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# Effect of prestressing on the first flexural natural frequency of beams

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**Abstract.** In this paper the effect of prestressing force on the first flexural natural frequency of beams is studied. Finite element technique is used to model the beam-tendon system, and the prestressing force is applied in the form of initial tension in the tendon. It is shown that the effect of prestressing force on the first natural frequency depends on bonded and unbonded nature of the tendon, and also on the eccentricity of tendon. For the beams with bonded tendon, the prestressing force does not have any appreciable effect on the first flexural natural frequency. However, for the beams with unbonded tendon, the first natural frequency significantly changes with the prestressing force and eccentricity of the tendon. If the eccentricity of tendon is small, then the first natural frequency decreases with the prestressing force and if the eccentricity is large, then the first flexural natural frequency increases with the prestressing force. Results of the present study clearly indicate that the first natural frequency can not be used as an easy indicator for detecting the loss of prestressing force, as has been attempted in some of the past studies.

**Keywords:** prestressed beam; flexural natural frequency; bonded and unbonded tendon; prestressing force.

## 1. Introduction

In the dynamic analysis of prestressed beams, which are widely used in bridges, knowledge of the flexural natural frequencies is of vital importance. In this context, one important question is: Does the prestressing force affect the flexural natural frequencies of beams? Many researchers have studied the influence of prestressing force on the flexural natural frequencies of beams, and there are differences in their conclusions.

Saiidi *et al.* (1994) have studied the effect of prestressing on the flexural natural frequencies of concrete beams. They started with the argument that prestressing will induce axial force in the beam and hence, with the increase in the prestressing force, the flexural rigidity of beam will reduce, which in turn, will cause decrease in the natural frequencies. However, in the field and laboratory experiments, Saiidi *et al.* (1994) found that the natural frequencies increase with the prestressing force. For this disparity between the experimental results and their argument, they opined that the presence of prestressing force causes closure of micro-cracks in the concrete, which increases the

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flexural rigidity and natural frequencies. The paper of Saiidi *et al.* (1994) has been discussed by three different groups of research workers. In the first discussion, Asta and Dezi (1996) pointed out that Saiidi's approach to consider the prestressing force as external axial force is not correct. Further, for a simply supported prestressed beam with bonded tendon passing through the center of beam, Asta and Dezi (1996) have derived the following expression for the flexural natural frequencies

$$\omega_n^2 = \frac{n^4\pi^4}{mL^4}\left[\left(E_b - \frac{N}{A_b}\right)I_b + \left(E_c + \frac{N}{A_c}\right)I_c\right] \tag{1}$$

where, $I_b$, $A_b$, $L$, $E_b$, and $m$ respectively denote moment of inertia, cross sectional area, span length, elastic modulus, and mass per unit length of the beam; $I_c$, $A_c$ and $E_c$ denote moment of inertia, cross sectional area and elastic modulus of the tendon and $N$ is prestressing force.

From this expression it is evident that, for the realistic values of various parameters of beams and tendons, the prestressing force will not have any significant effect on the natural frequencies of the beams. It may be reiterated here that the above formula is derived for simply supported prestressed beams with concentric, bonded tendon. In the second discussion on Saiidi's paper, Deak (1996) pointed out that if the tendon is unbonded and attached to the beam only at the ends, then only, the prestressing force could be treated as external axial force. In the third discussion, Jain and Goel (1996) opined that since the tendon becomes an integral part of the system, tension in the tendon cannot be treated as an external force, and hence, the prestressing force will not affect the flexural natural frequencies of beams.

Another interesting study on the effect of prestressing force on the flexural natural frequencies of beams is by Miyamoto *et al.* (2000). In this study, a formula for the flexural natural frequencies is derived for the prestressed concrete beams with unbonded external tendon of trapezoidal profile. Moreover, some experiments are also performed and it is noted that for less eccentric tendon profiles, the flexural natural frequencies decrease with the increase in the prestressing force. However, for the tendons with higher eccentricity, the prestressing force has only marginal effect on the flexural natural frequencies.

