

Design of tensegrity structures using artificial neural networks

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Abstract. This paper focuses on the application of artificial neural networks (ANN) for optimal design of tensegrity grid as light-weight roof structures. A tensegrity grid, 2 m × 2 m in size, is fabricated by integrating four single tensegrity modules based on half-cuboctahedron configuration, using galvanised iron (GI) pipes as struts and high tensile stranded cables as tensile elements. The structure is subjected to destructive load test during which continuous monitoring of the prestress levels, key deflections and strains in the struts and the cables is carried out. The monitored structure is analyzed using finite element method (FEM) and the numerical model verified and updated with the experimental observations. The paper then explores the possibility of applying ANN based on multilayered feed forward back propagation algorithm for designing the tensegrity grid structure. The network is trained using the data generated from a finite element model of the structure validated through the physical test. After training, the network output is compared with the target and reasonable agreement is found between the two. The results demonstrate the feasibility of applying the ANNs for design of the tensegrity structures.

Keywords: tensegrity; finite element method (FEM); strain; artificial neural network (ANN); roof.

1. Introduction

‘Tensegrity’ is a relatively new and revolutionary concept in structural engineering. A tensegrity structure typically consists of a set of discontinuous compression members tied together by tensile members (generally cables), in a continuous manner. The term ‘tensegrity’ was formally coined and patented by Fuller (1962) as a contraction of the two words, ‘tension’ and ‘integrity’. The most recent definition of tensegrity structures has been given by Motro (2003) as, “a system in a stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components.” Large literature can be found on tensegrity structures related to form finding, static and dynamic analysis, fabrication and applications (Stern 1999, Quirant *et al.* 2003, Fest *et al.* 2004). Fig. 1 shows a typical simple tensegrity structure, consisting of four struts and twelve cables, with the top square inscribed in the bottom square, such that the nodes are the apices of a half cuboctahedron. This configuration is called ‘half-cuboctohedron’ and this paper explores designing a roof structure by joining such units.

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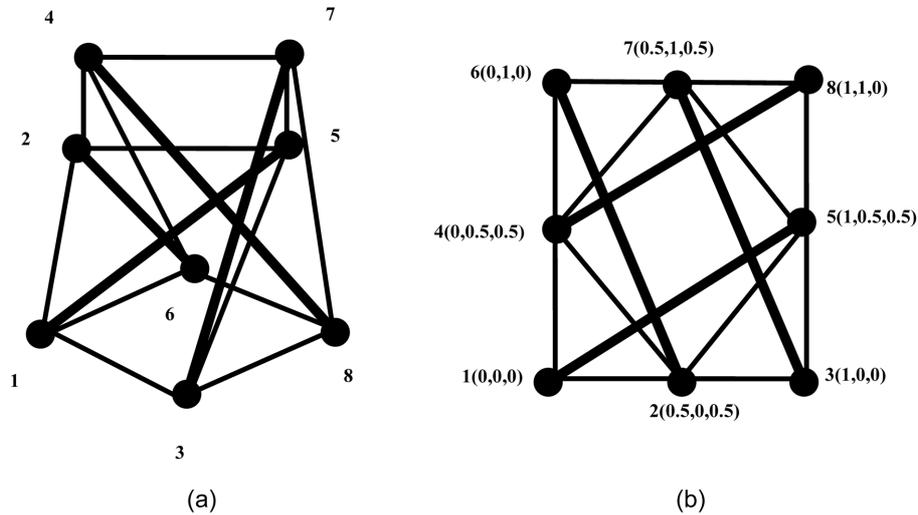


Fig. 1 Half cubo-octahedron unit; (a) Perspective view, (b) Top view (spatial coordinates are shown in brackets)

ANN constitutes a form of computing inspired by the biological structure of neurons inside the human brain. ANN can be expressed as a mathematical model composed of a large number of processing elements arranged in several layers. Although McCulloch and Pitts (1943) introduced the concept of neural networks in the 1940s, their real life applications, especially in the Civil Engineering discipline, commenced in the late 1980s only (e.g., Flood 1989). Since then, ANNs have been widely used to model numerous phenomena in structural mechanics (Civalek 2004, Cakiroglu *et al.* 2005). Applications of the ANNs on tensegrity structures, however, have been very limited. Domer *et al.* (2003) integrated the dynamic relaxation method with ANN and trained Stuttgart neural net simulator (SNNS) to generate the nodal displacements corresponding to various activation functions. Panigrahi *et al.* (2005) trained an ANN for carrying out the form finding of a three bar tensegrity structure using the force density method (Sheck 1974).

