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Vibration and stability of fluid conveying pipes with stochastic parameters

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Abstract. Flexible cantilever pipes conveying fluids with high velocity are analysed for their dynamic response and stability behaviour. The Young's modulus and mass per unit length of the pipe material have a stochastic distribution. The stochastic fields, that model the fluctuations of Young's modulus and mass density are characterized through their respective means, variances and autocorrelation functions or their equivalent power spectral density functions. The stochastic non self-adjoint partial differential equation is solved for the moments of characteristic values, by treating the point fluctuations to be stochastic perturbations. The second-order statistics of vibration frequencies and mode shapes are obtained. The critical flow velocity is first evaluated using the averaged eigenvalue equation. Through the eigenvalue equation, the statistics of vibration frequencies are transformed to yield critical flow velocity statistics. Expressions for the bounds of eigenvalues are obtained, which in turn yield the corresponding bounds for critical flow velocities.

Key words: fluid pipes; stochastic; dynamics; stability

1. Introduction

Flow-induced vibrations in closed conduits such as pipe lines in hydroelectric and nuclear power plants, pipe lines in process industries, suction and pressure pipes before and after pumps, fuel feeding lines of aerospace vehicles and rockets, tube arrays in steam generators, etc. need to be studied using the kinetic method of stability investigation. Significant research activity has taken place in this area and detailed investigations are available in abundance (Ashley and Haviland 1950, Benjamin 1961, Gregory and Paidoussis 1966, Paidoussis and Issid 1974, Ariaratnam and Sri Namachchivaya 1986, Chen 1972).

The dynamic response of such coupled systems has been shown to be highly sensitive even to a small fluctuation in the design variables (Pauli and Seyranian 1983, Rajan, *et al.* 1986). In real life mechanical industry, many factors like non-uniform material density, hardness of workpiece, machining and manufacturing errors, variations in sizes of bolts, rivets, etc., lead to different levels of uncertainty in respect of system parameters. As a result, the probabilistic description of strength parameters, material properties, geometric boundary conditions and exter-

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nal loadings is to be adopted that leads to a safer and reliable design of such important systems.

Further, the usage of modern construction materials like RCC in civil engineering and fibre reinforced composites in aerospace engineering has further underlined the need for a probabilistic description of system parameters. New analysis procedures have been developed on a probabilistic framework (Vanmarcke 1983, Shinozuka and Lenoe 1976, Ibrahim 1987) and these employ the stochastic process modeling of system parameters. As far as random eigenproblems are considered, only self-adjoint systems have been analysed to obtain the response moments (Soong and Cozzarelli 1976, Vom Scheidt and Purkert 1983, Collins and Thomson 1969, Shinozuka and Astill 1972, Boyce 1968). In the area of stochastic stability analysis, research activity has been directed along the following two lines: (1) Deterministic systems subjected to random loading in time which is a classical random vibration problem (Kozin 1988, Herrmann 1969, Plaut and Infante 1970) and (2) Stochastically parametered and conservatively loaded systems (Collins and Thomson 1969, Shinozuka and Astill 1972). The present authors have investigated the dynamics and stability of both self-adjoint and non-self adjoint structural systems (Anantha Ramu and Ganesan 1992, 1993, Ganesan, *et al.* 1992, 1993). Regarding the flow induced vibrations, the published work in this context deals only with the dynamic stability of fluid conveying pipes with a stochastic flow velocity (Ariaratnam and Sri Namachchivaya 1986) and the random fluctuations of material properties have not been considered in the analysis.

In this paper, a probabilistic analysis is presented for tubular cantilevers conveying a fluid, when the Young's modulus and the mass per unit length of the pipe have a stochastic distribution. The fluctuations of Young's modulus and mass per unit length of the pipe are treated to constitute, independent one-dimensional univariate, homogeneous real stochastic fields in space.

