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Analytical model for high-strength concrete columns with square cross-section

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Abstract. In the present paper a mechanical model to predict the compressive response of high strength short concrete columns with square cross-section confined by transverse steel is presented. The model allows one to estimate the equivalent confinement pressures exercised by transverse steel during the loading process taking into account of the interaction of the stirrups with the inner core both in the plane of the stirrups and in the space between two successive stirrups. The lateral pressure distributions at hoop levels are obtained by using a simple model of elastic beam on elastic medium simulating the interaction between stirrups and concrete core, including yielding of steel stirrups and damage of concrete core by means of the variation in the elastic modulus and in the Poisson's coefficient. Complete stress-strain curves in compression of confined concrete core are obtained considering the variation of the axial forces in the leg of the stirrup during the loading process. The model was compared with some others presented in the literature and it was validated on the basis of the existing experimental data. Finally, it was shown that the model allows one to include the main parameters governing the confinement problems of high strength concrete members such as: - the strength of plain concrete and its brittleness; - the diameter, the pitch and the yielding stress of the stirrups; - the diameter and the yielding stress of longitudinal bars; - the side of the member, etc.

Keywords: compression; high strength concrete; confinement stress-strain curves.

1. Introduction

The use of high strength concrete (HSC) has increased in the last decades especially addresses to the realization of high-rise buildings, long shear span bridges and off-shore platforms.

HSC with cylinder strengths in the range of 60 MPa to 100 MPa can now be produced economically wherever superplasticizer and high quality aggregate are available, and with very high strength when reactive powder, silica fume, etc. are utilised. Although HSC is characterized by high performances attributes such as high module, high strength to density ratio and improved durability, it is also characterized by higher brittleness compared to normal strength concrete (NSC).

It is well known starting from 1970 that if normal strength concrete members are confined by transverse steel and longitudinal bars, increases in the bearing capacity and in the corresponding strain are observed (see e.g., Amahd and Shah, 1982, Park *et al.* 1982, Mander *et al.* 1988). Studies

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produces in the last decades for HSC members (Cusson and Paultre 1994, Cusson and Paultre 1995, Foster *et al.* 1998, Razvi and Saatcioglu 1999, Liu and Foster 2000) focused the attention on the principal parameters governing the confinement in the concrete core such as: - concrete grade; - shape of the transverse cross-section of the members to be reinforced in relation to the type and grade of transverse and longitudinal steel (hoops, spirals, ties, jackets, etc.); - size of the specimens; - rate of loading; etc.

From mentioned studies it is evident that for a correct evaluation of the confinement effects in R.C. members has to be considered properly several factors such as: - concrete strength; - volume of confining reinforcement; - the actual stresses in ties; -non uniform confining pressures; -section geometry; -size and grade of steel utilised, etc. In the case of members with square or rectangular cross-section confined by transverse steel (stirrups and ties) many questions arises on the confinement effects especially referring to the evaluation of effective stress in transverse steel and on the confinement pressures distribution developed at maximum compressive strength of confined concrete necessary to predict accurately the compressive behaviour of confined concrete. Specifically, despite the many advantages of HSC two important questions have to be addressed properly: - the smaller confinement efficiency of ties in HSC columns; the second order effects on slender HSC columns.

From theoretical point of view many studies were addressed to predict the compressive response of NSC (see e.g., Mander *et al.* 1988) and more recently of HSC confined members (Cusson and Paultre 1995, Razvi and Saatcioglu 1999, Mau *et al.* 1998). From these studies it emerges that widely accepted model for rectilinear confined HSC does not exist.

In the present paper, after a brief presentation of the most common models utilised in the literature for the study of confinement effects in HSC members with square cross-section, a mechanical model is proposed able to predict the whole compressive response of confined HSC members and a comparison with existing models and with available experimental data is made.

2. Case of study and references models

Several models are available in the literature to analyse the confinement effects produced by transverse steel reinforcements on compressed high strength concrete (HSC) members with circular, square or rectangular cross-section reinforced with longitudinal and transverse bars (e.g. Cusson and Paultre 1995 and Razvi and Saaatcioglu 1999). These models allow one to evaluate the strength and strain enhancements due to transverse steel and give the stress-strain curves in compression including also the post-peak response.

Basic studies on the effect of confinement induced by transverse steel in normal strength concrete have as their starting point the experimental researches carried out by Richard *et al.* (1929). These studies have shown that if a uniform lateral pressure is applied to a cylindrical concrete specimen, a proportional strength increase is observed and a simple linear relationship can be assumed in the form

$$\frac{f_{cc}}{f_{co}} = 1 + k_1 \cdot \frac{f_1}{f_{co}} \tag{1}$$

f_{cc}, f_{co} being, respectively, the compressive strength of the confined and unconfined concrete, f_1 the lateral uniform confinement pressure and k_1 an empirical coefficient depending on the concrete type and assumed to be equal to 4.1.

It has to be remind that Eq. (1) was derived referring to a compressed cylindrical specimen subjected to an axial load and to a lateral uniform confinement pressure, but it can also be utilised in the case of passive confinement induced by transverse steel in which non-uniform confinement pressures develop. In this case, in order to utilise Eq. (1) a preliminary analysis to determine the effective confinement pressures induced by transverse steel is required and the equivalent uniform confinement pressure must be estimated.

For HSC elements, confined by transverse steel, Razvi and Saatcioglu (1999) highlight the fact that it is still possible to use Eq. (1), but with k_1 variable with the Poisson coefficient ν_c and assuming higher values when the confinement effect is maximum.

Razvi and Saatcioglu (1999), based on results of regression analyses of experimental data available in the literature, showed that the relationship between k_1 and f_1 is not linear and they assumed

$$k_1 = 6.7 \cdot f_1^{-0.17} \tag{2}$$

Similarly, Cusson and Paultre (1995) have shown that non linear relationships between f_{cc} and f_1 can be adopted in the form

$$\frac{f_{cc}}{f_{co}} = 1 + 2.1 \cdot \left(\frac{f_1}{f_{co}}\right)^{0.7} \tag{3}$$

Mander *et al.* (1988) estimate the increase in maximum compressive strength and in ductility due to the confinement effect induced by transverse steel by introducing the concept of "effective lateral confinement pressure" f_{1e} replacing f_1 (uniform confinement pressures) exercised by the transverse steel for different shapes of the transverse cross-section.

The starting point to determine f_{1e} is the consideration that the effective confinement pressure distribution due to the interaction between concrete core and transverse steel effectively develops on a confined concrete core A_e reduced with respect to the whole transverse cross-section A_{cc} (it is the cross area purged of the area of the longitudinal bars). Moreover, determining the uniform lateral confinement pressure f_1 by means of simple equilibrium conditions, the effective confinement pressure is obtained reducing f_1 through a confinement coefficient k_e, defined as $k_e = A_e/A_{cc} \leq 1$.

Therefore, the effective confinement pressure is obtained by means of the following expression

$$f_{1e} = f_1 \cdot k_e \tag{4}$$

For the members of Fig. 1, Mander *et al.* (1988) consider in the calculus of k_e and f_1 that the effective confinement lateral pressures act along a curved surface, which in the vertical sections is represented by a second-degree parabola with initial tangent 45° between two successive hoops.

Denoting with s' the net spacing between two successive hoops (it is evaluated purging the whole distance measured in the hoop axis of the diameter of the hoops), it is possible to obtain

$$k_e = \frac{A_e}{A_{cc}} = \frac{\left(1 - \sum_{i=1}^{i=4} \frac{(w_i)^2}{6 \cdot L^2}\right) \cdot \left(1 - \frac{s'}{2 \cdot L}\right)^2}{1 - \rho_{cc}} \tag{5}$$

Finally, the lateral confinement pressure f_1 is determined by utilizing equilibrium considerations on

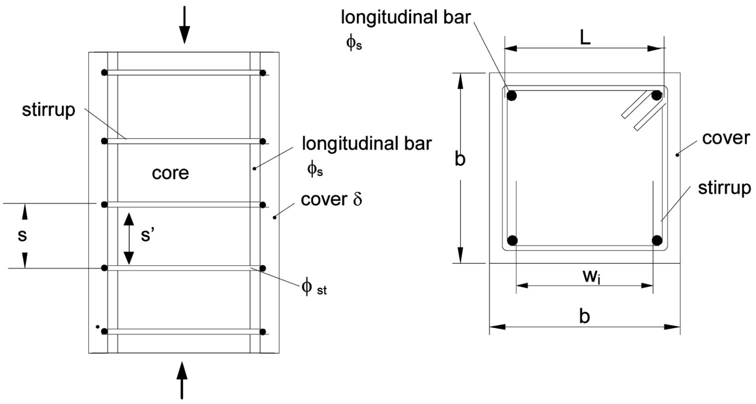

Fig. 1 Details of column under investigation

half the cross-section considered as a rigid body and by assuming the hoops to have yielded

$$f_1 = \frac{1}{2} \cdot \rho_s \cdot f_y \tag{6}$$

f_y being the yielding stress of the hoops and ρ_s the geometrical ratio of stirrups in the pitch s defined as $\rho_s = 2 \cdot A_{st}/L \cdot s$.

