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Stress concentration and deflection of simply supported box girder including shear lag effect

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Abstract. The shear lag has been studied for many years. Nevertheless, existing research gives a variety of stress concentration factors. Unlike the elementary beam theory, the application of load is not unique in reality. For example, concentrated load can be applied as point load or distributed load along the height of the web. This non-uniqueness may be a reason for the discrepancy of the stress concentration factors in the existing studies. The finite element method has been often employed for studying the effect of the shear lag. However, not many researches have taken into account the influence of the finite element mesh on the shear lag phenomenon, although stress concentration can be quite sensitive to the mesh employed in the finite element analysis. This may be another source for the discrepancy of the stress concentration factors. It also needs to be noted that much less studies seem to have been conducted for the shear lag effect on deflection while some design codes have formulas. The present study investigates the shear lag effect in a simply supported box girder by the three-dimensional finite element method using shell elements. The whole girder is modeled by shell elements, and extensive parametric study with respect to the geometry of a box girder is carried out. Not only stress concentration but also deflection is computed. The effect of the way load is applied and the dependency of finite element mesh on the shear lag are carefully treated. Based on the numerical results thus obtained, empirical formulas are proposed to compute stress concentration and deflection that includes the shear lag effect.

Keywords: shear lag; simply supported box girders; stress concentration; deflection; three-dimensional finite element analysis.

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1. Introduction

In the elementary beam theory, the normal stress in the longitudinal direction produced by bending deformation is assumed to be proportional to the distance from the neutral axis and therefore uniform across the flange width. However, as a flange gets wider, this assumption becomes invalid: the normal stress distribution is not uniform in the wide flange, but the stress takes the maximum value at the flange-web intersection in general, decreasing toward the middle of the flange. This phenomenon is called the shear lag.

The shear lag has been studied for many years. Timoshenko and Goodier (1970) have documented one of the earliest researches due to von Karman. The best-known achievement in the past is probably the one due to Reissner (1941, 1946). While these early researches are analytical, a numerical means, the finite element method in particular, is often utilized in the recent studies. A concise but excellent literature review of research on the shear lag is available in Tenchev (1996).

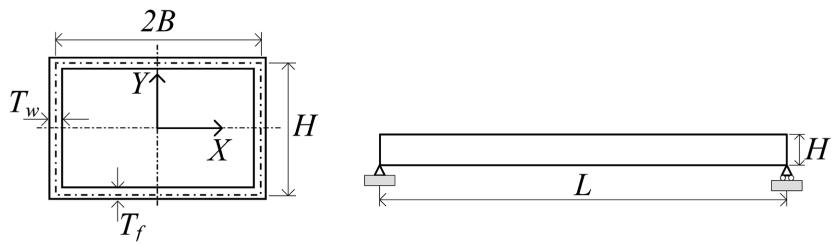
Although much research has been done for the problem and several design codes have already provided formulas to account for the shear lag effect (British Standards Institution 1982, Japan Road Association 2002, Eurocode 3 2003), discrepancies in numerical results are observed in the literature. This seems to be attributable to the factors that have considerable influence on numerical results but have been overlooked. For example, concentrated load is not necessarily point load: it can be applied as distributed load along the height of the web as is done by Tenchev (1996). Such non-uniqueness in loading can be a reason for the discrepancy of the stress concentration factors in the existing studies. It is also noteworthy that shear lag effect on deflection has not been studied much while some design codes have formulas for it (British Standards Institution 1982, Japan Road Association 2002).

The three-dimensional finite element analysis of a simply supported box girder by shell elements is carried out to study the shear lag effect in the present study. Two loading conditions of concentrated load at the mid-span and uniformly distributed load along the beam length are employed. Multiple ways to apply those loads are considered. Much attention is paid to finite element mesh as well, so as to minimize discretization error. Note that not many researchers have explicitly addressed how the discretization error is controlled in their shear lag study by the finite element method. To the best of the authors' knowledge, the work of Lee and Wu (2000) is one of the very few numerical studies where the discretization error in the finite element analysis is carefully treated.

The normal stress in the longitudinal direction in the flange is of interest for investigating the shear lag effect on stress. The stress in the mid-span cross section is focused on in particular, since the largest stress is expected. The vertical displacement at the mid-span is also computed to see the shear lag effect on deflection. An extensive parametric study is conducted, based on which empirical formulas are proposed. In all the analyses, a finite element program, MARC (1994), is used.

