analysis for pile buckling and shows that the present approaches may overestimate the buckling loads for slender piles. No solutions are available for buckling of piles subjected to downdrag



(a) (b) (c) (d)

Fig. 6 Model and definition sketch.

forces since such a phenomenon is very unlikely.

5. Problem definition and solution

A PD of flexural stiffness EI_d fully penetrates (Fig. 6a) into a soft clay of thickness L and is anchored into the drainage blanket at the top and to a stiff layer at the bottom. The axial force $P(z)$ in the drain at depth z is

$$P(z)=\int_0^z \tau(z)c\cdot dz \tag{2}$$

where $\tau(z)$ is the shear stress mobilized at the drain/soil interface, $c=2(B+t)$, the perimeter of the drain, and B and t are the drain width and thickness respectively. The total axial force P_m is

$$P_m=\int_0^L \tau(z)c\cdot dz \tag{3}$$

where L is the length of the drain. The variation (Fig. 6b) of the axial force with depth is postulated as

$$P(z)=P_m(z/L)^p \tag{4}$$

where p is an exponent. $p=0$ implies axial load is constant with depth while $p=1.0$ corresponds to a linear increase of downdrag force with depth. Since the displacements of the consolidating layer at the top are significantly large, the downdrag forces would be large there and p values are considered to range most likely between 0.1 and 0.25.

When the drain tends to bend, the soil mobilizes its lateral resistance. The resistance offered by the soil can be accounted for by the simpler Winkler type of response (Davisson 1963, Reddy and Valsangkar 1970) or the more general continuum approach (Madhav and Davis 1975). In this paper, the former approach is adopted (Fig. 6c) for its simplicity. The horizontal coefficient of subgrade reaction, $k_{sh}(z)$, of the normally consolidated soil, is considered to vary with depth (Fig. 6d), as

$$k_{sh}(z)=k_{sh0}\{1+\mu(z/L)^s\} \tag{5}$$

where k_{sh0} is the subgrade modulus at $z=0$, μ - a parameter signifying the increase in the modulus at depth $z=L$, and s - an exponent. For linear increase of modulus with depth $s=1.0$ while $s=0$ signifies a constant value of the modulus.

The PD - soil system is treated as a beam-column embedded in a Winkler medium (Fig. 6c) subjected to an axial force that varies with depth. The equation governing its response following Davisson (1963) is

$$EI_d\frac{d^4w}{dz^4}+\frac{d}{dz}\left\{P(z)\frac{dw}{dz}\right\}+k_{sh}(z)B\cdot w=0 \tag{6}$$

where w is the lateral or horizontal displacement of the drain. Combining Eqs. (4), (5) and (6) and simplifying, one gets

$$\frac{d^4w}{dZ^4}+\lambda Z\frac{d^2w}{dZ^2}+\frac{\lambda p}{Z^{(1-p)}}\frac{dw}{dZ}+\beta(1+\mu Z^s)\cdot w=0 \tag{7}$$

where $\lambda=P_m L^2/EI_d$, $Z=z/L$ and $\beta=k_{sh0}BL^4/EI_d$. The drain is considered to be hinged at both ends. The displacements and moments are zero, i.e.,

$$\text{at } Z=0, \text{ and } Z=1,\ w=0 \text{ and } \frac{d^2w}{dZ^2}=0 \tag{8}$$

Eqs. (7) and (8) form a set of homogeneous equations the solution of which gives the critical or buckling loads λ_{cr}. The finite difference approach (Madhav and Davis 1975) is chosen to obtain the critical loads. Discretizing the drain into N elements each of size, L/N, Eq. (7), in finite difference form is

$$\begin{aligned}&\{w_{i-2}-4w_{i-1}+6w_i-4w_{i+1}+w_{i+2}\}+\lambda Z_i^p\{w_{i-1}-2w_i+w_{i+1}\}/N^2\\&+\{\lambda p/2N^3\cdot Z_i^{(1-p)}\}\{w_{i+1}-w_{i-1}\}+\beta(1+\mu Z_i^s)w_i/N^4=0\end{aligned} \tag{9}$$

