Structural Engineering and Mechanics, Vol. 29, No. 6 (2008) 659-671
DOI: http://dx.doi.org/10.12989/sem.2008.29.6.659

An absolute displacement approach for modeling of sliding structures

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(Received August 7, 2006, Accepted December 11, 2007)

Abstract. A procedure to analyse the space frame structure fixed at base as well as resting on sliding bearing using total or absolute displacement in dynamic equation is developed. In the present method, the effect of ground acceleration is not considered as equivalent force. Instead, the ground acceleration is considered as a known value in the acceleration vector at degree of freedom corresponding to base of the structure when the structure is in non-sliding phase. When the structure is in sliding phase, only a force equal to the maximum frictional resistance is applied at base. Also, in this method, the stiffness matrix, mass matrix and the damping matrix will not change when the structure enters from one phase to another. The results obtained from the present method using absolute displacement approach are compared with the results obtained from the analysis of structure using relative displacement approach. The applicability of the analysis is also demonstrated to obtain the response of the structure resting on sliding bearing with restoring force device.

Keywords: absolute displacement; relative displacement; sliding bearing; harmonic ground acceleration; El Cenrto earthquake; space frame structure.

1. Introduction

Earthquakes are one of the most destructive forces that nature unleashes on earth. They not only cause loss of life and property but also shake the morale of the people. In recent years, structural engineers are giving more and more importance to the design of structures to absorb the energy input from earth vibration. The integrity of a structure can be protected from the earthquake forces either through the concept of “resistance” or “isolation”. In designing the structure by resistance, it is assumed that the earthquake forces can be transmitted directly to the structure and each member of the structure is required to design to resist the forces that may be induced by the earthquakes. In the other approach of “isolation”, the transmission of earthquake induced forces to the structure is prevented using certain isolation devices. Rubber bearings are the most commonly used base isolation devices. However, in recent years, the sliding bearings in the form of rollers, graphite powder and dry sand with some coefficient of friction are found increased applications. These types of bearings are found to be very effective in reducing structural response. The most attractive features of the sliding bearings are its effectiveness for a wide range of frequency in puts. In this type of bearing, the force transmitted to the structure across the isolation interface can be limited by

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keeping the coefficient of friction as low as practical. However, this results in large sliding and residual displacements, which may be difficult to incorporate in structural design. The practical effectiveness of sliding bearings can be enhanced further by adding suitable restoring mechanism to reduce the displacements to manageable levels. Restoring force devices also help the structure to bring back to its original position. Restoring force devices are in the form of high-tension springs, laminated rubber bearings or friction pendulum systems. In friction pendulum system, the restoring force is by gravity. In this system, the sliding surface takes a concave spherical shape so that the sliding and restoring mechanisms are integrated in one unit.

The structure resting on sliding surface will have two phases namely non-sliding phase and sliding phase. Because of the sliding and non-sliding phases exist alternatively, the dynamic behaviour of sliding structure is highly non linear. Hence various types of models are proposed for the analysis of structures resting on sliding bearing. Westermo and Udwadia (1983) used the simplest model, a rigid block sliding on a bed to study the response of a sliding structure subjected to harmonic support motions. Mostaghel *et al.* (1983) have studied the effectiveness of sliding structure subjected to harmonic support motion. A mathematical model with two degrees of freedom, one degree of freedom for super structure and the other degree of freedom for sliding foundation was used. They also used separate equations for each phase, i.e., one equation for non-sliding phase and the other equation for sliding phase. Using the same type of model, Mostaghel and Tanbakuchi (1983) studied the response of the sliding structure subjected to El Centro (1940) earthquake support motion. Qamaruddin *et al.* (1986) have carried out numerical and experimental studies to study the seismic response of brick and masonry buildings to evaluate the effectiveness of sliding bearing as a isolation technique. In order to overcome the difficulties of using separate equations for sliding and non-sliding phases, Yang *et al.* (1990) proposed a model using only one equation for both sliding as well as non-sliding phases. In this approach, the sliding bearing was modeled using a fictitious spring attached to the foundation floor. The spring was assumed to be bilinear with a very large stiffness in the non-sliding phase and zero stiffness in the sliding phase. Zayas *et al.* (1990) analysed the two degree freedom structure resting on sliding bearing with friction pendulum system assuming different equations for sliding and non-sliding phases. Jangid and Londhe (1998), Jangid (2000) and Bhasker and Jangid (2001) studied the response of a structure with multi degree of freedom resting on sliding bearing assuming different equations for non-sliding and sliding phases. Pranesh and Ravi (2000) studied the effectiveness of a variable frequency pendulum isolator. Two degree freedom sliding structure with separate equations for sliding and non-sliding phases is used for the analysis. Vafai *et al.* (2001) analysed the multi degree of freedom structure on sliding supports by replacing a fictitious spring in the model of Yang *et al.* (1990) by a link with a rigid-perfectly plastic material. Shakib and Fuladgar (2003) analysed a three dimensional single story structure resting on sliding support considering all the three components of El Centro (1940) earthquake. Calio *et al.* (2003) investigated the response of a multistory sliding structure with friction pendulum system. Krishnamoorthy and Saumil (2005) analysed the space frame structure resting on sliding bearing with six degree of freedom at each node using a fictitious spring to model the sliding surface as proposed by Yang *et al.* (1990).