For high strength concrete members (HSC) confined by transverse steel Razvi and Saatcioglu (1999) observed that f_1 is related to the effective working stress in the hoops reached at the maximum compressive strength of the concrete. Also observed that in several cases and especially when transverse steel with high yielding stress is utilized, its value is lower than the yielding stress f_y, therefore, the use of Eq. (6) overestimates the confinement pressure.

For this reason Razvi and Saatcioglu (1999) suggest utilizing Eq. (6), but substituting the yielding stress f_y with the effective f_s deduced by regression analyses of experimental data and assumed as

$$f_s = E_s \cdot \left(0.0025 + 0.04 \cdot \sqrt[3]{\frac{k_2 \cdot \rho_s}{f_{c0}}}\right) \le f_y \tag{7}$$

E_s being the elastic modulus of steel.

In Eq. (7) k_2 is a semi-empirical coefficient taking into account the effectiveness of the arrangement of the longitudinal bars and transverse steel both in the cross-section and along the height. Razvi and Saatcioglu (1999) suggest to adopt the following expression for k_2

$$k_2 = 0.15 \cdot \sqrt{\frac{L}{s} \cdot \frac{L}{s_1}} \le 1 \tag{8}$$

s_1 being the distance between two adjacent longitudinal bars (see Fig. 2). The k_2 coefficient takes into account that confinement pressure in the plane of cross-section and in the space among two successive stirrups is not uniform (see Fig. 2).

Cusson and Paultre (1995) have shown that suppose transverse steel yielded at maximum compressive strength course wrong previsions of bearing capacity especially in HSC members lightly confined or when high strength steel bars are utilised. Therefore, they suggest to adopt Eq. (6) replacing f_y with the actual stress f_s at maximum compressive strength, the latter determined by using non linear iterative procedure. Moreover, Cusson and Paultre (1995) to take into account

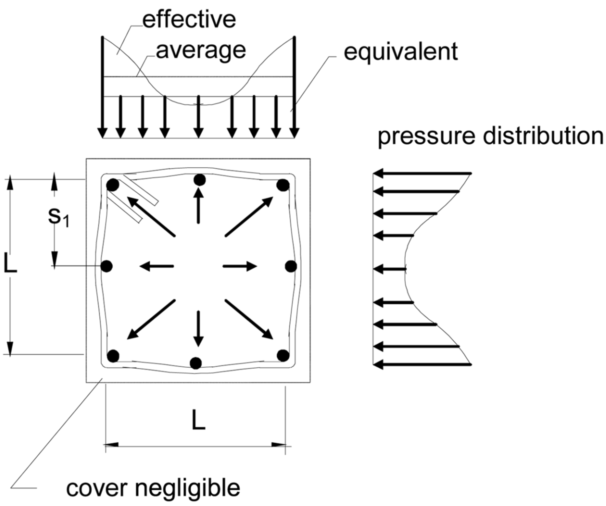

Fig. 2 Effective confinement pressure

of not uniform confinement pressures exercised by transverse steel in the plane of the stirrups and in the space between two successive stirrups adopt the k_e coefficient originally proposed by Mander *et al*. (1988).

Some other authors consider the not uniform confinement pressures distribution between two successive stirrups introducing a reduction factor k_{sv} of the average confinement pressured, the latter, determined at hoop level (see Eq. (6)).

e.g. Sheikh and Uzumeri (1980) suggest for k_{sv} the following expression

$$k_{sv} = \left(1 - \frac{s}{4 \cdot L}\right)^2 \tag{9}$$

Tim and Yip (1999) suggest analogously adopting

$$k_{sv} = 1 - \frac{s}{2 \cdot L} \tag{10}$$

Braga *et al*. (2006) based on a simplified analytical model adopt an analytical expression for k_{sv} able to take into account of L, s and also of the diameter of stirrups ϕ_{st} and of longitudinal bars ϕ_l adopting the following expression

$$k_{sv} = \frac{45 \cdot \left(\frac{\phi_1}{s}\right)^3}{45 \cdot \left(\frac{\phi_1}{s}\right)^3 + \frac{\phi_{st}}{\phi_l} \cdot \frac{\phi_{st}}{L}} \tag{11}$$

More recently Teerawong *et al*. (2004) performing 3-D finite element analyses demonstrate that the effective confining stress at hoop level $\sigma_{h,\,eff}$ can be related to the average effective confining stress σ_{eff} assuming

$$k = \frac{\sigma_{h,\,eff}}{\sigma_{eff}} = 1 + 3.1342 \cdot \frac{s}{B'_c} \tag{12}$$

Moreover for the evaluation of $\sigma_{h,eff}$ Teerawong *et al*. (2004) refer to the model of Mau *et al.* (1998) which gives a reduction coefficient of average confinement pressures at hoop level with the expression

$$f = 1 - 0.575 \cdot \frac{s}{a} \tag{13}$$

a being $B_c'/2$.

In the following sections for comparison between analytical model it will be adopted a reduction factor of average confinement pressure expressed according to Teerawong *et al*. (2004) by *kxf*, the k_e coefficient according to Mander *et al*. (1988) and k_2 according to Razvi and Saatcioglu (1999).

To determine the whole response of compressed members in terms of stress-strain (σ–ε) curves for NSC members a well-known model is that proposed by Mander *et al*. (1988), based on the use of the analytical stress-train curve originally given by Popovics (1973) in the form

$$\sigma = f_{cc} \cdot \frac{\left(\frac{\varepsilon}{\varepsilon_{cc}}\right) \cdot r}{r - 1 + \left(\frac{\varepsilon}{\varepsilon_{cc}}\right)^r} \tag{14}$$

with $x = \varepsilon/\varepsilon_{cc}$ and ε_{cc}, the axial strain of confined concrete corresponding to the maximum compressive strength f_{cc}, calculated by means of

$$\varepsilon_{cc} = \varepsilon_{c0} \cdot \left[1 + 5 \cdot \left(\frac{f_{cc}}{f_{co}} - 1\right)\right] \tag{15}$$

The coefficient r of Eq. (11) is evaluated by means of $r = \frac{E_c}{E_c - \frac{f_{cc}}{\varepsilon_{cc}}}$, with $E_c = 5700 \cdot \sqrt{f_c'}$ (MPa) the tangent initial modulus of elasticity of NSC concrete.

The model of Mander *et al*. (1988) is based essentially on knowledge of three fundamental parameters (f_{cc}, ε_{cc} and E_c), but it is not suited to accurately predicting the response of confined HSC members, because it assumes that the confinement pressure is constant during the loading process and is equal to the value producing yielding of transverse steel for low strain values too. Moreover, Eq. (14) is not able to capture the brittle stress-strain response of HSC matrices. Razvi and Saatcioglu (1999) propose adopt Eq. (14) for the ascending branch of the stress-strain curves in compression of confined HSC members but with the following modification: - the initial tangent elasticity modulus E_c is evaluated with the expression (MPa)

$$E_c = 6900 + 3320 \cdot \sqrt{f_c'} \text{ (MPa)} \tag{16}$$

For the evaluation of the compressive strain ε_{cc} at peak stress the following expression of empirical nature is assumed

$$\varepsilon_{cc} = \varepsilon_{c0} \cdot \left[1 + 5 \cdot \frac{k_1 \cdot f_{1e}}{f_{co}}\right] \tag{17}$$

with k_1 given by Eq.(3) and ε_{c0} given b:

$$\varepsilon_{c0} = 0.0028 - 0.0008 \cdot \frac{40}{f_{co}} \tag{18}$$

For the post-peak response of HSC members Razvi and Saatcioglu (1999) suggest to adopt a linear σ–ε relationship, intercepting the point of coordinates (ε_{cc}, f_{cc}), and that having ordinate 0.2 f_{cc} and abscissa ε_{085} defined as

$$\varepsilon_{c085} = \varepsilon_{c0} + 0.0018 \cdot \left(\frac{40}{f_{co}}\right)^2 \tag{19}$$

Cusson and Paultre (1995) adopt for HSC confined compressed members two different σ–ε relationships: the first one from zero stress up to the peak stress expressed through Eq. (8) and the second one from the peak stress up to 50% of f_{cc} corresponding to a strain ε_{C50C} and in which a linear variation of stresses was supposed.