2. Mathematical formulation

The equation of small lateral motions, for a tubular cantilever of length L, mass per unit length m and flexural rigidity EI, conveying a fluid of mass per unit length M that flows axially with velocity U and discharges at the free end can be written as (Refer Fig. 1)

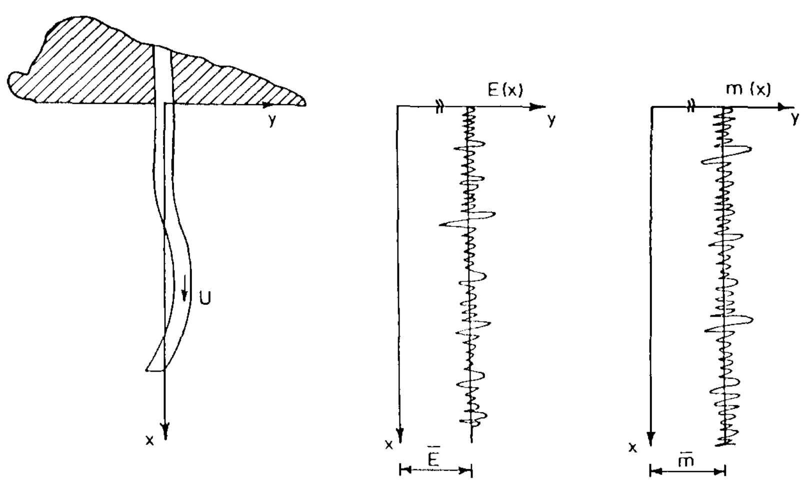

Fig. 1 Cantilever pipe conveying fluid

$$EI\frac{\partial^4 y}{\partial x^4}+[MU^2-(M+m)(L-x)g]\frac{\partial^2 y}{\partial x^2}+2MU\frac{\partial^2 y}{\partial x\partial t}+(M+m)g\frac{\partial y}{\partial x}+(M+m)\frac{\partial^2 y}{\partial t^2}=0 \quad (1)$$

subject to the boundary conditions,

$$y(o)=y'(o)=y''(L)=y'''(L)=0 \quad (2)$$

The primes in the above equation and in the sequel, denote differentiation with respect to x.

When the fluctuations of Young's modulus and mass per unit length of the pipe material are considered to constitute random fields in space, as in the authors' earlier publications (Anantha Ramu and Ganesan 1992, 1993, Ganesan, *et al.* 1992, 1993)

$$E(x)=\bar{E}[1+a(x)] \quad (3)$$

$$m(x)=\bar{m}[1+b(x)] \quad (4)$$

where $a(x)$ and $b(x)$ are two, independent, one-dimensional, univariate, homogeneous, real, zero-mean stochastic fields in space.

The solution of the above mentioned differential equation is assumed to be of the form,

$$y(x,\ t)=Y(x)\ T(t) \quad (5)$$

where it is implicit that the spectrum is discrete. $Y(x)$ and $T(t)$ are the space dependent normal modes and time dependent amplitudes respectively. Substitution of this solution into the differential equation and the boundary conditions yields a differential equation for any $Y_n(x)$ which can be solved by the standard perturbation procedure. The details of such a procedure as exactly as in the earlier publication (Anantha Ramu and Ganesan 1992) and hence they are not repeated here. It is sufficient to note that after substituting

$$G=\frac{L^2}{EI},\ \frac{\dot{T}_n(t)}{T_n}\ GL=\mu_n \text{ and } \lambda_n=\frac{mL^4\omega_n^2}{EI} \text{ and } \zeta=\frac{M}{\bar{m}},\ \tau=\frac{x}{L},\ \ddot{T}_n(t)=\omega_n^2 T_n(t) \text{ and } \frac{\partial}{\partial t}=(\cdot),$$

the following the perturbational expansions are employed for the output parameters:

$$\lambda_n=\lambda_{on}+\alpha\lambda_{1n}+\beta\lambda_{2n}+\cdots \quad (6)$$

$$\mu_n=\mu_{on}+\alpha\mu_{1n}+\beta\mu_{2n}+\cdots \quad (7)$$

and

$$Y_n(\tau)=Y_{on}(\tau)+\alpha Y_{1n}(\tau)+\beta Y_{2n}(\tau)+\cdots \quad (8)$$

