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Discrete singular convolution for buckling analyses of plates and columns

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Abstract. In the present study, the discrete singular convolution (DSC) method is developed for buckling analysis of columns and thin plates having different geometries. Regularized Shannon's delta (RSD) kernel is selected as singular convolution to illustrate the present algorithm. In the proposed approach, the derivatives in both the governing equations and the boundary conditions are discretized by the method of DSC. The results obtained by DSC method were compared with those obtained by the other numerical and analytical methods.

Keywords: buckling; plates; discrete singular convolution; columns; circular plate; skew plate.

1. Introduction

Plates of arbitrary shape have been widely used in civil, aerospace, mechanical engineering and marine industries. Knowledge of the buckling characteristics of these structures is important. The buckling analysis of beams and plates may be either analytical or numerical. The analytical or rigorous approach consists of methods for seeking direct solutions to the governing differential equations of plates. It is well known that the analytical solution of problems can be obtained for only a certain simple cases. Generally, analytical solution cannot be found. Consequently, approximate numerical methods are the only alternative that can be employed. Recently, the method of discrete singular convolution (DSC) proposed by Wei (2001) has been increasingly applied to solve many engineering and sciences problems. As stated by Wei (2001a) singular convolutions (SC) are a special class of mathematical transformations, which appear in many science and engineering problems, such as the Hilbert, Abel and Radon transforms. In fact, these transforms are essential to many practical applications, such as computational electromagnetic, signal and image processing, pattern recognition, topography, molecular potential surface generation and dynamic

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simulation. It was stated that by Wei (2001, 2001a) DSC not only provides a rigorous justification for a number of informal manipulations in physical science and engineering, but also opens a new area of mathematics, which in turn gives impetus in many other mathematical disciplines, such as operator calculus, differential equations, functional analysis, harmonic analysis and transformation theory (Zhao and Wei 2002).

This paper deals with the application of the DSC method for the buckling analysis of columns, rectangular, skew, and circular plates with clamped and simply supported boundary conditions. To the author knowledge, it is the first time the DSC method has been successfully applied to plate problems for the analysis of buckling. Results are compared with existing solutions available from other analytical and numerical methods. In the present paper, details of the DSC method are not given; interested readers may refer to the works of Wei *et al.* (2002, 2002a), Zhao *et al.* (2002, 2005), and Civalek (2007, 2007a, 2007b). In the present study, a new computational algorithm, the discrete singular convolution (DSC), is introduced for solving the buckling problems of columns and plates. Plates of different shapes such as rectangular, circular, square, and skew subjected to different boundary conditions are selected to demonstrate the accuracy of the method.

2. Discrete singular convolution (DSC)

The method of discrete singular convolution (DSC) is an effective and simple approach for the numerical verification of singular convolutions, which occur commonly in mathematical physics and engineering. The discrete singular convolution method has been extensively used in scientific computations in past ten years. For more details of the mathematical background and application of the DSC method in solving problems in engineering, the readers may refer to some recently published reference (Wei 2000, Wei *et al.* 2001, Zhao *et al.* 2002). The mathematical foundation of the DSC algorithm is the theory of distributions and wavelet analysis. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. Following notations given by Wei (2001), a singular convolution can be defined by Wei (2001)

$$F(t) = (T*\eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx \tag{1}$$

where $T(t-x)$ is a singular kernel. The mathematical property or requirement of $f(x)$ is determined by the approximate kernel T_α. Recently, the use of some new kernels and regularizer such as delta regularizer (Wei *et al.* 2002) was proposed to solve applied mechanics problem. The Shannon's kernel is regularized as (Zhao *et al.* 2005)