Combining Eqs. (9) for the nodes 2 to n, with the boundary conditions (Eq. (8)), a set of $(N-1)$ simultaneous equations are obtained as

$$[A]\{w\}+\lambda[C]\{w\}=0 \tag{10}$$

where $[A]$ and $[C]$ are square matrices of size $(N-1)$, consisting of coefficients of the terms in Eq. (9) and are given in the Appendix. Premultiplying Eq. (10) with $-[C]^{-1}$, the negative inverse of $[C]$, one gets

$$[A^*]\{w\}=\lambda\{w\} \tag{11}$$

where $[A^*]=-[C]^{-1}[A]$. Eq. (11) is of the standard form, the eigenvalues of which give the buckling loads, and the eigenvectors the buckling shapes of the drain. The smallest eigenvalues λ_{cr} is the normalized load at which the drain buckles or gets kinked.

6. Results

The deformation of a plastic drain along with the compression of very soft soil is modelled as a phenomenon similar to buckling of a pile or column under axial load. In this case the drain is subjected to a downward pull all along its length with the consolidation of the soft soil. It is assumed in the analysis that the downward force increases nonlinearly with depth (Fig. 6b) with the exponent $p<1.0$. The lateral resistance offered by the soft soil is represented by Winkler springs whose stiffness (Fig. 6d) increases linearly with depth ($s=1.0$).

The governing equation in finite difference form (Eq. (9)) is solved for different values of N, the number of elements into which the drain is discretized. A value of N equal to 48 is adopted to obtain results with a minimum of error and within reasonable computer time. The first eigenvalue and the first eigenvector give the least buckling load and the corresponding shape of the deformed drain respectively.

Since the parameters K_{sh0} and P_{cr} are normalized with EI_d, the flexural stiffness of the drain, the normalized stiffness parameter β and the variation of λ_{cr} are very large for the usual range of properties of soft soils. It is found preferable and illustrative to present the variation of λ_{cr}/β with β for different values of p, in Figs. 7 and 8. In all the cases λ_{cr}/β decreases with increasing values of β. For $p=1.0$, (uniform axial force with depth), and $\mu=0.0$ (constant subgrade modulus with depth), λ_{cr}/β decreases (Fig. 7a) from 0.096 to 0.00013 for β increasing from 10^3 to 10^9. For nonhomogeneous ground ($\mu>0$), the critical buckling loads are larger but the trend of their

