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Technical Note

Parametric resonance of a rotating taper pre-twisted beam with cracks

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1. Introduction

During actual service, the rotating speed of a rotating beam system is subjected to a small range of variations under external disturbance is unavoidable. Few studies focus on the parametric instability of a taper pre-twisted beam with a crack. Cracks frequently appear in rotating machinery due to manufacturing flaws or cyclic fatigue during operation. Especially in turbo-disks, numerous cracks can be observed after severe operating conditions as Perkins and Bache (2005). In this article, the Galerkin method is used to derive the discrete equation of this system. Finally, by using the method of multiple scales perturbation to specify the regions of instability in this cracked beam system is considered.

2. Equations of motion

The taper pre-twisted beam structure at a rotating speed Ω is shown in Fig. 1. It consists of a rigid hub with radius R_h and the length of cantilever beam is L. The thickness and breadth at the root of the beam are t_0 and b_0. The transverse flexible deflections of the beam in the rotational plane and the out-off rotational plane are denoted by components $v_s(r, t)$ and $u_s(r, t)$ respectively. r denotes the arbitrary position.

2.1 Twisted beam without crack

By using Hamilton's principle, the equations of motion of the taper pre-twisted beam can be derived as

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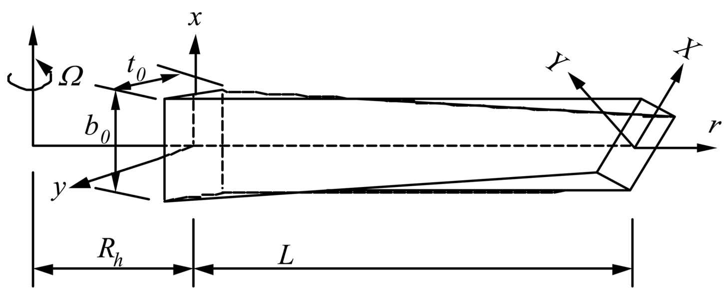

Fig. 1 Geometry of the rotating taper pre-twisted beam

$$\rho A\ddot{u} - \rho\Omega^2\left[\int_r^L A(r+R_h)dr(u)'\right]' + E(I_{yy}(u)'' + I_{xy}(v)'')'' = 0 \tag{1}$$

$$\rho A\ddot{v} - \rho\Omega^2\left\{Av + \left[\int_r^L A(r+R_h)dr(v)'\right]'\right\} + E(I_{xx}(v)'' + I_{xy}(u)'')'' = 0 \tag{2}$$

A and E are the area and Young Modulus respectively. I_{xx}, I_{xy} and I_{yy} are the area moments of inertia. The corresponding boundary condition is the same as a cantilever beam.

In this article, the rotational speed Ω_0 is considered to be perturbed with a small perturbation speed $f(t)$ in this article, as Young (1991). Galerkin's method employed, the equations of motion of the non-constant rotating beam can then be expressed in matrix form as

$$[m]\begin{Bmatrix}\ddot{p}\\ \ddot{q}\end{Bmatrix} + [k]\begin{Bmatrix}p\\ q\end{Bmatrix} = (2\Omega_0 f + f^2)[d]\begin{Bmatrix}p\\ q\end{Bmatrix} \tag{3}$$

2.2 Beam with a crack

For convenience, a dimensionless parameter $\bar{r} = r/L$ is considered. In this article, the crack is located at $\bar{r} = \bar{r}^*$ of the beam. $\bar{\gamma} = a/t_0(1-\beta\bar{r}^*)$ denotes the crack depth ratio. The total strain energy of the defective beam also consists of the released energy U^c caused by the cracks. According to Kuang and Huang (1999), the released energy caused by a crack for a beam is derived as

$$U^c = 3E(1-\mu^2)t_0(1-\beta\bar{r}^*)\int_0^1 I_{xx}Q(\bar{\gamma})(v'')^2\delta(\bar{r}-\bar{r}^*)d\bar{r} \tag{4}$$

2.3 Instability analysis for a cracked beam system

After modal analysis and complex calculation, the above Eq. (3) can then be rewritten as

$$[I]\{\ddot{u}\} + [A]\{u\} = -\left(\frac{f}{\Omega_0} + \frac{f^2}{2\Omega_0^2}\right)[D]\{u\} \tag{5}$$