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)}\exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right];\ \sigma > 0 \tag{2}$$

where Δ is the grid spacing. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (2) is practically and has an essentially compact support for numerical interpolation. Eq. (2) can also be used to provide discrete approximations to the singular convolution kernels of the delta type (Zhao *et al.* 2005)

$$f^{(n)}x \approx \sum_{k=-M}^{M} \delta_\Delta(x-x_k)f(x_k) \tag{3}$$

where $\delta_\Delta(x-x_k) = \Delta\delta_\alpha(x-x_k)$ and superscript (n) denotes the nth-order derivative, and $2M+1$ is the computational bandwidth which is centred around x and is usually smaller than the whole computational domain. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x-x_M, x+x_M]$. This can be expressed as (Wei 2001)

$$\left.\frac{d^n f(x)}{dx^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta^{(n)}_{\Delta,\sigma}(x_i-x_k) f(x_k)\,; \quad (n = 0, 1, 2, \ldots) \tag{4}$$

where superscript n denotes the nth-order derivative with respect to x.

3. Buckling of plates and columns

3.1 Buckling of linear elastic columns

The non-dimensional governing differential equation for buckling behavior of an elastic column is given by

$$EI\frac{d^4W}{dX^4} + PL^2\frac{d^2W}{dX^2} = 0 \tag{5}$$

in which, $X = x/L$, $W = w/L$ and $\lambda = PL^2/EI$. Eq. (5) can be given by applying the DSC as

$$\sum_{k=-M}^{M} \delta^{(4)}_{\Delta,\sigma}(k\Delta x) W_{k,j} + \lambda \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x) W_{k,j} = 0. \quad \text{for} \quad j = 0, 1, \ldots, N \tag{6}$$

Numerical applications have been done for a linearly elastic beam under three different boundary conditions, namely simply supported-simply supported (S-S), clamped-simply supported (C-S), clamped-clamped (C-C). Following, DSC form of clamped and simply supported boundary conditions are given.

$$\text{For clamped supported: } W_{1,j} = 0 \text{ and } \sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta x) W_{1,N} = 0 \tag{7a, 7b}$$

$$\text{For simply supported: } W_{1,j} = 0 \text{ and } \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x) W_{1,N} = 0 \tag{8a, 8b}$$

Eq. (6) can be rewritten as,

$$\sum_{k=-M}^{M} \delta^{(4)}_{\Delta,\sigma}(k\Delta x) W_{k,j} + \lambda \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x) W_{k,j} = 0 \text{ for } j = 3, 4, \ldots, (N-2) \tag{9}$$

Wei *et al*. (2002, 2002a) and Zhao *et al*. (2002) proposed a practical method to incorporate the boundary conditions. More recently, Zhao *et al*. (2005) applied the iteratively matched boundary method to impose the free boundary conditions for solid mechanic problem. In this paper, details of imposing of boundary conditions in DSC method are not given in detail; interested readers may refer to the works of Wei *et al*. (2002, 2002a), Zhao *et al*. (2002), and Civalek (2007b, 2008). By the DSC rule, the governing equations and the corresponding boundary conditions can be replaced by a system of simultaneously linear algebraic equations in terms of the displacements at all the sampling points. Thus, the buckling load of column under a given axial load P can be found by solving resulting eigenvalue equations.

3.2 Buckling of circular plates

Consider a thin, circular plate of uniform thickness subject to a uniform compressive radial load F_c distributed around the edge of the plate. There were two different governing equations in the literature (Bert *et al.* 1994). Both the third order and the fourth order differential equation can be used to obtain the buckling load. In this study, only third-order governing equation is used. This equation is given by

$$\frac{\partial^3 u}{\partial r^3} + \frac{1}{r}\left(\frac{\partial^2 u}{\partial r^2}\right) - \frac{1}{r^2}\left(\frac{\partial u}{\partial r}\right) = -\frac{F_c a^2}{D}\left(\frac{\partial u}{\partial r}\right) \tag{10}$$

where, a is known as the outside radius of the plate and D denotes the flexural rigidity of plates, and it is given as $D = Eh^3/12(1 - \nu^2)$, ν is the Poisson' ratio, E is the modulus of elasticity of the plate material, h is the uniform plate thickness, u is the displacement in the z direction. Applying the DSC method to Eq. (10), one obtains

$$\sum_{k=-M}^{M} \delta^{(3)}_{\Delta,\sigma}(k\Delta x)U_{k,j} + \frac{1}{R_i}\sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x)U_{k,j} - \frac{1}{R_i^2}\sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta x)U_{k,j} + \frac{F_c a^2}{D}\sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta x)U_{k,j} \tag{11}$$