In the present study, a $2\text{ m} \times 2\text{ m}$ size grid has been fabricated as a cohesive unit by joining four individual half-cubooctahedron units of $1\text{ m} \times 1\text{ m}$ size along the bottom cables. The detailed experimental investigations consisting of material characterization, fabrication, instrumentation, destructive testing and numerical modelling have been described in related publication (Panigrahi *et al.* 2007). In this paper, the possibility of applying the ANNs for designing tensegrity grid structures by means of a multilayered feed forward back propagation algorithm is mainly covered.

2. Material characterization, fabrication and instrumentation

In this study, galvanized iron (GI) pipes of medium type, conforming to IS 1239-I (1990), are used as compression members. Cables fabricated from 0.25 mm diameter galvanized high carbon steel wires conforming to IS 1835 (1976) are employed as tensile elements. Each cable consists of six strands with nineteen wires in accordance with IS 3459 (1977).

Both the struts and the cables are tested in the laboratory for Young's modulus and ultimate strength (Panigrahi *et al.* 2007). For the strut, the Young's modulus was determined as $2.05 \times 10^5\text{ N/}$

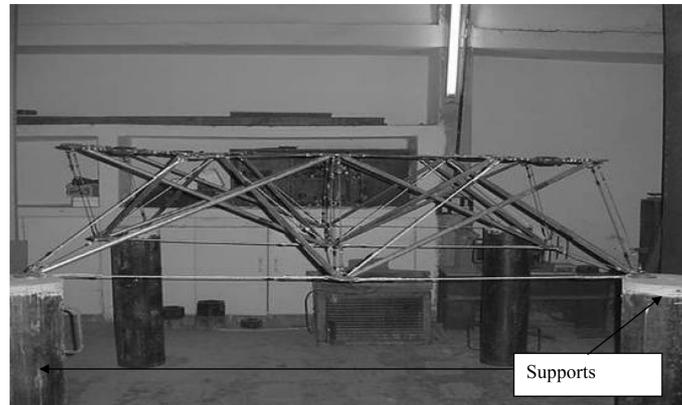


Fig. 2 Fabrication of tensegrity grid



Fig. 3 Detail of central bottom joint

mm^2 and the ultimate stress as 410 N/mm^2 . Similarly, for the stranded wire, these values were found as $0.954 \times 10^5 \text{ N/mm}^2$ and 1119.6 N/mm^2 respectively. It is observed that cables possess much higher strength, several times in magnitude, as compared to the struts.

A dismantlable tensegrity grid of size $2 \text{ m} \times 2 \text{ m}$ was fabricated using the GI pipes and the cables by joining the four half-cuboctohedron units shown in Fig. 1. The erected grid is shown in Fig. 2. The bottom cables measured 1 m in length, the top and the side cables 0.707 m in length and the struts 1.224 m in length, centre to centre of joints. Cable mode of erection was adopted i.e. assembling and dismantling is facilitated through a cable by means of a turnbuckle. This arrangement makes sure that the resulting structure is 'dismountable'. A typical detail of the central bottom joint is shown in Fig. 3. This tensegrity grid is different in fabrication as compared to other similar grids reported earlier in the literature (e.g., Hanaor 1993, Quirant *et al.* 2003, Fest *et al.* 2004), which were typically fabricated by joining the individual single units at the common nodes. The proposed structure, however, was achieved by joining the individual units along the inner bottom cables rather than the nodes. As a result, the numbers of cables is lesser by four as compared to conventional approach.

All members of one unit (quarter) of the grid were instrumented with electrical strain gauges (ESGs), four on each pipe (5 mm gauge length) and two on each cable (2 mm gauge length). Two

linear variable displacement transducers (LVDT) were also positioned; one under the central bottom node and the other under a side bottom node, for measuring the vertical displacements. The average prestress force in struts was computed as 2.58 kN from the measured strain data. The structure was loaded in quasi-static mode by gradually increasing the vertical loads in the form of iron weights and concrete cubes. The final failure occurred due to the failure of a cable connection. In addition, several cables are found to be slack. More details of the test are covered in Panigrahi *et al.* (2007).