In the experiments conducted by Saiidi *et al.* (1994) and Miyamoto *et al.* (2000) concrete beams were used, wherein, due to prestressing, micro-cracks in the concrete get closed, which increases the flexural rigidity. Thus, the effect of prestressing gets intermixed with that of the increase in flexural rigidity. Notwithstanding this intermixing, it is interesting to note that in the field and laboratory experiments, Saiidi *et al.* (1994) have observed an increase in the flexural natural frequencies with the prestressing force. Whereas, Miyamoto *et al.* (2000) who used unbonded tendons, have found a slight decrease in the natural frequencies. From this observation, and from the comments made by Asta and Dezi (1996) and Deak (1996), it appears that the nature of the tendon, i.e., whether the tendon is bonded or unbonded, and its eccentricity will influence the effect of prestressing on the flexural natural frequencies of beams. However, so far in the literature, there is no detailed study on the influence of bonded and unbonded nature of the tendon on the flexural natural frequencies of prestressed beams. In this context, it may however be noted that, there are studies on the effect of bonded and unbonded nature of tendon on the static behavior of prestressed beams (Mattock *et al.* 1971, Ramos and Aparcio 1996, Ariyawardena and Ghali 2002). In these studies, the effect of bonded and unbonded nature of tendon on the ultimate load carrying capacity of prestressed beams is highlighted. The bonded tendons are generally used in prestressing of concrete beams. Unbonded tendons are used in concrete beams and also in the strengthening and rehabilitation of old bridges.

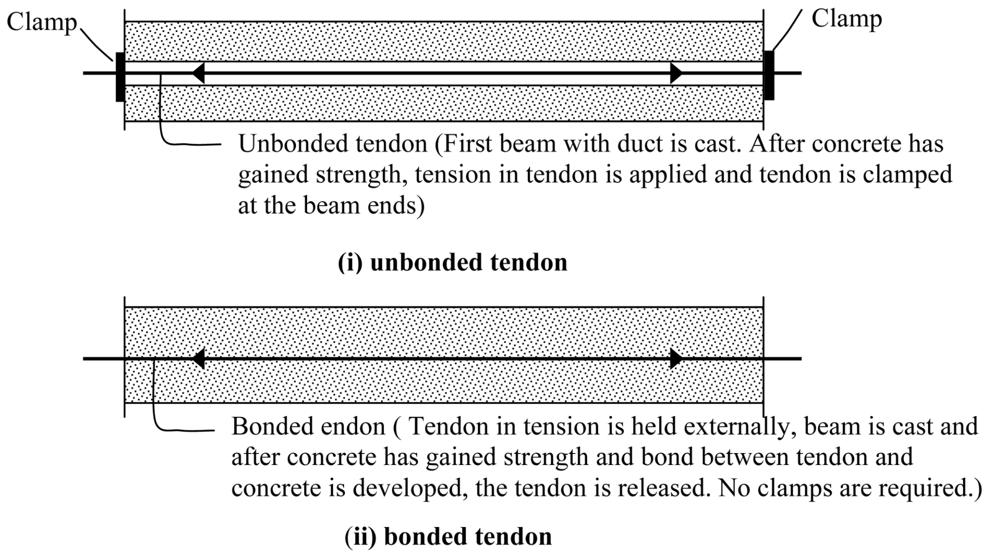


Fig. 1(a) Schematic representation of unbonded and bonded tendon in a concrete beam

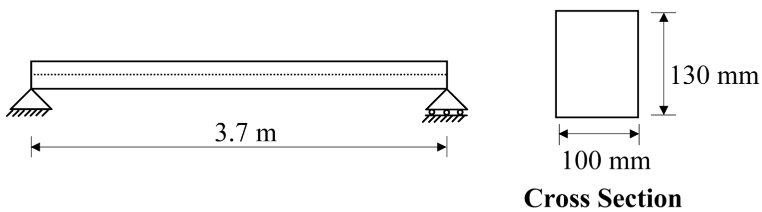


Fig. 1(b) Details of beam geometry

Moreover, in steel bridges, prestressing is done using unbonded tendons (Belena 1977, Triostky 1990, Ronghe and Gupta 2002). Schematic representation of a concrete beam with unbonded and bonded tendon is shown in Fig. 1(a). In unbonded case, the tendon which is in tension, passes through a hollow duct and is clamped to the beam at the end points only. In this case, prestressing force gets transmitted to beam through end points only. In bonded case, the tendon in tension is temporarily held externally and beam is cast, and after concrete has gained enough strength and the bond between concrete and tendon is fully developed, the tendon is released, thereby transferring the prestressing force to the beam. In this case, the prestressing force gets transferred uniformly all along the length of the beam and there are no clamps at the ends.