Cusson and Paultre (1995) suggest adopting

$$\varepsilon_{cc} = \varepsilon_{co} + 0.21 \cdot \left(\frac{f_{1e}}{f_{co}}\right)^{1.7} \tag{20}$$

And ε_{C50C} expressed through

$$\varepsilon_{C50C} = \varepsilon_{C50U} + 0.15 \cdot \left(\frac{f_{1e}}{f_{co}}\right)^{1.1} \tag{21}$$

With ε_{C50U} strain value for unconfined concrete assumed 0.004.

Based on the abovementioned considerations, in the following sections a new analytical model will be presented that is able to predict the whole σ–ε relationship for HSC compressed members confined by transverse steel and longitudinal bars.

3. Proposed model

The case examined here is the one related to a short member having a square cross-section (already shown in Fig. 1) and confined by transverse closed steel stirrups with diameter ϕ_s and area A_{st} placed at pitch s with a cover δ. Longitudinal bars are placed at the four corners of the cross-section of sided L. No size effect (important in full scale members) was considered, and negligible cover was supposed.

The model here proposed essentially is addressed to determine the confinement pressures induced by transverse steel in the space between two successive stirrups and therefore the equivalent confinement pressures to utilize for the determination of the maximum compressive strength and strain capacities by Eqs. (1), (2), (20).

A simplified analysis is carried out to explain in a simplified way the confinement effects in the concrete core due to the presence by transverse stirrups, which can be pointed out numerically by a non linear finite element approach as made in Teerawong *et al.* (2004) or as made in Braga *et al.* (2006) utilising a continuum elastic model.

The model proposed here refers to the geometrical model shown in Fig. 3, representing a three dimensional prismatic concrete member having square cross-section of side L and confined by transverse stirrups. If the concrete member is loaded axially, and maintains its prismatic shape, it

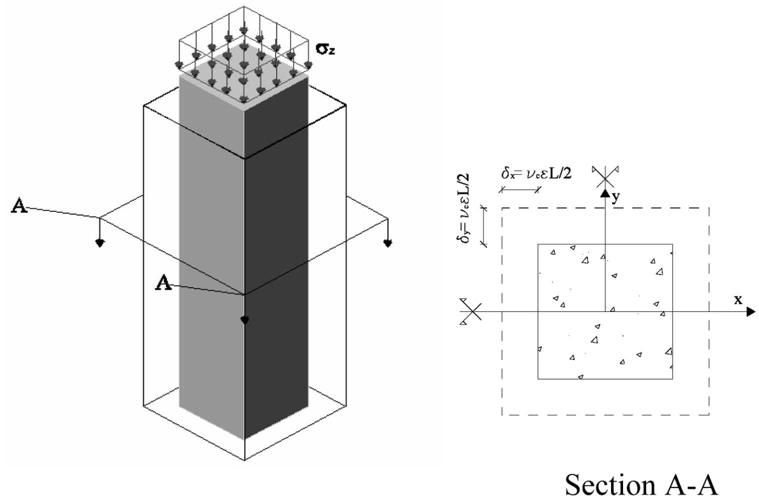

Fig. 3 Geometrical model for compressed concrete members

tends to be subjected to an axial strain ε and a lateral strain with corresponding lateral displacement of concrete core (without confinement pressure) expressed as $\delta = \nu_c \cdot \varepsilon \cdot L/2$, ν_c being the elastic Poisson coefficient of the concrete core. This displacement δ is partially reduced by the presence of the transverse stirrups. Because of the adhesion between concrete surface and transverse steel, shear stress distribution arises at the interfaces with significant values up to complete cover spalling process occurs, with intensification of axial stresses in the stirrups close to the corners of the cross-section. Moreover, the interaction between stirrups and concrete core produces in the direction perpendicular to the stirrups, non uniform distribution of confinement pressures and axial forces variable in the stirrups. Following only confinement pressures will be considered, while shear stresses will be neglected for simplicity and increases in maximum strength is mainly due to confinement pressures. Moreover axial forces will be supposed constant along the perimeter of the stirrup.

In the next sections the focus will be on the determination of the confinement pressures at hoop level, therefore it will be on the determination of confinement pressures in the concrete core between two successive stirrups.

3.1 Confinement pressures at hoop levels

In the case of compressed prismatic concrete member it can be assumed that the section is in a plane state of deformation, with the normal stresses in the plane of the cross-section and σ_z parallel to the vertical axis of the member. It is possible to further simplify the three dimensional model of Fig. 3 by assuming a plane model (see Fig. 4) and considering, for the symmetry of the system, only one quarter of the transverse cross-section. Particularly it is considered the plane of the stirrup in which is enclosed a concrete shell in a plane state of strain.

The model is further simplified considering that, when lateral expansion occurs in the concrete member, the displacement along the corner of the diagonal direction in the concrete shell can be assumed the one corresponding to the lateral elongations along the two beams of length $L/2$ supported by unilateral elastic springs (see Fig. 4).

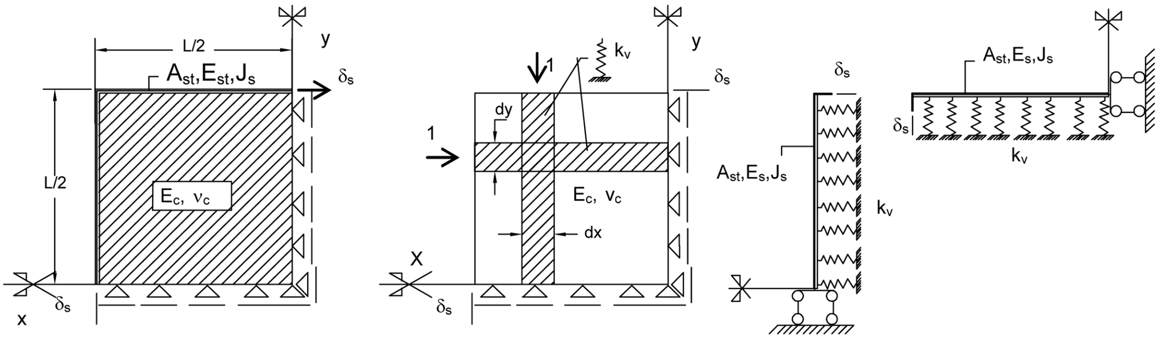

Fig. 4 Modelling of column ties as beams on an elastic foundation

The stiffness of the springs, analogously to the problem of elastic beam on elastic soil, has dimension of a stress divided by a length. Its value is obtained as $k_v = 2 \cdot E_c/L \cdot (1 - \nu_c)$, E_c is the modulus of elasticity of concrete. This expression was derived (in a simplified way) considering the axial stiffness of a beam of base *dx* in *x* direction and *dy* in *y* direction, unit depth and length *L*/2 further increased by considering the term $(1 - \nu_c)$ to take into account of the plane problem (see Fig. 4).

These spring as shown in Fig. 4 act in the direction perpendicular to the beam axis (leg of the stirrup) and simulate the interaction at hoop level between stirrup and concrete core.

The effect of axial forces of the stirrups was taken into account by imposing (as it will show in the following section) that the displacement at the end of the beam (corner of cross-section) is equal to the elongation of the stirrup's leg in the perpendicular direction denoted as δ_s.

The two elastic beams simulating the stirrups have flexural stiffness proportional to the quantity $E_s \cdot J_s$, J_s is the moment of inertia of transverse cross-section of stirrups assumed to be $J_s = \pi \cdot \phi_{st}^4/64$ with ϕ_{st} the diameter of stirrups simulating the contact base of stirrup with the beam on elastic springs.

The beam is subjected to distortion in the points of contact between the hoops and longitudinal bars equal to δ_s corresponding in the confined concrete core (it is the volume enclosed in the plane of stirrups and between two successive stirrups at pitch *s*) to a force *F* (F_x in *x* direction equal for the equilibrium to F_y in *y* direction) producing the $w(x)$ deflection of the beam.