3. Finite element analysis

The grid structure whose fabrication is described in the preceding section was modelled using finite element method (FEM) using the preprocessor of ANSYS 9 (2004). Geometric non-linearity has been considered due to the fact that large deflections were observed during the experiment. The detailed procedure for geometric nonlinear analysis of prestressed cable networks using matrix displacement approach has been described by Argyris and Scharpf (1972). All the cable and the strut elements were considered as 3D spar elements with three degrees of freedom in translation at each node. The material has been assumed to be linearly elastic and isotropic. Fig. 4 shows the finite element model of the structure. All the four bottom corner nodes i.e., 1, 3, 6 and 8, were restrained against translation along each coordinate axis. The prestress force measured in the struts in the self stressed equilibrium configuration upon erection was used to determine the initial strain in the cables. The axial force in top cable, F_a , bottom cable, F_b and leg tie, F_t can be expressed in terms of strut force F_s , based on Stern (1999) and Panigrahi *et al.* (2007) as

$$F_t = F_a = 0.578F_s \quad (1)$$

$$F_b = 0.41F_s \quad (2)$$

From these equations, the average prestress force in the top, the bottom and the tie cables can be computed as 1.50 kN, 1.06 kN and 1.49 kN respectively, considering the strut force $F_a = 2.58$ kN as reference. The model was simulated with external loads as applied during the experiment,

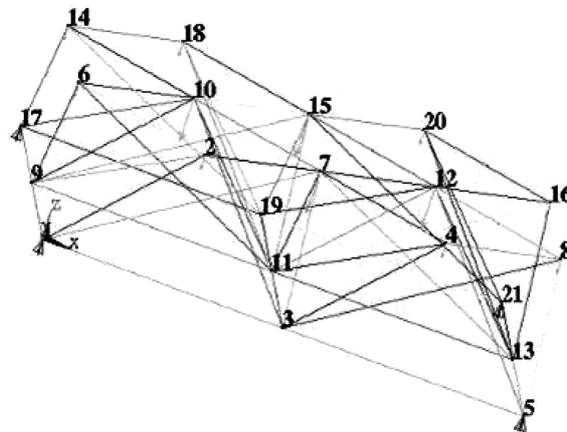


Fig. 4 Finite element model of tensegrity grid structure 2 m × 2 m size

distributed equally among all top nodes as concentrated loads.

Fig. 5 compares the results obtained using FEM with those obtained experimentally. It is observed that key displacements and key member forces obtained through FEM match well with the experimental results. This provides a validation to the finite element model of the structure, which will be used for parametric study and training ANN in the next sections.