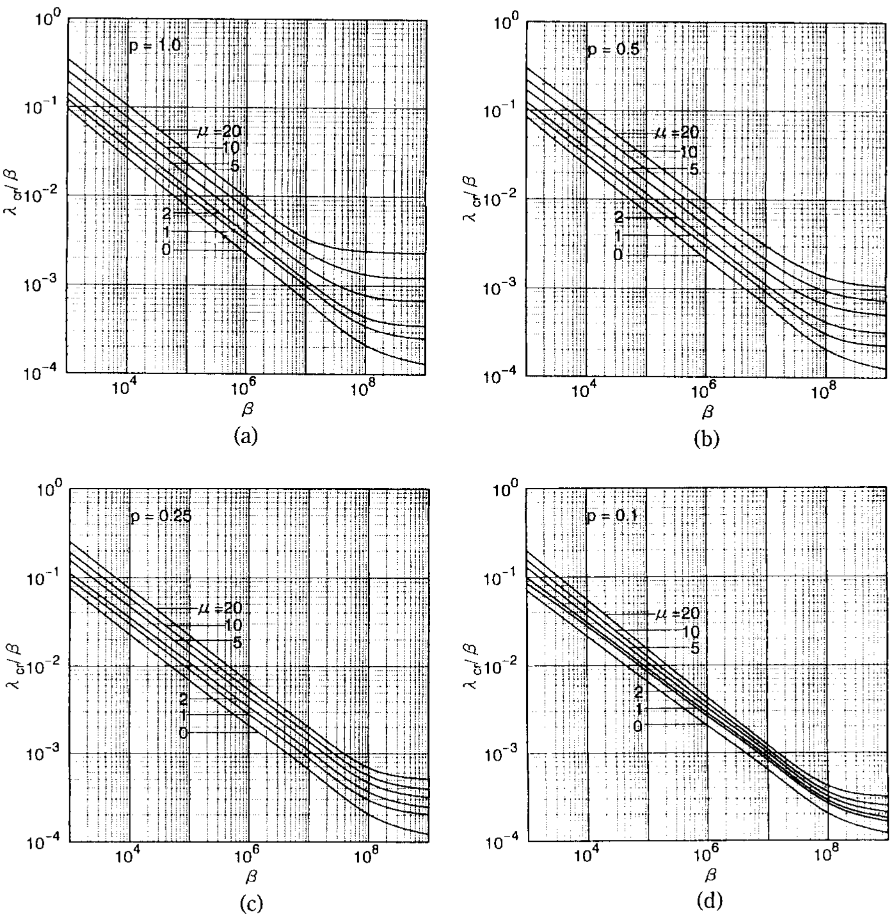


Fig. 7 Variation of λ_{cr}/β with μ.

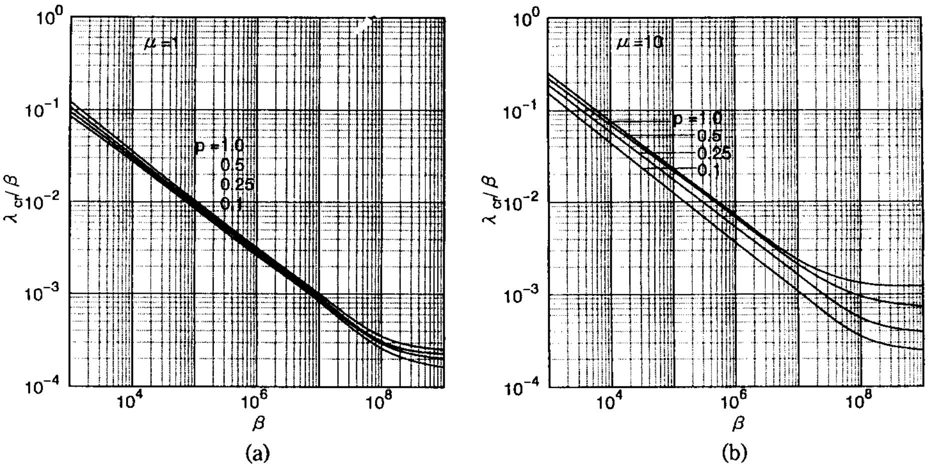


Fig. 8 Variation of λ_{cr}/β with p.

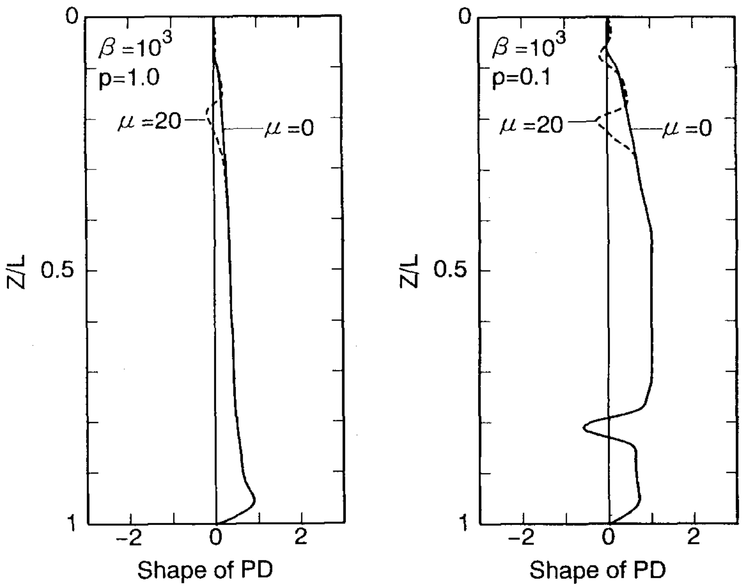
variation with β is the same. For larger values of μ, λ_{cr}/β is independent of β for higher values of β indicating a linear variation of λ_{cr} with β. The range of β over which λ_{cr}/β is constant increases with β. Similar results are observed (Fig. 7b, 7c and 7d) for $p<1$, in which case, the axial loads vary nonlinearly with depth and the axial load at any depth is higher than the value corresponding to $p=1.0$. The critical buckling loads are smaller for $p<1.0$ compared to those for $p=1.0$.