Following the perturbation procedure outlined in (Anantha Ramu and Ganesan 1992), the generating solution is obtained by equating the terms associated with perturbing parameters of power zero, as

$$\begin{aligned} &Y''''_{on}(\tau)+MU^2GY''_{on}(\tau)-MGLg(1-\tau)Y''_{on}(\tau)-\bar{m}GLg(1-\tau)Y''_{on}(\tau)\\ &\quad+2MU\mu_{on}Y'_{on}(\tau)+\{M+\bar{m}\}GgLY'_{on}(\tau)=-(1+\zeta)\lambda_{on}Y_{on}(\tau) \end{aligned} \quad (9)$$

subjected to the boundary condition,

$$Y_{on}(0)=0$$

$$Y'_{on}(0)=0$$

$$\bar{E}IY''_{on}(\tau)|_{\tau=1}=0$$

$$\bar{E}IY'''_{on}(\tau)|_{\tau=1}=0 \tag{10}$$

In a similar manner, the differential equations for $Y_{in}(\tau)$, $i=1, 2$ can be obtained by considering terms of like powers in α and β. Thus the mode shapes $Y_{in}(\tau)$, $i=0, 1, 2$ are obtained. Further, the equations for obtaining λ_{1n} and λ_{2n} in terms of input parameter randomness can also be obtained (See Appendix I)

Using the generating solution, $Y_{on}(\tau)$ and λ_{on} and subsequently λ_{1n} and λ_{2n} can be obtained. Back substitution of these λ_{1n} and λ_{2n} into the differential equations for Y_{1n} and Y_{2n} will yield their solutions when solved for boundary conditions. Thus, complete solutions for vibration frequencies and mode shapes can be obtained for any sample set of values of $a_i(\tau)$ and $b_i(\tau)$ from the ensemble $a(\tau)$ and $b(\tau)$ respectively. But, our interest is to get the response moments which can yield the statistical information about the dynamic response parameters. To this end, expressions for λ_{on}, λ_{1n} and λ_{2n} are substituted in the perturbation expressions Eq. (6).

3. Eigensolution statistics

The mean value of any vibration frequency is obtained after some simplification, as

$$<\lambda_n>=\lambda_{on} \tag{11}$$

since the random fields $a(\tau)$ and $b(\tau)$ have zero means. So the vibration frequency obtained from the generating solution becomes the mean value of λ_n and it is independent of any random input.

The covariance between any two normalized vibration frequencies λ_i and λ_j is given by (using functions defined in Appendix II)

$$\begin{aligned}Cov(\lambda_i, \lambda_j)=&(1+\zeta)^{-2}F_1(Y_{oi}, Y_{oj})\left\{\int_0^1\int_0^1 R_{aa}(\tau_1-\tau_2)F_2(Y_{oi}, Y_{oj})d\tau_1 d\tau_2\right.\\&\int_0^1\int_0^1 R_{bb}(\tau_1-\tau_2)F_3(Y_{oi}, Y_{oj})d\tau_1 d\tau_2\\&+(\bar{m}GLg)^2\int_0^1\int_0^1(1-\tau_1)(1-\tau_2)R_{bb}(\tau_1-\tau_2)F_4(Y_{oi}, Y_{oj})d\tau_1 d\tau_2\\&+(\bar{m}GLg)^2\int_0^1\int_0^1 R_{bb}(\tau_1-\tau_2)F_5(Y_{oi}, Y_{oj})d\tau_1 d\tau_2+\cdots\end{aligned} \tag{12}$$

Expansion functions for the mode shapes that correspond to λ_i and λ_j are identified by the subscripts i and j.

The variance of any eigenvalue λ_n is thus given by setting $i=j=n$ in the above expression. If the correlation functions of the input stochastic fields are denoted by $\rho_{aa}(\tau_1-\tau_2)$ and $\rho_{bb}(\tau_1-\tau_2)$ respectively, so that $-1\leq\rho_{aa}(\tau_1-\tau_2)\leq+1$ and $-1\leq\rho_{bb}(\tau_1-\tau_2)\leq+1$, the above equation can be given in terms of normalized spectral density functions, normalized with respect to the respective variances. Thus, the variance of any eigenvalue is obtained to be

$$\begin{aligned}Var(\lambda_n)=&(1+\zeta)^{-2}H_1(Y_{on})\\&\left\{\sigma_a^2\int_0^1\int_0^1\rho_{aa}(\tau_1-\tau_2)F_2(Y_{on}, Y_{on})d\tau_1 d\tau_2+\sigma_b^2\int_0^1\int_0^1\rho_{bb}(\tau_1-\tau_2)F_3(Y_{on}, Y_{on})d\tau_1 d\tau_2\right.\end{aligned}$$

$$+(\bar{m}GLg)^2\sigma_b^2\int_0^1\int_0^1\rho_{bb}(\tau_1-\tau_2)(1-\tau_1)(1-\tau_2)F_4(Y_{on},\ Y_{on})d\tau_1 d\tau_2$$

$$+(\bar{m}GLg)^2\sigma_b^2\int_0^1\int_0^1\rho_{bb}(\tau_1-\tau_2)F_5(Y_{on},\ Y_{on})d\tau_1 d\tau_2+\cdots\cdots\Big\} \tag{13}$$

4. Bounds for eigenvalue statistics

Having established the covariances and variances in terms of normalized spectral density functions (or correlation functions), one can proceed to establish the bounds for covariances and variances. Here, the bounds for variances are established, for clarity.