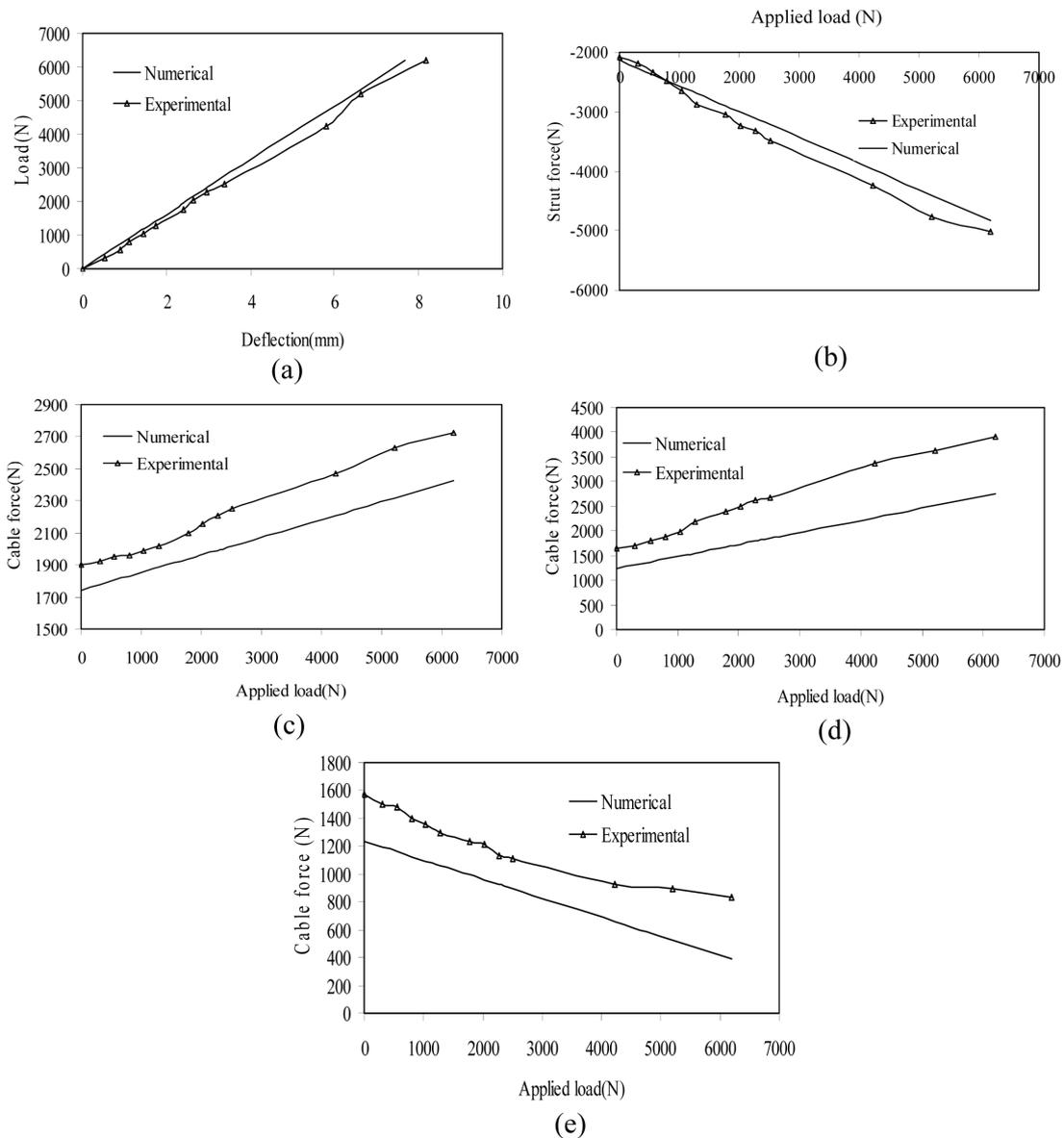


Fig. 5 Comparison between numerical and experimental results; (a) Deflection at node 3, (b) Force in strut connecting nodes 3 and 10, (c) Force in bottom cable connecting nodes 3 and 11, (d) Force in leg cable connecting nodes 3 and 7, (e) Force in top cable connecting nodes 7 and 2

4. Parametric study of tensegrity structures

After validating the finite element model of $2\text{ m} \times 2\text{ m}$ grid, it is extended to $4\text{ m} \times 4\text{ m}$, $6\text{ m} \times 6\text{ m}$, $8\text{ m} \times 8\text{ m}$ grids to study the effect of rigidity ratio (the ratio of the axial stiffness of the strut to that of the cable), the height of the structure and the support conditions. The prestress force in the self stressed equilibrium configuration is considered as 1.5 kN for the top cable as the reference. In general, due to high flexibility associated with the tensegrity structures, deflection is expected to govern the design. The analysis is first carried out on $8\text{ m} \times 8\text{ m}$ grid, whose 3D model is shown in Fig. 6. The maximum deflection of the structure was determined for a uniformly distributed load of 0.4 kN/m^2 (applied at a load step of 0.005 kN/m^2), for four different heights: 0.5 m , 0.6 m , 0.7 m and 0.8 m . The cross sectional area of the struts was fixed as 160.28 mm^2 and the necessary cross sectional area of cables varied so as to achieve the rigidity ratios of 10, 20, 30, 40 and 50. The support conditions have considerable influence on the deflection. In the first case, the structure was supported on four corners with all three degrees of freedom locked. In the second case, additional supports at the intermediate nodes on the periphery were provided in the vertical direction.

Typical variation of the maximum deflection for different heights corresponding to a rigidity ratio 10 is shown in Fig. 7 for the structure supported on four corners only. It is observed that with increase in height, the maximum deflection decreases appreciably. Comparing the results for a height of 0.8 m with 0.5 m , a reduction of 33% to 60% is observed. Fig. 8 similarly shows maximum deflection for the $8 \times 8\text{ m}$ grid structure for different rigidity ratios corresponding to a height of 0.8 m . The increase in deflection is substantial with increase in the rigidity ratio, since increase in rigidity ratio implies reduction in the area of cable elements. Fig. 9 shows a comparison of the maximum deflection for the grid structure with and without intermediate supports at the central periphery nodes. The deflection is found to reduce considerably for the case with intermediate supports. Typically, the deflection reduces from 86.5 mm to 20.6 mm for a load of 400 N/m^2 .