Fig. 8a and 8b bring out the effect of the rate of increase of axial load with depth, p, on the λ_{cr}/β versus β variations. For $\mu=1.0$, (smaller nonhomogeneity), the effect of p varying from 1.0 to 0.1 is relatively small compared to the case for $\mu=10.0$ (high nonhomogeneity). In the former case (Fig. 8a) the reductions in λ_{cr}/β for p decreasing from 1.0 to 0.1, range from 0.74 to 0.67 while in the latter case (Fig. 8b), the range is from 0.65 to 0.2.

The first eigenvector represents the buckled shape of the drain. Figs. 9, 10 and 11 are for stiff ($\beta=10^3$), medium ($\beta=10^6$) and flexible $\beta=(10^9)$ drains. A stiff drain with axial load increasing linearly with depth exhibits typical buckling near the tip where the axial load is maximum (Fig. 9a) for $\mu=0$ (homogeneous soil) while a local buckle gets superimposed near the top where the stiffness is relatively small compared to the stiffness at $z=L$ for $\mu=20$ (nonhomogeneous soil). If $p=0.1$, i.e., nonlinear variation of axial load with depth, a kink (Fig. 9b) is noted at $z/L=0.8$ for $\mu=0$, while typical kinking type deformation is observed in the top 0 to 0.25 H depth of the drain for $\mu=20$.

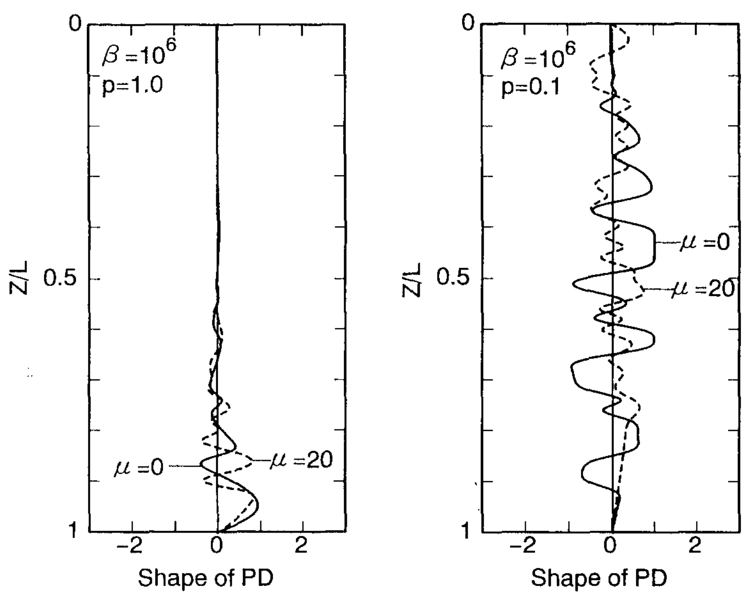
For a medium stiff drain ($\beta=10^6$) and $p=1.0$, the drain remains (Fig. 10a) straight over the top 80% of its length and buckles in the lower 20% of the length for all μ values. For $p=1$, buckling and kinking (Fig. 10b) can be noted over the full length of the drain for $\mu=0$ and 20. For very flexible drains ($\beta=10^9$) and linearly increasing axial load ($p=1.0$) buckling and kinking extend (Fig. 11a) to the lower 0.25 L of the drain for $\mu=0$ (constant subgrade modulus with depth) and over the lower half for $\mu=20$ (nonhomogeneous soil). Kinking and buckling extend upwards with increasing values of μ. Most interestingly, kinking in the top half of the drain (Fig. 11b) results for $\mu=20.0$ and for $p=0.1$ while buckling and kinking occur from $z/L=0.5$ for $p=1.0$.

It would be very desirable for the predictions to be compared with the actual observed shape. In the absence of precise knowledge of the properties of the soil, especially of the modulus of subgrade reaction and its variation with depth, it is difficult to make the comparisons. The



(a) (b)

Fig. 9 Deformed shape of stiff PD.