It is now very clear that the variance of any eigenvalue is function of normalized spectral densities s_{aa} and s_{bb}, for a system with known variances of material properties E and m. If each of the stochastic fields describing the material property fluctuations has a perfect correlation regardless of the physical separation, i.e., $\rho_{aa}(\tau_1-\tau_2)=\rho_{bb}(\tau_1-\tau_2)=1$, the normalized spectral densities $s_{aa}(f)$ and $s_{bb}(f)$ are dirac delta functions and further $s_{aa}(f)$ and $s_{bb}(f)$ concentrate around the point $f=0$. In this case, the variance of any eigenvalue becomes as,

$$Var(\lambda_n)=\frac{1}{(1+\zeta)^2\Big\{\int_0^1 Y_{on}^2(\tau)d\tau\Big\}^2}$$

$$\Big\{\sigma_a^2\Big[\int_0^1\Big[Y''_{on}(\tau)\Big]^2 d\tau\Big]^2+\lambda_{on}^2\sigma_b^2\Big[\int_0^1 Y_{on}^2(\tau)d\tau\Big]^2$$

$$+(\bar{m}GLg)^2\sigma_b^2\Big[\int_0^1(1-\tau)Y''_{on}(\tau)Y_{on}(\tau)d\tau\Big]^2+(\bar{m}GLg)^2\sigma_b^2\Big[\int_0^1 Y'_{on}(\tau)Y_{on}(\tau)d\tau\Big]^2+\cdots\Big\} \tag{14}$$

The other extreme is to consider a perfectly random case, which is known to be a white noise. In that case, the correlation function is a spike function at the zero separation distance and the normalized spectral density function is a straight line parallel to the wave frequency axis. If the two fields are simultaneously considered to be the white noise fields, we have,

$$S_{aa}(f)=S_{ao}=\frac{1}{2fu}$$

$$S_{bb}(f)=S_{bo}=\frac{1}{2fu} \qquad \text{as } fu\rightarrow\infty$$

Two different cutoff frequencies can also be used, i.e. f_{u1} and f_{u2} for s_{aa} and s_{bb} respectively (The limiting case that $f_u\rightarrow 0$ is what we discussed previously). The corresponding correlation functions are, in the limit,

$$\rho_{aa}(\tau)=\delta(0);\ \ \rho_{bb}(\tau)=\delta(0)$$

Using these, the variances of eigenvalues can be found out easily. But, these correlation functions result in infinite total power in the wave frequency domain of the random fields. So, a realistic model is to account the spectral density function as a finite power whitenoise or band-limited whitenoise. In such cases, we have

$$\rho_{aa}(\tau_1-\tau_2)=2\cdot s_o\frac{\sin f_u(\tau_1-\tau_2)}{(\tau_1-\tau_2)} \quad \text{and} \quad \rho_{bb}(\tau_1-\tau_2)=2\cdot s_o\frac{\sin f_u(\tau_1-\tau_2)}{(\tau_1-\tau_2)} \tag{15}$$

i.e., sinc functions and not exponential functions. Now, it can be shown that, with $i=j=n$