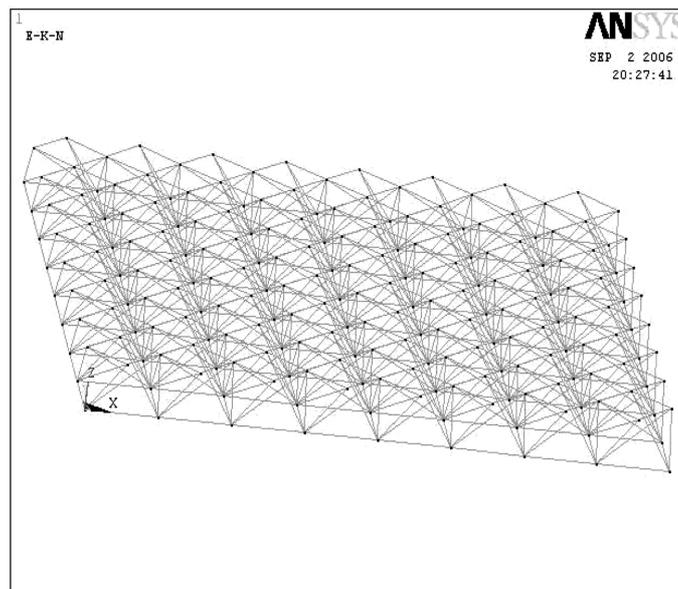


Fig. 6 Finite element model of $8 \times 8\text{ m}$ grid structure

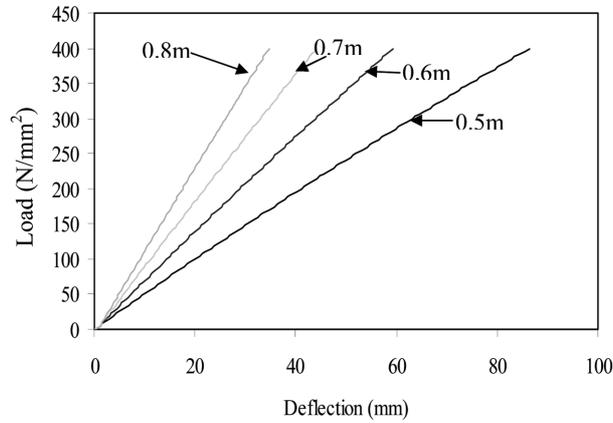


Fig. 7 Maximum deflections for 8×8 m grid structure for different heights

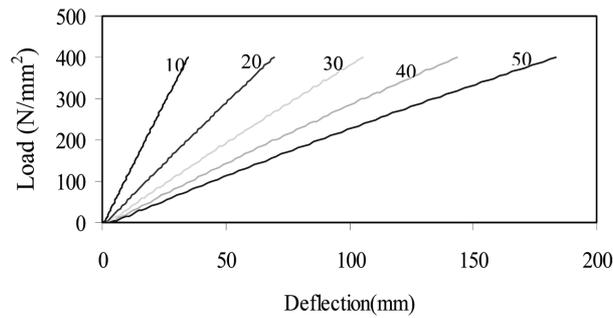


Fig. 8 Maximum deflections for 8×8 m grid structure for different rigidity ratio

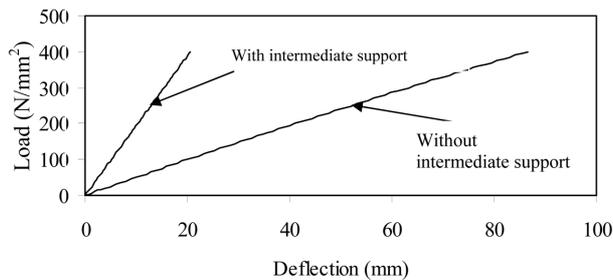


Fig. 9 Comparison of maximum deflections in 8×8 m grid structure for different support conditions

5. Application of ANN for design of tensegrity structure

Figs. 7, 8 and 9 illustrate that deflection rather than strength is likely to be the criterion for designing the tensegrity grid structures. The deflection mainly depends upon the grid size, the cross sectional area of tension and compression members, height of the structure, the loads and whether the supports have been provided on peripheral intermediate nodes. The influence of these parameters has already been illustrated in the previous section through a numerical case study. In

this section, the possibility of applying ANN based on backpropagation approach for design of the tensegrity structure is described.