The equilibrium equation of the elastic beam of inertia J_s on elastic springs in a deformed configuration, in term of lateral displacements *w*, is governed by the following differential equation.

$$\frac{d^4w}{dx^4} + \frac{k_v}{E_s \cdot J_s}(\delta - w) = 0 \qquad (22)$$

By introducing the β parameter defined as

$$\beta = \sqrt[4]{\frac{k_v \cdot \phi_{st}}{4 \cdot E_s \cdot J_s}} \qquad (23)$$

Eq. (22) can be rearranged in the homogeneous equation in the form

$$\frac{d^4w}{dx^4} + 4 \cdot \beta^4 \cdot (\delta - w) = 0 \qquad (24)$$

It has to be observed that the β parameter assumes the role of relative confined core-stirrups stiffness and in can be expressed in explicit form by

$$\beta = 1.79 \cdot \phi_{st} \cdot \sqrt[4]{\frac{1}{E_s \cdot \phi_{st}} \cdot \frac{E_c}{1-\nu_c} \cdot \frac{1}{L}} \tag{25}$$

The solution of the differential equation for the deflection curve is

$$w(x) = \delta + A \cdot \cosh\beta x \cdot \cos\beta x + B \cdot \sinh\beta x \cdot \sin\beta x + C \cdot \sinh\beta x \cdot \cos\beta x + D \cdot \cosh\beta x \cdot \sin\beta x \tag{26}$$

After the reference system with the axis of the abscissa coinciding with the axis of the transverse stirrups bars and originating in the section $L/2$ is assumed, taking into account the symmetry conditions, Eq. (26) can be simplified as follows

$$w(x) = \delta + A \cdot \cosh\beta x \cdot \cos\beta x + B \cdot \sinh\beta x \cdot \sin\beta x \tag{27}$$

For calculation of the constants A and B in the extremity section ($x = +-L/2$) defined by the transverse legs of stirrups, the following boundary conditions are imposed: the displacement of the bar is equal to the displacement of the hoop; the rotation of the bar is zero

$$w(x)|_{x = L/2} = \delta_s; \quad \left.\frac{dw(x)}{dx}\right|_{x = L/2} = 0 \tag{28}$$

By imposing the boundary conditions it results

$$\begin{cases} A = -2 \cdot (\delta - \delta_s) \cdot \dfrac{\sinh\left(\frac{\beta \cdot L}{2}\right) \cdot \cos\left(\frac{\beta \cdot L}{2}\right) + \cosh\left(\frac{\beta \cdot L}{2}\right) \cdot \sin\left(\frac{\beta \cdot L}{2}\right)}{\sinh(\beta \cdot L) + \sin(\beta \cdot L)} \\ B = 2 \cdot (\delta - \delta_s) \cdot \dfrac{\sinh\left(\frac{\beta \cdot L}{2}\right) \cdot \cos\left(\frac{\beta \cdot L}{2}\right) - \cosh\left(\frac{\beta \cdot L}{2}\right) \cdot \sin\left(\frac{\beta \cdot L}{2}\right)}{\sinh(\beta \cdot L) + \sin(\beta \cdot L)} \end{cases} \tag{29}$$

Hence, for $-L/2 \le x \le L/2$, it is possible to find in the pattern of the confinement pressure $q(x)$ due to the presence of the stirrup through the expression

$$q(x) = k_v \cdot [\delta - w(x)] \tag{30}$$

Adopting the proposed model it is possible to obtain the deformation shape at hoop level of the cross-section and the confinement pressures distribution acting on the legs of the stirrup as shown in Fig. 5.

If the confinement pressure is integrated in the plane of the stirrup it is possible to introduce the equivalent uniform confinement pressure in the plane of the stirrups defined as

$$\bar{q} = \frac{1}{L} \cdot \int_{-L/2}^{L/2} q(x)dx = \frac{k_v}{L \cdot \beta} \cdot \left[(A+B) \cdot \cosh\left(\frac{\beta \cdot L}{2}\right) \cdot \sin\left(\frac{\beta \cdot L}{2}\right) + (A-B) \cdot \cos\left(\frac{\beta \cdot L}{2}\right) \cdot \sinh\left(\frac{\beta \cdot L}{2}\right)\right] \tag{31}$$

It has to be observed that A and B depend on δ and δ_s. the latter depending on the level of the

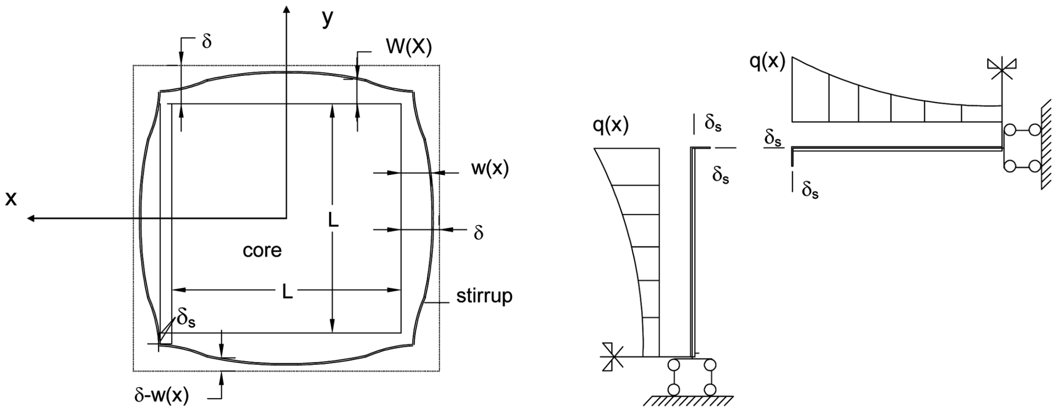

Fig. 5 Deformed shape of transverse stirrup and reference simplified physical model

axial force F acting on it. Its value will be determined in the next section by imposing the equilibrium of the confinement pressures in the volume of concrete core enclosed In the pitch s.

3.2 Equivalent confinement pressures between two stirrups

The equivalent uniform confinement pressures $\bar{q}$ at hoop level (see Eq. (31)) do not reflect the discontinuities of confinement pressures between two successive stirrups therefore a reducing effectiveness coefficient (see e.g. Eqs. (9), (10), (11)) has to be introduced.

This coefficient reflects the circumstance that the confinement pressures distribution induced by transverse steel is not uniform, not only in the plane of the stirrup, but also in the space between two successive stirrups. The analytical determination of the distribution of these pressures is a difficult problem to solve in prismatic members taking into account of non linear behavior of constituent materials. In the case of cylindrical members confined by circular hoops it was recently demonstrate that for elastic behavior of concrete an exact solution of the mechanical problems exists (Mau *et al.* 1998), while form prismatic members with square cross-section approximate solution based on the use of finite element method are generally proposed (see e.g. Mau *et al.* 1998 or Teerawong *et al.* 2004) or finally as shown by Braga *et al.* (2006) utilizing a continuum model based on the knowledge of the Airy' Functions for members in plane state of strain to determine the confinement pressures and shear stresses distribution at hoop level and confinement pressures distribution between two successive stirrups.

In the present paper it was supposed that in a generic point at distance z from the leg of the stirrup the confinement pressures distribution can be expressed by an analytical equation $A(z)$ giving at a generic point enclosed between two successive stirrups the confinement pressures expressed by means of

$$p(x, z) = q(x) \cdot A(z) \tag{32}$$

Fig. 6 shows the qualitative 3-D variation of confinement pressures generated by stirrups expressed by Eq. (32) with $A(z)$ assumed as exponential function.

By integrating $p(x, z)$ in the space s the resultant of confining pressures acting in the plane of

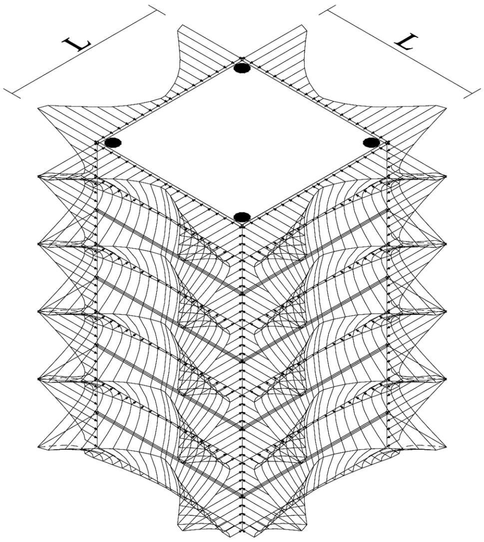

Fig. 6 3-D variation of confinement pressures generated by stirrups

sides L and s can be expressed as

$$R = \int_{-s/2}^{s/2} \int_{-L/2}^{L/2} q(x) \cdot A(z) \cdot dx \cdot dz = \int_{-L/2}^{L/2} k_v \cdot [\delta_r - w(x)]dx \cdot \int_{-s/2}^{s/2} A(z) \cdot dz = L \cdot s \cdot \bar{q} \cdot k_{sv} \quad (33)$$

To determine R the knowledge of the expression of $A(z)$ function is not necessary if k_{sv} is assumed as

$$k_{sv} = \int_{-s/2}^{s/2} A(z) \cdot dz = e^{-\frac{3}{2} \cdots \frac{s}{L}} \quad (34)$$

This choice for k_{sv} is in agreement with the analytical models available in to the literature (see Eqs. (9), (10), (11)) and also it allows to consider the limit conditions corresponding to the circumstances that for $s = 0$ results $k_{sv} = 1$ (maximum confinement effect) and for $s \geq 2 \cdot L$ the value of k_{sv} its negligible (such is obtained utilizing Eqs. (9), (11) and confirmed experimentally in the literature). Moreover, Eq. (34) is the integral of an exponential function, the latter is a possible function to reproduce the variation of the confinement pressures distribution in the space between two successive stirrups as also recently observed by Braga *et al*. (2006).