$$\begin{aligned}Var(\dot{\lambda}_n)=&(1+\zeta)^{-2}H_1(Y_{on})\Big\{2\sigma_a^2 s_o\int_0^1\int_0^1\frac{\sin f_u(\tau_1-\tau_2)}{(\tau_1-\tau_2)}[Y''_{on}(\tau_1)]^2[Y''_{on}(\tau_2)]^2 d\tau_1 d\tau_2\\&+2\lambda_{on}^2\cdot\sigma_b^2\cdot s_o\int_0^1\int_0^1\frac{\sin f_u(\tau_1-\tau_2)}{(\tau_1-\tau_2)}Y_{on}^2(\tau_1)Y_{on}^2(\tau_2)d\tau_1 d\tau_2\\&+2(\bar{m}GLg)^2\sigma_b^2\cdot s_o\int_0^1\int_0^1\frac{\sin f_u(\tau_1-\tau_2)}{(\tau_1-\tau_2)}(1-\tau_1)(1-\tau_2)Y''_{on}(\tau_1)Y_{on}(\tau_1)\\&Y''_{on}(\tau_2)Y_{on}(\tau_2)d\tau_1 d\tau_2+2(\bar{m}GLg)^2\sigma_b^2\cdot s_o\int_0^1\int_0^1\frac{\sin f_u(\tau_1-\tau_2)}{(\tau_1-\tau_2)}Y'_{on}(\tau_1)Y_{on}(\tau_1)\\&Y'_{on}(\tau_2)Y_{on}(\tau_2)d\tau_1 d\tau_2+\cdots\cdots\Big\}\end{aligned} \tag{16}$$

However, exponential correlation with one parameter can also be assumed wherein the first order autoregressive models could be accommodated to evaluate the bounds. Now, it is very clear that the limiting case of this sinc correlation yields the lower bound for eigenvalue statistics, that is zero.

5. Statistics of critical loads

From the results obtained in the previous sections, one can get the statistical information about the vibration frequencies and mode shapes. To ascertain the stability limits of the system, using the more general kinetic method, the nature of the vibration frequencies is important. Depending upon the nature of the real and imaginary parts of the vibration frequencies, conclusions about the stability of the system can be drawn. For a stochastically parametered system, for a sample set of random parameters, a sample set of eigensolution can be obtained. A set of values of the system parameters obtained from individual realizations of the stochastic fields involved, thus constitutes a deterministic occurrence of the vibration frequencies. So, the conditions for the stable or unstable states of the system can be expressed exactly for a deterministic occurrence of vibration frequencies which in turn yields a deterministic occurrence of flow velocities. Since, the critical flow velocity itself is a random quantity when the system parameters are constituting an ensemble, only for a particular and individual set of realizations of critical flow velocities and system control parameters stability conclusions can be drawn.

To solve the stability problem of the system, the averaged eigenvalue equation is sought. For any set of individual realizations of system parameters, the equation giving the vibration frequency as a function of flow velocity can be obtained from the solution of generating differential equation and the boundary conditions for Y_{in}. This averaged equation yields averaged, complex eigenvalue $\bar{\omega}$, consisting of an averaged real part $\bar{\omega}_r$ and an averaged imaginary part $\bar{\omega}_{im}$. Here, it is assumed that $\bar{\omega}$ is that particular vibration frequency, which has got the maximum real part. Employing the kinetic method of stability investigation for the averaged system and further noting that any set of averaged parameters is an individual realization of the ensemble of the system parameters, we can write the following conclusions regarding the stability of the system:

For $\bar{\omega}_r<0$ system is stable;
For $\bar{\omega}_r=0$ system is critical;
For $\bar{\omega}_r>0$ and $\bar{\omega}_{im}\neq 0$, system is unstable and the type of instability is flutter.
For $\bar{\omega}_r>0$ and $\bar{\omega}_{im}=0$, system is unstable and the type of instability is divergence, i.e., Euler buckling.

The particular and deterministic flow velocity (which is the lowest) corresponding to the instability of the system is the averaged critical flow velocity. This is the mean value of the critical flow velocity which is a random variable.

To obtain the variance, covariance, etc., of the critical flow velocity, the eigenvalue equation is considered:

$$U_c=F_1(\omega)=F_2(\lambda)$$

If the individual and point statistics of vibration frequencies are known, it is possible to derive the statistical description of critical flow velocities either through explicit analytical relationships F_1 and F_2 or through Monte Carlo simulations. So, if the monotonic unique relationship between the vibration frequencies and critical flow velocity exists as F_1 or F_2, an unique inverse relationship can be obtained as,

$$\omega=F_3(U_c) \text{ and } \lambda=F_4(U_c)$$

where

$$F_3(\cdot)=F_1^{-1}(\cdot) \text{ and } F_4(\cdot)=F_2^{-1}(\cdot)$$

In such cases, closed form analytical relationship can be written for the probability density function of U_c, using the standard procedures of transformation of random variables, as

$$f_{U_c}(U)=f_\omega(\omega)\frac{d}{dU_c}F_3(U_c) \text{ and } f_{U_c}(U)=f_\lambda(\lambda)\frac{d}{dU_c}F_4(U_c)$$