5.1 Back-propagation neural networks

A back propagation neural network generally consists of three or more layers of neurons, essentially one input layer, one output layer and one or more hidden layers. The first phase of operation of the feedforward-back propagation ANN is called feed forward, during which the output $H_h(t)$ of a neuron in the hidden layer, corresponding to a given input vector $I(t)$, for i^{th} training pattern, is (Shanker 2005)

$$H_h(t) = f[Net_h(t)] \quad (3)$$

where

$$Net_h(t) = \sum_i W_{hi}(t) \cdot I_i(t) \quad (4)$$

and $f(x)$ is a differentiable activation function. W_{hi} represents the weight associated with $I_i(t)$. Similarly, the output of the unit O in the output layer, $O_o(t)$ is given by

$$O_o(t) = f[Net_o(t)] \quad (5)$$

where

$$Net_o(t) = \sum_h W_{oh} \cdot f \left[\sum_i W_{hi}(t) \cdot I_i(t) \right] \quad (6)$$

The second phase of operation is called the error back-propagation. The error function $E(o)$ is defined by the sum of squares of the differences between the desired output, $T_o(t)$ and the network output $O_o(t)$, as

$$E(O) = \frac{1}{2} \sum [T_o(t) - O_o(t)]^2 \quad (7)$$

where, the summation is extended over all the training patterns. For the hidden layer to output layer connections, the adaptive rule of the weight W_{oh} can be determined by generalized delta rule as

$$W_{oh}(t + \Delta) = W_{oh}(t) + \Delta W_{oh} \quad (8)$$

where

$$\Delta W_{oh} = -\eta \frac{\partial E(O)}{\partial W_{oh}} = -\eta \sum_h \Delta_o(t) \cdot H_h(t) \quad (9)$$

and $\Delta_o(t)$ is given by

$$\Delta_o(t) = \frac{df(Net_o)}{dNet_o} [T_o(t) - O_o(t)] \quad (10)$$

The coefficient η is called the learning rate. Similarly, the adaptive rule for the input layer to hidden layer connections, W_{hi} , can be expressed as

$$\Delta W_{hi} = -\eta \frac{\partial E(o)}{\partial W_{hi}} = -\eta \sum_h \Delta_h(t) \cdot I_i(t) \quad (11)$$

where

$$\Delta_h(t) = \frac{df(Net_h)}{dNet_h} \sum_o W_{ho} \cdot \Delta_o(t) \quad (12)$$

By applying the differentiation process successively, the error back propagation rules can be extended to networks with any number of hidden layers. Their weights will be continuously adjusted until the outputs attain a desired accuracy. The accuracy of the trained network depends upon the number of hidden layers, the number of neurons in hidden layer and the number, distribution and format of the training patterns.

5.2 ANN architecture selection, testing and training

For the present problem, that is the design of tensegrity structures, a multilayered back propagation based ANN has been chosen in MATLAB 7 (2007) environment. The input consisted of five neurons, representing the load intensity, grid size, height of the structure, support conditions and the limiting deflection. The output consisted of one node only, representing cross sectional area of cable. Cross sectional area of strut has been assumed to have a constant value of 160.28 mm². For training and testing, the data sets were generated by the static analysis of grids of size 4 m, 6 m and 8 m using FEM, as described in the previous section. A load intensity of 0.4 kN/m² was considered, with a load step of 0.005 kN/m². The height of the structure was varied from 0.5 m to 0.8 m at an interval of 0.1 m. Keeping the strut area constant, the cross sectional area of cable was chosen corresponding to the rigidity ratio of 10, 20, 30, 40 and 50 (i.e., 34.514, 17.252, 11.505, 8.629 and 6.903 mm² respectively). In the first case, the structure was considered supported on four corners only. In the network input, this condition is denoted by 0. In the second case, additional supports were provided on the bottom central nodes lying on periphery. This condition is denoted by 1 in the network input. The maximum deflection on the central bottom node was again determined using FEM as illustrated in the previous section. For testing, the data has been generated for a load step of 0.00625 kN/m², for 5 m size grid, which is not included in the training set. For training purpose, the data sets corresponding to deflections higher than span/100 have been ignored, since such large deflections are not permissible in majority of international codes of practice in structural engineering. The architecture used for the present study is shown in Fig. 10. One hidden layer with 17 neurons was found to produce the best results after a number of trial and error. All data were normalized between 0 and 1 and were shuffled randomly before training. Log sigmoid was used as activation function for the input and the hidden layer, where as purelin function is used for the output layer.