Fig. 7 shows the variation of the k_{sv} coefficient with s/L obtained with Eq. (34) and also the analogous coefficients given by Eqs. (9), (10), (11). In the comparison of Fig. 7, Eq. (11) was utilized referring to two longitudinal bars having diameter 20 mm *e* and stirrups having 8 mm diameter. The comparison shows a good agreement between the mentioned models above.

To determine the F force on the stirrups it is necessary to consider the equilibrium between the confinement pressure in the space of sides L and s and the axial forces F in the legs of the stirrup (see Fig. 8) resulting

$$R = 2 \cdot F = L \cdot s \cdot \bar{q} \cdot k_{sv} \quad (35)$$

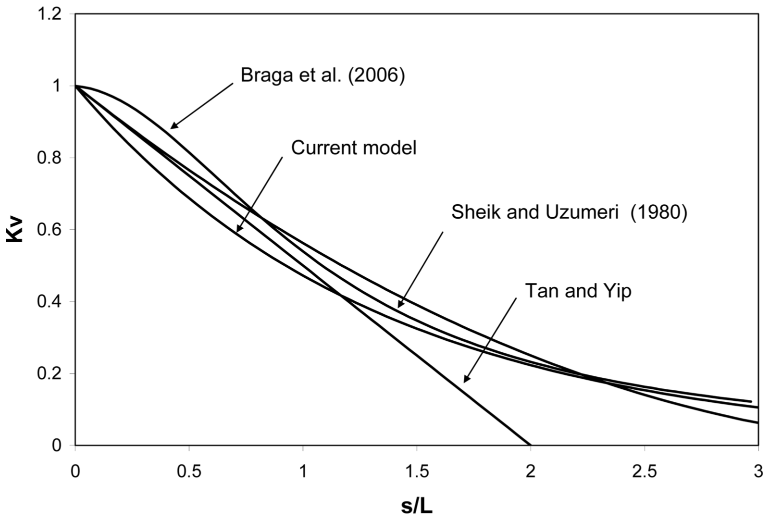

Fig. 7 k_{sv} efficiency vertical factor

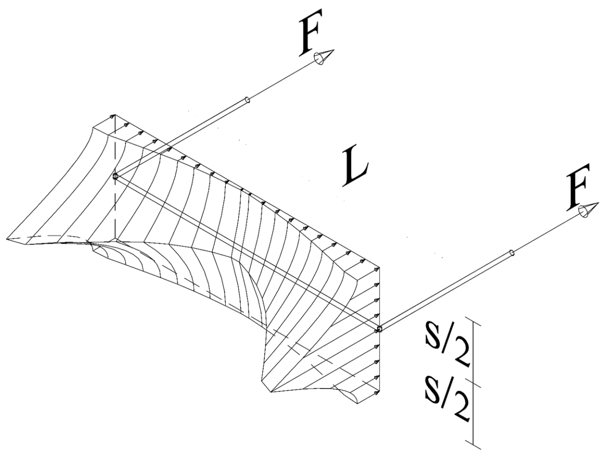

Fig. 8 Equilibrium of internal forces at hoop levels

In the elastic range the F force is related to the elongation δ_s by the relation

$$\delta_s = \frac{F \cdot L}{2 \cdot E \cdot A_{st}} \tag{36}$$

E being the modulus of steel equal to E_s in the elastic range or E_h in the hardening phase.

To determine the F force on the stirrups, for each axial shortening value, it is possible utilizes Eq. (35) by means of Eq. (36) and Eq. (32) resulting

$$F = \frac{\frac{k_v}{\beta} \cdot [(A_1 - A_2) \cdot C + (A_1 + A_2) \cdot D] \cdot k_{sv}}{1 - \frac{1}{E_s \cdot A_{st}} \cdot (A_1 - A_2) \cdot (C + D) \cdot k_{sv}} \cdot \nu_c \cdot \varepsilon \cdot L \tag{37}$$

Adopting the position

$$C = \cosh\left(\frac{\beta \cdot L}{2}\right) \cdot \sin\left(\frac{\beta \cdot L}{2}\right) \quad (38)$$

$$D = \cos\left(\frac{\beta \cdot L}{2}\right) \cdot \sinh\left(\frac{\beta \cdot L}{2}\right) \quad (39))$$

$$A_1 = \frac{\sinh\left(\frac{\beta \cdot L}{2}\right) \cdot \cos\left(\frac{\beta \cdot L}{2}\right) + \cosh\left(\frac{\beta \cdot L}{2}\right) \cdot \sin\left(\frac{\beta \cdot L}{2}\right)}{\sinh(\beta \cdot L) + \sin(\beta \cdot L)} \quad (40)$$

$$A_2 = \frac{\sinh\left(\frac{\beta \cdot L}{2}\right) \cdot \cos\left(\frac{\beta \cdot L}{2}\right) - \cosh\left(\frac{\beta \cdot L}{2}\right) \cdot \sin\left(\frac{\beta \cdot L}{2}\right)}{\sinh(\beta \cdot L) + \sin(\beta \cdot L)} \quad (41)$$

If during the loading process F reaches the yielding value and if elastic-plastic behavior of transverse steel is supposed it values remains constant and equal to $F_y = f_y\, A_{st}$.

It must be borne in mind that the transverse stirrups are subject to tension and bending moment (shear forces are neglected) that are proportional to the axial stiffness and to a transversal load that, in the absence of concrete cover favours its yielding.

During the loading process it has to be observed that with the increasing in the deflection of the stirrups out of the plane the bending moment and the axial forces increase therefore is has to be cheeked that the maximum flexural moment in the leg of the stirrups do not exceed the plastic moment determined e.g. as suggested in Chen and Sohal (1995) in the presence of axial forces F, procedure mentioned in details in Teerawong *et al*. (2004).

Fig. 9 shows the variation in the confinement pressure given by Eq. (30) multiplied by Eq. (34) with the variation in the x position along the leg of the stirrup (x = 0 is in the middle of the stirrup leg). In the same graph also the analogous distribution of confinement pressures obtained with the

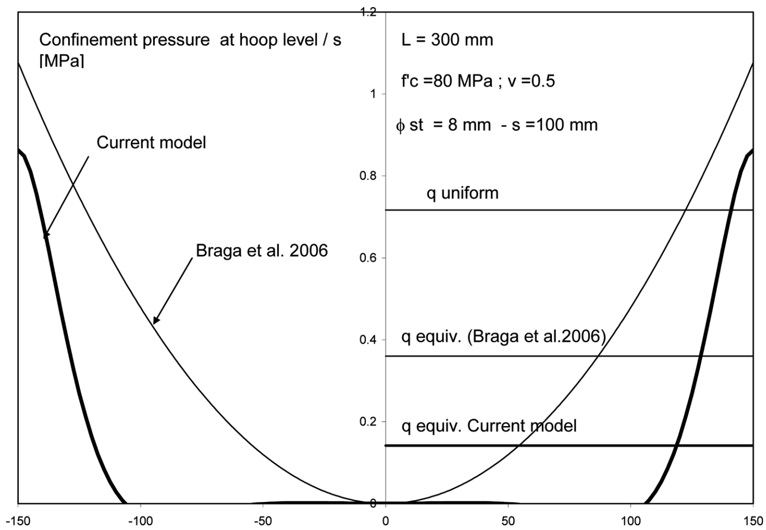

Fig. 9 Confinement pressures distribution at hoop levels according to the proposed model