The aim of present paper is to study the effect of bonded and unbonded nature of the tendon on the first flexural natural frequency of prestressed beams. In the dynamic analysis of bridges, where prestressed beams are used, usually the first flexural vibration mode is of prime importance. Hence, in the present paper, only the first flexural natural frequency is considered. The effect of eccentricity of the tendon on the first flexural natural frequency is also studied. Finite element approach is used to model the beam-tendon system and the prestressing force is applied in the form of initial tension in the tendon.

## 2. Details of beam-tendon system and analysis procedure

The beam-tendon system used by Saiidi *et al.* (1994) in laboratory experiments is considered. This is a concrete beam with elastic modulus of $21.3 \times 10^6$ kN/m$^2$, mass density of 2.4 t/m$^3$ and has geometry as shown in Fig. 1(b). Steel tendon of 13 mm diameter with elastic modulus of $20.0 \times 10^7$ kN/m$^2$ and mass density of 7.85 t/m$^3$ is used.

The beam-tendon system is modeled using finite element software, NISA (1994), wherein, the beam is modeled using Kirchoff beam element (NKTP 39 of NISA element library) and the tendon is modeled using 3-D spar element (NKTP 14 of NISA element library). Beams and tendons are discritized into 100 finite elements. The Prestressing force is applied by giving initial tension in the tendon. First, nonlinear static analysis, including the geometric nonlinearity, is performed and the geometric stiffness matrix of the beam-tendon system is obtained. In the next step, using this stiffness matrix, the first flexural natural frequency of beam-tendon system is obtained. By varying the level of prestressing force, its effect on the first flexural natural frequency is studied. It may be noted that in this type of modeling of the beam-tendon system, the prestressing force becomes an integral part of the system and is not treated as external axial force. Since the flexural mode of vibration in the vertical plane is of interest, out-of-plane motion of the beam-tendon system is constrained.

Before proceeding further, it will be appropriate to ascertain the correctness of the above described procedure for finding the first flexural natural frequency of prestressed beams. For this purpose, this procedure is used to obtain the first flexural natural frequency of a simply supported beam subjected to an external axial force, and results are compared with the following analytical solution from Timoshenko *et al.* (1974)

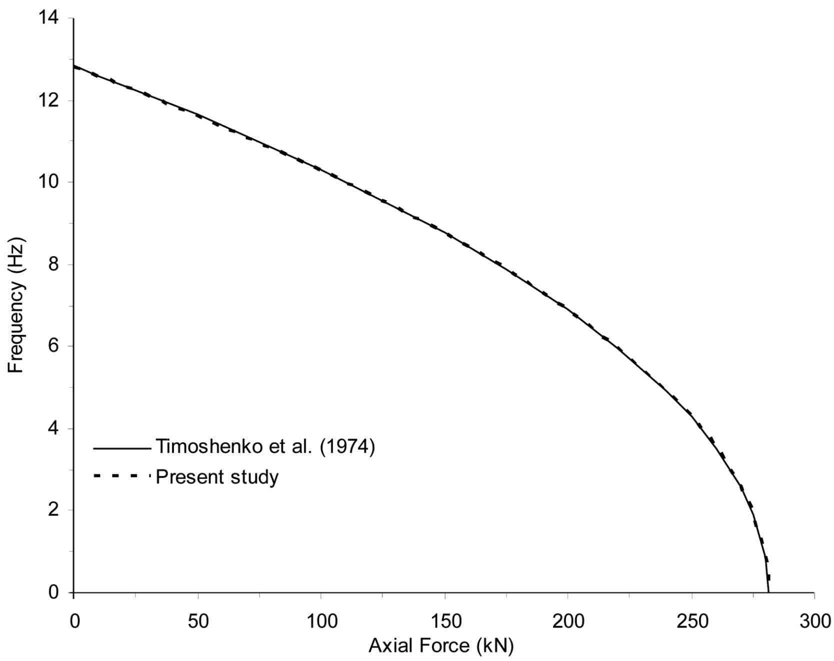


Fig. 2 Effect of external axial force on first flexural natural frequency of beam

$$\omega_n^2 = \left(\frac{n\pi}{L}\right)^4 \frac{E_b I_b}{m} - \left(\frac{n\pi}{L}\right)^2 \frac{N}{m} \tag{2}$$

This comparison is shown in Fig. 2, wherein, it is seen that the results obtained using the above procedure compare very well with the analytical solution. This ascertains the validity of the above described procedure for obtaining the first natural frequency of prestressed beams.