In a similar manner, the distribution functions are obtained as,

$$F_{U_c}(U_1, U_2, \cdots U_n)=P[\{F_1(\omega_1)\leq U_1\}, \{F_1(\omega_2)\leq U_2\}, \cdots\cdots\{F_1(\omega_n)\leq U_n\}]=F_\omega(F_3(U_1), F_3(U_2)\cdots\cdots F_3(U_n))$$

and

$$F_{U_c}(U_1, U_2, \cdots, U_n)=P[\{F_2(\lambda_1)\leq U_1\}, \{F_2(\lambda_2)\leq U_2\}, \cdots\cdots\{F_2(\lambda_n)\leq U_n\}]=F_\lambda(F_4(U_1), F_4(U_2)\cdots\cdots F_4(U_n))$$

When the probability density functions or probability distribution functions are obtained from these equations, the n^{th} moment of critical flow velocities can readily be calculated. To obtain the bounds for critical flow velocities, the bounds for variances of eigenvalues are obtained as shown in the previous section. Those values are used as the input for the nonlinear transformation to get the bounds for critical flow velocity statistics.

6. Numerical example

A cantilever pipe of unit length which has a stochastically distributed Young's modulus and mass per unit length is being considered. Flow velocity of the fluid that flows through the pipe is deterministic. The theory for deterministic material properties and fluid flow properties is given in Gregory and Paidoussis (1966), Paidoussis and Issid (1974). To determine the conditions of stability for such systems, two methods have been suggested in Gregory and Paidoussis (1966). Method I is adopted for the deterministic case and for the averaged problem in the present case. So,

$$\beta=\frac{UM}{(M+m)U_j};\ u=\left(\frac{M}{EI}\right)^{1/2}(UU_j)^{1/2}L;\ \omega=\left(\frac{M+m}{EI}\right)^{1/2}\Omega L^2$$

The notations are in accordance with that of Gregory and Paidoussis (1966).

Five different correlation models are considered and these correlation models are the most commonly observed and efficient ones.

(1) The triangular correlation function:

$$\rho(\tau_1-\tau_2)=1-\frac{\tau_1-\tau_2}{a},\qquad |\tau_1-\tau_2|\le a$$
$$=0\qquad |\tau_1-\tau_2|\ge a$$

where a is a constant.

(2) The first-order autoregressive correlation function:

$$\rho(\tau_1-\tau_2)=\exp[-|\tau_1-\tau_2|/b],\ b=\text{constant}=f(\varepsilon).$$

where ε=correlation length, corresponding to the prescribed level of statistical dependence.

(3) Second-order autoregressive correlation model:

$$\rho(\tau_1-\tau_2)=\left[1+\frac{|\tau_1-\tau_2|}{c}\right]\exp[-|\tau_1-\tau_2|/c],\ c \text{ is a constant.}$$

(4) Gaussian correlation model:

$$\rho(\tau_1-\tau_2)=\exp[-(|\tau_1-\tau_2|/d)^2],\ d \text{ is a constant.}$$

(5) Finite power white noise field:

Table 1 Variances of λ_1 for $u_c=5.5$ and $\beta=0.2$, when E and m are random fields. $a=b=c=d=15$

Input variance x 10	Correlation models				
$\sigma_a^2=\sigma_b^2=\sigma^2$	Triangular	First-order AR	Second-order AR	Gaussian	Finite power white noise $f_u=10$
0.1	400.7474	400.9061	360.5261	400.7474	400.7474
0.2	801.4940	801.8114	720.4780	801.4940	801.4940
0.3	1202.2409	1202.7171	1077.7997	1202.2409	1202.2409
0.4	1602.9683	1603.6032	1436.5708	1602.9683	1602.9683
0.5	2003.7104	2004.5040	1795.6957	2003.7104	2003.7104
0.6	2404.4319	2405.3843	2154.4466	2404.4319	2404.4319
0.7	2805.1705	2806.2815	2513.4997	2805.1705	2805.1705
0.8	3205.9092	3207.1790	2872.5712	3205.9092	3205.9092
0.9	3606.6477	3608.0763	3231.3147	3606.6477	3606.6477
1.0	4007.3846	4008.9719	3590.1656	4007.3846	4007.3946