The regularity condition at the centre of the plate is given in DSC form

$$\sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta x)U_{1,N} = 0 \tag{12}$$

The boundary conditions for a clamped outside edge are also given in DSC form

$$U_{N,j} = 0 \quad \text{and} \quad \sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta x)U_{N,j} = 0 \tag{13a, 13b}$$

Similarly, the boundary conditions for a simply supported outside edge are

$$U_{N,j} = 0 \quad \text{and} \quad \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x)U_{N,j} + \nu\frac{1}{R}\sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta x)U_{N,j} = 0 \tag{14a, 14b}$$

where the repeated index j means summation from 1 to N. We only keep the discretized equations for $j = 2$ to $(N - 2)$ in Eq. (11) because there is one boundary condition at $R = 0$ and there are two boundary conditions at $R = 1$. Consequently, we solve the remaining eigenvalue problem to obtain the natural frequencies.

3.3 Buckling analysis of rectangular plates

The governing differential equation of buckling of a thin rectangular plate is given by following non-dimensional form

$$\frac{\partial^4 U}{\partial X^4} + 2k^2 \frac{\partial^4 U}{\partial X^2 \partial Y^2} + k^4 \frac{\partial^4 U}{\partial Y^4} = H_x \frac{a^2}{D} \frac{\partial^2 U}{\partial X^2} \tag{15}$$

Where u is the transverse displacement of the midsurface of the plate, U is the dimensionless mode function of the deflection, $X = x/a$, $Y = y/b$ are the dimensionless coordinates, a and b are the dimensions of the plate parallel to x-axis and y-axis, $k = a/b$ is ratio of the plate edge length or aspect ratio, and H_x is the uniaxial compression load. D denotes the flexural rigidity of plates and it's given as $D = Eh^3/12(1 - \nu^2)$, ν is the Poisson' ratio, E is the modulus of elasticity, h is the uniform plate thickness. DSC form of Eq. (15) is

$$\sum_{k=-M}^{M} \delta^{(4)}_{\Delta,\sigma}(k\Delta x) U_{i+k,j} + 2k^2 \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x) U_{i+k,j} \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x) U_{i,k+j}$$
$$k^4 \sum_{k=-M}^{M} \delta^{(4)}_{\Delta,\sigma}(k\Delta x) U_{i,j+k} = H_x \frac{a^2}{D} \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x) U_{i+k,j} \tag{16}$$

The boundary conditions for a plate clamped on all four edges (C-C-C-C) are the displacement and rotation must be zero on edge

$$U(X,0) = U(X,1) = 0 \quad \text{and} \quad U(0,Y) = U(1,Y) = 0 \qquad (17a,\ 17b)$$

$$\frac{\partial U}{\partial Y}(X,0) = \frac{\partial U}{\partial Y}(X,1) = 0 \quad \text{and} \quad \frac{\partial U}{\partial X}(0,Y) = \frac{\partial U}{\partial X}(1,Y) = 0 \qquad (18a,\ 18b)$$

The boundary conditions for a plate simply supported on all four edges (S-S-S-S) are the displacement and moment must be zero on edge

$$U(X,0) = U(X,1) = 0 \quad \text{and} \quad U(0,Y) = U(1,Y) = 0 \qquad (19a,\ 19b)$$

$$\frac{\partial^2 U}{\partial Y^2}(X,0) = \frac{\partial^2 U}{\partial Y^2}(X,1) = 0 \quad \text{and} \quad \frac{\partial^2 U}{\partial X^2}(0,Y) = \frac{\partial^2 U}{\partial X^2}(1,Y) = 0 \qquad (20a,\ 20b)$$

Substituting the discredited boundary conditions into Eq. (16) gives the following typical eigenvalue equation as similar to the Eq. (9) for beam

$$[S]\{U\} - \Omega^2\{U\} = 0 \tag{21}$$

The eigenvalues, the buckling loads, of the [S] matrix are obtained by inverse iteration with shifting (Bathe 1982). Thus, solving Eq. (21) yields buckling loads.