(a) (b)

Fig. 10 Deformed shape of medium stiff PD.

horizontal coefficient of subgrade reaction of the soil, in particular of soft sensitive soil, gets affected by the effect of disturbance caused by the drain installation itself. Some typical drain configurations predicted for $\beta=10^6$, $\mu=2$ and $p=1.0$ and 0.1 (Fig. 12a) and $p=0.25$ and $\mu=0$ and 20 (Fig. 12b) compare well with the observed ones. Specifically the shape predicted for

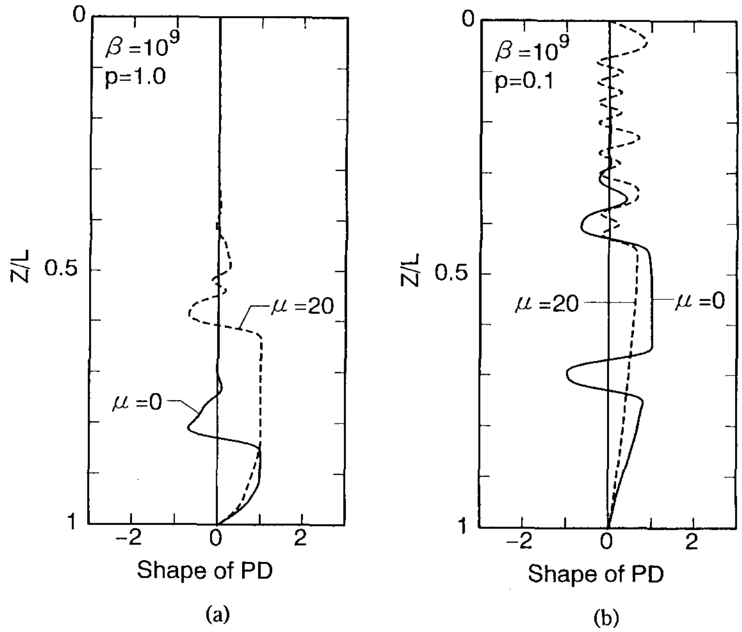


Fig. 11 Deformed shape of flexible PD.

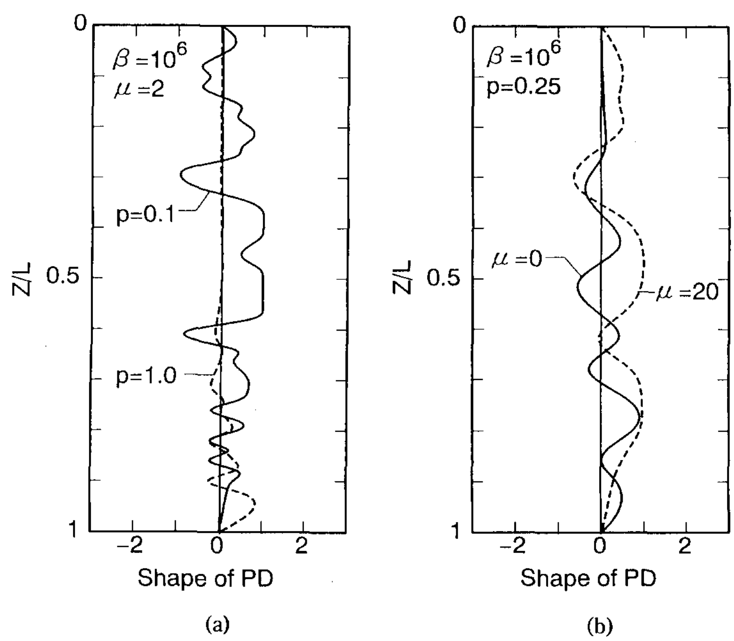


Fig. 12 Prediction of observed shape of deformed PD.

$\beta = 10^6$, $\mu = 2$ and $p = 0.1$ is very similar to the one shown in Fig. 1 and the shape predicted for $\beta = 10^6$, $p = 0.25$ and $\mu = 0$ is close to the ones exhibited by PD-C (Fig. 2a, Kamon, *et al.* 1991) and castle board drain (Fig. 2f) of Jimbo (1991). The shape of PD-MB (Fig. 2e, Miura, *et al.* 1991) is close to the prediction for $\beta = 10^6$, $p = 0.25$ and $\mu = 20$.

7. Conclusions

While considerable literature is available on the behavior of soft clays treated with vertical drains, very little is known about the behavior of the drain as it deforms along with the soil. However, it is well established that the long term performance of the drain, particularly the hydraulic characteristics depend on its integrity which is affected by the mechanical properties and deformations. The flexural stiffness of commercially available drains is found to range between 0.4 and 0.73 Nm^3.