Table 2 Variances of λ_1 for $u_c=5.5$ and $\beta=0.2$, when E is a random field. $a=b=c=d=15$

Input variance x 10	Correlation models				
$\sigma_a^2=\sigma^2$	Triangular	First-order AR	Second-order AR	Gaussian	Finite power white noise $f_u=10$
0.1	397.5819	397.7402	357.6639	397.5819	397.5819
0.2	795.1661	795.4827	714.6736	795.1661	795.1661
0.3	1192.7491	1193.2240	1069.1285	1192.7491	1192.7491
0.4	1590.3321	1590.9653	1425.0166	1590.3321	1590.3321
0.5	1987.9152	1988.7066	1781.2708	1987.9152	1987.9152
0.6	2385.4983	2386.4480	2137.1540	2385.4983	2385.4983
0.7	2783.0505	2784.1585	2493.3270	2783.0505	2783.0505
0.8	3180.6292	3181.8955	2849.4912	3180.6299	3180.6290
0.9	3578.2077	3579.6323	3205.3526	3578.2077	3578.2077
1.0	3975.7863	3977.3692	3561.3219	3975.7863	3975.7863

Table 3 Variances of λ_1 for $u_c=5.5$ and $\beta=0.2$, when E and m are random fields. $a=b=c=d=10$

Input variance x 10	Correlation models				
$\sigma_a^2=\sigma_b^2=\sigma^2$	Triangular	First-order AR	Second-order AR	Gaussian	Finite power white noise $f_u=10$
0.1	396.1960	396.5507	349.4371	396.1960	396.1960
0.2	792.3908	793.1003	698.2487	792.3908	792.3908
0.3	1188.5861	1189.6504	1046.8082	1188.5861	1188.5861
0.4	1584.7630	1586.1819	1395.3901	1584.7630	1584.7630
0.5	1980.9538	1982.7275	1743.8411	1980.9538	1980.9538
0.6	2377.1234	2379.2519	2092.5713	2377.1234	2377.1234
0.7	2773.3105	2775.7938	2441.0245	2773.3105	2773.3105
0.8	3169.3467	3172.1847	2789.5625	3169.3467	3169.3467
0.9	3565.5149	3568.7077	3137.9751	3565.5149	3565.5149
1.0	3961.6816	3965.2291	3486.6390	3961.6816	3916.6816

$$\rho(\tau_1-\tau_2)=2\cdot S_o\,\frac{\sin f_u(\tau_1-\tau_2)}{(\tau_1-\tau_2)}$$

where S_o is the strength of white noise and f_u is the upper cut-off frequency of the power spectral density given by

$$S(f)=S_o\begin{cases}\dfrac{\sigma^2}{2f_u}, & |f|<f_u\\ 0, & |f|>f_u\end{cases}$$

First both E and m are treated to be random fields. The variances of normalized frequencies λ for different correlation structures of the input fields and input variances are given in Table 1. Then, only E is treated random and the results are given in Table 2. In these two cases, $a=b=c=d=15$ and $S_o=0.001$. Subsequently, for $a=b=c=d=10$ and $S_o=0.001$, the results are

Table 4 Variances of critical flow velocity when $\tilde{\omega}_c$ is normally distributed and $\bar{\omega}_c=45$

Input variance $\sigma^2_{\tilde{\omega}_c}$	Variance of u_c when $p=18$	Variance of u_c when $p=25$
1.0	0.1818	2.3071×10^{-2}
1.1	0.2380	3.0518×10^{-2}
1.2	0.3078	3.8940×10^{-2}
1.3	0.3939	4.8279×10^{-2}
1.4	0.4996	6.0486×10^{-2}
1.5	0.6306	7.4279×10^{-2}
1.6	0.7909	9.0515×10^{-2}
1.7	0.9908	0.1102
1.8	1.2437	0.1320
1.9	1.5747	0.1577

given in Table 3 when E and m are random. This study shows the effect of randomness in different system parameters and the type of correlation of the random fields, on the vibration frequency statistics.