In all the above models adopted for the analysis of structures resting on sliding bearings, the dynamic equations are expressed in terms of relative displacement with respect to ground or in terms of relative displacement with respect to the base mass. Ie. the displacement at each floor of the structure is expressed as the difference between the displacement at that floor and the displacement at the ground surface or at base of structure. The acceleration and velocity are also expressed in terms of relative acceleration and relative velocity. When the displacements are expressed in terms of relative displacement with respect to base mass, then for non-sliding phase, since the structure is moving along with ground and the relative displacement at base of the structure is equal to zero, the dynamic equations are formulated using only the equations corresponding to the super structure neglecting the equations of base mass. When the structure is in sliding phase, the equations corresponding to both super structure as well as base mass are considered. Thus, two separate sets of equations, one corresponding to sliding phase and the other corresponding to non-sliding phase are used in this approach. When the displacements are expressed in terms of relative displacement with respect to ground, the sliding bearing is modeled as fictitious spring with high (or infinite) stiffness in non-sliding phase and zero stiffness in sliding phase. Hence, the dynamic equation will not change when the structure passes from one phase to another where as the stiffness matrix and damping matrix changes when the structure enters from one phase to another in this approach. In both the above approaches, the effect of ground acceleration due to earthquake is considered as an equivalent force equal to the product of the mass and ground acceleration. This force will be acting at each floor of the structure when the structure is in non-sliding phase. When the structure is in sliding phase, in addition to this force, a force equal to the maximum frictional resistance acting opposite to the direction of sliding is applied at base. The response quantities obtained from the analysis are the relative displacements, relative velocity and relative acceleration. The absolute or total acceleration may be obtained adding the relative acceleration to the ground acceleration where as it is not possible to obtain the absolute velocity and absolute displacement.

In the present method, a different approach to model the structure resting on sliding bearing is used. In this approach, the displacement is expressed in terms of total (or absolute) displacement instead of relative displacement. The effect of ground acceleration is not considered in terms of equivalent forces as in the case of relative displacement approach. Instead the value of ground acceleration is considered as a known value in acceleration vector at degree of freedom corresponding to base of the structure when the structure is in non-sliding phase. Ie., when the structure is in non-sliding phase, only the acceleration is acting at base and no forces are acting on the structure. During sliding phase, since there is no contact between the base of the structure and ground, the ground acceleration will not transmit to the structure and hence acceleration will not be acting at the base of the structure but a force equal to the maximum frictional resistance will be acting at the base of the structure. This force is acting opposite to the direction of sliding. Thus, in the case of non-sliding phase, the acceleration equal to the ground acceleration will be acting at base whereas in the case of sliding phase only the force equal to the maximum frictional resistance is acting at base. The stiffness matrix, mass matrix and damping matrix will not change when the structure passes from one phase to another. Hence, it is not necessary to compute the stiffness matrix, damping matrix and mass matrix when the structure passes from one phase to another. The response quantities obtained from the analysis are the absolute displacement, absolute acceleration and absolute velocity. The displacement of the ground, ground acceleration and ground velocity can also be obtained. The relative acceleration, relative velocity and relative displacement can also be obtained knowing the ground acceleration, ground velocity and ground displacement.

2. Analytical modeling

The space frame structure is divided into number of elements consisting of beams and columns connected at nodes. Each element is modeled using two noded frame element with six degrees of freedom at each node i.e., three translations along X, Y and Z axes and three rotations about these axes. For each element, the stiffness matrix $[k]$, consistent mass matrix $[m]$ in local direction and transformation matrix $[T]$ is obtained. The mass matrix and stiffness matrix from local direction are transformed to global direction as proposed by Paz (1991). The overall mass matrix $[M]$ and stiffness matrix $[K]$ for the whole structure is then obtained by assembling mass matrix $[m]$ and stiffness matrix $[k]$ of each element. The overall dynamic equations of equilibrium for the entire structure can be expressed in matrix notations as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \quad (1)$$

where $[M]$, $[C]$ and $[K]$ are the overall mass, damping, and stiffness matrices for the structure including base mass. The damping of the superstructure is assumed as Rayleigh type and the damping matrix $[C]$ is determined using the equation $[C] = \alpha[M] + \beta[K]$ where α and β are the Rayleigh constants. These constants can be determined if the damping ratio for each mode are known. For sliding bearing, sliding friction is assumed to be of the Coulomb type and the coefficients of static and dynamic friction are taken to be the same. $\{\ddot{u}\}, \{\dot{u}\}$ and $\{u\}$ are the total or absolute acceleration, absolute velocity and absolute displacement vectors at nodes and $\{F(t)\}$ is the nodal load vector. $\{u\} = \{u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}, u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}, \ldots.. u_n, v_n, w_n, \theta_{xn}, \theta_{yn}, \theta_{zn}\}$ where n is the number of nodes.

2.1 Analysis of structure fixed at base

The methods adopted for the analysis of structure fixed at base were in terms of relative displacement, relative velocity and relative acceleration instead of absolute displacement, absolute acceleration and absolute velocity as in the present method. Also, the effect of ground acceleration due to earthquake was considered as an equivalent force which is equal to the product of mass and ground acceleration. Hence, the force vector in Eq. (1) is equal to $\{F(t)\} = -[M]\{I\}\ddot{u}_g$. Since the relative displacement at base is equal to zero when the structure is fixed at base, the equations were formulated only with respect to super structure neglecting the equations corresponding to the base mass. The response quantities obtained from the analysis are the relative displacement, relative acceleration and relative velocity.