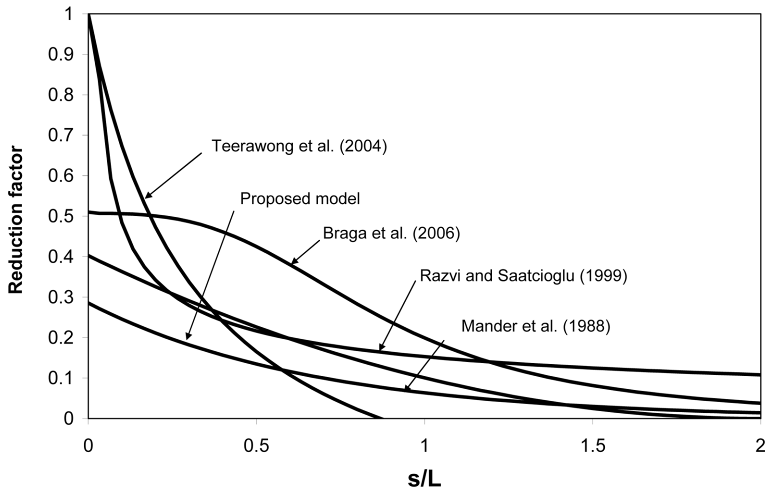

Fig. 10 Variation of reduction factors with *s/L*

model proposed by Braga *et al.* (2006) and uniform equivalent pressure given by Eq. (31) are given. The uniform confinement pressures were obtained supposing that for the case examined stirrups do not yield and assuming form equilibrium consideration between uniform confinement pressures and axial forces in the leg of the stirrups supposed to be in elastic range resulting $f_1 = \frac{E_s \cdot \pi \cdot \phi_{st}^2}{s \cdot L} \cdot \delta$.

Data utilised were L = 300 mm, s = 100 mm, v_c = 0.5, E_c = 2/3 × 35 GPa and E_s = 210 GPa and ϕ_{st} = 8 mm. It is interesting to observe that non linear variation of confinement pressures is activated and there is a significant difference between equivalent and uniform confinement pressures. It has to be observed that differences are obtained among the proposed model and the one proposed by Braga *et al.* (2006) model. The proposed model is more conservative because does not take into account of the shear stress-distribution which were considered in the model of Braga *et al.* (2006) to determine the axial force in the leg of the stirrups and therefore in the confinement pressures distribution. It has to be observed moreover that the proposed model allows one to consider that when ϕ_{st} increases and s spacing tends to zero value almost uniform confinement pressures are obtained while adopting the model proposed by Braga *et al.* (2006) the shape of confinement pressures does not change.

In the graph of Fig. 10 is shown the comparison of the proposed model with some of the other mentioned. In it is shown the variation with *s/L* of the: - reduction factor proposed by Mander *et al.* (1988) (k_e factor); - $f \cdot k$ factor adopted by Teerawong *et al.* (2004); - k_2 factor proposed by Razvi and Saatcioglu (1999); - reduction factor deduced with the model proposed by Braga *et al.* (2006); and $\bar{q}/f_1 \cdot k_{sv}$ factor assumed in the present paper. The comparison shows that although all the models mentioned take into account of the non uniform confinement pressure in the plane of the stirrups and between two successive stirrups substantial differences are observed.

3.3 Stress-strain curves of confined concrete and proposed incremental approach

To determine the compressive response of confined concrete in term of stress-strain curves it was adopted the σ–ε curves proposed by Sargin (1971) in the form rearranged by La Mendola and Papia (2002).

The relationship assumed is

$$\frac{\sigma}{f_{cc}} = \frac{A \cdot \frac{\varepsilon}{\varepsilon_{cc}} + (D-1) \cdot \left(\frac{\varepsilon}{\varepsilon_{cc}}\right)^2}{1 + (A-2) \cdot \frac{\varepsilon}{\varepsilon_{cc}} + D \cdot \left(\frac{\varepsilon}{\varepsilon_{cc}}\right)^2} \tag{42}$$

In which $A = E_{c0}/E^*_{sec}$ and $E^*_{sec} = f_{cc}/\varepsilon_{cc}$ and D was assumed 1.1 for concrete with $f_{co} > 45$ MPa.

The variation law of the ν_c coefficient with the axial strain ε was given in Elwi and Murray, (1979) expressed by

$$\nu = \nu_0 \cdot \left[1 + 1.38 \cdot \frac{\varepsilon}{\varepsilon_{cu}} - 5.36 \cdot \left(\frac{\varepsilon}{\varepsilon_{cu}}\right)^2 + 8.59 \cdot \left(\frac{\varepsilon}{\varepsilon_{cu}}\right)^3\right] \tag{43}$$

ν_0 being the elastic Poisson ratio assumed equal to 0.20, $\varepsilon_{cu} = 2\ \varepsilon_{c0}$ the ultimate strain of unconfined concrete.

The initial modulus of elasticity E_{c0} was adopted according to Cusson and Paultre (1995) as

$$E_{c0} = 6900 + 3320\sqrt{f_{c0}}\ (\text{MPa}) \quad \text{per } f_{c0} \geq 45\text{MPa} \tag{44}$$

In particular to plot the complete stress-strain curves depending on the axial shortening and on the confinement pressures a numerical procedure was adopted. The starting point is the assumption of Eq. (42), but referring to a curve intertwining with several curves given by Eq. (42), each pertaining to a level of confining pressure corresponding to the current axial and lateral strain values.

In particular the procedure is based on the following steps: - an initial value of axial shortening ε is assumed; - the lateral displacements $\delta = \nu_c \cdot \varepsilon \cdot L/2$ is computed assuming a fixed variation law of ν_c with ε (Eq. (43)); - the axial forces in the transverse steel is computed on the basis of the secant elasticity modulus of the concrete core and of the Poisson coefficient by using Eq. (43); - the effective confinement pressure is calculated by considering the average confinement pressure; - the compressive strength of the confined concrete is calculated using Eqs. (1), (2); - ε_{cc}, and finally σ are determined by means of Eqs. (15), (43); - repeating this procedure for all possible values of axial strain the complete stress-strain curve is plotted.

The whole stress was finally determined including also the contribution of longitudinal bars supposed with elastic-plastic behavior.

Fig. 10 shows typical stress-strain curves for short compressed members with square cross-section of side L = 250 mm cast with high strength concrete (80 MPa) and reinforced with stirrups having 4 mm diameter at pitch 50 mm and for three different grade of steel of 400, 800 and 1200 MPa, respectively. All curves are dimensionless. The stress and the strain are referred to the maximum strength and to the corresponding strain of unconfined concrete. The axial force in the stirrups is dimensionless with respect to the yielding force. From the graph it emerges clearly the influence of the grade of steel on the confinement effects in high strength concrete for given pitch and diameter of stirrups, resulting that increasing the yielding stress stirrups do not yield and the increasing in the confinement effects are not proportional with the increasing in the yielding stress, as widely confirmed experimentally in the literature.

4. Comparison with experimental results

Although a large number of experimental data are available in the literature for the compressive behaviour of high strength columns confined by transverse steel (see e.g. Cusson and Paultre 1994), in this section we refer to the experimental results given by Lima and Giongo (2004) and by Hong *et al*. (2006). This choice was related to the fact that both the experimental researches referred to high strength concrete members with square cross-section confined by single stirrups and in the presence of longitudinal bars. Mainly the interest is because quite different strength of concrete and grade of steel are considered and measurement of strain in the legs of the stirrups during the test were made. Table 1 gives the geometrical and mechanical properties of concrete specimens.