3.4 Buckling analysis of skew plates

It is known that, there are no closed- form solutions for the buckling behavior of skew plates. Therefore, numerical methods must be utilized to solve the problem. Consider a thin isotropic skew plate. The governing differential equations for skew plates under uniaxial compression F_x along the x direction and its differential quadrature form are given respectively (Wang *et al.* 1994)

$$u_{xxxx} - (4k\cos\theta)u_{xxxy} + 2k^2(1 + 2\cos^2\theta)u_{xxyy} - (4k^3\cos\theta)u_{xyyy} + k^4 u_{yyyy} = -\frac{a^2}{D}F_x \sin^4(\theta/4)u_x \tag{22}$$

Numerical applications have been done for a thin skew plate under two different boundary conditions, namely simply supported-simply supported (S-S), clamped-clamped (C-C). Simply supported boundary conditions are

$$u = 0 \quad \text{and} \quad u_{xx} - 2u_{xy}\cos\theta = 0 \quad \text{at} \quad x = 0, a \tag{23a}$$

$$u = 0 \quad \text{and} \quad u_{yy} - 2u_{xy}\cos\theta = 0 \quad \text{at} \quad y = 0, b \tag{23b}$$

The clamped boundary conditions are

$$u = 0 \quad \text{and} \quad u_x = 0 \quad \text{at} \quad x = 0, a \tag{24a}$$

$$u = 0 \quad \text{and} \quad u_y = 0 \quad \text{at} \quad y = 0, b \tag{24b}$$

After the applying the DSC algorithm, the discretized form of Eq. (22) can be given by

$$\begin{aligned}
&\sum_{k=-M}^{M} \delta^{(4)}_{\Delta,\sigma}(k\Delta x)\,U_{i+k,j} - 4k\cos\theta \sum_{k=-M}^{M} \delta^{(3)}_{\Delta,\sigma}(k\Delta x)\,U_{i+k,j} \sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta y)\,U_{i,k+j} \\
&+ 2k^2(1+2\cos^2\theta) \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x)\,U_{i+k,j} \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta y)\,U_{i,k+j} \\
&- 4k^3\cos\theta \sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta x)\,U_{i,k+j} \sum_{k=-M}^{M} \delta^{(3)}_{\Delta,\sigma}(k\Delta y)\,U_{i,k+j} \\
&+ k^4 \sum_{k=-M}^{M} \delta^{(4)}_{\Delta,\sigma}(k\Delta y)\,U_{i,k+j} = -\frac{a^2}{D} F_x \sin^4(\theta/4) \sum_{k=-M}^{M} \delta^{(4)}_{\Delta,\sigma}(k\Delta x)\,U_{i+k,j}
\end{aligned} \tag{25}$$

4. Results and comparisons

Table 1 and Table 2 summarize numerical results of buckling loads by DSC for the case of linear elastic columns with three different boundary conditions. The buckling load by Chajes (1974) using the finite difference (FD) method is also presented in Table 1 for comparison. It is shown in Table 1 that DSC results using five grid points are more accurate than the FD for five grid points. From Table 2, one can conclude that for all numerical methods, the solutions converge as the grid number is increased. A reasonably converged solution may be achieved for 5 grids by DSC. In addition to this, a reasonably converged solution may be obtained for 9 grid points using FEM. The best solution is obtained for $N = 7$ grid points by using DSC method.