However, in the present method, the equations are formulated in terms of absolute displacement, absolute velocity and absolute acceleration. Also, when the structure is fixed at base, the structure moves with ground and hence the total acceleration, total velocity and total displacement at base of the structure is equal to the ground acceleration, ground velocity and ground displacement. Hence, acceleration equal to the ground acceleration is applied at base of the structure instead of considering the effect of ground acceleration as an equivalent force. Thus, the acceleration, $\ddot{u}_b$, at base of the structure is known and is equal to the ground acceleration, $\ddot{u}_g$, and the force vector $\{F(t)\} = 0.0$. The vector $\{\ddot{u}\}$ in Eq. (1) consists of known values of acceleration at degrees of freedom corresponding to the base of the structure and unknown values of acceleration corresponding to the degrees of freedom of the super structure. Accelerations at the degrees of freedom other than the degrees of freedom at base, displacements and velocity at all the degrees of freedom including the degrees of freedom corresponding to base can be determined from Eq. (1). The response quantities obtained from the analysis are the absolute acceleration, absolute displacement and absolute velocity instead of relative acceleration, relative velocity and relative displacement. Hence, the present method can be used in situations where absolute displacement and absolute velocity are needed rather than relative displacement and relative velocity.

2.2 Analysis of structure resting on sliding bearing

When the structure is resting on sliding type of bearing with a coefficient of friction equal to, μ, then the mobilized frictional force, F_x, at base will be resisted by the frictional resistance, F_s, which acts against the direction of mobilized frictional force. When the mobilized frictional force, F_x, at base is less than the frictional resistance, F_s, (i.e., $|F_x| < F_s$) the structure will not have relative movement at base and this phase of structure is known as non – sliding phase. However, when the mobilized frictional force, F_x, is equal to or more than the frictional resistance, F_s (i.e., $|F_x| \geq F_s$) the structure starts sliding at base and this phase of the structure is known as sliding phase. When the structure is in sliding phase and whenever reverses its direction of motion (when the relative velocity at base is equal to zero) then the structure may again stop its movement at base and may enter the non-sliding phase or may slide in opposite direction.

Two different models were proposed by number of researchers to model the structure resting on sliding bearing. Ie. in non-sliding phase, since the relative displacement of the base of the structure is equal to zero, the equations were formulated in terms of only the relative displacement of the super structure as explained above for the analysis of structure fixed at base. When the structure is in sliding phase, the equations were formulated considering the relative displacement of the base mass and super structure. In another approach as adopted by Yang *et al.* (1990), Vafai *et al.* (2001) and Krishnamoorthy and Saumil (2005) the sliding bearing is modeled as a spring with very high value of stiffness or infinite stiffness during non-sliding phase and zero value of stiffness in sliding phase.

The following procedure is adopted in the present analysis to model the non-sliding and sliding phases in the case of structure resting on sliding bearing.

2.2.1 Non-sliding phase

In the non sliding phase, the frictional resistance, F_s, between the base of the structure and sliding surface is greater than the mobilized frictional force, F_x, of the system and the structure moves with the ground. Hence, the structure during this phase can be modeled as the structure fixed at base and hence the method adopted for the analysis of structure fixed at base as already explained is used for this phase. Ie. the acceleration, $\ddot{u}_b$, of the base is equal to the ground acceleration, $\ddot{u}_g$, and the force vector $\{F(t)\} = 0.0$. Accelerations at the degrees of freedom other than the degrees of freedom at base, displacements and velocity at all the degrees of freedom including the degrees of freedom corresponding to base is determined from Eq. (1) as already explained.

2.2.2 Sliding phase

When the mobilized frictional force, F_x, over comes the frictional resistance, F_s, between the base of the structure and sliding surface, the system enters the sliding phase and starts sliding. The mobilized frictional force, F_x, under the base of the structure is equal to, F_s, and remains constant during this phase. Thus a force equal to the frictional resistance is acting at the base during this phase. Since there is no contact between the base of the structure and ground, the ground acceleration is not transmitted to the structure and hence the acceleration at base of the structure is not equal to the ground acceleration. The acceleration vector $\{\ddot{u}\}$ in this phase is not known at all degrees of freedom including the degrees of freedom corresponding to the base of the structure. As in the case of non-sliding phase, the super structure is not subjected to any force. However, a force equal to the frictional resistance is acting at the base of the structure. This force acts opposite to the direction of sliding at base. Hence the force vector in Eq. (1) is equal to

$$\{F(t)\} = -\{F_{x\max}\}\operatorname{sgn}(\dot{u}_{rb})$$

where, $\{F_{x\max}\}$ is the vector with zeros at all locations except those corresponding to the horizontal degrees of freedom at base of structure. At these degrees of freedom, the vector $\{F_{x\max}\}$ will have values equal to F_s. $\operatorname{sgn}(\dot{u}_{rb})$ is the sign of the relative velocity at base and its value is equal to +1 or −1 depending on the direction of sliding.