## 3. Results

### 3.1 Beams with straight concentric tendon

Simply supported beam with straight, concentric bonded and unbonded tendons is considered. In the finite element model, the beam as well as the tendon is discritized into 100 finite elements. In the case of beam with bonded tendon, all the beam and tendon elements have common nodes, whereas, for the beam with unbonded tendon, the beam and tendon elements have common nodes only at the end supports. Results on the effect of prestressing force on the first flexural natural frequency of beam with bonded and unbonded tendon are shown in Fig. 3. It is seen that the effect of prestressing force on the first flexural natural frequency is drastically different for beam with bonded and unbonded tendons. For the beam with bonded tendon, there is practically no change in the first flexural natural frequency with the prestressing force. These results match well with those obtained using the analytical expression (Eq. (1)) of Asta and Dezi (1994). For the beam with unbonded tendon, the first flexural natural frequency significantly decreases with the prestressing

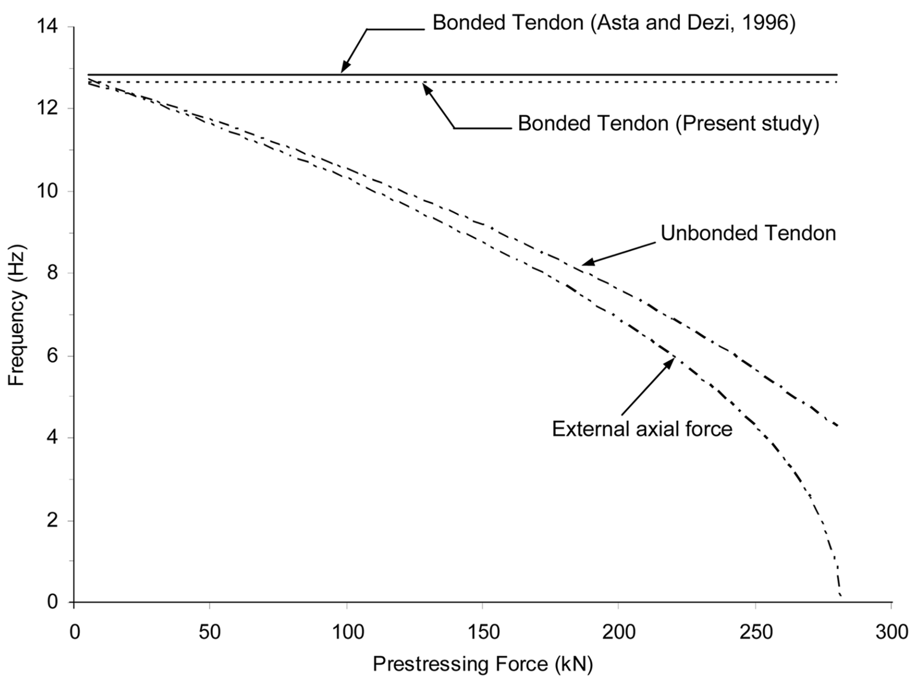


Fig. 3 Effect of prestressing force on first flexural natural frequency of beam with straight concentric tendon

force. This decrease is qualitatively similar to the one observed due to external axial force. However, the rate of decrease in the frequency for the case of unbonded concentric tendon is lower than that for the case of external axial force (Fig. 3). Here, the prestressing force is varied from 5 kN to 280 kN. For the prestressing force less than 5 kN, the first natural frequency of unbonded tendon becomes less than that of the beam. For prestressing force higher than 280 kN, which is very close to the buckling load of 281.2 kN, results did not show good convergence in the nonlinear static analysis.

### 3.2 Beams with straight eccentric tendon

Straight eccentric tendons of eccentricity $e/D$ = 0.25 and $e/D$ = 0.75 are considered. Here, $e$ is the distance of the tendon from the center-line of the beam and $D$ is the depth of the beam. In the finite element model of the beam with bonded eccentric tendon, all the nodes of tendon elements are attached to the beam nodes with rigid links (Fig. 4(a)), whereas, for the beam with unbonded eccentric tendon, only end support nodes of the tendon and beam elements are connected by rigid links (Fig. 4(b)). The variation of the first flexural natural frequency with the prestressing force is