To study the mechanics of the drains, the drain-soil system is treated as a beam-column supported laterally by a Winkler medium. The axial load along with the drain increases nonlinearly with depth while the horizontal coefficient of subgrade reaction of the soft soil increases linearly with depth. The homogeneous set of equations governing the mechanics of drain-soil system are derived and solved to obtain the critical buckling loads and the buckled shape of the drain as eigenvalues and eigenvectors respectively. The normalized critical buckling load varies significantly with the normalized subgrade modulus, the rate of increase of axial load along the length of the drain and the increase of subgrade modulus with depth. A stiff drain deforms predominantly by buckling while a flexible drain tends to form kinks and folds in the top half of the drain. The derived shapes of the drains are consistent with those reported in literature.

Notations

B	width of the drain
C_x	influence coefficient
c	perimeter of the drain
E	Young's modulus of drain
EI_d	flexural stiffness of drain
F	force
I_d	moment of inertia of drain
k_{sh}	coefficient of subgrade reaction
L	length of drain
N	number of elements
P	axial load in the drain
P_{cr}	critical or buckling load
p	exponent signifying increase of load with depth
s	exponent signifying increase of K_{sh} with depth
t	thickness of drain
w	lateral displacement
Z	$=z/L$-normalized depth
z	depth
β	$=K_{sh0}BL^4/EI_d$
δ	deflection
λ	normalized critical or buckling load
μ	parameter signifying increase of K_{sh} with depth
τ	shear stress at drain-soil interface

References

Ali, F.H. (1991), "The flow behavior of deformed prefabricated vertical drains", *Geotextiles and Geomem-*

brances, **10**, 235-248.
Crawford, C.B., Fannin, R.J., de Boer, L.T. and Kern, C.B. (1992), "Experiences with prefabricated vertical (wick) drain at Vernon", *B.C., Can. Geotech. J.*, **29**, 67-79.
Davisson, M.T. (1963), "Estimating buckling loads for piles", *Proc. 2nd Pan American Conf. SMFE*, Brazil, **1**, 351-371.
Garassino, A., Jamiolkowski, M., Lancellotta, R. and Tonghini, M. (1975), "Behavior of pre-loading embankments on different vertical drains", *Proc. 5th ECSMFE*, **1**, 213-218.
Hansbo, S. (1981), "Consolidation of fine-grained soils by prefabricated drains", *Proc. 10 ICSMFE*, Stockholm, **3**, 677-682.
Holtz, R.D. and Christopher, B.R. (1987), "Characteristics of prefabricated drains for accelerating consolidation", *Proc. 9th ECSMFE*, Dublin, **2**, 903-906.
Jamiolkowski, M., Lancellota, R. and Wolski, W. (1983), General Report, Specialty Session 6, *Proc. 8th ECSMFE*, Helsinki, **3**, 1201-1226.
Jimbo, N. (1991), "Paper drains and plastic board drains", Special Lecture, Saga Univ., P.T.-5 (in Japanese).
Kamon, M., Pradhan, T.B.S. and Suwa, S. (1991), "Evaluation of design factor of prefabricated band shaped drains", *ICGECD, GEO-COAST '91*, Yokohama, 329-334.
Kerr, A.D. (1988), "On the buckling of slender piles", *Soils and Foundations*, **28**(2), 144-148.
Koda, E., Szyaansky, A. and Wolski, W. (1989), "Behavior of geodrains in organic subsoil", *Proc. 12th ICSMFE*, Rio de Janeiro, **2**, 1377-1380.
Kremer, R.H.J., Oostveen, J.P., Van Weele, W.F. and Meyvogel, I.J. (1983), "The quality of vertical drainage", *Proc. 8th ECSMFE*, Helsinki, **2**, 721-726.
Lawrence, C.A. and Koerner, R.M. (1988), "Flow behavior of kinked strip drains", *In Proc. Symp. Geosynthetics for Soil Improvement*, GE Div., ASCE, 22-39.
Madhav, M.R. and Davis, E.H. (1975), "Buckling of finite beams in elastic continuum", *J. Engrg. Mech. Div. ASCE*, **100**(EMD), 1227-1236.
Miura, N., Park, Y.M. and Fukuhara, S. (1991), "Experiment on the drainage properties of plastic drains", *Proc. 26th JS SMFE* Annual meeting, Nogano, 2009-10 (in Japanese).
Miura, N., Park, Y.M. and Madhav, M.R. (1993), "Fundamental study on drainage performance of plastic board drains", *Proc. JSCE*, **481**(III-25), 31-40.
Nakamura, I., Okawa, H. and Kotani, K. (1985), *Construction of Paper Drains of Super Length*, Kisokou, 57-64 (in Japanese).
Oostveen, J.P. and Troost, G.H. (1990), "Discharge index tests on vertical drains", *Proc. 4th Int. Conf. Geotextiles, Geomembranes and Related Products*, The Hague, **1**, 345-350.
Pradhan, T.B.S., Kamon, M. and Suwa, S. (1991), "Design method for the evaluation of discharge capacity of prefabricated band shaped drains", *Proc. 9th Asian Reg. Conf. SMFE*, Bangkok, **1**, 523-526.
Reddy, A.S. and Valsangkar, A.J. (1970), "Buckling of fully and partially embedded piles", *J. Soil Mech. and Founds., ASCE*, **96**(SM6), 1951-1965.
Rixner, J.J., Kramer, S.R. and Smith, A.D. (1986), *Prefabricated Vertical Drains*, I, Summary of Research Effort, FHWA/RD-86/169.
Shinsha, H., Hayashi, K. and Watari, Y. (1987), "Effect of band-shaped plastic drains on the progress of consolidation", *Proc. 22nd JSSMFE Annual Meeting*, 1791-1792 (in Japanese).
Tanaka, H. (1990), "Settlement of ground improved by band drains at the development of the Tokyo International Airport", *Proc. 10th SEAGC*, Taipei, 133-138.
Toakley, A.R. (1965), "Buckling loads for elastically supported structs", *J. Engrg. Mech. ASCE*, **91**(EM3), 205-231.
Van Zanten, V. (1986), "The guarantee of the quality of vertical drainage systems", *Proc. 3rd Int. Conf. Geotextiles*, Vienna, **2**, 651-655.