Thus during sliding phase, only the force at base of the structure is acting and acceleration, displacement and velocity at each degree of freedom including those corresponding to the base of the structure are obtained from Eq. (1). It is to be noted that the Eq. (1) is used to obtain the displacement, acceleration and velocity for the sliding as well as non-sliding phase without changing the mass matrix, stiffness matrix and damping matrix.

The velocity and displacement of the ground can also be obtained using the time history of the ground acceleration. The relative displacement, relative velocity and relative acceleration at base of the structure can then be obtained using the displacement, velocity and acceleration of ground and absolute displacement, absolute velocity and absolute acceleration of base. The relative velocity is required to check whether the structure enters the non-sliding phase or not when it is in sliding phase.

2.3 Solution of the dynamic equation

The dynamic equations in the sliding phase as well as in non-sliding phase are solved in the incremental form using Newmark's method. In this method, knowing the displacement, velocity and acceleration at time t, the displacement, velocity and acceleration at time $t + \Delta t$ can be obtained. Owing to its unconditional stability, the constant average acceleration scheme (with $\beta = 1/4$ and $\gamma = 1/2$) is adopted. A time step Δt equal to 4×10^{-5} seconds is used in the analysis.

3. Numerical examples

In this section, three examples are presented. In example 1, the results obtained from the present analysis are compared with the results presented by Vafai *et al*. (2001). In example 2, the response of the four story space frame structure resting on sliding bearing is obtained using the present analysis. In example 3, the applicability of the present analysis is demonstrated to obtain the response of a space frame structure resting on sliding bearing with restoring force device.

3.1 Comparison of the present analysis with the results presented by Vafai et al. (2001)

Fig. 1(a) shows a four story plane frame structure resting on sliding bearing considered by Vafai *et al*. (2001). The response of the structure was obtained by Vafai *et al*. (2001) considering only one displacement (horizontal) at each floor. The sliding bearing was modeled as a fictitious rigid link with infinite stiffness during non-sliding phase and zero stiffness during sliding phase. To compare the results obtained from present analysis with the results presented by Vafai *et al*. (2001), the same frame is analysed considering one displacement at each floor. Fig. 2(a) shows the absolute

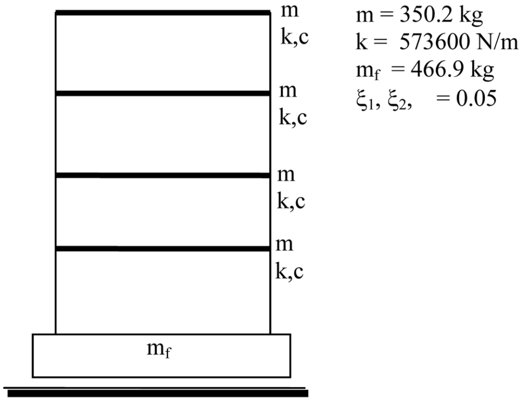

Fig. 1(a) Four story plane frame structure

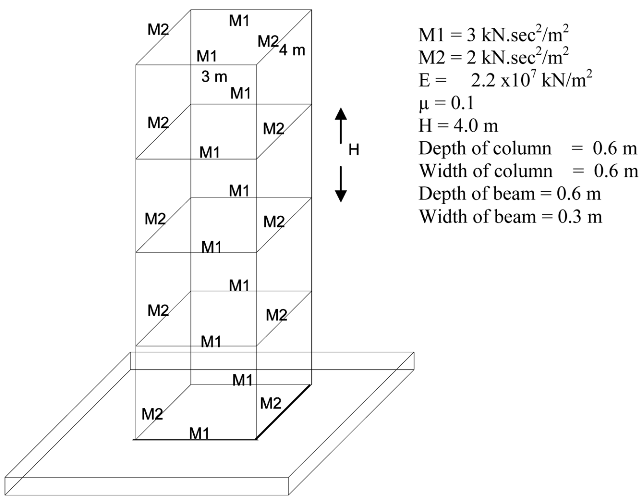

Fig. 1(b) Four story space frame structure

acceleration and base shear obtained from the present analysis and reported by Vafai *et al*. (2001) when the structure is subjected to a harmonic ground acceleration of intensity $a_o\sin(\omega t)$ with a_o = 0.5 g and ω = 10.472 rad/sec when the structure is fixed at base. It can be observed from Fig. 2(a) that the absolute acceleration and base shear obtained from the present analysis agree very well with the absolute acceleration and base shear presented by Vafai *et al*. (2001). The response obtained from the present analysis and reported by Vafai *et al*. (2001) when the structure is resting on sliding bearing are shown in Fig. 2(b). The coefficient of friction of base material μ is equal to 0.1. As observed from the figure the absolute acceleration and base shear obtained from the present analysis agree well with the absolute acceleration and base shear reported by Vafai *et al*. (2001). However, the sliding displacement (relative displacement of the structure at base with ground) obtained from the present analysis is slightly more than the sliding displacement reported by Vafai *et al*. (2001). This may be due to the fact that the sliding displacement is quite sensitive to the initial condition of sliding and non-sliding phases of the system.