To find out the variances of critical flow velocities, a Monte Carlo simulation technique is formulated using the frequency equation relating the normalized parameters u and ω with the exponents of the trial solution,

$$Y_o(\tau)=Ae^{i\alpha\tau}$$

where,

$$\bar{u}=\left(\frac{M}{EI}\right)^{1/2}(UU_j)^{1/2}L,$$

$$\bar{\omega}=\left(\frac{M+m}{\mathrm{EI}}\right)^{1/2}\Omega L^2$$

constant c in Eqs. (23) in Gregory and Paidoussis (1966) is taken to be zero. The variances of critical flow velocity U_c when $\omega_c=\bar{\omega}_c=(1+\omega_c)$ and ω_c is normally distributed with variance 1 are given in Table 4, for 200 simulations.

7. Conclusions

The foregoing has described an effective means of integrating the concepts of probability theory and structural dynamics to analyse coupled systems with uncertain parameters. A general method of analysis for stochastically parametered fluid conveying tubular cantilever pipes is demonstrated. Fluctuations of material properties, Young's modulus and mass density of the pipe material, are modeled through continuous independent one dimensional invariate real homogeneous spatial random fields. Complete eigenvalue statistics are obtained in terms of input parameter statistics through approximate closed-form relationships. Critical flow velocity statistics are also obtained. To enhance the practical use, bounds for eigenvalue statistics and critical velocity statistics are obtained. These avoid the difficulty of extracting the appropriate correlation functions of the input random fields from experimental data and so remains as a useful design input.

Numerical results are presented employing five different and commonly-used correlation models. Sensitivity of response moments to the distribution of correlation properties has been obtained through these numerical results. Such an information is critical while selecting appropriate spectral distributions to model the field data, for a fixed value of accuracy. A particular value of operational safety and design life dictates this value of accuracy through the sensitivity of response moments obtained earlier.

The general treatment permits direct simplifications to such cases as the vibration of the classical Euler column and stability of Beck's column, etc. Bounds and covariances are directly derived herein, unlike the limited scope of earlier works like in Collins and Thomson (1969), Shinozuka and Astill (1972) even as they treat simple conservative systems. So the present work also provides the complete covariance structure of both frequencies of oscillation as well as buckling loads of conservatively-loaded columns and beam-columns.

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Appendix I

$$\lambda_{1n}=\frac{\begin{aligned}\Big\{&-a(0)Y''_{on}(0)Y'_{on}(0)-a(1)Y''_{on}(1)Y_{on}(1)+\int_0^1 a(\tau)[Y''_{on}(\tau)\cdot Y_{on}(\tau)]''d\tau\\ &-2\int_0^1 a(\tau)[Y'''_{on}(\tau)\cdot Y_{on}(\tau)]'d\tau+\int_0^1 a(\tau)Y'''_{on}(\tau)\cdot Y_{on}(\tau)d\tau\Big\}\end{aligned}}{(1+\zeta)\int_0^1 Y^2_{on}(\tau)d\tau}$$

In a similar manner, we can get,

$$\lambda_{2n}=\frac{\left\{-\lambda_{on}\int_0^1 b(\tau)Y^2_{on}(\tau)d\tau+m\bar{G}Lg\int_0^1(1-\tau)b(\tau)\cdot Y''_{on}(\tau)\cdot Y_{on}(\tau)d\tau-\bar{m}GgL\int_0^1 b(\tau)Y'_{on}(\tau)\cdot Y_{on}(\tau)d\tau)\right\}}{(1+\zeta)\int_0^1 Y^2_{on}(\tau)d\tau}$$

Appendix II

$$F_1(Y_{oi},\ Y_{oj})=1/\{(\int_0^1 Y^2_{oi}(\tau)d\tau)\int_0^1 Y^2_{ij}(\tau)d\tau)\}$$

$$F_2(Y_{oi},\ Y_{oj})=[Y''_{oi}(\tau_1)]^2[Y''_{oj}(\tau_2)]^2$$

$$F_3(Y_{oi},\ Y_{oj})=[Y^2_{oi}(\tau_1)Y^2_{oj}(\tau_2)]\lambda_{oi}\lambda_{oj}$$

$$F_4(Y_{oi},\ Y_{oj})=Y''_{oi}(\tau_1)Y_{oi}(\tau_1)Y''_{oj}(\tau_2)Y_{oj}(\tau_2)$$

$$F_5(Y_{oi},\ Y_{oj})=Y'_{oi}(\tau_1)Y_{oi}(\tau_1)Y'_{oj}(\tau_2)Y_{oj}(\tau_2)$$

$$H_1(Y_{on})=\left\{\int_0^1 Y^2_{on}(\tau)d\tau\right\}^2$$