Table 3 tabulates the critical buckling loads obtained by DSC method for circular plates with clamped and simply supported boundary conditions. Table 3 includes both the different numerical solutions and the exact solutions. Authors used the finite element method (FEM) for this problem before (Civalek 1998). Results obtained from finite element method are indicated by FEM. HDQ results are obtained by using the third order differential equations for $N = 7$ grid points (Bert *et al.* 1994). For $N = 9$, Bert and *et al.* (1994) obtained the buckling loads. These results are also given in

Table 3 for comparison. For N = 9, HDQ and DQ results are also obtained once again for this problem. The obtained buckling loads are compared with those calculated with the DQ, HDQ, FEM, and the exact method. The DSC results are generally in agreement with the results produced from the analytical (Iyengar 1988) and the DQ results (Bert *et al*. 1994). Buckling loads of simply supported circular plates with different values of Poisson's ratios are given in Table 4. It is shown in this table that, when the number of grid points is larger than 16, the DSC results are independent of grid. It is also shown that the increasing value of Poisson's ratios always increases the buckling load. Table 5 summarizes numerical results of non-dimensionalized buckling loads by DQ and DSC for cases of square plates with four different support conditions. As can be seen, the DSC results compare very well with the analytical solutions from references (Iyengar 1988) for only 9 × 9 grid

Table 1 Comparison of buckling loads for different columns

	Finite difference Chajes, 1982 (N = 5)	Exact Iyengar, 1988	DSC (N = 5)	DSC (N = 9)
C-C	41.360	39.478	40.434	39.480
S-S	11.548	9.869	10.469	9.868
C-S	22.296	20.142	20.965	20.142

Table 2 Comparison of column buckling loads for different numerical methods

Support Conditions	Chajes, 1982 (N = 5)	Civalek, 2004					Present results		Exact Iyengar, 1988
		FEM (N = 5)	FEM (N = 7)	FEM (N = 9)	HDQ (N = 5)	HDQ (N = 7)	DSC (N = 5)	DSC (N = 7)	
C-C	41.360	40.254	39.984	39.614	39.547	39.478	40.434	39.480	39.478
S-S	11.548	10.376	9.816	9.897	9.851	9.869	10.469	9.869	9.869
C-S	22.296	21.946	20.664	20.285	20.205	20.141	20.965	20.143	20.142

Table 3 Critical buckling load of the circular plates ($\nu = 0.30$; $\overline{N}_{cr} = N_{cr}a^2/D$)

Support conditions	DSC (N = 9)	FEM (N = 11) Civalek, 1998	DQ (N = 9)	HDQ (N = 7)	HDQ (N = 9)	Iyengar, 1988
S-S	4.29	4.12	4.20	4.19	4.20	4.20
C-C	14.67	14.74	14.68	14.66	14.68	14.68

Table 4 Buckling loads of simply supported circular plates with different values of Poisson's ratios ($\overline{N}_{cr} = N_{cr}a^2/D$)

ν	DSC (N = 11)	DSC (N = 16)	DSC (N = 21)
0.0	3.485	3.405	3.403
0.1	3.712	3.650	3.648
0.2	3.911	3.816	3.815
0.3	4.213	4.201	4.189
0.4	4.415	4.441	4.436
0.5	4.688	4.680	4.674
0.6	5.237	5.115	5.112

Table 5 Buckling loads of square plates ($\nu = 0.30$; $\bar{N}_{cr} = N_{cr}a^2/D$)

Support conditions	DQ (7 × 7) Civalek, 2004	HDQ (7 × 7) Civalek, 2004	DSC (9 × 9)	Exact Iyengar, 1988
S-S-S-S	4.19	4.18	4.20	4.20
C-C-C-C	14.57	14.62	14.66	14.68
C-S-C-S	63.78	64.85	66.29	66.32
S-C-S-C	6.82	7.05	7.68	7.69

points. It is observed that by increasing the number of grid points within the range $5 \le N_x = N_y \le 9$, the DSC results approach monotonically the corresponding exact results.