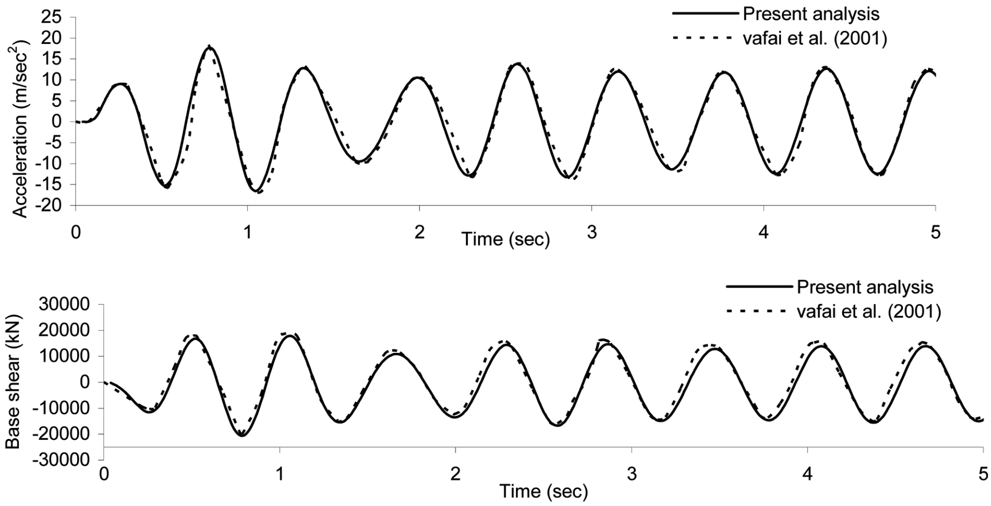

Fig. 2(a) Response of a four storey plane frame structure fixed at base

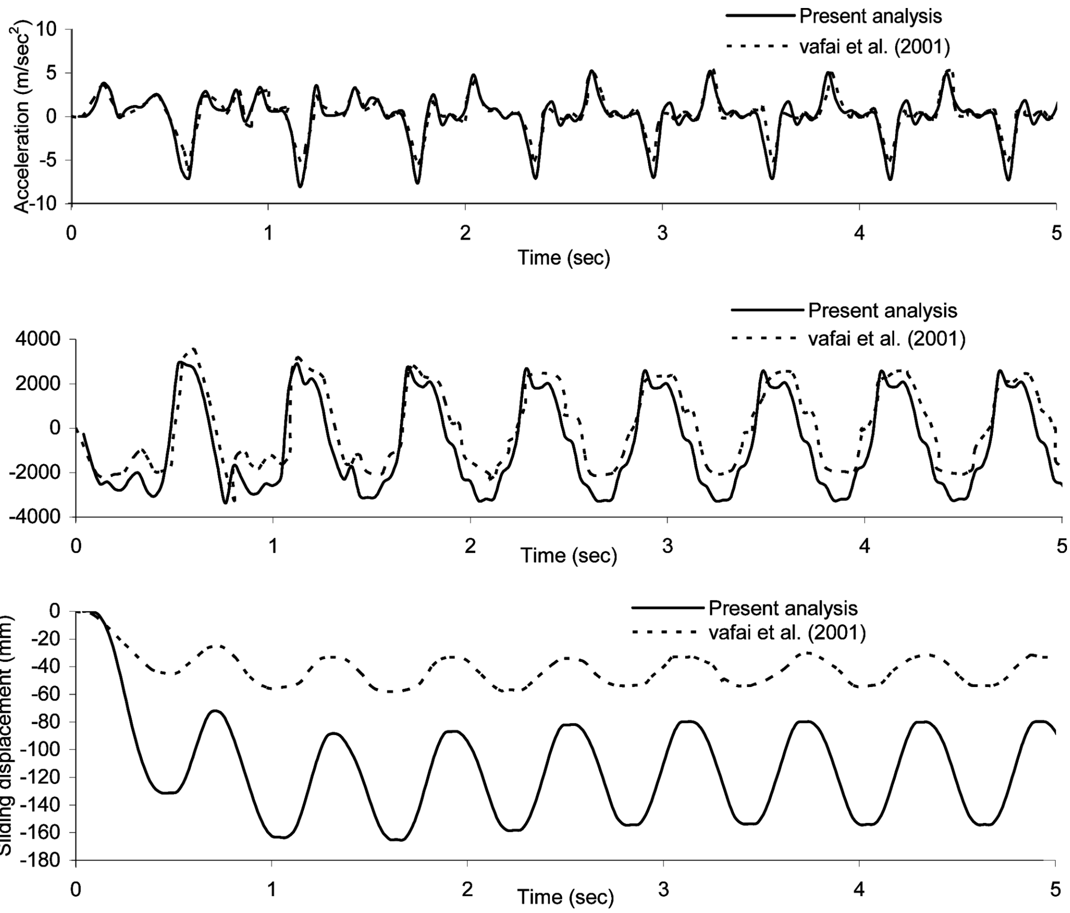

Fig. 2(b) Response of a four storey plane frame structure isolated at base

3.2 Analysis of four story space frame structure resting on sliding bearing

Fig. 1(b) shows a four story space frame structure resting on sliding bearing considered for the analysis. The geometric and material properties of the structure are shown in Fig. 1(b). The natural period of the structure is equal to 0.5 sec. The damping ratio of the structure considered for the analysis is equal to 5%. For this damping ratio, the values of Rayleigh constants α and β are found