2. Analysis model

Simply supported box girders under concentrated load at mid-span or uniformly distributed load are analyzed. The symbols employed in the present study for describing the structural geometry are illustrated in Fig. 1. For those box girders, the stress concentration factors at the mid-span can be



(a) Cross section (b) Side view

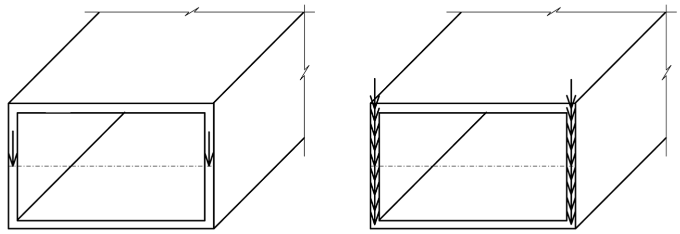
Fig. 1 Structural geometry of box girder

Table 1 K_c in literature

Literature	K_c	
	Concentrated load	Distributed load
Tenchev (1996)	1.31	1.07
British (1982)	1.39	1.05
Japan (2002)	1.23	1.09
Eurocode 3 (2003)	-	1.05
Lee *et al.* (2000)	1.56	1.05
Song *et al.* (1990)	1.34	1.04
Sedlacek *et al*. (1993)	1.35	1.05
Tahan *et al.* (1997)	1.58	1.05

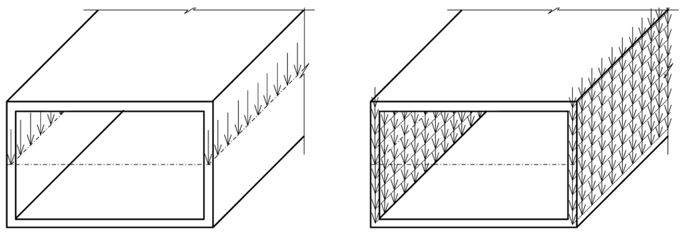
evaluated by the formula given in the design codes (British Standards Institution 1982, Japan Road Association 2002, Eurocode 3 2003). The factors can be obtained also by the results in the literature (Tenchev 1996, Lee and Wu 2000, Song and Scordelis 1990, Sedlacek and Bild 1993, Tahan *et al.* 1997). For a box girder with H/L = 0.1, B/H = 1.0 and T_f/T_w = 1.0, those values are summarized in Table 1 where K_c stands for the stress concentration factor defined by the ratio of the maximum normal stress in the flange to that of the elementary beam theory. Significant discrepancy is recognized, especially under concentrated load. As Table 1 may prove, it is not an easy task to evaluate a rigorous stress distribution having the shear lag phenomenon. Generally, the largest normal stress in a flange occurs at the edge and sharp decrease is observed toward the middle of the flange. Because of the large stress gradient, K_c can be very sensitive to a mathematical model set up for the analysis.

The existing studies may be classified into two groups based on the type of analysis: analytical approach (Timoshenko and Goodier 1970, Reissner 1941, Reissner 1946, Song and Scordelis 1990, Sedlacek and Bild 1993, Tahan *et al*. 1997, Hwang *et al*. 2004) and finite element approach (Tenchev 1996, Lee and Wu 2000, Moffatt and Dowling 1975). The former often needs to introduce assumptions so as to simplify a problem and yield a solution. On the other hand, the finite element method requires few assumptions in principle. However, due to the limitation of computer capacity, the shear lag is investigated in the two dimensional framework of a plane stess problem in some finite element analysis (Tenchev 1996, Lee and Wu 2000) while the behavior in accordance with the elementary theory of bending is assumed in some other finite element analysis (Moffatt and Dowling 1975).



(a) Load C-1 (b) Load C-2

Fig. 2 Concentrated load: (a) Load C-1, (b) Load C-2



(a) Load D-1 (b) Load D-2

Fig. 3 Distributed load

In general, the less the assumptions imposed on the analysis are, the closer to the reality the mathematical model becomes. To this end, in the present study, an entire box girder is modeled as it is, using 4-node shell elements: the shear lag problem is not reduced to a plane-stress problem and no beam assumptions are implemented.

In the three-dimensional finite element model, unique load in the beam theory can be applied in various ways. Herein the load applications that may cause local effects on the stress distribution in the flange are avoided. As for concentrated load, therefore, two loading models shown in Fig. 2 are adopted: Load C-1 is concentrated load at the middle of the web and Load C-2 is uniformly distributed load along the height of the web. Two loading models shown in Fig. 3 are considered for distributed load: Load D-1 is uniformly distributed load along the centerline of the web and Load D-2 is uniformly distributed load not only along the beam axis but also along the web height of every cross section. To be noteworthy, although the stress concentration may be influenced by the way the load is applied, researchers other than Tenchev (1996) have not described their loading condition explicitly, to the best knowledge of the authors.