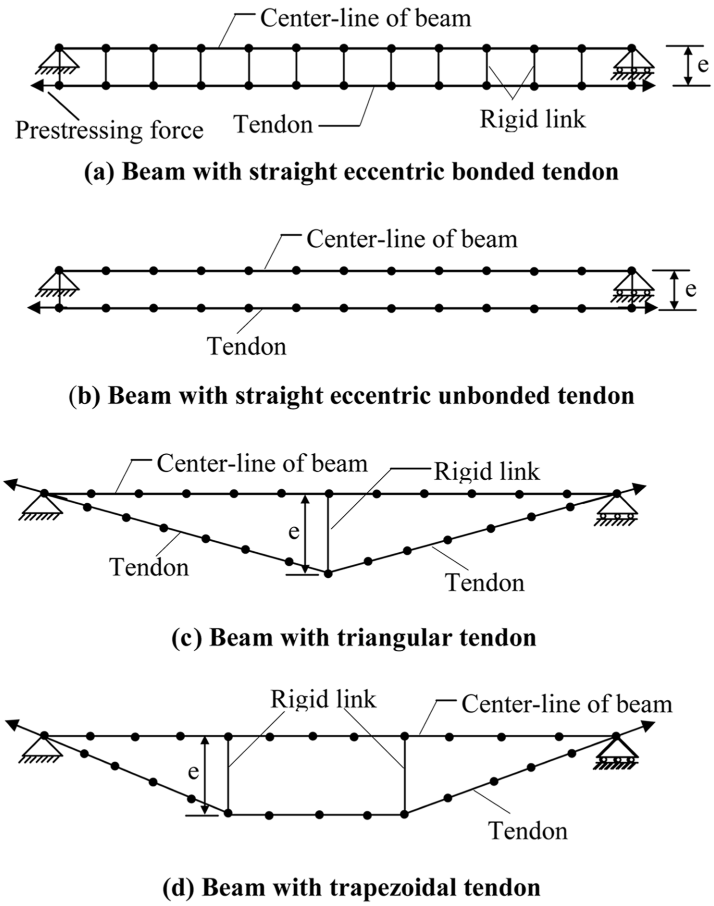


Fig. 4 Finite element models of prestressed beams with different tendon profiles

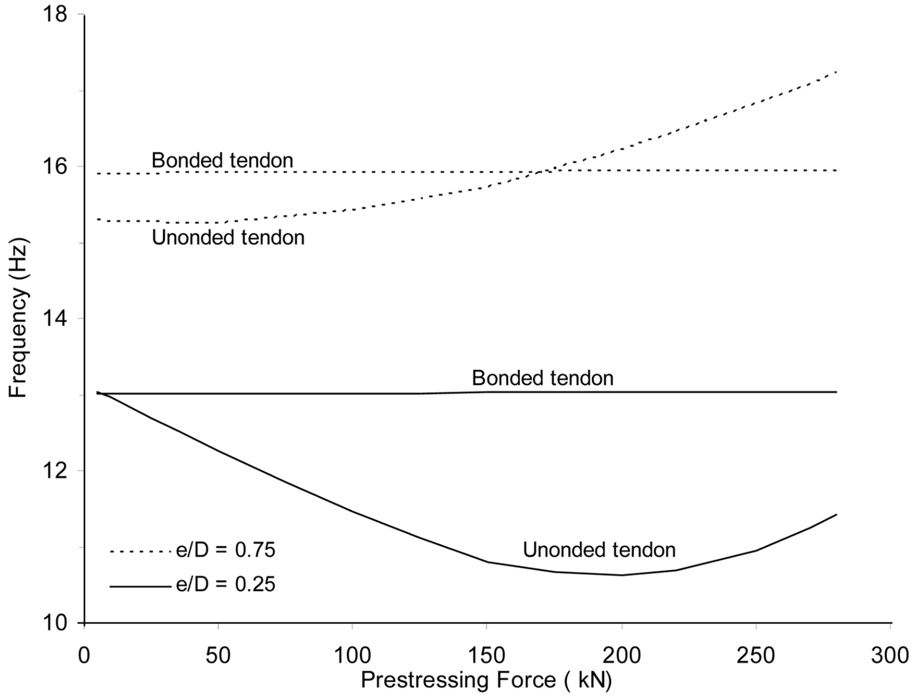


Fig. 5 Effect of prestressing force on first flexural natural frequency of beam with straight eccentric tendon

shown in Fig. 5. It is seen that for the beam with bonded tendon, the frequency does not change with the prestressing force. Moreover, for the beam with bonded tendon of higher eccentricity (*e/D* = 0.75), the natural frequency is more than that for the beam with tendon of lower eccentricity (*e/D* = 0.25).