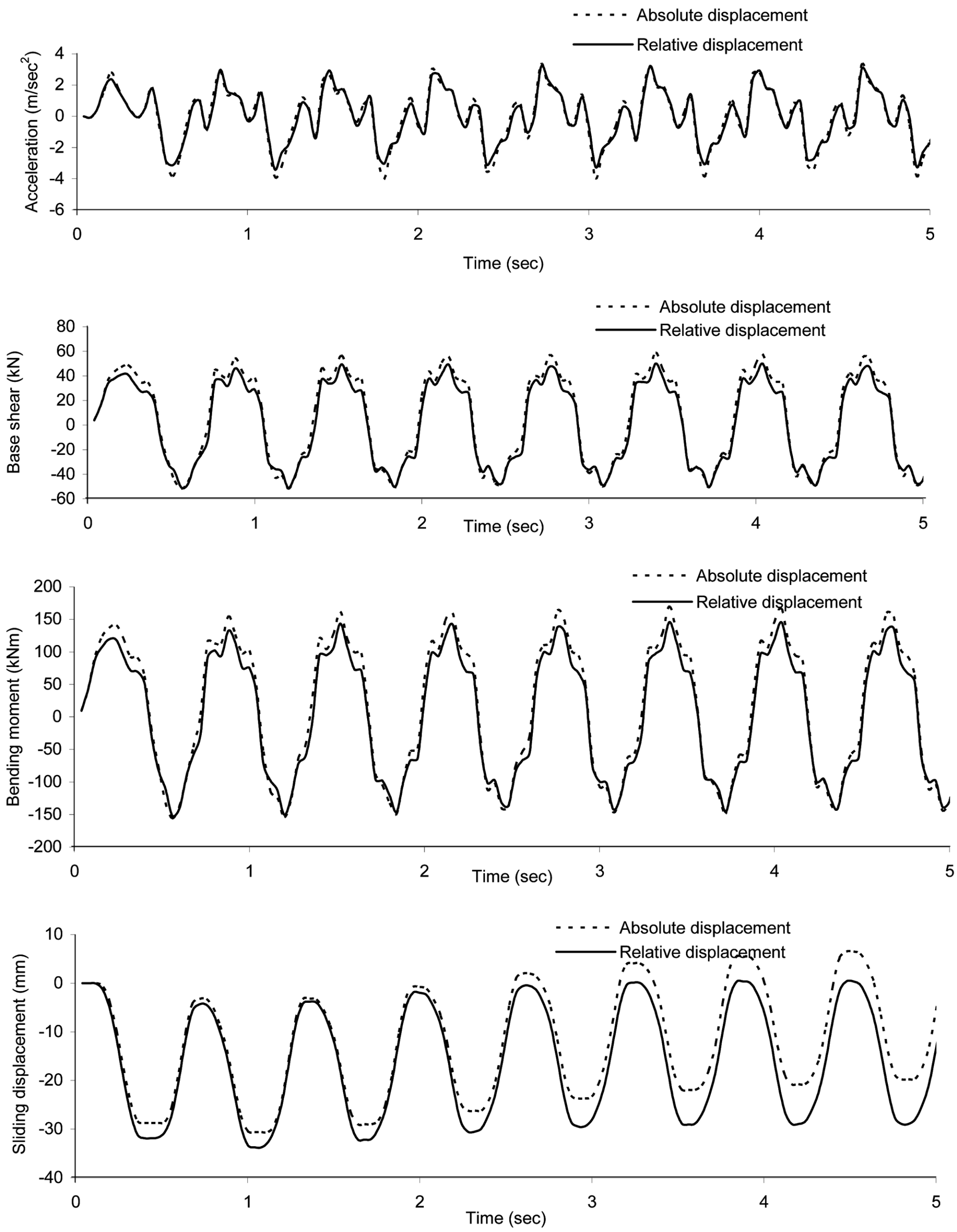

Fig. 3 Response of a four storey space frame structure subjected to harmonic ground acceleration

to be equal to 0.64 and 0.0039 respectively. The response of the structure is obtained when it is subjected to the harmonic ground acceleration of intensity 0.2$g$sin(ωt) with ω = 11 rad/sec and to El Centro (1940) earth quake ground acceleration.

Fig. 3 shows the response of the structure obtained from the present analysis when harmonic ground acceleration is acting at base. The response of the structure obtained from the present analysis when the structure is subjected to El Centro (1940) earthquake is shown in Fig. 4. The response obtained from the analysis based on relative displacement approach is also shown in Fig. 3