3. Numerical evaluation

3.1 Stress concentration

The structural model described in the previous chapter is analyzed by the finite element method, using shell elements. Although the finite element method is very versatile and powerful, the results may depend largely on finite element mesh employed in the analysis, which is especially so when stress concentration is dealt with. Because of this, we first study the influence of finite element

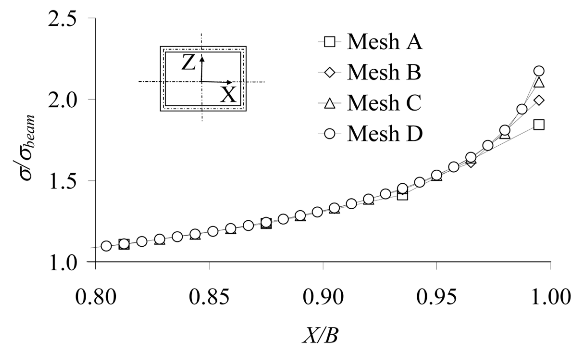


Fig. 4 Normal stress distribution in upper flange

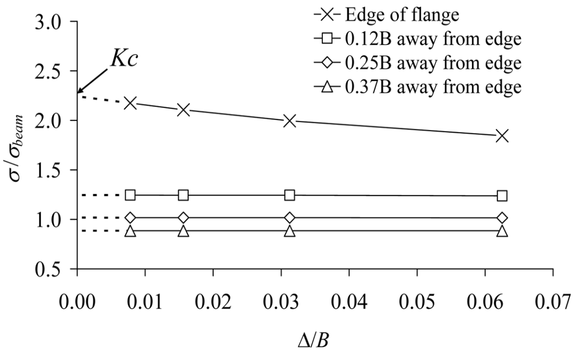


Fig. 5 Variation of normal stress with respect to representative element size

mesh on the stress concentration. Fig. 4 shows the normal stress distributions in an upper flange at the mid-span. In this figure, σ is the normal stress obtained by the present finite element analysis while σ_{beam} is the normal stress due to the beam theory. Needless to say, σ_{beam} is constant across the flange width. This is the result for a box girder (H/L = 0.2, B/H = 1.0, T_f/T_w = 1.0) under Load C-2 by four finite element meshes, Meshes A to D. All the elements in each mesh are rectangular and from Meshes A to D, the size of an element is made finer in a consistent way. It is noted that although due to symmetry only a quarter of the box girder is analyzed, the numbers of elements used for the quarter model amount to 800, 3,200, 12,800 and 51,200 for Meshes A to D, respectively. Fig. 4 illustrates not only the shear lag phenomenon but also the dependence of the stress distribution on the finite element mesh: as expected, the dependence is stronger at the edge of the flange where the largest stress concentration takes place. At the same time, the tendency of stress convergence is observed, as the size of the finite element becomes smaller.

Fig. 5 gives the variation of the normal stress in the flange with respect to a representative element size Δ: the four symbols of the same kind represent the stresses obtained by the four finite element meshes, Meshes A to D. Importantly, the four lines in Fig. 5 become almost straight as Δ gets small, which is in accordance with the description of Cook *et al.* (1989): the strain error is proportional to element size. In a linear analysis, "strain" in this statement can be replaced by "stress". The linear extrapolation can then be used to estimate the converged stress when the element size vanishes, as indicated by the dotted lines in Fig. 5. This extrapolation method is called "multimesh extrapolation" by Cook *et al.* (1989). The converged stress ratio σ/σ_{beam} at the edge of the flange, i.e., the point indicated by an arrow in Fig. 5, gives the value of K_c that we seek in the present finite element analysis.

3.2 Deflection

Using the same finite element meshes as above, similar study is made for deflection. Fig. 6 shows the variation of the deflection ratio w/w_{beam} at the mid-span, i.e., the ratio of the deflection due to the finite element analysis w to the deflection due to the beam theory w_{beam} at the mid-span, with respect to the square of a representative element size Δ. To be noteworthy, the line in Fig. 6 becomes almost straight as Δ^2 becomes small, which is in accordance with the description of Cook *et al.* (1989): the deflection error is proportional to the square of element size. Just like in the evaluation of K_c, the linear extrapolation can then be used to estimate the converged value of