Buckling loads obtained for square plates are presented in Table 6 together with the exact solutions (Iyengar 1988), finite element (Civalek 1998), and harmonic differential quadrature (Civalek 2004). Results obtained from the finite element method are indicated by FEM. Reasonably accurate results can be achieved by using only 9 × 9 grid points for DSC. From the table, the convergence of the DSC method is seen to be very good. It is also shown in this table that, DSC method produces better convergent solutions than the FEM when a similar number of grid points are used. In case of the rectangular plate; the obtained results are presented for aspect ratios of $k = a/b$ = 1/5, 2/5, 4/5, 5/5. Six different type plate configurations are taken into consideration. Table 7 presents the non-dimensional buckling load for rectangular plates. The buckling coefficients for clamped skew plates are listed in Table 8 together with those given by Wang *et al.* (1994). The numerical results had been obtained for k = 1. Four different skew angles θ are taken into consideration. HDQ and DQ results are obtained using various number of grid points for comparison. Reasonably accurate results can be achieved by using 9 grid points in HDQ and DQ. However, a reasonably converged solution may be obtained for 11 grid points using FEM. The non-dimensional buckling coefficients of skew plates with simply supported boundary conditions obtained by DSC are listed in Table 9. The number of discrete points considered along the non-dimensional *X*- and *Y*-axes was taken to be seven and eleven of these cases.

Table 6 Comparison of buckling loads of square plates ($\nu = 0.30$; $\bar{N}_{cr} = N_{cr}a^2/D$)

Support conditions	Civalek, 2004				DSC (7 × 7)	DSC (9 × 9)	FEM Civalek, 1998	FEM Civalek, 1998	Iyengar, 1988
	HDQ (5 × 5)	HDQ (7 × 7)	HDQ (9 × 9)	HDQ (11 × 11)					
S-S-S-S	4.12	4.18	4.20	4.20	4.85	4.20	4.12	4.16	4.20
C-C-C-C	14.18	14.62	14.68	14.68	15.11	14.66	13.98	14.56	14.68
C-S-C-S	64.01	64.85	66.30	66.32	67.03	66.29	68.79	68.34	66.32
S-C-S-C	6.93	7.05	7.67	7.69	7.95	7.68	-	7.63	7.69

Table 7 Non-dimensional buckling load of rectangular plates for various aspect ratios

a/b	C-C-C-C	C-C-C-S	C-C-S-S	C-S-S-S	C-S-C-S	S-C-S-S
1/5	40.38	40.65	20.85	21.01	40.13	10.55
2/5	44.13	42.91	23.73	22.96	42.66	13.64
4/5	70.01	59.66	42.01	36.10	54.99	33.58
5/5	99.12	78.33	61.68	48.64	66.33	56.02

Table 8 Buckling coefficients of clamped skew plates [$a/b = 1$; $N_{cr} = (F_x a^2)/D$]

Skew angle (θ)	DQ (9 × 9)	DQ (11 × 11)	HDQ (9 × 9)	DSC (9 × 9) Present study	FEM (11 × 11) Civalek, 1998	Wang *et al.* 1994(DQ) (11 × 11)
45	21.35	20.29	21.01	19.95	18.65	20.23
60	12.89	13.49	12.94	12.65	11.73	13.54
75	11.82	10.90	11.13	10.88	12.44	10.84
90	9.96	11.23	10.02	11.07	13.08	10.07

Table 9 Buckling coefficients of simply supported skew plates [$a/b = 1$; $N_{cr} = (F_x a^2)/D$]

Skew angle (θ)	DSC (7 × 7)	DSC (9 × 9)	DSC (13 × 13)
45	12.13	9.92	9.05
60	6.16	5.73	5.11
75	5.85	4.87	4.48
90	5.07	4.56	4.03

5. Conclusions

In the present study, DSC method was introduced to study the buckling analysis of plates and columns. The discretizing and programming procedures are straightforward and easy. Several test examples for different plate shapes have been selected to demonstrate the convergence properties, accuracy and simplicity in numerical implementation of DSC procedures. This has verified the accuracy and applicability of the DSC method to the class of problem considered in this study.

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