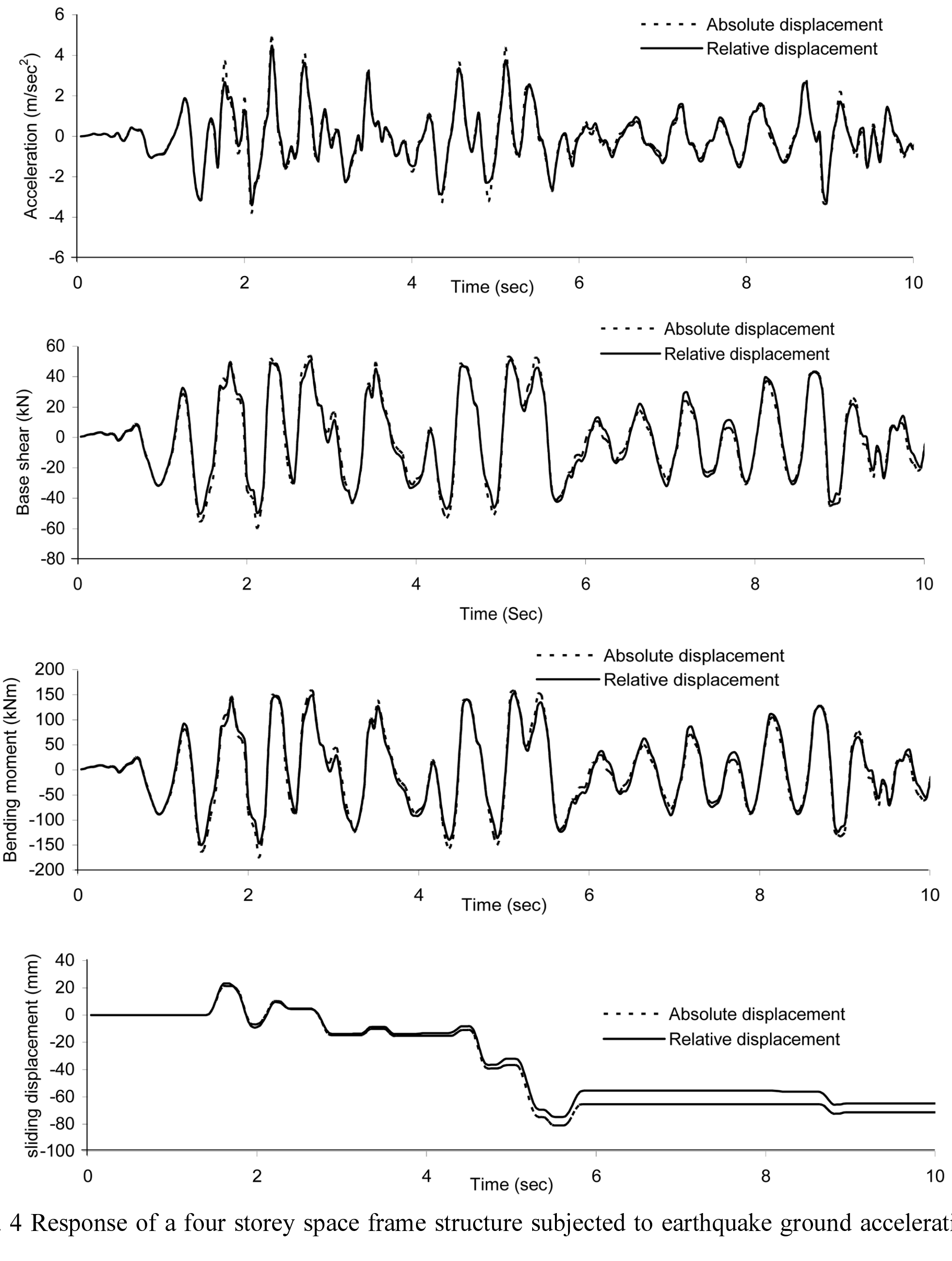

Fig. 4 Response of a four storey space frame structure subjected to earthquake ground acceleration

and Fig. 4. The method proposed by Yang *et al*. (1990) considering the sliding bearing as a fictitious spring is adopted to model the bearing in this method. It can be observed from the figure that the response obtained from the present analysis using absolute displacement agree well with the response obtained from the analysis using relative displacement for both cases of harmonic ground acceleration and earthquake ground acceleration.