Let the rotation speed be perturbed by a very small periodic excitation $f(t)$, which can be represented by a Fourier series of harmonic components. After calculation, the dynamic equation of the cracked beams system can be rewritten as

$$[I]\{\ddot{u}\} + [A]\{u\} = -\varepsilon\left(\bar{f} + \frac{\varepsilon}{2}\bar{f}^2\right)[D]\{u\} \tag{6}$$

where $\varepsilon = |F_M|/\Omega_0$ and $\bar{f}(t) = f(t)/|F_M|$, ε is the relative disturbance order and $|F_M|$ is the maximum magnitude of components of a Fourier series excitation. By using the multiple scales perturbation method, the unstable region can be solved.

3. Numerical results and discussion

The beam is specified with the following non-dimensional parameters: $(R_h/L) = 0.2$, $(b_o/L) = 0.1$, $(t_0/L) = 0.02$, $\alpha = \beta = 0.25$, $\theta = 45^\circ$ and $\bar{r}^* = 0$. In the meanwhile, α and β are the taper ratios and θ is the pre-twisted angle for the beam. For convenience, a number of non-dimensional parameters, i.e., $\bar{\omega}_0 = \omega/\omega_0$ and $\bar{\Omega}_0 = \Omega_0/\omega_0$, are also used. The frequency ω_0 is defined herein as $\omega_0 = 0.01\sqrt{E/\rho L^2}$.

To determine the stability of a rotating cracked beam system, a simplified harmonic perturbation speed as $f(t) = 2\cos\omega t$ is imposed on the steady speed Ω_0. Fig. 2 displays the instability of a system with and without cracks. In engineering applications, most investigators pay great attention to the lowest unstable zones in the system. Therefore, the variations in the lowest unstable zone of a beam system with or without a crack are illustrated in this figure. The lowest unstable region, near $2\bar{\omega}_1$, shifts to a lower frequency domain as a crack exists in the system. Obviously, the lowest unstable zone, near $2\bar{\omega}_1$, is also enlarged when the beam system has a crack. It is clear that cracks not only alter the dynamics but also significantly change the instability of a rotating beam system. The rotational speed $\bar{\Omega}_0$ of the beam may also affect the unstable region as shown in Fig. 3. The additional stiffness introduced from the centrifugal force increase the natural frequencies. It can be observed that the first unstable band is enlarged and shifted toward a higher frequency. The lowest unstable region enlarges in a system with higher rotating speed.

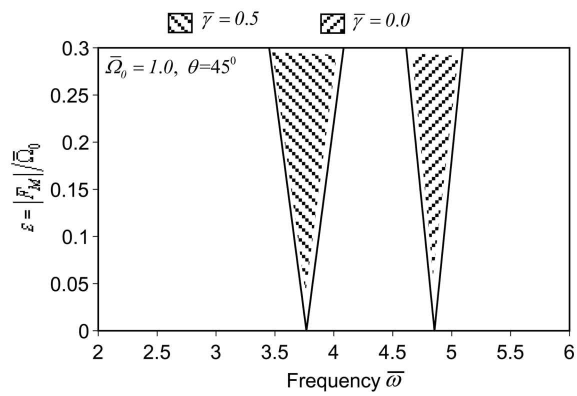

Fig. 2 The lowest unstable region of a rotating beam system with and without a crack

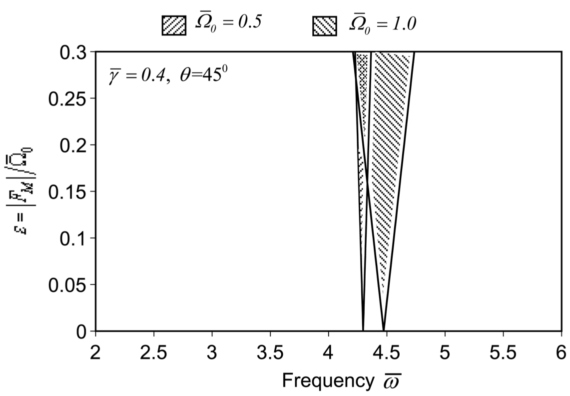

Fig. 3 The lowest unstable region of the beam system with different rotating speed

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