In the case of beam with unbonded tendon of eccentricity, *e/D* = 0.25, the frequency first decreases with the prestressing force and then increases (Fig. 5). However, for *e/D* = 0.75, the frequency continuously increases with the prestressing force. Due to the eccentricity of the tendon, beam is subjected to moment, which causes the stiffening effect. Hence, for higher eccentricity of the tendon, the natural frequency continuously increases with the prestressing force.

### *3.3 Beams with triangular and trapezoidal tendon profiles*

Unbonded tendons with triangular and trapezoidal profiles (Figs. 4(c), 4(d)) are commonly used in the external prestressing of bridges. The variation of the first flexural natural frequency with the prestressing force is shown in Figs. 6 and 7 respectively for triangular and trapezoidal tendon profiles. These results are given for eccentricities of *e/D* = 0.25, *e/D* = 0.75 and *e/D* = 1.25. It is seen that the prestressing force does not have significant effect on the first natural frequency, though the natural frequency shows slight reduction with the prestressing force for tendons with low eccentricity. It is also seen that the first natural frequency is higher for beams with tendons of larger eccentricities.

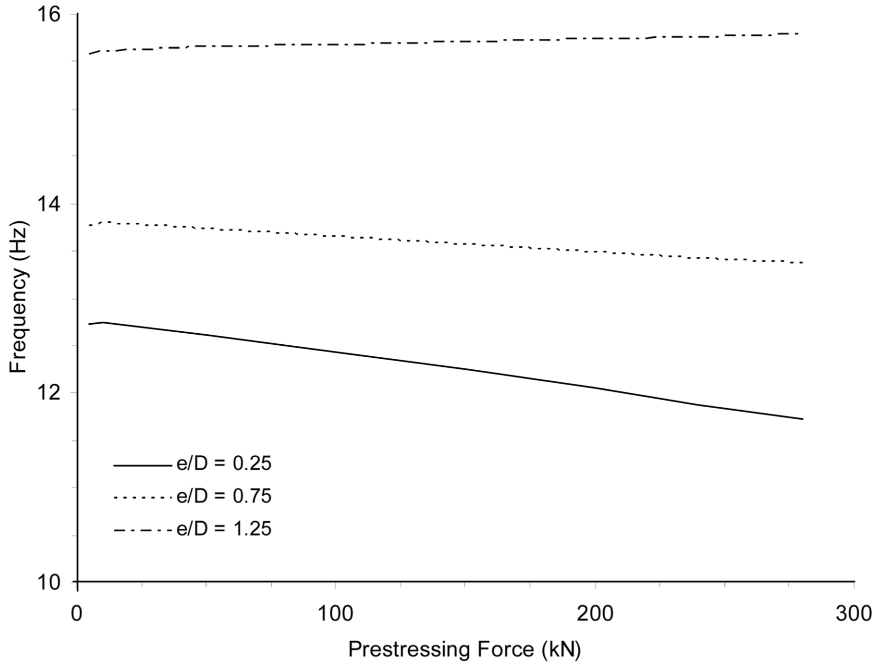


Fig. 6 Effect of prestressing force on first flexural natural frequency of beam with triangular tendon profile