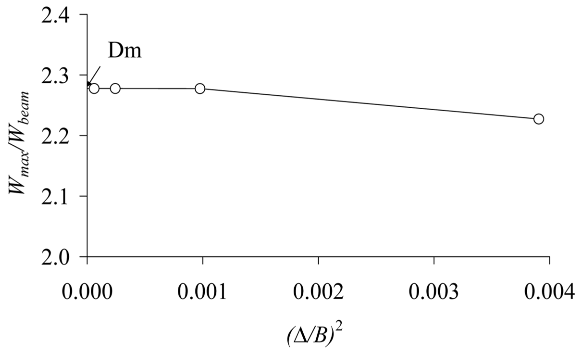


Fig. 6 Variation of deflection with respect to representative element size

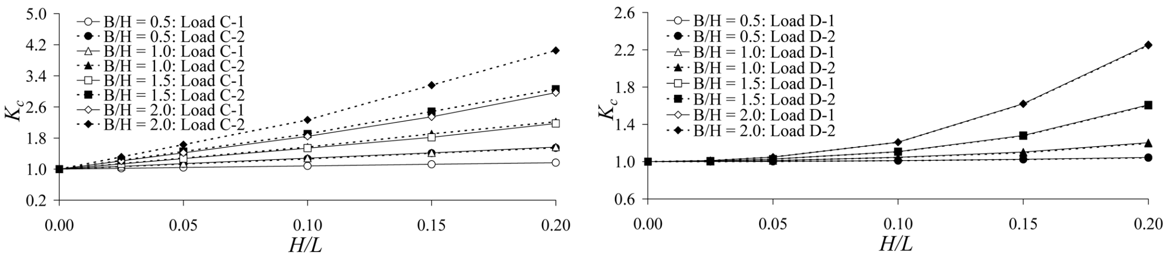
w/w_{beam} when the element size vanishes, as indicated by the dotted line in Fig. 6. The converged value of w/w_{beam}, i.e., the point indicated by the arrow in Fig. 6, gives the deflection magnification factor D_m that is sought in the present finite element analysis.

4. Parametric study

Based on the modeling and the numerical evaluations described above, three-dimensional finite element analysis is conducted so as to reveal the influence of the parameters that characterize the geometry of a box girder. In particular, the following values are considered: H/L = 0.025, 0.05, 0.10, 0.15, 0.20; B/H = 0.5, 1.0, 1.5, 2.0; T_f/T_w = 0.5, 1.0, 1.5, 2.0. The combination of all these values results in 80 box girders different from each other in geometry. In this paramaetric study, as explained earlier, multiple loadings are applied to each girder and multiple finite element meshes are used to eliminate discretization error by the multimesh extrapolation method for every girder under a specific loading condition.

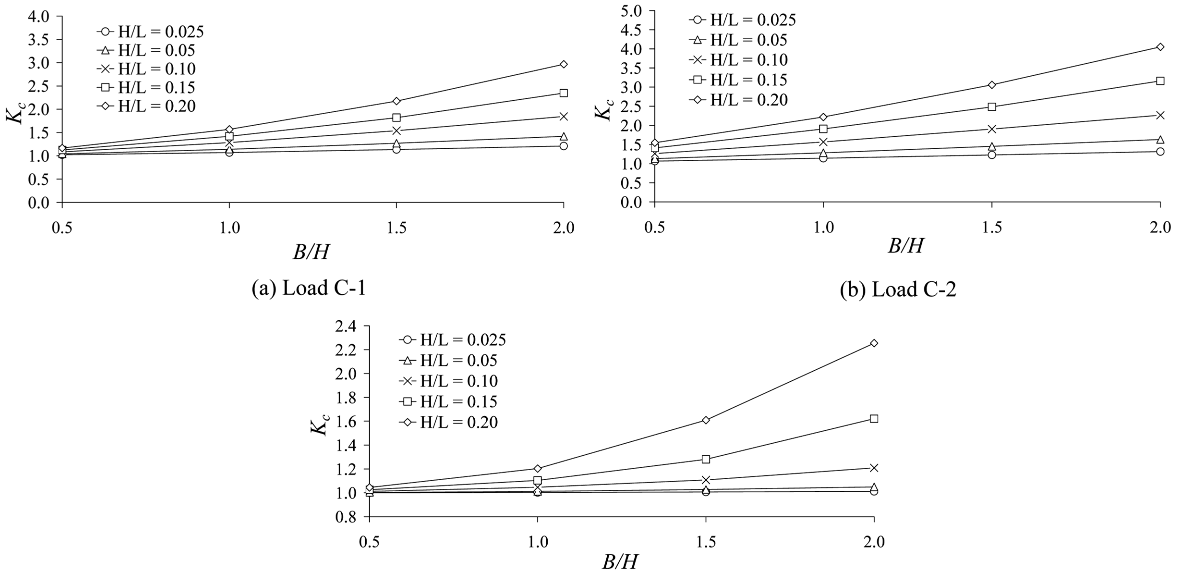
4.1 Stress concentration factor K_c

As a typical example of the present numerical results, Fig. 7 shows the variation of K_c with respect to H/L for the cross sections with T_f/T_w = 1.0 under concentrated load (Loads C-1 and C-2) and uniformly distributed load (Loads D-1 and D-2). Load C-2 induces larger K_c than Load C-1 consistently and the difference is considerable. Load D-1 yields larger K_c than Load D-2, but the



(a) Concentrated load (b) Distributed load

Fig. 7 Variation of K_c with respect to H/L (T_f/T_w = 1.0)