The ANN was trained for a total of 8567 data sets and tested for 7409 sets. The mean square error is shown in Fig. 11. The validations of key data points are shown in Table 1. and the test

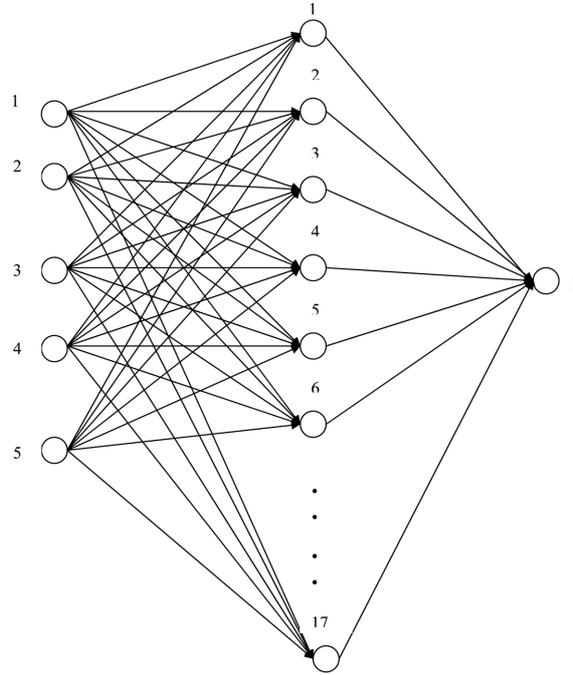


Fig. 10 Architecture of ANN

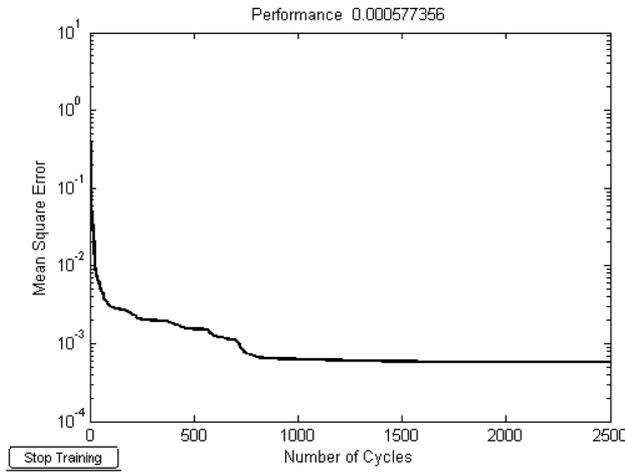


Fig. 11 Mean Square Error of the network

pattern in Fig. 12. The test pattern shows the maximum deviation as 10%. In majority cases, a much lower deviation, typically less than 1% has been observed. Such small deviations are acceptable in civil engineering practice. It may additionally be noted that a deviation is as high as 40% can observed in case of small loads. However, since the associated deflection is negligible (typically less than 0.6 mm), these cases will not be critical for design. Although the ANN was not trained for 5 m grid, the area of cable size predicted is reasonably close to target area of cable as shown in Table 1.