Appendix

The matrices $[A]$ and $[C]$ are as follows:

$$[A]=\begin{pmatrix}
5+T_2 & -4 & 1 & 0 & 0 & 0 & \cdots & & & & & \\
-4 & 6+T_3 & -4 & 1 & 0 & 0 & \cdots & & & & & \\
1 & -4 & 6+T_4 & -4 & 1 & 0 & \cdots & & & & & \\
\cdots & & & & & & & & & & & \\
0 & 0 & 0 & \cdots & \cdots & 1 & -4 & 6+T_i & -4 & 1 & 0 & 0 \\
\cdots & & & & & & & & & & & \\
\cdots & & & & & & & 1 & -4 & 6+T_{n-2} & -4 & 1 \\
\cdots & & & & & & & 0 & 1 & -4 & 6+T_{N-1} & -4 \\
\cdots & & & & & & & & 0 & 1 & -4 & 5+T_N
\end{pmatrix}$$

with $T_i=\beta(1+\mu Z_i^s)/N^4$, $Z_i=(i-1)L/N$ and $i=2$ to N.

$$[C]=\begin{pmatrix}
-2U_2 & U_2+V_2 & 0 & 0 & \cdots & & & & \\
U_3-V_3 & -2U_3 & U_3+V_3 & 0 & \cdots & & & & \\
0 & U_4-V_4 & -2U_4 & U_4+V_4 & \cdots & & & & \\
\cdots & & & & & & & & \\
0 & 0 & 0 & 0 & U_i-V_i & -2U_i & U_i+V_i & 0 & 0 \\
\cdots & & & & & & & & \\
\cdots & & & & & & U_{N-1}-V_{N-1} & -2U_{N-1} & U_{N-1}+V_{N-1} \\
\cdots & & & & & & 0 & U_N-V_N & -2U_N
\end{pmatrix}$$

with $U_i=Z_i^p/N^2$ and $V_i=p/(2N^3Z_i^{(1-p)}$.