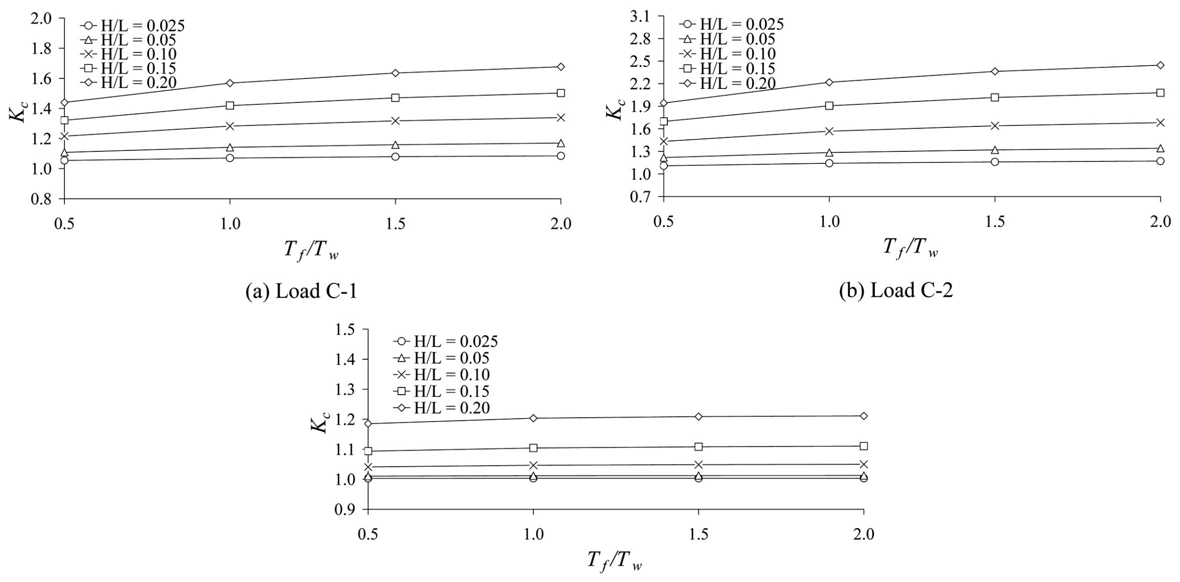


(a) Load C-1

(b) Load C-2

(c) Load D-1

Fig. 8 Variation of K_c with respect to B/H (T_f/T_w = 1.0)



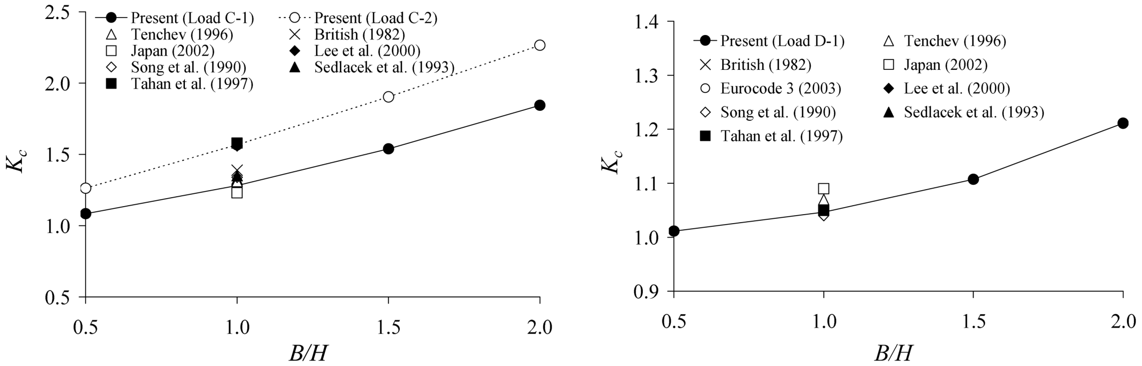
(a) Load C-1

(b) Load C-2

(c) Load D-1

Fig. 9 Variation of K_c with respect to T_f/T_w (B/H = 1.0)

difference appears insignificant. Thus, it is decided that while both Loads C-1 and C-2 are applied in the case of concentrated load, only Load D-1 needs to be considered for distributed load in the present finite element analyses. To show the influences of B/H and T_f/T_w on K_c, Figs. 8 and 9 are presented.



(a) Concentrated load (b) Distributed load

Fig. 10 Comparison of K_c (H/L = 0.1, T_f/T_w = 0.5)

The general trends regarding the effect of the geometrical parameters on K_c observed in Figs. 7 to 9 can be summarized as follows:

1. K_c tends to grow with the increase of H/L.
2. The influence of H/L on K_c is very small for B/H equal to 0.5 under Loads C-2 and D-1.
3. K_c tends to grow with the increase of B/H.
4. The influence of B/H on K_c is very small for H/L equal to 0.025 in the case of concentrated load and equal to or smaller than 0.05 in the case of distributed load.
5. K_c tends to grow with the increase of T_f/T_w.
6. The influence of T_f/T_w on K_c is very small for H/L equal to or smaller than 0.05 in the case of concentrated load. Under distributed load, the influence of T_f/T_w on K_c is very small for the entire range of T_f/T_w considered herein.