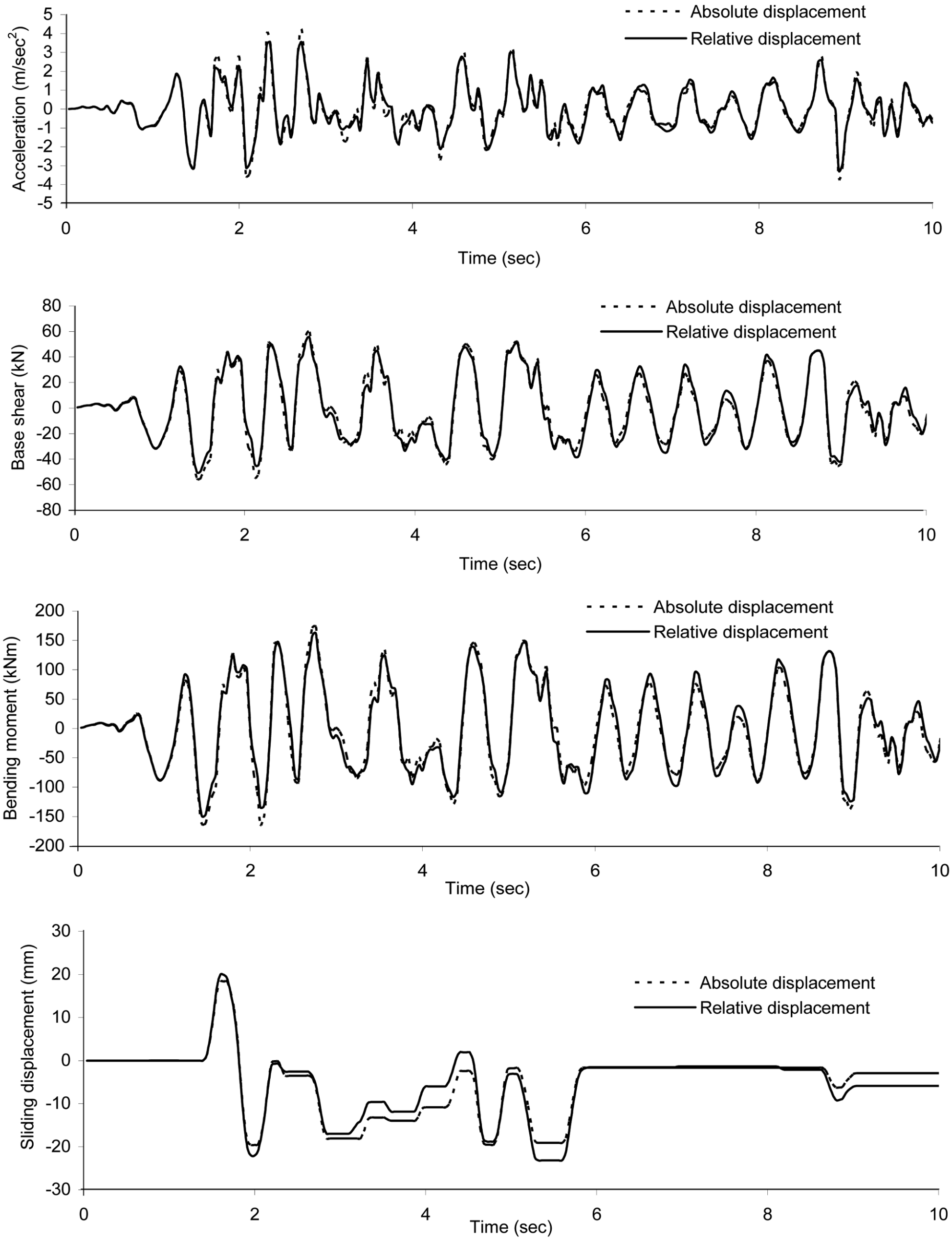

Fig. 5 Response of a four storey space frame structure with restoring force device subjected to earthquake ground acceleration

3.3 Analysis of four story space frame structure resting on sliding bearing with restoring force device

The applicability of the present method is also demonstrated when the structure is resting on sliding bearing with restoring force device. The restoring force device in the sliding bearing is used to reduce the sliding displacement and to restore the structure (to bring the structure to its original position at the end of earthquake). The restoring force devices may be in the form of laminated rubber bearings, high tension springs or friction pendulum system which restores the structure due to gravity. The method adopted for the analysis of structure resting on sliding bearing with restoring force device is same as the analysis of structure resting on sliding bearing without restoring force device when the structure is in non-sliding phase where as during sliding phase the force vector $\{F(t)\}$ is modified to consider the force in the restoring force device and it is equal to

$$\{F(t)\} = -\{F_{\max}\}\mathrm{sgn}(\dot{u}_{rb}) - \{F_r\}$$

$\{F_r\}$ is the vector with zeros at all locations except those corresponding to the horizontal degrees of freedom at base of structure. At these degrees of freedom the value of Fr is equal to the force in restoring force device. This force is equal to the product of the stiffness of restoring force device and relative displacement at base of the structure.

The structure shown in Fig. 1(b) with a restoring force device of stiffness 745 kN/m (isolation period, T_b = 1.5 rad/sec) is analysed when it is subjected to a ground acceleration due to El Centro (1940) earthquake. The response of the structure obtained from the present analysis using absolute displacement and obtained from the analysis using relative displacement is shown in Fig. 5. The response obtained from the present analysis using absolute displacement agrees closely with the response obtained from the analysis using relative displacement. It can also be observed from the Fig. 4 and Fig. 5 that the sliding displacement of the structure at the end of earthquake is almost equal to zero when the structure is resting on sliding bearing with restoring force device where as it is equal to about 60 mm when the structure is resting on sliding bearing with out restoring force device. i.e., the structure resting on sliding bearing with restoring force device comes to its original position where as the structure resting on sliding bearing without restoring force device shifts to new position after the earthquake.

4. Conclusions

The structure fixed at base as well as isolated at base is analysed. The dynamic equations are formulated in terms of absolute displacement instead of relative displacement. The response of the structure obtained from the present analysis using absolute displacement is compared with the response of the structure obtained from the analysis using relative displacement. Based on the analysis it can be concluded that the results obtained from the present analysis using the approach based on absolute displacement agree well with the results obtained from the analysis using the approach based on relative displacement. Hence, the proposed method of analysis can be used to analyse the structure subjected to ground acceleration due to earthquake. Also, the method can be used to obtain the absolute displacement, absolute acceleration and absolute velocity instead of relative displacement, relative acceleration and relative velocity. The mass matrix, stiffness matrix and damping matrix will not change when the structure passes from one phase to another. Hence, the method is simple as compared with other methods since it is not necessary to compute the mass matrix, stiffness matrix and damping matrix at each time interval.

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