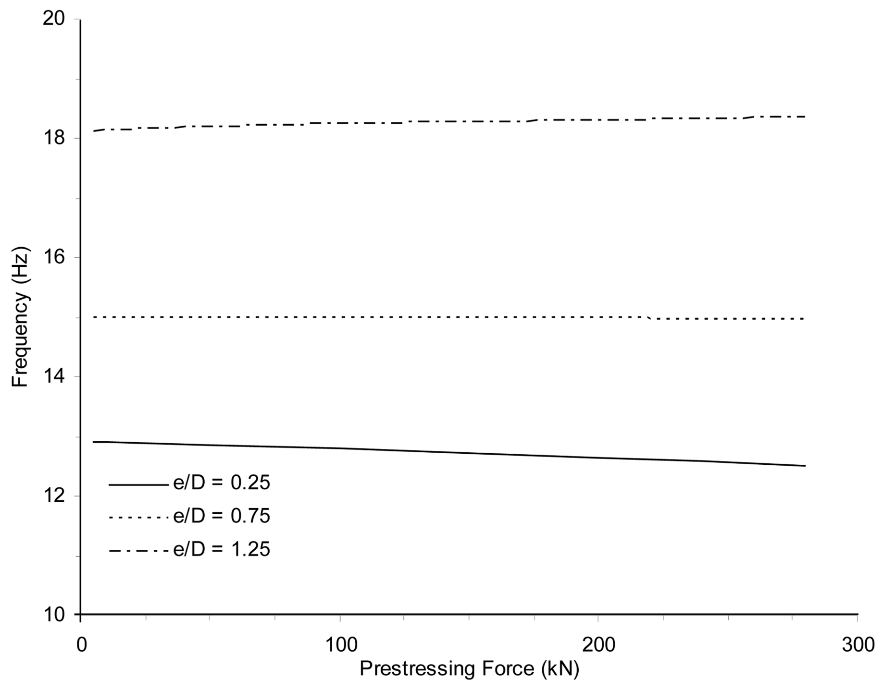


Fig. 7 Effect of prestressing force on first flexural natural frequency of beam with trapezoidal tendon profile

## 4. Discussion and conclusions

Results of the present study reveal that the effect of the prestressing force on the first flexural natural frequency of prestressed beams depends on three factors: i) whether tendon is bonded or unbonded; ii) the eccentricity of the tendon; and iii) the longitudinal profile of the tendon. If the tendon is straight, concentric (i.e., passing through the center-line of beam), and is bonded to the beam along its length, then the prestressing force does not have any appreciable effect on the first flexural natural frequency (Fig. 3). However, if the straight, concentric tendon is unbonded (i.e., attached to beam only at its ends), then, the prestressing force causes softening effect and the first flexural natural frequency significantly reduces with the prestressing force (Fig. 3). This reduction is qualitatively similar to the reduction due to external axial force, but quantitatively, this reduction is less than the reduction caused by external axial force (Fig. 3). This is due to the fact that, presence of the tendon, which is in tension, increases the stiffness of the beam-tendon system.

If straight tendon is placed eccentrically, then, for the case of bonded tendon, the prestressing force does not have any effect on the first natural frequency (Fig. 5). However, if straight, eccentric tendon is unbonded, then, the prestressing force influences the first flexural natural frequency and this influence depends on the eccentricity of the tendon. The prestressing force in the eccentrically placed tendon induces moment in the beam, which causes stiffening effect and there is an interplay between the softening effect due to the prestressing force and the stiffening effect due to moment. If the eccentricity is low, then the softening effect is predominant than the stiffening effect, hence, the first flexural natural frequency reduces with the prestressing force (Fig. 5; $e/D$ = 0.25). However, for large eccentricity, the stiffening effect is predominant and the frequency increases with the prestressing force (Fig. 5; $e/D$ = 0.75).

In the tendons with triangular and trapezoidal profiles, the eccentricity is invariably present, and it varies along the length. For these tendon profiles also, the effect of prestressing force, depends on the eccentricity of tendon (Figs. 6 and 7). For low eccentricity, there is a slight reduction in the first flexural natural frequency, whereas, for the higher eccentricity, there is practically no reduction. It is interesting to note that for the same eccentricity, there is higher reduction in the natural frequency for the triangular profile than the trapezoidal profile. This is due to the fact that in the trapezoidal profile, the extent of bonding is more since it is attached to the beam at two intermediate locations, whereas, the tendon with triangular profile is attached at only one intermediate location. Thus, if the extent of bonding between the tendon and the beam is more, then the softening effect of the prestressing force reduces.

In some of the past studies (Kato and Simida 1986, Mirza *et al.* 1990, Singh 1991, Mo and Hwang 1996), efforts have been made to use the natural frequencies and dynamic response characteristics as indicators for detecting the loss of prestressing force or damages in the beam. However, from the results of the present study, it is quite clear that, the prestressing force can have entirely different effect on the first flexural natural frequency of the beam, depending on the bonded and unbonded nature, eccentricity and tendon profile. Thus, the natural frequency is not an easy indicator for detecting the loss of prestressing force. Results presented in the present study are for an example beam-tendon system. For any other beam-tendon system, qualitatively the results will remain same, though there could be some changes depending on the relative values of the rigidity of the beam and tendon. Numerical results presented in this study need experimental verification. Such experimental study shall be performed on the steel beams rather than concrete beams, so as to avoid the increase in the stiffness of beams due to closure of micro cracks in the presence of prestressing force.

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