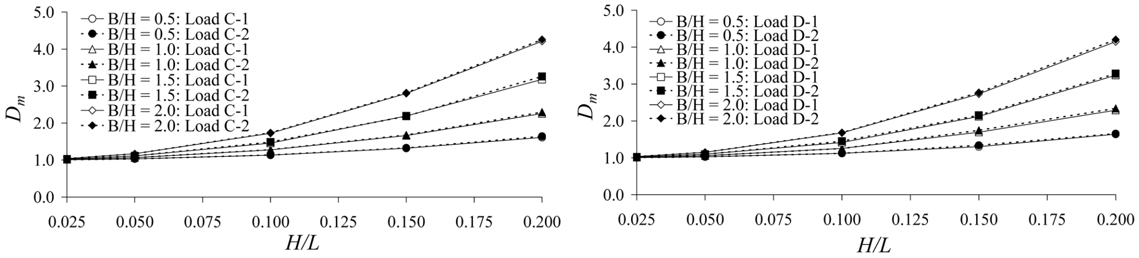
Fig. 10 shows the present numerical results of K_c together with those in Table 1. In concentrated load, K_c due to Tenchev (1996) is the closest to the present result with Load C-1, while Lee *et al.* (2000) and Tahan *et al.* (1997) give the results very close to the present result with Load C-2. In the distributed load, discrepancy is smaller with K_c due to Japan (2002) being the most different from the present result.

4.2 Deflection magnification factor D_m

Fig. 11 shows a typical variation of D_m with respect to H/L under concentrated load (Loads C-1 and C-2) and uniformly distributed load (Loads D-1 and D-2). Although Load C-2 yields larger D_m than Load C-1, the values due to the two loadings are close to each other. Load D-2 gives larger D_m than Load D-1, but the difference is very small. Thus, the way of applying load is insignificant for D_m for both concentrated and distributed loads. In the following numerical analyses, therefore, only Loads C-2 and D-2 are considered. For the influence of B/H and T_f/T_w on D_m, Figs. 12 and 13 are presented.

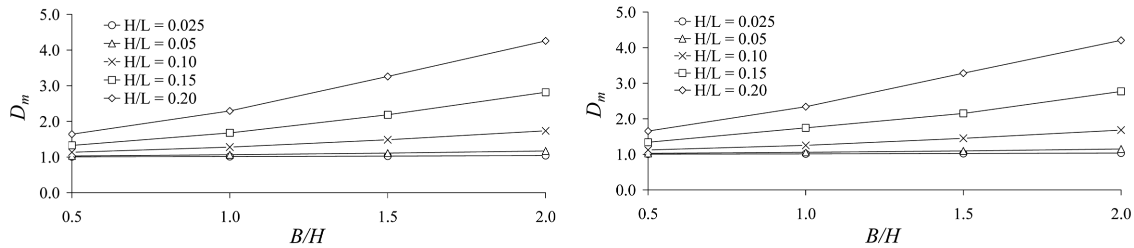
The general trends observed in Figs. 11 to 13 can be summarized as follows:

1. D_m tends to grow with the increase of H/L.
2. D_m tends to grow with the increase of B/H.



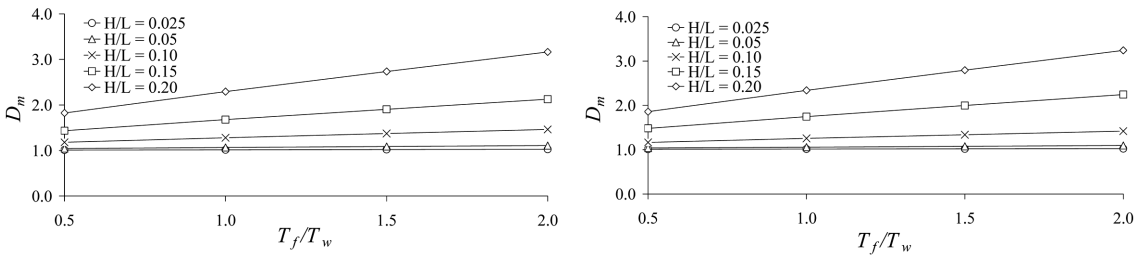
(a) Concentrated load (b) Distributed load

Fig. 11 Variation of D_m with respect to H/L (T_f/T_w = 1.0)



(a) Concentrated load (b) Distributed load

Fig. 12 Variation of D_m with respect to B/H (T_f/T_w = 1.0)

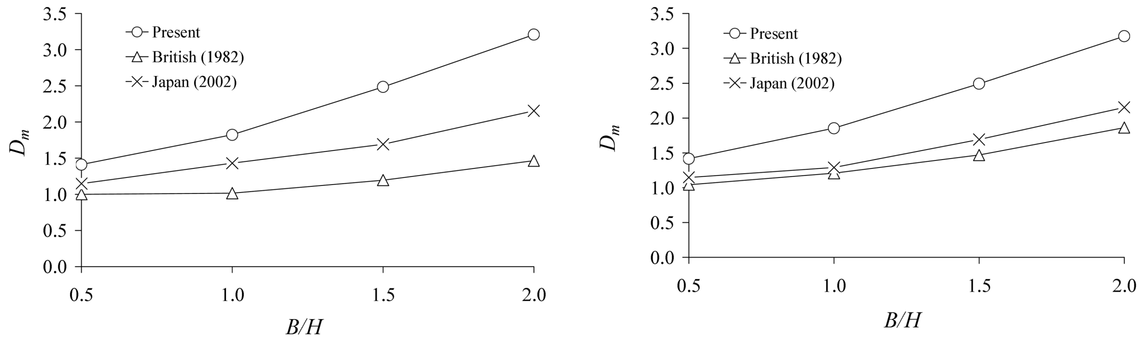


(a) Concentrated load (b) Distributed load

Fig. 13 Variation of D_m with respect to T_f/T_w (B/H = 1.0)