Fig. 11 refers to experimental data given in Lima and Giongo (2006) and referred to maximum

Table 1 Geometrical and mechanical characteristics of confined specimens

Ref.	L (mm)	ϕ_{st} (mm)	s (mm)	ρ_s (%)	f_y (MPa)	$\rho_s f_y$ (MPa)	f_c (MPa)	ε_{c0} (%)
Lima and Giongo (2004)								
	123.8	6.3	150	0.33	656	2.16	91	0.33
	123.8	6.3	50	1.00	656	6.56	91	0.33
	123.8	6.3	150	0.33	656	2.16	68	0.28
	123.8	6.3	50	1.00	656	6.56	68	0.28
Hong *et al*. (2006)								
	250	6.0	50	0.96	317	3.20	40.8	0.22
	250	6.4	25	1.92	1288	24.7	40.8	0.22
	250	6.4	25	1.92	1288	24.7	72.0	0.25
	250	6.4	25	1.92	1288	24.7	100.0	0.29

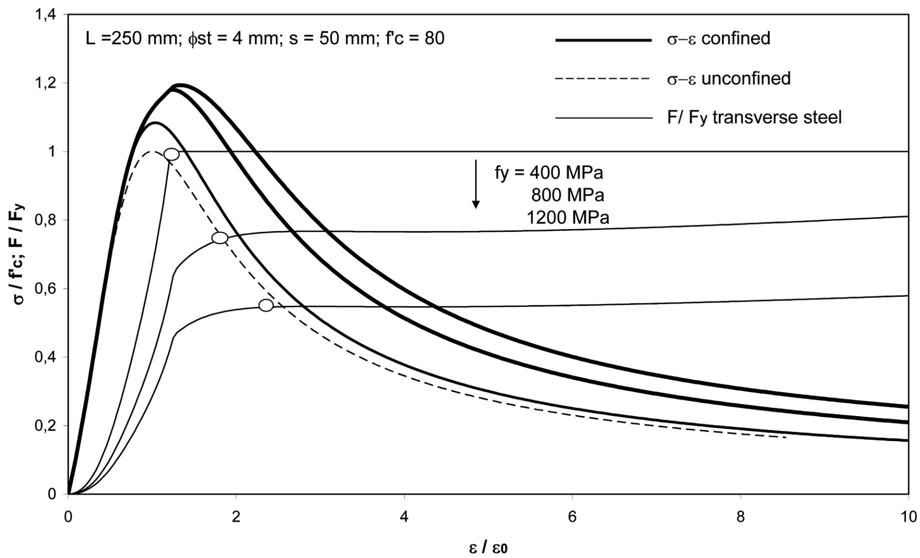

Fig. 11 Dimensionless stress-strain curves in compression

stress and stresses corresponding to 85% and 50% of the peak strength. In the same graph analytical curves obtained by using the proposed model are given. The comparison between curves shows a good agreement. From the analytical results it emerges, as already observed experimentally in Hong *et al*. (2006) that an increase in the volumetric ratio ρ_s increases the peak strength and strain values and the post-peak ductility is enhanced too, while the increase in compressive strength reduces ductility.

The comparison shows the ability of the model to predict the strength and strain enhancement due to confinement effect considering the effects of the cross-section shape and the mechanical properties of the constituent materials.

Fig. 12 shows the predicted stress in the hoops with the variation in the yielding stress and the experimental values given in Lima and Giongo (2004).

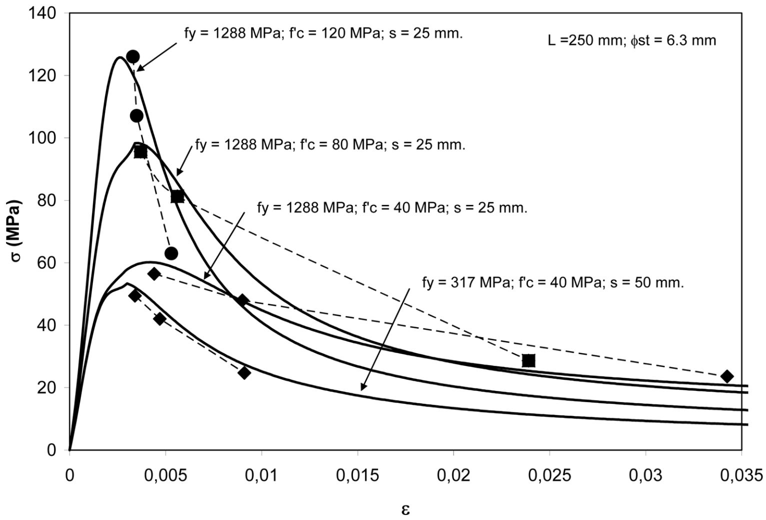

Fig. 12 Stress-strain curves: Proposed model and comparison with Hong *et al*. (2006) data

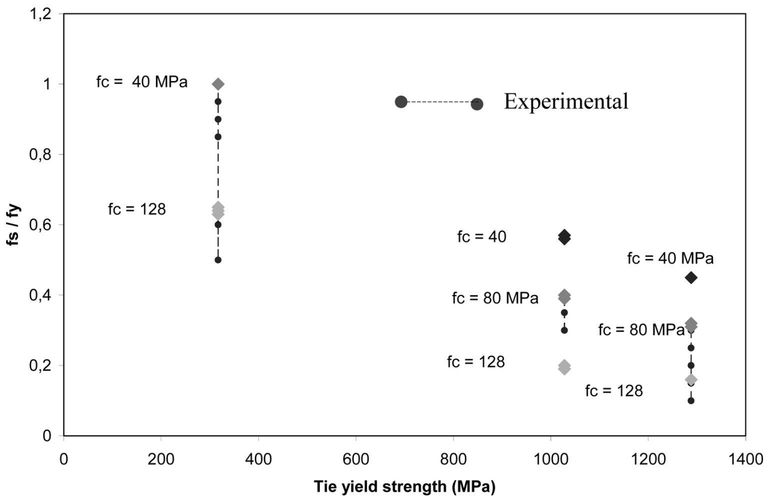

Fig. 13 f_s/f_y variation with f_y in lateral ties at the peak strength

Also in this case good agreement is observed and it emerges clearly that when yield strength increases maximum allowable stress in stirrups at concrete failure decreases, especially with the increase in the concrete strength.

Fig. 13 refers to the experimental data given in Hong *et al.* (2006) and mentioned in Table 1. All cases examined in Fig. 14 refer to members with square cross-section of side 123.8 mm with square cross-section confined with close stirrups having 6.3 mm diameter and 656 MPa yielding stress. Cases examined are different for concrete strength (68 and 91 MPa) and pitch of stirrups (150 and 50 mm). In the same graphs also prediction with the proposed model and with the other model mentioned and given in the literature (Mander *et al.* 1988, Cusson and Paultre 1995, Ravzi *et al.* 1999) are given. The comparison shown that the model of Cusson and Paultre (1995) is the best to fit almost all cases examined while, the model of Mander *et al.* (1988), as expected, gives the worst prediction for the case of high strength concrete. The model proposed here gives acceptable prediction of the experimental data and also it has the advantages that are supported by a mechanical model with clear physical meaning.