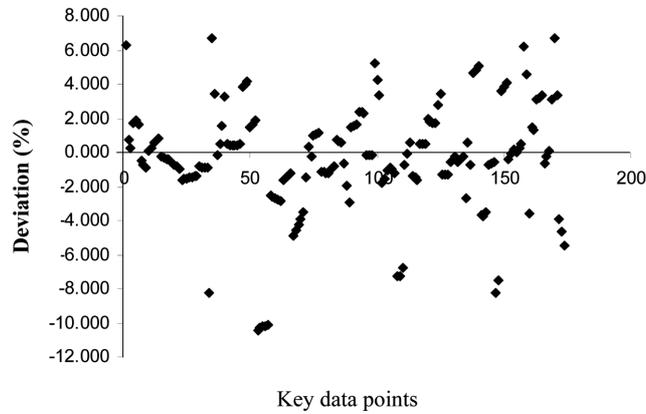


Fig. 12 Percentage deviations for the test pattern

Table 1 Typical test results of the proposed ANN

Load (N/m ²)	Height (m)	Grid size (m)	Support condition	Deflection (mm)	Target area of cable (mm ²)	Area predicted by ANN (mm ²)	Error (%)
6.25	0.5	4	0	0.0863451	34.514	32.334	6.317
18.75	0.5	4	0	0.2587435	34.514	34.417	0.280
37.5	0.5	4	0	0.5175472	34.514	33.878	1.843
381.25	0.5	4	0	5.3066319	34.514	34.775	-0.756
237.5	0.6	4	0	2.507334	34.514	34.476	0.111
256.25	0.6	4	0	2.698735	34.514	34.326	0.544
343.75	0.6	4	0	6.8975751	17.257	17.543	-1.658
368.75	0.6	4	0	7.3909101	17.257	17.517	-1.506
381.25	0.6	4	0	7.6378586	17.257	17.506	-1.443
387.5	0.6	4	0	7.7614032	17.257	17.501	-1.415
393.75	0.6	4	0	7.8849947	17.257	17.497	-1.389
312.5	0.8	4	0	2.2491414	34.514	34.642	-0.370
356.25	0.8	4	0	2.5295809	34.514	34.865	-1.018
365	0.5	5	0	11.643194	34.514	38.109	-10.416
400	0.5	5	0	25.042894	17.257	17.756	-2.893
250	0.5	5	0	23.024003	11.506	11.679	-1.502
210	0.8	5	0	15.637048	6.903	7.240	-4.884
230	0.8	5	0	16.981392	6.903	7.146	-3.526
6.25	0.5	6	0	1.2534991	34.514	35.026	-1.485
18.75	0.5	6	0	2.1200292	34.514	34.606	-0.267
231.25	0.6	6	0	23.34754	17.257	17.450	-1.117
156.25	0.7	6	0	18.274996	11.505	11.645	-1.213
31.25	0.7	6	0	6.349604	8.629	8.684	-0.639
43.75	0.7	6	0	8.0877088	8.629	8.879	-2.901
356.25	0.8	6	0	11.709202	34.514	34.165	1.010
393.75	0.8	6	0	35.589396	11.505	11.435	0.612
231.25	0.8	6	0	28.262938	8.629	8.505	1.443

Table 1 Continued

Load (N/m ²)	Height (m)	Grid size (m)	Support condition	Deflection (mm)	Target area of cable (mm ²)	Area predicted by ANN (mm ²)	Error (%)
206.25	0.8	6	0	31.524494	6.903	6.742	2.336
31.25	0.5	8	0	12.114203	17.257	17.563	-1.774
43.75	0.5	8	0	17.055327	17.257	17.437	-1.044
381.25	0.6	8	0	56.451772	34.514	34.579	-0.187
6.25	0.7	8	0	1.2050416	34.514	32.705	5.240
18.75	0.7	8	0	2.5240377	34.514	33.363	3.335
337.5	0.7	8	0	74.732595	17.257	17.414	-0.912
356.25	0.7	8	0	79.098293	17.257	17.469	-1.229
93.75	0.6	8	0	68.08762	6.903	7.370	-6.769

6. Conclusions

This paper has presented a new approach for designing tensegrity grids for roof structures using ANN. The behaviour of the structure has been modelled using FEM, which was validated experimentally. The effect of rigidity ratio, structure height and support conditions on maximum deflection has been studied in detail. These parameters have profound influence on the deflection, which is the main design criterion for the tensegrity structure. A new multilayer feed forward back propagation ANN based model has been proposed for designing tensegrity based grid structures. The ANN has been trained from the data generated by analyzing different tensegrity based 3D grid structure, using FEM, by varying the loads, heights, rigidity ratios and the support conditions. Testing of the ANN showed good performance, with the error within reasonable limits.

The tensegrity structures are well known to exhibit non-linear behaviour resulting from geometric non-linearity. Hence, the ability of the ANN to model tensegrity structures reasonably well clearly establishes their robustness in modelling non-linear structural engineering problems. Though half cuboctahedron configuration has been considered in the present study, the ANN can also be trained for other configurations. Use of ANN can drastically reduce the analysis and design effort and thus can be a valuable aid to the structural engineers.

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Notation

The following symbols are used in this paper

- a : length of top cable
 b : length of top cable
 $E(o)$: error function
 F_a : force in top cable
 F_b : force in bottom cable
 F_s : force in strut
 F_t : force in leg tie
 $f(x)$: a differentiable activation function
 $H_h(t)$: output vector of neuron
 $I(t)$: input vector
 L_t : length of leg tie
 L_s : length of strut
 $O_o(t)$: network output
 $T_o(t)$: desired output
 W_{oh} : adaptive rule of weights for hidden layer to output layer connection
 W_{hi} : adaptive rule of weights for input layer to hidden layer connection
 η : called the learning rate