3. The influence of B/H is very small for H/L equal to or smaller than 0.05.
4. D_m tends to grow with the increase of T_f/T_w.
5. The influence of T_f/T_w is very small for H/L equal to or smaller than 0.05.

Some design codes (British Standards Institution 1982, Japan Road Association 2002) provide formulas for the shear lag effect on deflection. Fig. 14 presents D_m calculated through those formulas together with the present numerical results. While D_m due to the design formulas exhibits the similar tendencies to that obtained by the present finite element analysis, discrepancy is evident: D_m due to the design formulas considerably underestimates D_m by the finite element analysis. The difference between the D_m values due to the formulas in the two design codes is also obvious: D_m due to Japan (2002) is consistently larger than that due to British (1982).



(a) Concentrated load (b) Distributed load

Fig. 14 Comparison of D_m (H/L = 0.2, T_f/T_w = 0.5)

5. Empirical formulas

5.1 Stress concentration factor K_c

A regression analysis based on the present numerical results gives the following formulas for K_c:

Concentrated load (Load C-1):

$$K_c = a_1(H/L) + 1 \tag{1}$$

where

$$a_1 = b_1(B/H)^{c_1} \tag{2}$$

$$b_1 = 0.832\ln(T_f/T_w) + 2.77 \tag{3}$$

$$c_1 = -0.034\ln(T_f/T_w) + 1.744 \tag{4}$$

Concentrated load (Load C-2):

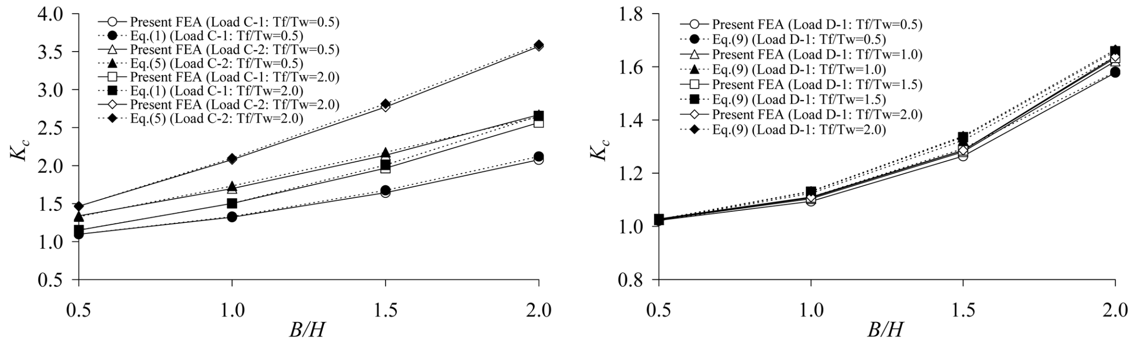
$$K_c = a_2(H/L) + 1 \tag{5}$$

where

$$a_2 = b_2(B/H)^{c_2} \tag{6}$$

$$b_2 = 1.756\ln(T_f/T_w) + 6.101 \tag{7}$$

$$c_2 = 0.053\ln(T_f/T_w) + 1.202 \tag{8}$$



(a) Concentrated load (b) Distributed load

Fig. 15 K_c due to proposed formulas and finite element analysis (H/L = 0.15)

Distributed load (Load D-1):

$$K_c = a_3(H/L)^2 + 1 \tag{9}$$

where

$$a_3 = b_3(B/H)^{c_3} \tag{10}$$

$$b_3 = 1.225\ln(T_f/T_w) - 0.494(T_f/T_w) + 6.001 \tag{11}$$

$$c_3 = -0.041\ln(T_f/T_w) - 0.006(T_f/T_w) + 2.371 \tag{12}$$

Fig. 15 illustrates some comparisons between K_c due to the empirical formula and the present finite element analysis (FEA). Good agreement is observed in the figure. The overall accuracy of the proposed formula for each loading condition is calculated as the mean square error by the following equation (Tenchev 1996)

$$\bar{\varepsilon} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^2} \tag{13}$$

where

$$\varepsilon_i = \frac{K_{cEmp} - K_{cFEA}}{K_{cFEA}} \times 100(\%) \tag{14}$$

where N is the number of the present finite element results for a loading condition, and $K_{c\,Emp}$ and $K_{c\,FEA}$ are the K_c values obtained from the proposed empirical formulas and the present finite element analysis, respectively. Since in the present study, the combination of the geometrical parameters has required 80 box girders to be analyzed for each loading, N in Eq. (13) is equal to 80. Using Eq. (13), the mean square error is found to be 1.83%, 2.99% and 2.33% for Loads C-1, C-2 and D-1, respectively.