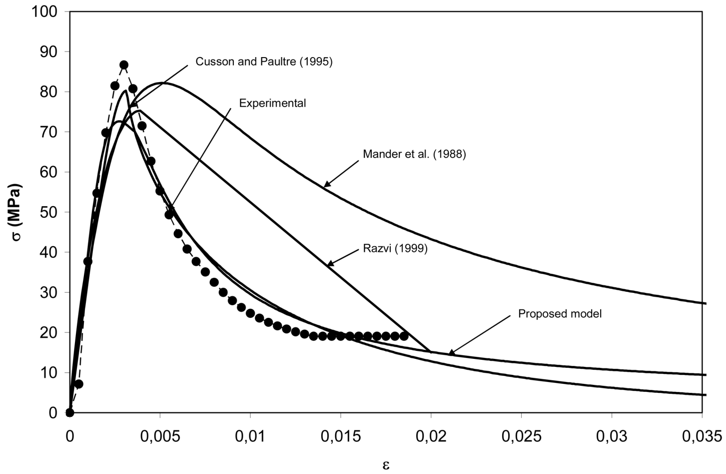

Fig. 14(a) Stress-strain curves in compression: s = 150 mm f_{co} = 68 MPa

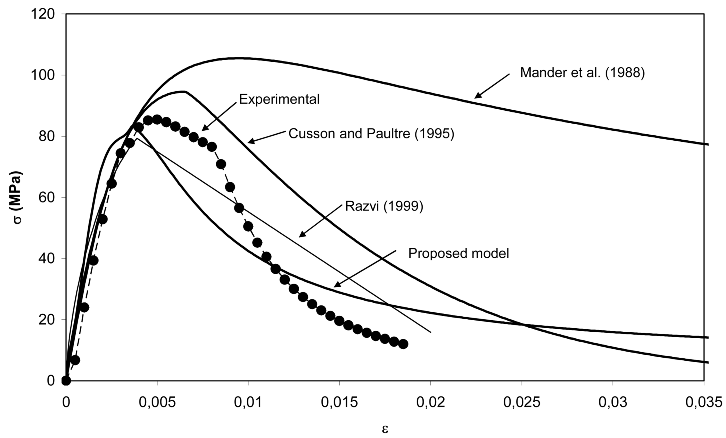

Fig. 14(b) Stress-strain curves in compression: s = 50 mm f_{co} = 68 MPa

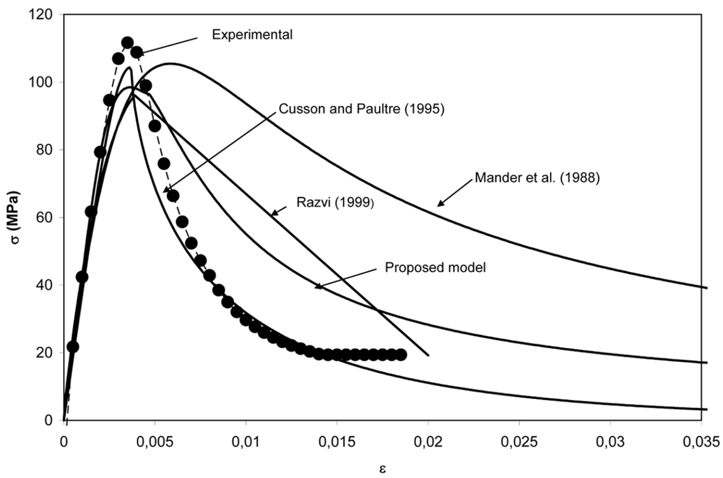

Fig. 14(c) Stress-strain curves in compression: s = 150 mm f_{co} = 91 MPa

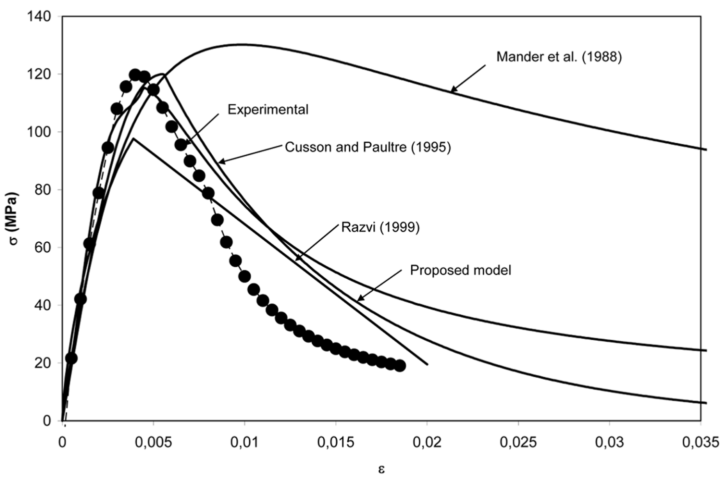

Fig. 14(d) Stress-strain curves in compression: s = 50 mm f_{co} = 91 MPa

5. Conclusions

A mechanical model to predict the compressive response of high strength short concrete columns with square cross-section confined by transverse steel is presented. The model allows one to estimate the equivalent confinement pressures exercised by transverse steel during the loading process taking into account of their interaction with the concrete core (both in the plane of the stirrups and in the space between two successive stirrups) and of the variation of the axial forces in the leg of the stirrup during the loading process. The model is able to take into account of the main parameters governing the confinement problems of high strength concrete members such as: - the strength of plain concrete and its brittleness; - the yielding stress of stirrups and their diameter and pitch; - the diameter and the yielding stress of longitudinal bars; - the side of the member, etc.

Moreover, from the application of the model it emerges that:

- confinement pressures distribution due to transverse steel are not uniform in the plane of the cross-section with maximum values across the corners;
- confinement pressures generated in the plane of the stirrups propagate decreasing intensity along the eight of the member;
- interaction between single stirrups is observed influenced by the pitch, and by the diameter of stirrups and by the concrete characteristics;
- yielding of transverse steel do not occurs at maximum compressive strength if lightly confinements and high strength steel are utilised;
- increasing the yield strength of steel do not significantly improve the confinement effect;
- for given diameter, pitch and grade of the steel confinement effects strongly depend on the concrete strength.

Finally, comparison between analytical results and experimental data given in the literature shows good agreement.

References

Ahmad, S.M. and Shah, S.P. (1982), "Stress-strain curves of concrete confined by spiral reinforcement", *ACI Struct. J.*, **79**(6), 484-490.

Braga, F., Gigliotti, R. and Laterza, M. (2006), "Analytical stress-strain relationship for concrete confiend by steel stirrusp and/or FRP jackets", *J. Struct. Eng.,* ASCE, **132**(9), 1402-1419.

Chen, W.F. and Sohal, I. (1995), "Plastic design and second-order analysis of steel frames", Book published by Springer-Verlag, New York, 509 pp.

Cusson, D. and Paultre, P. (1994), "High-strength concrete columns confined by rectangular ties", *J. Struct. Eng.,* ASCE, **120**(3), 783-804.

Cusson, D. and Paultre, P. (1995), "Stress-strain model for confined high strength concrete", *J. Struct. Eng.,* ASCE, **121**(3), 468-477.

Cusson, D. and Paultre, P. (1995), "Stress-strain model for confined high-strength concrete", *J. Struct. Eng.,* ASCE, **121**(3), 468-477.

Elwi, A.A. and Murray, D.W. (1979), "A 3D hypoelastic concrete constitutive relationship", *J. Eng. Mech.,* ASCE, **105**, 623-641.

Fafitis, A. and Shah, S.P. (1985), "Lateral reinforcement for high-strength concrete columns", *ACI Spec. Publ.* SP 87-12, American Concrete Institute (ACI), 213-232.

Foster, S.J., Liu, J. and Sheikh, S.A. (1998), "Cover spalling in HSC columns loaded in concentric compression", *J. Struct. Eng.*, ASCE, **124**(12), 1431-1437.

Hong, K., Han, S.H. and Yi, S.T. (2006), "High-strength columns confined by low-volumetric-ratio lateral ties", *Eng. Struct.*, **28**, 1346-1353.

La Mendola, L. and Papia, M. (2002), "General stress-strain model for concrete or masonry response under uniaxial cyclic compression", *Struct. Eng. Mech.*, **14**(4), 435-454.

Lima Junior, H.C. and Giongo, J.S. (2004), "Steel-fibre high-strength concrete prisms confined by rectangular ties under concentric compression", *Mater. Stuct.*, **37**, 689-697.

Liu, J. and Foster, S.J. (2000), "Strength of tied high-strength concrete columns loaded in concentric compression", *ACI Stuct. J.*, **97**(1), 149-156.

Mander, J.B., Priestley, M.J.N. and Park, R. (1988), "Observed stress-strain behavior of confined concrete", *J. Struct. Eng.,* ASCE, **114**(8), 1827-1849.

Mander, J.B., Priestley, M.J.N. and Park, R. (1988), "Theoretical stress-strain model for confined concrete", *J. Struct. Eng.*, ASCE, **114**(8), 1804-1826.

Mau, S.T., Elwi Alaa, E. and Zhou Si-Zhu. (1988), "Analytical study of spacing of lateral steel and column

confinement", *J. Struct. Eng*., ASCE, **124**(3), 262-269.
Park, R., Priestley, M.J.N. and Gill, W.D. (1982), "Ductility of square-confined concrete columns", *Proceedings,* ASCE, **108**(4), 929-950.
Popovics, S. (1973), "A numerical approach to the complete stress-strain curve of concrete", *Cement Concrete Res.*, **3**(5), 583-599.
Razvi, R. and Saatcioglu, M. (1999), "Circular high-strength concrete columns under concentric compression", *ACI Struct. J.*, **96**(5), 817-825.
Razvi, S. and Saatcioglu, M. (1999), "Confinement model for high-strength concrete", *J. Struct. Eng*., ASCE, **125**(3), 281-288.
Richart, F.E., Brandtzaeg, A. and Brown, R.L. (1929), "The failure of plain and spiral reinforced concrete in compression", Engineering Experiment Station; Bulletin N.190, University of Illinois, Urbana, USA.
Sargin, M. (1971), "Stress-strain relationship for concrete and the analysis of structural concrete sections", Solid Mechanics Division, University of Waterloo, Ontario.
Sheikh, S. and Uzumeri, S.M. (1982), "Analytical model for concrete confinement in tied columns", *J. Struct. Eng.,* ASCE, **108**(12), 2703-2722.
Tan, T.H. and Yip, W.K. (1999), "Behaviour of axially loaded concrete columns confined by elliptical hoops", *ACI Struct. J.*, **96**(6), 967-971.
Teerawong, J., Lukkunaprasit, P. and Senjuntichai, T. (2004), "Strength enhancement in confined concrete with consideration of flexural flexibilities of ties", *Struct. Eng. Mech.*, **18**(2), 151-166.