5.2 Deflection magnification factor D_m

A regression analysis based on the present numerical results gives the following formulas for D_m:

Concentrated load (Load C-2):

$$D_m = a_1(B/H)^{2.3} + a_2(T_f/T_w) + a_3(B/H)(T_f/T_w) + a_4 \tag{15}$$

where

$$a_1 = 4.12(H/L)^{1.77} \tag{16}$$

$$a_2 = -83.53(H/L)^{4.5} \tag{17}$$

$$a_3 = 41.43(H/L)^{2.33} \tag{18}$$

$$a_4 = 76.53(H/L)^3 - 18.09(H/L)^2 + 1.54(H/L) + 0.97 \tag{19}$$

Distributed load (Load D-2):

$$D_m = a_1(B/H)^{1.6} + a_2(T_f/T_w) + a_3(B/H)(T_f/T_w) + a_4 \tag{20}$$

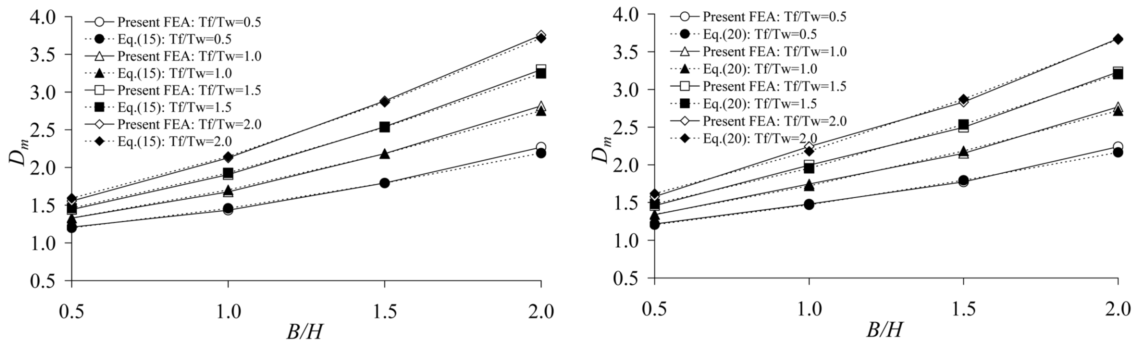
where

$$a_1 = 7.78(H/L)^{1.8} \tag{21}$$

$$a_2 = -579.02(H/L)^4 + 204.73(H/L)^3 - 21.89(H/L)^2 + 0.97(H/L) - 0.01 \tag{22}$$

$$a_3 = 50.46(H/L)^{2.48} \tag{23}$$

$$a_4 = 39.6(H/L)^3 - 8.77(H/L)^2 + 0.25(H/L) + 1 \tag{24}$$



(a) Concentrated load (b) Distributed load

Fig. 16 D_m due to proposed formulas and finite element analysis (H/L = 0.15)

Fig. 16 illustrates some comparisons between D_m due to the empirical formula and the present finite element analysis (FEA). Good agreement is observed in the figure.

The overall accuracy of the proposed formula for each loading condition is calculated as the mean square error in the same way as in the case of K_c by Eq. (13) where ε_i is now the error computed by the following equation

$$\varepsilon_i = \frac{D_{mEmp} - D_{mFEA}}{D_{mFEA}} \times 100(\%) \tag{25}$$

where $D_{m\,Emp}$ and $D_{m\,FEA}$ are the D_m values obtained from the proposed empirical formula and the present finite element analysis, respectively. The mean square error thus obtained turns out to be 1.06% and 1.58% for concentrated and distributed loads, respectively.

6. Conclusions

Extensive parametric study with respect to the geometry of a box girder has been carried out by the three-dimensional finite element analysis so as to reveal the shear lag effect in a simply supported box girder on both stress concentration and deflection. Shell elements have been used to model the entire box girder. The effect of the way of applying load and the dependency of the stress concentration on the finite element mesh have been carefully treated, while many existing research works seem to have paid little attention to these aspects of analysis modeling.

Based on the present numerical results thus obtained, empirical formulas have been proposed to compute the stress concentration factor K_c and the deflection magnification factor D_m. It has been confirmed that the proposed formulas can yield those two factors in good agreement with the present finite element results.

The present study indicates that the real stress and deflection can be much larger than those due to the beam theory. It is also shown that the existing studies including design codes may yield the stress and deflection quite different from each other and the present result as well. It is therefore hoped that the proposed formulas would be of some help to improve the current situation.

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